

A New Decision Making Approach for Winning Strategy Based on Muti Soft Set Logic

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P A P E R I N F O	A B S T R A C T
<p>Chronicle: Received: 03 February 2020 Revised: 28 March 2020 Accepted: 29 May 2020</p>	<p>We introduce a new concept of certainty and coverage of a parameter of the soft set and present a new decision making approach for the soft set over the universe using the certainty of a parameter. Also, we point out the drawbacks of the reduct definition by pointing out the delusion of Proposition 14 given by Herawan et al. [20] and provide the revised definition of the reduct of the multi soft set.</p>
<p>Keywords: Certainty; Coverage. Flow Graph. Decision Making. Multi-Valued Information System. Multi Soft Set.</p>	

1. Introduction

Many practical problems involve data that contain uncertainties. These uncertainties may be dealt with existing theories such as fuzzy set theory [1] and rough set theory [2]. In 1999, Molodstov [3] pointed out the difficulties of these theories and he posited the concept of soft set theory. Maji et al. [4] made a theoretical study of soft set in 2003. Soft set theory has rich potential for applications. In [5], Maji et al. presented an application of soft sets in decision making problems and extend the concept into fuzzy soft sets in [6]. In the year 2010, Cagman et al. gave the *uni – int* decision making algorithm using soft sets [7] and soft matrices [8]. Feng et al. [9] extended Cagman and Enginoglu’s *uni – int* decision making algorithm. They introduced several new soft decision making methods including $uni - int^k$, $uni - int^t$ and $int^m - int^n$ decision making methods. Also, Han et al. [10] initiated the pruning method for solving $int^m - int^n$ decision making method. Feng et al. [11] introduced the concept of discernibility matrix in 2014 and using this, they provided a decision making algorithm for soft sets. Also, in the same year, Dauda et al. [12] presented a decision making algorithm of soft sets using AND and OR operations. In 2020, Wang et al. [13] introduced a novel plausible reasoning based on intuitionistic fuzzy propositional logic and Meng et al. [14] proposed an inequality approach with the quasi-ordered set to evaluate the performances of decision making units. Recently many authors studied the concepts of decision making in terms of fuzzy set and intuitionistic fuzzy sets [15, 16, 17, 18]. In this paper, we define a new definition of certainty of a parameter of the soft set and with the help of this definition, we present a new decision making approach for the soft set over the universe which is a partition of objects. The

standard soft set deals with a binary-valued information system. For a multi-valued information system, Herawan et al. [19] introduced a concept of multi soft sets in 2009. Also, they [20] introduced the reduct concept in multi soft sets using the value class of the multi soft matrix. In this paper, we point out the delusion of proposition 14 of [20] and provide the revised definition of the reduct of the multi soft set.

Definition 1. [20]. The idea of multi soft sets is based on a decomposition of a multi-valued information system $S = (U, A, V, f)$ into $|A|$ number of binary valued information systems $S = (U, A, V_{\{0,1\}}, f)$ where $|A|$ denotes the cardinality of A . Consequently, the $|A|$ binary valued information systems, define multi soft sets $(F, A_m) = \{(F, a_i) \mid 1 \leq i \leq |A|\}$.

Definition 2. [20]. Matrix $M_{a_i}, 1 \leq i \leq |A|$ is called matrix representation of the soft set (F, a_i) over universe U . The dimension of matrices is defined by $\dim(M_{a_i}) = |U| \times |V_{a_i}|$. All entries of $M_{a_i} = [a_{ij}]$ is belong to a set $\{0,1\}$ where $a_{ij} = \begin{cases} 0 & \text{if } |f(u, \alpha)| = 0 \\ 1 & \text{if } |f(u, \alpha)| = 1 \end{cases}$ where $1 \leq i \leq |U|, 1 \leq j \leq |V_{a_i}|, u \in U$ and $\alpha \in V_{a_i}$. The collection of all matrices representing (F, A_m) is denoted by M_A . That is, $M_A = \{M_{a_i} \mid 1 \leq i \leq |A|\}$.

Definition 3. [20]. Let $M_{a_i} \in M_A$ be a matrix representation of a multi soft set (F, A_m) over U . The value class of M_{a_i} , that is, class of all value sets of M_{a_i} , denoted $C_{M_{a_i}}$ is defined by $C_{M_{a_i}} = \{\{u \mid |f(u, \alpha_1)| = 1\}, \dots, \{u \mid |f(u, \alpha_{|V_{a_i}|})| = 1\}\}, 1 \leq i \leq |V_{a_i}|, u \in U$ and $\alpha \in V_{a_i}$. Clearly, $C_{M_{a_i}} \subseteq \wp(U)$.

Definition 4. [20]. Let $M_{a_i} = [a_{kl}], 1 \leq k \leq |U|, 1 \leq l \leq |V_{a_i}|$ and $M_{a_j} = [a_{mn}], 1 \leq m \leq |U|, 1 \leq n \leq |V_{a_j}|$ be two matrices in M_A . The AND operation between M_{a_i} and M_{a_j} is defined as $M_{a_i} \text{ AND } M_{a_j} = M_{a_{ij}} = [a_{pq}]$ with $\dim(M_{a_{ij}}) = |U| \times (|V_{a_i}| \times |V_{a_j}|)$ where $a_{p1} = \min\{a_{k1}, a_{m1}\}, a_{p2} = \min\{a_{k1}, a_{m2}\}, \dots, a_{p(|V_{a_i}| \times |V_{a_j}|)} = \min\{a_{k|V_{a_i}|}, a_{m|V_{a_j}|}\}$.

Proposition 1. Let M_A be a multi soft matrix over U representing multi soft set (F, A_m) . A set of attributes B of A is a reduct for A if only if $C_{AND M_b} = C_{AND M_a}$.

2. Soft Sets

In this section, we define a new definition for soft set over the universe.

Definition 5. Let (F, E) be a soft set over U . Suppose U is partitioned into t classes, namely, U_1, U_2, \dots, U_t . If $U_1 = \{u_1, u_2, \dots, u_i\}, U_2 = \{u_{i+1}, u_{i+2}, \dots, u_j\}, \dots, U_t = \{u_{k+1}, u_{k+2}, \dots, u_n\}$, then the soft set tabular representation of (F, E) as follows.

Table 1. Tabular representation of soft set.

U	e_1	e_2	\dots	e_m
u_1	$f(u_1, e_1)$	$f(u_1, e_2)$	\dots	$f(u_1, e_m)$
u_2	$f(u_2, e_1)$	$f(u_2, e_2)$	\dots	$f(u_2, e_m)$
\dots	\dots	\dots	\dots	\dots
u_i	$f(u_i, e_1)$	$f(u_i, e_2)$	\dots	$f(u_i, e_m)$
u_{i+1}	$f(u_{i+1}, e_1)$	$f(u_{i+1}, e_2)$	\dots	$f(u_{i+1}, e_m)$
u_{i+2}	$f(u_{i+2}, e_1)$	$f(u_{i+2}, e_2)$	\dots	$f(u_{i+2}, e_m)$
\dots	\dots	\dots	\dots	\dots
u_j	$f(u_j, e_1)$	$f(u_j, e_2)$	\dots	$f(u_j, e_m)$
\dots	\dots	\dots	\dots	\dots
u_{k+1}	$f(u_{k+1}, e_1)$	$f(u_{k+1}, e_2)$	\dots	$f(u_{k+1}, e_m)$
u_{k+2}	$f(u_{k+2}, e_1)$	$f(u_{k+2}, e_2)$	\dots	$f(u_{k+2}, e_m)$
\dots	\dots	\dots	\dots	\dots
u_n	$f(u_n, e_1)$	$f(u_n, e_2)$	\dots	$f(u_n, e_m)$

where $f(u_i, e_j) = 1$ if $u_i \in F(e_j)$ and $f(u_i, e_j) = 0$ otherwise.

Definition 6. Let (F, E) be a soft set over U with the partitions U_1, U_2, \dots, U_l . Then we define the following.

- The support of $e \in E$ is defined as $supp(e) = \sum_{U_i \in U} supp_{U_i}(e)$ where $supp_{U_i}(e) = |\{u_j \in U_i \mid f(u_j, e) = 1\}|$ and $supp(A_1, A_2)$ is the number of occurrences of A_2 with respect to the parameter A_1 .
- The coverage of $A_1 \Rightarrow A_2$ is defined by $cov(A_1, A_2) = \frac{supp(A_1, A_2)}{|U|}$.
- The certainty of $A_1 \Rightarrow A_2$ is defined by $cer(A_1, A_2) = \sum_{U_i \in U} cer(U_i, A_1, A_2)$ where $cer(U_i, A_1, A_2) = \frac{supp_{U_i}(A_1, A_2)}{|U_i|}$.

Example 1. Consider the soft set (F, E) over the universe $U = \{U_1, U_2, U_3\}$ where $E = \{e_1, e_2, e_3, e_4, e_5\}$, $U_1 = \{u_1, u_2, u_3\}$, $U_2 = \{v_1, v_2, v_3, v_4\}$ and $U_3 = \{w_1, w_2, w_3\}$ whose tabular representation is given below.

Table 2. Tabular representation of (F, E).

U	e ₁	e ₂	e ₃	e ₄	e ₅
u ₁	1	0	1	0	0
u ₂	0	1	1	0	1
u ₃	0	0	1	1	0
v ₁	1	0	1	0	1
v ₂	0	1	0	1	1
v ₃	1	0	0	0	0
v ₄	0	1	1	1	0
w ₁	1	0	1	0	1
w ₂	0	0	1	1	1
w ₃	1	0	0	0	1

Then $cer(U_1, e_1, 1) = \frac{1}{3} = 0.333$, $cer(U_1, e_1, 0) = \frac{2}{3} = 0.667$, $cer(U_2, e_1, 1) = \frac{2}{4} = 0.5$, $cer(U_2, e_1, 0) = \frac{2}{4} = 0.5$, $cer(U_3, e_1, 1) = \frac{2}{3} = 0.667$, $cer(U_3, e_1, 0) = \frac{1}{3} = 0.333$. Hence $cer(e_1, 1) = 1.5$ and $cer(e_1, 0) = 1.5$. And $cer(U_1, e_2, 1) = \frac{1}{3} = 0.333$, $cer(U_1, e_2, 0) = \frac{2}{3} = 0.667$, $cer(U_2, e_2, 1) = \frac{2}{4} = 0.5$, $cer(U_2, e_2, 0) = \frac{2}{4} = 0.5$, $cer(U_3, e_2, 1) = \frac{0}{3} = 0$, $cer(U_3, e_2, 0) = \frac{3}{3} = 1$. Thus, we have $cer(e_2, 1) = 0.833$ and $cer(e_2, 0) = 2.167$. Also, $cer(U_1, e_3, 1) = \frac{3}{3} = 1$, $cer(U_1, e_3, 0) = \frac{0}{3} = 0$, $cer(U_2, e_3, 1) = \frac{2}{4} = 0.5$, $cer(U_2, e_3, 0) = \frac{2}{4} = 0.5$, $cer(U_3, e_3, 1) = \frac{2}{3} = 0.667$, $cer(U_3, e_3, 0) = \frac{1}{3} = 0.333$. Therefore, $cer(e_3, 1) = 2.167$ and $cer(e_3, 0) = 0.833$. Now, $cer(U_1, e_4, 1) = \frac{1}{3} = 0.333$, $cer(U_1, e_4, 0) = \frac{2}{3} = 0.667$, $cer(U_2, e_4, 1) = \frac{2}{4} = 0.5$, $cer(U_2, e_4, 0) = \frac{2}{4} = 0.5$, $cer(U_3, e_4, 1) = \frac{1}{3} = 0.333$, $cer(U_3, e_4, 0) = \frac{2}{3} = 0.667$. Hence $cer(e_4, 1) = 1.166$ and $cer(e_4, 0) = 1.834$. And $cer(U_1, e_5, 1) = \frac{1}{3} = 0.333$, $cer(U_1, e_5, 0) = \frac{2}{3} = 0.667$, $cer(U_2, e_5, 1) = \frac{2}{4} = 0.5$, $cer(U_2, e_5, 0) = \frac{2}{4} = 0.5$, $cer(U_3, e_5, 1) = \frac{3}{3} = 1$, $cer(U_3, e_5, 0) = \frac{0}{3} = 0$. Thus, $cer(e_5, 1) = 1.833$ and $cer(e_5, 0) = 1.167$.

Also, $cov(e_1, 1) = \frac{5}{10} = 0.5$ and $cov(e_1, 0) = \frac{5}{10} = 0.5$, $cov(e_2, 1) = \frac{3}{10} = 0.3$ and $cov(e_2, 0) = \frac{7}{10} = 0.7$, $cov(e_3, 1) = \frac{7}{10} = 0.7$ and $cov(e_3, 0) = \frac{3}{10} = 0.3$, $cov(e_4, 1) = \frac{4}{10} = 0.4$ and $cov(e_4, 0) = \frac{6}{10} = 0.6$, $cov(e_5, 1) = \frac{6}{10} = 0.6$ and $cov(e_5, 0) = \frac{4}{10} = 0.4$. Then the flow graph associated with certainty and coverage is given in the following Fig. 1.

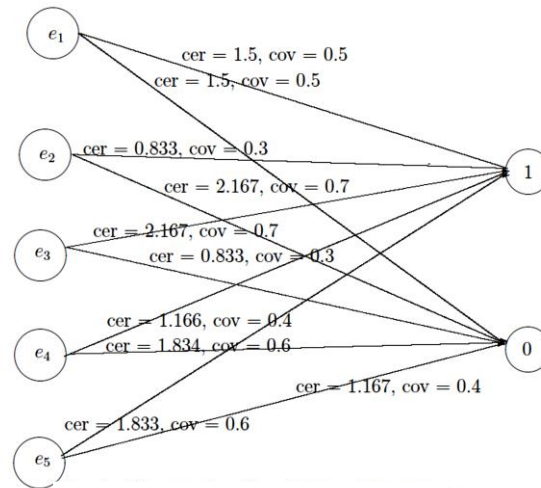


Fig. 1. Flow graph with certainty and coverage.

From the Fig. 1, the approximate graph for the flow graph is given as follows.

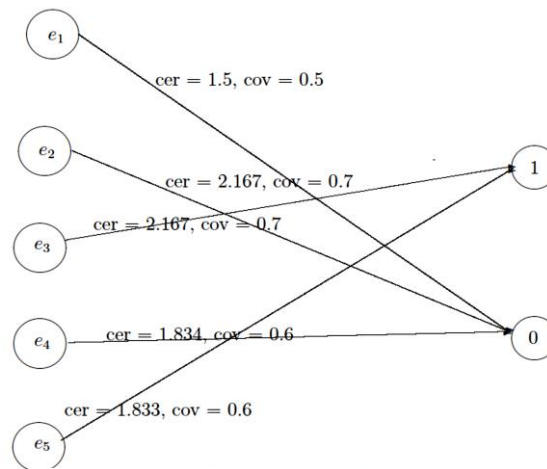


Fig. 2. Approximate graph.

3. Experimental Results

In this section, we illustrate the proposed approach through an example of a data set. Let (F, E) be a soft set over the universe $U = \{U_1, U_2, U_3, U_4\}$ and $E = \{e_1, e_2, e_3, e_4, e_5\}$ where e_1 stands for the Party 'A', e_2 stands for the Party 'B', e_3 stands for the Party 'C', e_4 stands for the Party 'D', e_5 stands for the Party 'E' and $U_1 = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$ is the set of people who are politicians, $U_2 = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\}$ is the set of formers, $U_3 = \{w_1, w_2, w_3, w_4\}$ is the set of government employees and $U_4 = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$ is the set of students.

Table 3. Tabular representation for the given soft set (F, E) .

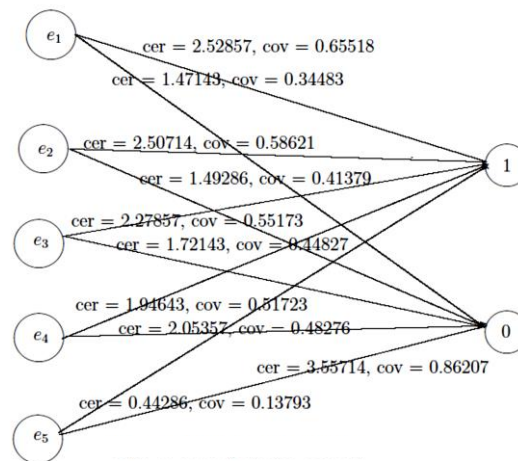
U	e₁	e₂	e₃	e₄	e₅
u_1	0	1	1	0	0
u_2	1	1	0	0	0
u_3	0	1	0	1	0
u_4	1	0	1	1	0
u_5	0	1	0	1	1
u_6	1	1	1	0	0
u_7	0	1	0	1	0
v_1	1	0	1	1	0
v_2	0	1	1	0	0
v_3	1	0	1	0	1
v_4	1	1	0	1	0
v_5	0	0	1	0	1
v_6	1	0	1	1	1
v_7	1	1	0	0	0
v_8	0	1	0	1	0
v_9	1	0	0	1	0
v_{10}	0	0	1	0	0
w_1	0	1	1	0	0
w_2	1	0	1	0	0
w_3	1	1	1	0	0
w_4	0	1	0	1	0
x_1	1	0	0	1	0
x_2	1	1	0	1	0
x_3	1	0	1	0	0
x_4	1	0	0	1	0
x_5	1	1	0	0	0
x_6	1	1	1	0	0
x_7	1	0	1	1	0
x_8	1	1	1	1	0

Then the certainty and coverage of each party is given in the following Table 4.

Table 4. Certainty and coverage of parties.

E	Certainty	Coverage
$(e_1, 1)$	2.52857	0.65518
$(e_1, 0)$	1.47143	0.34483
$(e_2, 1)$	2.50714	0.58621
$(e_2, 0)$	1.49286	0.41379
$(e_3, 1)$	2.27857	0.55173
$(e_3, 0)$	1.72143	0.44827
$(e_4, 1)$	1.94643	0.51723
$(e_4, 0)$	2.05357	0.48276
$(e_5, 1)$	0.44286	0.13793
$(e_5, 0)$	3.55714	0.86207

From Fig. 3, branches of the flow graph represent the parties together with their certainty and coverage factors. For instance, the $(e_1, 1)$ has the certainty factor 2.52857 and coverage factor 0.65518.

**Fig. 3.** Flow graph for Table 4.

The flow graph gives a clear insight into the winning strategy of all parties. We can replace flow graph shown in Figure by "approximate" flow graph shown in Fig. 4. From the Fig. 4, we can conclude that the Parties A, B and C are the winning parameters whose the coverage of 0.65518, 0.58621 and 0.55173, respectively.

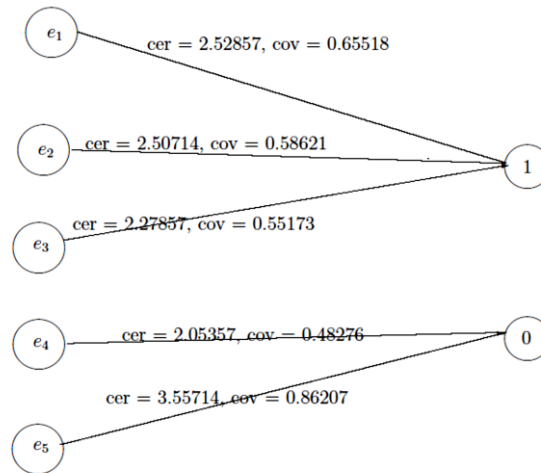


Fig. 4. Approximate flow graph.

4. Erratum

In [10], the authors defined the reduct of a multi soft set and gave a characterization for reduct of a multi soft set. The following Example shows that reduct of a multi soft set under the Proposition 1 need not be unique.

Example 2. Consider the multi-valued information system given in Example 15 of [10]. Then the matrices representing the multi soft set (F, A_m) is $M_A = \{M_{a_1}, M_{a_2}, M_{a_3}, M_{a_4}\}$ where

$$M_{a_1} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, M_{a_2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}, M_{a_3} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, M_{a_4} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

Here M_{a_3} AND $M_{a_4} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$ and $C_{1 \leq j \leq 4}^{AND M_{a_j}} = \{\{1\}, \{2\}, \{3, 4\}, \{5\}\}.$

Now, M_{a_2} AND $(M_{a_3}$ AND $M_{a_4}) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

Therefore, $C_{M_{a_2} \text{ AND } M_{a_3} \text{ AND } M_{a_4}} = \{\{1\}, \{2\}, \{3,4\}, \{5\}\}$.

$$\text{Also, } M_{a_1} \text{ AND } (M_{a_3} \text{ AND } M_{a_4}) = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Therefore, $C_{M_{a_1} \text{ AND } M_{a_3} \text{ AND } M_{a_4}} = \{\{1\}, \{2\}, \{3,4\}, \{5\}\}$. Hence $\{a_1, a_3, a_4\}, \{a_2, a_3, a_4\}$ are reductions of (F, A_m) . Also, in [10], the authors determined the reductions of (F, A_m) and gave the reductions as $\{a_1, a_2, a_3\}$ and $\{a_3, a_4\}$. That is, the reductions are not unique.

Based on the Example 2, we have the following Lemma.

Lemma 1. Suppose E_1 and E_2 are two members of the value class of $AND M_{a_i}$. Then for any $e_1 \in E_1, e_2 \in E_2, \{e_1, e_2\}$ will not be a subset of any value class of $AND M_{a_i}$.

Proof. Suppose $e_1 \in E_1$ and $e_2 \in E_2$. Since $E_1 \neq E_2, e_{1i} = 1$ and $e_{2j} = 1$ for some i and j . Suppose $\{e_1, e_2\} \subseteq F$ where F is a value class of $AND M_{a_i}$. Then $e_{1k} = e_{2k} = 1$ for some k . By the definition of "AND" product, $e_{11} = e_{21} = 1$ in the matrix $AND M_{a_i}$. That is, e_1 and e_2 belong to the same value class of $AND M_{a_i}$ which is a contradiction.

The following Theorem shows that superset of a reduct set is again a reduct set.

Theorem 1. If B is a reduction of the multi soft set (F, A_m) , then $B \cup C$ is also a reduction of (F, A_m) where $C \subseteq A - B$.

Proof. Suppose B is a reduction of (F, A_m) . Then $C_{AND M_{a_i}}^{a_i \in B} = C_{AND M_{a_i}}^{a_i \in A}$. That is, the number of value classes of $AND M_{a_i}$ and the number of value classes of $AND M_{a_i}$ are equal. Also, by the definition of

“AND” product, the number of value classes of $AND M_{a_i}$ is less than or equal to the number of value classes of $AND M_{a_i}$. Therefore, by Lemma 1, for any $C \subseteq A - B$, the value classes of $AND M_{a_i}$ and the value classes of $AND M_{a_i}$ are equal. Hence $C_{a_i \in B \cup C}^{AND M_{a_i}} = C_{a_i \in B}^{AND M_{a_i}}$.

By the above Theorem, whenever B is a reduction of the multi soft set (F, A_m) , $B \cup C$ is also a reduction of (F, A_m) . In Example 15 of [10], the authors gave reduction of the multi soft set as $\{a_1, a_3, a_4\}, \{a_2, a_3, a_4\}, \{a_1, a_2, a_3\}$ and $\{a_3, a_4\}$. But by Theorem 1, any set containing a reduction set is a reduct set and hence the whole set A is a reduct set. Thus, we remove the redundancy, we modify the reduct definition for multi soft set as follows.

Definition 6. Let M_A be a multi soft set over U representing multi soft set (F, A_m) . A set of attributes B of A is a reduct for A if B is a minimal subset of A such that $C_{b \in B}^{AND M_b} = C_{a \in A}^{AND M_a}$.

Example 3. Consider the multi soft set as in Example 4.1. Here $\{a_1, a_2, a_3\}$ and $\{a_3, a_4\}$ are reductions of (F, A_m) but $\{a_1, a_3, a_4\}$ and $\{a_2, a_3, a_4\}$ are not reductions of (F, A_m) .

5. Conclusion

In this paper, we have presented a new decision making approach for the soft set over the universe with partition of objects using the certainty and coverage of a parameter. Also, we have pointed out the misconception of the reduct definition given by Herawan et al. [20].

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