J. Fuzzy. Ext. Appl

Journal of Fuzzy Extension & Applications
www.journal-fea.com

A New Approach For Ranking Of Intuitionistic Fuzzy Numbers

Suresh Mohan *, ***Arun Prakash Kannusamy, Vengataasalam Samiappan***
Department of Mathematics, Kongu Engineering College, Erode, Tamil Nadu, India.

P A P E R   I N F O

Chronicle:
Received: 08 October 2019
Revised: 16 December 2019
Accepted: 08 January 2020

Keywords:
Intuitionistic Fuzzy Sets.
Intuitionistic Fuzzy Numbers.
Trapezoidal Intuitionistic.
Fuzzy Numbers.
Triangular Intuitionistic Fuzzy Numbers.
Magnitude of Intuitionistic Fuzzy Number.

A B S T R A C T

The concept of an intuitionistic fuzzy number (I F N) is of importance for representing an ill-known quantity. Ranking fuzzy numbers plays a very important role in the decision process, data analysis and applications. The concept of an intuitionistic fuzzy number (IFN) is of importance for quantifying an ill-known quantity. Ranking of intuitionistic fuzzy numbers plays a vital role in decision making and linear programming problems. Also, ranking of intuitionistic fuzzy numbers is a very difficult problem. In this paper, a new method for ranking intuitionistic fuzzy number is developed by means of magnitude for different forms of intuitionistic fuzzy numbers. In Particular ranking is done for trapezoidal intuitionistic fuzzy numbers, triangular intuitionistic fuzzy numbers, symmetric trapezoidal intuitionistic fuzzy numbers, and symmetric triangular intuitionistic fuzzy numbers. Numerical examples are illustrated for all the defined different forms of intuitionistic fuzzy numbers. Finally some comparative numerical examples are illustrated to express the advantage of the proposed method.

1. Introduction

Atanassov [1] introduced the concept of Intuitionistic Fuzzy Sets (IFS) which is a generalization of the concept of fuzzy set. In IFS the degree of non-membership denoting the non-belonging of an element to a set is explicitly specified along with the degree of membership.

In many real world problems, due to insufficiency in the information available, the evaluation of membership values is not possible up to our satisfaction. Also the evaluation of non –membership values is not always possible and there remains an indeterministic part in which hesitation survives. A fuzzy number plays a vital role in representation of such unknown quantity. Following this concept, the generalized concept of intuitionistic Fuzzy Number (IFN) introduced by Grzegrorzewski [5] in 2003 receives high attention and different definitions of IFN's have been proposed. Grzegrorzewski [6] defined two families of metrics in the space of IFNs and proposed a ranking method of IFNs.

Mitchell [9] interpreted an IFN as an ensemble of fuzzy numbers and introduced a ranking method. Wang [18] gave the definition of intuitionistic trapezoidal fuzzy number and interval intuitionistic

* Corresponding author
E-mail address: mathssuresh84@gmail.com
DOI: 10.22105/jfea.2020.247301.1003

trapezoidal fuzzy number. Based on expected values, score functions and accuracy function of intuitionistic trapezoidal fuzzy numbers a new kind of ranking was proposed by Wang et al. in 2009. They also developed the Hamming distance of intuitionistic trapezoidal fuzzy numbers and Intuitionistic Trapezoidal Fuzzy Weighted Arithmetic Averaging (ITFWAA) operator, then proposed multi-criteria decision-making method with incomplete certain information based on intuitionistic trapezoidal fuzzy number.

In 2011, Salim Rezvani defined a new ranking technique for trapezoidal intuitionistic fuzzy numbers based on value-index and ambiguity –index of trapezoidal intuitionistic fuzzy numbers. Similar value-index and ambiguity – index based ranking method for triangular intuitionistic fuzzy numbers was given by Li et al. [7] in 2010. Li [8] proposed a ranking order relation of TIFN using lexicographic technique. Nayagam et al. [12] introduced TIFNs of special type and described a method to rank them which seems to be unrealistic. Nehi [11] put forward a new ordering method for TIFNs in which two characteristic values for IFN.

Symmetric trapezoidal intuitionistic fuzzy numbers are ranked with a special ranking function which has been applied to solve a class of linear programming problems in which the data parameters are symmetric trapezoidal intuitionistic fuzzy number by Parvathi et al. [14] in 2012. Dubey et al. in 2011 developed a ranking technique for special form of triangular intuitionistic fuzzy numbers.

This paper is organized as follows. In Section 2 some preliminary definitions and concepts regarding intuitionistic fuzzy numbers were presented. In Section 3, we define the magnitude of different forms of trapezoidal and triangular intuitionistic fuzzy numbers. Section 4 is devoted to the illustration of some numerical examples for the concepts defined in the Section 3 and also contains the comparative study of results obtained by the proposed method with other existing ranking methods. Section 5 concludes the paper by giving some advantages of the proposed method over other methods.

2. Preliminaries

Definition 1. [1] An IFS A in X is given by

$$A = \{(x, \mu_A(x), \nu_A(x)), x \in X\},$$

where the functions $\mu_A(x)\colon X \to [0,1]$ and $\nu_A(x)\colon X \to [0,1]$ define, respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set A, which is a subset of X, and for every $x \in X,\ 0 \le \mu_A(x) + \nu_A(x) \le 1$.

Obviously, every fuzzy set has the form $\{(x, \mu_A(x), \mu_{A^c}(x)), x \in X\}$.

For each IFS A in X, we will call $\Pi_A(x) = 1 - \mu(x) - \nu(x)$ the intuitionistic fuzzy index of x in A. It is obvious that $0 \le \Pi_A(x) \le 1, \forall\, x \in X$.

Definition 2. [11]. An IFS $A = \{(x, \mu_A(x), \gamma_A(x) | x \in X)\}$ is called IF-normal, if there exist at least two points $x_0, x_1 \in X$ such that $\mu_A(x_0) = 1, \gamma_A(x_1) = 1$, It is easily seen that given intuitionistic fuzzy set A is IF-normal if there is at least one point that surely belongs to A and at least one point which does not belong to A.

Definition 3. [11]. An IFS $A = \{(x, \mu_A(x), \gamma_A(x) | x \in X)\}$ of the real line is called IF-convex, if

$$\forall x_1, x_2 \in \mathbb{R}, \forall \lambda \in [0,1],$$
$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \mu_A(x_1) \wedge \mu_A(x_2),$$
$$\gamma_A(\lambda x_1 + (1-\lambda)x_2) \geq \gamma_A(x_1) \wedge \gamma_A(x_2).$$

Thus A is IF –convex if its membership function is fuzzy convex and its non-membership function is fuzzy concave.

Definition 4. [11]. An IFS $A = \{(x, \mu_A(x), \gamma_A(x) | x \in X)\}$of the real line is called an IFN if

– A is IF-normal,
– A is IF-convex,
– μ_A is upper semicontinuous and γ_A is lower semicontinuous,
– $A = \{(x \in X \,| \gamma_A(x) < 1\}$ is bounded.

Definition 5. [11]. A is a trapezoidal intuitionistic fuzzy number with parameters $b_1 \leq a_1 \leq b_2 \leq a_2 \leq a_3 \leq b_3 \leq a_4 \leq b_4$ and denoted by $A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$. In this case we will give

$$\mu_A(x) = \begin{cases} 0 & ; x < a_{1,} \\ \dfrac{x - a_1}{a_2 - a_1} & ;\ a_1 \leq x \leq a_2 \\ 1 & ; a_2 \leq x \leq a_3 \\ \dfrac{x - a_4}{a_3 - a_4} & ; a_3 \leq x \leq a_4 \\ 0 & ; a_4 < x. \end{cases}$$

$$\gamma_A(x) = \begin{cases} 0 & ; x < b_{1,} \\ \dfrac{x - b_2}{b_1 - b_2} & ;\ b_1 \leq x \leq b_2 \\ 1 & ; b_2 \leq x \leq b_3 \\ \dfrac{x - b_3}{b_4 - b_3} & ; b_3 \leq x \leq b_4 \\ 0 & ; b_4 < x. \end{cases}$$

In the above definition, if we let$b_2 = b_3$ ($and\ hence\ a_2 = a_3$), then we will get a triangular intuitionistic fuzzy number with parameters $b_1 \leq a_1 \leq (b_2 = a_2 = a_3 = b_{3)} \leq a_4 \leq b_4$ and denoted by $A = (b_1, a_1, b_2, a_4, b_4)$.

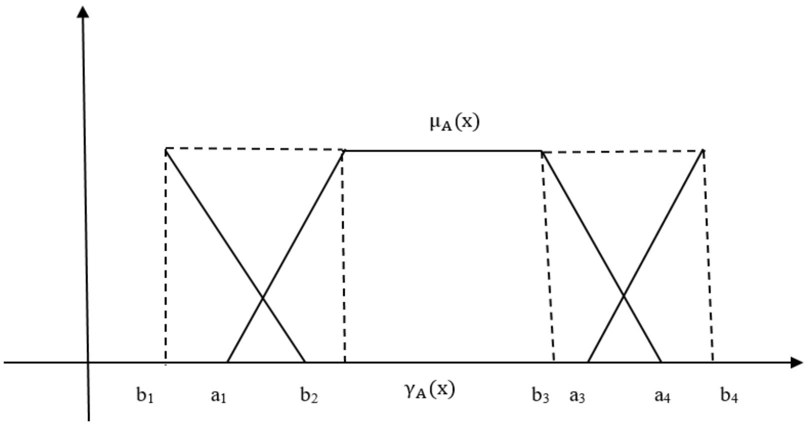

Fig. 1. *Trapezoidal intuitionistic fuzzy number.*

Definition 6. [7]. A TIFN $\tilde{a} = (\underline{a}, a, \overline{a}\,; w_{\tilde{a}}, u_{\tilde{a}})$ is a special IF set on the real number set R, whose membership function and non-membership function are defined as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{w_{\tilde{a}}(x-\underline{a})}{(a-\underline{a})} & \text{if } \underline{a} \le x < a \\ w_{\tilde{a}} & \text{if } x = a \\ \dfrac{w_{\tilde{a}}(\overline{a}-x)}{(\overline{a}-a)} & \text{if } a < x \le \overline{a} \\ 0 & \text{if } x < \underline{a} \text{ or } x > \overline{a} \end{cases}$$

$$v_{\tilde{a}}(x) = \begin{cases} \dfrac{[a-x+u_{\tilde{a}}(x-\underline{a})]}{(a-\underline{a})} & \text{if } \underline{a} \le x < a \\ u_{\tilde{a}} & \text{if } x = a \\ \dfrac{[x-a+u_{a}(\overline{a}-x)]}{(\overline{a}-a)} & \text{if } a < x \le \overline{a} \\ 0 & \text{if } x < \underline{a} \text{ or } x > \overline{a} \end{cases}$$

Where the values $w_{\tilde{a}}$ and $u_{\tilde{a}}$ represent the maximum degree of membership and the minimum degree of non-membership, respectively, such that they satisfy the conditions $0 \le w_{\tilde{a}} \le 1, 0 \le u_{\tilde{a}} \le 1, 0 \le w_{\tilde{a}} + u_{\tilde{a}} \le 1$.

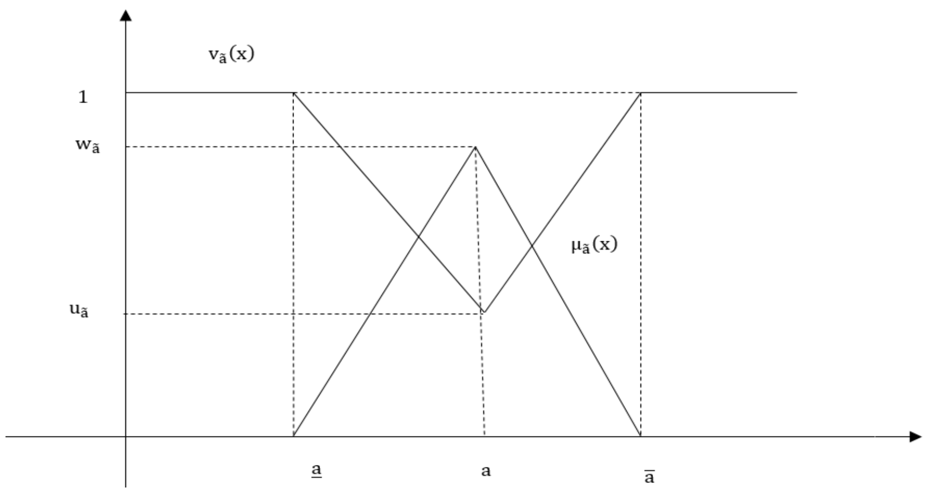

Fig. 2. *Triangular intuitionistic fuzzy number.*

Definition 7. [14]. An IFN $\tilde{A}$ in R is said to be a symmetric trapezoidal intuitionistic fuzzy numbers if there exists real numbers a_1, a_2, h, h' where $a_1 \le a_2, h \le h'$ and $h, h' > 0$ such that the membership and non-membership functions are as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-(a_1-h)}{h} & ; x \in [a_1-h, a_1] \\ 1 & ; x \in [a_1, a_2] \\ \dfrac{a_2+h-x}{h} & ; x \in [a_2, a_2+h] \\ 0 & ; \text{otherwise} \end{cases}$$

$$v_{\tilde{A}}(x) = \begin{cases} \dfrac{(a_1-x)}{h'} & ; x \in [a_1-h', a_1] \\ 0 & ; x \in [a_1, a_2] \\ \dfrac{x-a_2}{h'} & ; x \in [a_2, a_2+h'] \\ 1 & ; \text{otherwise} \end{cases}$$

Definition 8. [17]. A Generalized Triangular Intuitionistic Fuzzy Number (GTIFN) $\tilde{\tau}_a = \langle (a, l_\mu, r_\mu; w_a), (a, l_\gamma, r_\gamma; u_a) \rangle$ is a special intuitionistic fuzzy set on a real number set $\mathfrak{R}$, whose membership function and non-membership functions are defined as follows:

$$\mu_{\tilde{\tau}_a}(x) = \begin{cases} \dfrac{x-a+l_\mu}{l_\mu} w_a & ; a-l_\mu \le x < a \\ w_a & ; x = a \\ \dfrac{a+r_\mu-x}{r_\mu} w_a & ; a < x \le a+r_\mu \\ 0 & ; \text{otherwise} \end{cases}$$

$$v_{\tilde{\tau}_a}(x) = \begin{cases} \dfrac{(a-x)+u_a(x-a+l_\gamma)}{l_\gamma} & ; a-l_\gamma \le x < a \\ u_a & ; x = a \\ \dfrac{(x-a)+u_a(a+r_\gamma-x)}{r_\gamma} & ; a < x \le a+r_\gamma \\ 1 & ; \text{otherwise} \end{cases}$$

Where $l_\mu, r_\mu, l_\gamma, r_\gamma$ are called the spreads of membership and non-membership functions, respectively and a is called mean value. w_a and u_a represent the maximum degree of membership and minimum degree of non-membership respectively such that they satisfy the conditions $0 \le w_a \le 1, 0 \le u_a \le 1$ and $0 \le w_a + u_a \le 1$.

Definition 9. [13]**.** A TIFN is an intuitionistic fuzzy set in R with the following membership function $\mu_A(x)$ and non-membership function $\vartheta_A(x)$

$$\mu_A(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & , a_1 \le x \le a_2 \\ \dfrac{x-a_3}{a_2-a_3} & , a_2 \le x \le a_3 \\ 0, & \text{otherwise} \end{cases}$$

$$\vartheta_A(x) = \begin{cases} \dfrac{a_2 - x}{a_2 - a_1'} & , a_1' \le x \le a_2 \\ \dfrac{x - a_2}{a_3' - a_2} & , a_2 \le x \le a_3' \\ 1 & , \text{otherwise.} \end{cases}$$

Where $a_1' \le a_1 \le a_2 \le a_3 \le a_3'$ and $\mu_A(x) + \vartheta_A(x) \le 1$ or $\mu_A(x) = \vartheta_A(x)$ for all $x \in R$. This TIFN is denoted by $A = (a_1, a_2, a_3; a_1', a_2, a_3')$.

Definition 10. [18]. Let $\tilde{a} = \langle([a,b,c,d]; \mu_{\tilde{a}}), ([a_1,b,c,d_1]; \gamma_{\tilde{a}})\rangle$ be a trapezoidal intuitionistic fuzzy number whose membership and non-membership is given by

$$\mu_{\tilde{a}} = \begin{cases} \dfrac{x-a}{b-a}\mu_{\tilde{a}}, & a \le x < b \\ 1 & , \quad b \le x \le c \\ \dfrac{d-x}{d-c}\mu_{\tilde{a}}, & c < x \le d \\ 0 & , \text{otherwise} \end{cases}$$

$$\gamma_{\tilde{a}} = \begin{cases} \dfrac{b - x + \gamma_{\tilde{a}}(x - a_1)}{b-a} & , a_1 \le x < b \\ 0 & , \quad b \le x \le c \\ \dfrac{x - c + \gamma_{\tilde{a}}(d_1 - x)}{d_1 - c} & , \quad c < x \le d_1 \\ 1 & , \text{otherwise.} \end{cases}$$

Where $0 \le \mu_{\tilde{a}} \le 1, 0 \le \gamma_{\tilde{a}} \le 1, \mu_{\tilde{a}} + \gamma_{\tilde{a}} \le 1, a, b, c, d \in R$. When $b = c$, the intuitionistic trapezoidal fuzzy number becomes intuitionistic triangular fuzzy number.

3. New Approach for Ranking of Intuitionistic Fuzzy Numbers

In this section we define the concept of magnitude of an intuitionistic fuzzy number and discussed various methods for ranking the different forms of triangular intuitionistic fuzzy numbers and trapezoidal intuitionistic fuzzy numbers by means of magnitude**.**

Definition 11. Let $A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ be a Trapezoidal intuitionistic fuzzy number we define magnitude as follows:

$$\mathrm{Mag}(A) = \frac{1}{2}\int_0^1 (f_A(x) + g_A(x) + h_A(x) + k_A(x))\, f(r)dr. \quad (1)$$

where (r) is a non-negative and increasing weighting function on $[0,1]$ with

$$f(0) = 0, \qquad f(1) = 1 \text{ and } \int_0^1 f(r)dr = \frac{1}{2}.$$

In this paper we assume $f(r) = r$ for our convenience, we get magnitude of A as

$$\mathrm{Mag(A)} = \frac{1}{12}\,(a_1 + b_1 + a_4 + b_4 + 2(a_2 + a_3 + b_2 + b_3))\,. \qquad (2)$$

Using this definition of $Mag(A)$, we define the ranking procedure of any two trapezoidal intuitionistic fuzzy numbers as follows:

- $Mag(A) > Mag\ (B) iff\ A \succ B$.
- $Mag(A) < Mag\ (B) iff\ A \prec B$.
- $Mag(A) = Mag\ (B) iff\ A \backsim B$.

Remark 1. If $A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)\ and\ B = (b_1', a_1', b_2', a_2', a_3', b_3', a_4', b_4')$ be any two trapezoidal intuitionistic fuzzy numbers, then $Mag(A + B) = Mag\ A + Mag\ B$.

Definition 12. We define magnitude of a symmetric trapezoidal intuitionistic fuzzy number, $A = (b_1, a_1, a_2, a_2, a_3, a_3, a_4, b_4)$ using *Eq. (1)* as

$$\mathrm{Mag(A)} = \frac{1}{12}\,(a_1 + b_1 + a_4 + b_4 + 4(a_2 + a_3)). \qquad (3)$$

Remark 2. For any two symmetric trapezoidal intuitionistic fuzzy numbers $A = (a_1, a_2, h, h, a_1, a_2, h', h'), B = (a_1, a_2, k, k, a_1, a_2, k', k')$, we have

$$\mathrm{Mag(A)} = \mathrm{Mag(B)}. \qquad (4)$$

Remark 3. For any symmetric trapezoidal intuitionistic fuzzy number

$$A = (-a_1, a_1, h, h, -a_1, a_1, h', h'),\ \mathrm{Mag(A)} = 0. \qquad (5)$$

Definition 13. For a trapezoidal intuitionistic fuzzy number $A = (a_1, a_2, \alpha, \beta, a_1, a_2, \alpha', \beta')$

$$\mathrm{Mag(A)} = \frac{1}{12}\,(\beta - \alpha + 6(a_1 + a_2) + 2(\beta' - \alpha'). \qquad (6)$$

Definition 14. Let $A = (a_1, a_2, h, h, a_1, a_2, h', h')$ be a symmetric trapezoidal intuitionistic fuzzy number. Then its magnitude defined by

$$\mathrm{Mag}(A) = \frac{1}{2}(a_1 + a_2).. \quad (7)$$

Definition 15. Let $A = (a_1, b_1, c_1, d_1; a_1', b_1, c_1, d_1')$ be a trapezoidal intuitionistic fuzzy number, then

$$\mathrm{Mag}(A) = \frac{1}{12}(a_1 + d_1 + 2(a_1' + d_1') + 3(b_1 + c_1)). \quad (8)$$

Definition 16. If $A = (\overline{a}, a, \underline{a}; w_a, u_a)$ is a triangular intuitionistic fuzzy number , then

$$\mathrm{Mag}(A) = \frac{1}{12}\left[\frac{4a-2(\overline{a}+\underline{a})+3w_a(\underline{a}+\overline{a})}{w_a} + \frac{2(\overline{a}+a+\underline{a})-3u_a(a+\overline{a})}{(1-u_a)}\right] \quad (9)$$

Definition 17. Let $A = ((a, l_\mu, r_\mu; w_a), (a, l_\gamma, r_\gamma; u_a))$ be a triangular intuitionistic fuzzy number. Then

$$\mathrm{Mag}\ (A) = \frac{1}{12}\left\{\left(\frac{6aw_a - 3w_a(l_\mu - r_\mu) + 2(l_\mu - r_\mu)}{w_a} + \frac{6(a - au_a) + 3u_a(l_\gamma - r_\gamma) + 2(r_\gamma - l_\gamma)}{(1 - u_a)}\right)\right\}. \quad (10)$$

Definition 18. Let $A = \langle [a, b, c, d]; \mu_a, \gamma_a \rangle$ be a trapezoidal intuitionistic fuzzy number, then

$$\mathrm{Mag}(A) = \frac{1}{12}\left\{\frac{2(b - a + c - d) + 3\mu_a(a + d)}{\mu_a} + \frac{2(a + d) + (b + c) - 3\gamma_a(a + d)}{1 - \gamma_a}\right\}. \quad (11)$$

Definition 19. Consider a triangular intuitionistic fuzzy number of the form $A = (a_1, a_2, a_3; a_1', a_2, a_3')$, then

$$\mathrm{Mag}(A) = \frac{1}{12}\{a_1 + a_3 + 6a_2 + 2(a_1' + a_3')\} \quad (12)$$

4. Numerical Examples

This section illustrates some examples for comparative analysis of various existing ranking methods

Example 1. Consider two trapezoidal intuitionistic fuzzy numbers as follows: A= (0.2,0.4,0.6,0.8,0.11,0.12,0.13,0.15) and B=(0,0.1,0.2,0.3,0.4,0.5,0.6,0.7). In [11], Nehi used characteristic values of membership or non-membership functions to rank trapezoidal intuitionistic fuzzy numbers. The ranking procedure depends on the value of ‘k’. As ‘k’ varies in the interval$(0, \infty)$, the ranking also varies which leads to an unreasonable result. This can be seen from the following example.

Table 1. *Calculation of* $c_{\mu}^{k}(A)$.

a_1	a_2	a_3	a_4	b_1	b_2	b_3	b_4	k	$c_{\mu}^{k}(A)$
0.4	0.8	0.11	0.13	0.2	0.6	0.12	0.15	1	0.392
0.4	0.8	0.11	0.13	0.2	0.6	0.12	0.15	2	0.408

Table 2. *Calculation of* $c_{\mu}^{k}(B)$.

a_1	a_2	a_3	a_4	b_1	b_2	b_3	b_4	k	$c_{\mu}^{k}(B)$
0.1	0.3	0.4	0.6	0	0.2	0.5	0.7	1	0.350
0.2	0.4	0.5	0.7	0.1	0.3	0.6	0.8	2	0.450

From the table, we see that when k=1, $A > B$ and when k=2, $B > A$

Example 2. Consider two symmetric trapezoidal intuitionistic fuzzy numbers A=(23,25,1,1;23,25,3,3) and B = (5,7,2,2; 5,7,4,4) as in [15]. Here the ranking of STIFNs are obtained by a special ranking function by considering all the parameters of both membership and non- membership functions of given STIFNs. The values obtained by this method are similar to the proposed method.

Example 3. Consider two trapezoidal intuitionistic fuzzy numbers of the forms A = (0.2,0.3,0.4,0.5; 0.1,0.3,0.4,0.6) and B = (0.1,0.2,0.3,0.4; 0,0.2,0.3,0.5) discussed in [16]. Rezvani used value index of membership and non-membership functions separately to rank trapezoidal intuitionistic fuzzy numbers.

Example 4. Consider three triangular intuitionistic fuzzy numbers as below A=(0.592,0.774,0.910;0.6,0.4) , B=(0.769,0.903,1;0.4,0.5) and C=(0.653,0.849,0.956;0.5,0.2) as given in [7]. In the paper [7] Li used ratio ranking method to rank triangular intuitionistic fuzzy numbers and applied it to multi attribute decision making problem In the case of ration ranking method, the raking differs on the choice of λ. For the above IFN's we have

Table 3. *Ranking of IFN's for values of* λ.

S.No	λ	Ranking results
1	$[0, 0.1899)$	$A > C > B$
2	$(0.1899,0.9667)$	$C > A > B$
3	$(0.9667,1]$	$C > B > A$

So this leads to a conflicted state which yields an unreasonable result.

Example 5. Consider the same IFN's as in *example 4* and ranking developed in [8]. Here the ranking is done by the extended additive weighted method using the value-index and ambiguity-index. For the above numbers, we have the following ranking results as tabulated below from [8].

Table 4. *Ranking of IFN's for values of* λ.

S.No.	λ	Ranking results
1	[0,0.793]	$C > A > B >$
2	(0.793,1]	$A > C > B$

From the above table, we see that the ranking differs on the basis of given weight λ.

Example 6. Consider the two Generalized triangular fuzzy intuitionistic numbers $\tilde{\tau}_a = \big((5,1,2;0.6),(5,1.5,2.6;0.3)\big)$ and $\tilde{\tau}_b = \big((6,2,1;0.6),(6,2.1,1.5;0.4)\big)$ in [17]. If we use $R_{\mu(}\tilde{\tau}_a)$ to rank these numbers we obtain $\tilde{\tau}_a < \tilde{\tau}_b$. But when we rank in terms of $R_{\gamma}(\tilde{\tau}_a)$, we get $\tilde{\tau}_a > \tilde{\tau}_b$. Hence the ranking of generalized triangular intuitionistic fuzzy numbers varies with the use of membership and non-membership value in ranking. This is an unreasonable result. Therefore the proposed method which uses both membership and non-membership values as a whole is suitable for ranking such GTIFN's.

Example 7. Consider the two triangular intuitionistic fuzzy numbers as follows: A = {(14,15,17;0.9),(10,15,18;0)} and B = {(25,30,34;0.9),(23,30,38;0)}as in [4]. In this paper, Dubey used the concept of value and ambiguity of a triangular intuitionistic fuzzy numbers to rank the above numbers. The ranking obtained in [4] is similar to the proposed method.

Example 8. Consider 5 set of trapezoidal intuitionistic fuzzy number as in [18].

$$\widetilde{a_1} = \langle [0.407,0.539,0.683,0.814]; 0.727,0.21 \rangle.$$

$$\widetilde{a_2} = \langle [0.547,0.679,0.810,0.942]; 0.705,0.230 \rangle.$$

$$\widetilde{a_3} = \langle [0.424,0.572,0.704,0.868]; 0.697,0.252 \rangle.$$

$$\widetilde{a_4} = \langle [0.392,0.557,0.724,0.902]; 0.639,0.280 \rangle.$$

$$\widetilde{a_5} = \langle [0.411,0.555,0.699,0.831]; 0.812,0.137 \rangle.$$

In [18] ranking is done based on the comparison of score function values and accuracy function values of integrated intuitionistic fuzzy numbers. The ranking here in [18] differs from our proposed method.

Example 9. Consider two triangular intuitionistic fuzzy numbers as below $\tilde{A} = \{(2.68,3,3.71); (2.2,3,4.67)\}$ and $B = \{(2.75,6,9.375); (2.38,6,16.2)\}$ as in [13]. In this paper ranking is done by using the score function and the result obtained is similar to the proposed method.

The following table gives a comparative analysis of various ranking methods so far defined in intuitionistic fuzzy setting with the proposed method.

Table 5. *Comparative analysis of different ranking methods.*

S.No	Intuitionistic Fuzzy Numbers	Existing Method	Proposed Method
1	*A=(0.2,0.4,0.6,0.8,0.11,0.12,0.13,0.15)* *B=(0,0.1,0.2,0.3,0.4,0.5,0.6,0.7)*	$c_\mu^k(A) = 0.392$; $c_\mu^k(B) = 0.35$, $A \succ B$ [11]	$Mag(A) = 0.35$ $Mag(B) = 0.35$ $A \sim B$
2	*A=(23,25,1,1;23,25,3,3)* *B=(5,7,2,2;5,7,4,4)*	$\Re(A) = 49$ $\Re(B) = 13$, $A \succ B$ [15]	$Mag\ (A) = 24$; $Mag(B) = 6$, $A \succ B$
3	*A=(0.2,0.3,0.4,0.5;0.1,0.3,0.4,0.6)* *B= (0.1,0.2,0.3,0.4;0,0.2,0.3,0.5)*	$v_\mu(A) = 0.35$; $v_\mu(B) = 0.25$, $A \succ B$ [16]	$Mag(A) = 0.35$; $Mag(B) = 0.25$ $A \succ B$
4	*A=(0.592,0.774,0.910;0.6,0.4)* *B=(0.769,0.903,1;0.4,0.5)* *C=(0.653,0.849,0.956;0.5,0.2)*	$R(A,\lambda) = 0.4321$; $R(B,\lambda) = 0.3455$; $R(C,\lambda) = 0.3858$ $A \succ C \succ B$ [7]	$Mag(A) = 0.8282$; $Mag(B) = 0.9688$; $Mag(C) = 0.9322$, $B \succ C \succ A$
5	*A=(0.592,0.774,0.910;0.6,0.4)* *B=(0.769,0.903,1;0.4,0.5)* *C=(0.653,0.849,0.956;0.5,0.2)*	$v_\lambda(A) = 0.276$; $v_\lambda(B) = 0.224$; $v_\lambda(C) = 0.534$, $C > A > B$ [8]	$Mag(A) = 0.828$; $Mag(B) = 0.969$; $Mag(C) = 0.932$, $B > C > A$
6	$\tau_a = \big((5,1,2;0.6),(5,1.5,2.6;0.3)\big)$ $\tau_b = \big((6,2,1;0.6),(6,2.1,1.5;0.4)\big)$	$R_\gamma(\tau_a) = 3.98$, $R_\gamma(\tau_b) = 3.51$, $A \succ B$ [17]	$Mag(\tau_a) = 5.12$, $Mag\ (\tau_b) = 5.96$, $A \prec B$
7	$A = \{(14,15,17;0.9),(10,15,18;0)\}$ $B = \{(25,30,34;0.9),(23,30,38;0)\}$	$F(A,\lambda) = 13.76$; $F(B,\lambda) = 27.47$; $A \succ B$ [4]	$Mag(A) = 15.28$; $Mag(B) = 31.74$ $A \succ B$
8	$\widetilde{a_1} = \langle[0.407,0.539,0.683,0.814];0.727,0.21\rangle$ $\widetilde{a_2} = \langle[0.547,0.679,0.810,0.942];0.705,0.230\rangle$ $\widetilde{a_3} = \langle[0.424,0.572,0.704,0.868];0.697,0.252\rangle$ $\widetilde{a_4} = \langle[0.392,0.557,0.724,0.902];0.639,0.280\rangle$ $\widetilde{a_5} = \langle[0.411,0.555,0.699,0.831];0.812,0.137\rangle$	$S(\widetilde{a_1}) = 0.236$; $S(\widetilde{a_2}) = 0.261$; $S(\widetilde{a_3}) = 0.206$; $S(\widetilde{a_4}) = 0.153$; $S(\widetilde{a_5}) = 0.353$; $a_5 \succ a_2 \succ a_1 \succ a_3 \succ a_4$ [18]	$Mag(\widetilde{a_1}) = 0.611$; $Mag(\widetilde{a_2}) = 0.745$; $Mag(\widetilde{a_3}) = 0.640$; $Mag(\widetilde{a_4}) = 0.642$; $Mag(\widetilde{a_5}) = 0.625$; $a_2 \succ a_4 \succ a_3 \succ a_5 \succ a_1$
9	$\tilde{A} = \{(2.68,3,3.71);(2.2,3,4.67)\}$ *and* $\tilde{B} = \{(2.75,6,9.375);(2.38,6,16.2)\}$	$S(\tilde{A}) = 3.2175$ $S(\tilde{B}) = 7.645$ $\tilde{A} \prec \tilde{B}$ [13]	$Mag(\tilde{A}) = 3.1772$ $Mag(\tilde{B}) = 7.12$ $\tilde{A} \prec \tilde{B}$

5. Conclusions

In many of the existing ranking methods, ranking is done either by considering the membership or non-membership values only. But in the newly proposed method the ranking is done directly by taking both membership and non-membership values in a single formula. This ranking procedure is very simple and time consuming compared to the existing methods. We also illustrated the advantages of our method by means of suitable examples. The proposed ranking technique can be applied to multi-criteria decision making problems, linear programming problems, assignment problems, transportation, some management problems and industrial problems which are our future research works.

References

[1] Atanassov K. (1986). Intuitionistic fuzzy sets. *Fuzzy sets and systems*, *20*, 87-96.

[2] Abbasbandy, S., & Hajjari, T. (2009). A new approach for ranking of trapezoidal fuzzy numbers. *Computers & mathematics with applications*, *57*(3), 413-419.

[3] Allahviranloo, T., Abbasbandy, S., & Saneifard, R. (2011). A method for ranking of fuzzy numbers using new weighted distance. *Mathematical and computational applications*, *16*(2), 359-369.

[4] Dubey, D., & Mehra, A. (2011). Linear programming with triangular intuitionistic fuzzy number. *Proceedings of the 7th conference of the european society for fuzzy logic and technology* (pp. 563-569). Atlantis Press.

[5] Grzegrorzewski, P. (2003). The hamming distance between intuitionistic fuzzy sets. *Proceedings of the 10th IFSA world congress*, *Istanbul*, *Turkey* (Vol. 30, pp. 35-38).

[6] Li, D. F. (2010). A ratio ranking method of triangular intuitionistic fuzzy numbers and its application to MADM problems. *Computers & mathematics with applications*, *60*(6), 1557-1570.

[7] Li, D. F., Nan, J. X., & Zhang, M. J. (2010). A ranking method of triangular intuitionistic fuzzy numbers and application to decision making. *International journal of computational intelligence Systems*, *3*(5), 522-530.

[8] Mitchell, H. B. (2004). Ranking-intuitionistic fuzzy numbers. *International journal of uncertainty, fuzziness and knowledge-based systems*, *12*(03), 377-386.

[9] Mahapatra, G. S., & Mahapatra, G. S. (2010). Intuitionistic fuzzy fault tree analysis using intuitionistic fuzzy numbers. *International mathematical forum*, *5*(21), 1015-1024.

[10] Nehi, H. M. (2010). A new ranking method for intuitionistic fuzzy numbers. *International journal of fuzzy systems*, *12*(1).

[11] Nayagam, V.L.G., Venkateshwari, G., Sivaraman, G., (2008). Ranking of intuitionistic fuzzy numbers, *Proc. of international conference on fuzzy systems 2008, Fuzz-IEEE* (pp. 1971-1974).

[12] Nagoorgani, A., & Ponnalagu, K. (2012). A new approach on solving intuitionistic fuzzy linear programming problem. *Applied mathematical sciences*, *6*(70), 3467-3474.

[13] Parvathi, R., & Malathi, C. (2012). Arithmetic operations on symmetric trapezoidal intuitionistic fuzzy numbers. *International Journal of Soft Computing and Engineering*, 2, 268-273.

[14] Parvathi, R., & Malathi, C. (2012). Intuitionistic fuzzy simplex method. *International journal of computer applications*, *48*(6), 39-48.

[15] Rezvani, S. (2013). Ranking method of trapezoidal intuitionistic fuzzy numbers. *Annals of fuzzy mathematics and informatics*, *5*(3), 515-523.

[16] Seikh, M. R., Nayak, P. K., & Pal, M. (2012). Generalized triangular fuzzy numbers in intuitionistic fuzzy environment. *International journal of engineering research and development*, *5*(1), 08-13.

[17] Jianqiang, W., & Zhong, Z. (2009). Aggregation operators on intuitionistic trapezoidal fuzzy number and its application to multi-criteria decision making problems. *Journal of systems engineering and electronics*, *20*(2), 321-326.

[18] Wang, J. (2008). Overview on fuzzy multi-criteria decision-making approach. *Control and decision*, *23*(6), 601-606.