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Some Remarks on Neutro-Fine Topology

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Abstract

The neutro-fine topological space is a space that contains a combination of neutrosophic and fine sets. In this study, the various types of open sets such as generalized open and semi-open sets are defined in such space. The concept of interior and closure on semi-open sets are defined and some of their basic properties are stated. These definitions extend the concept to generalized semi-open sets. Moreover, the minimal and maximal open sets are defined and some of their properties are studied in this space. As well as, discussed the complement of all these sets as its closed sets. The basic properties of the union and intersection of these open sets are stated in some theorems. Only a few sets satisfy this postulates, and others are disproved as shown in the counterexamples. The converse of some theorems is proved in probable examples.

Keywords: Neutro-Fine-Generalized open sets, Neutro-Fine-Semi open sets, Neutro-Fine-Semi interior, Neutro-Fine-Semi closure, Neutro-Fine-Generalized semi open sets, Neutro-Fine minimal open set, Neutro-Fine maximal open sets.

1 | Introduction

The classical set theory developed by Zadeh [40] was termed as a Fuzzy Set (FS), whose elements amuse ambiguous features of true and false membership functions. The FS theory applied in the boundless area of a domain, while Atanassov [39] extended this theory as an Intuitionistic Fuzzy Set (IFS) theory. Later, Smarandache [21] explored a set that contains one more membership function called indeterminacy along with truth and falsity degrees as elements of the Neutrosophic Set (NS). Also, he generalized the NS on IFS [22] and recently proposed his work on attributes valued set, Plithogenic Set (PS) [23]. Nowadays, this set made an outstanding impact on many applications [1]-[4], [11]-[15], [16], [18], [19] and play a vital role in Decision Making (DM) problems [10], [17], [20] and Multi-Criteria DM (MCDM) problems [5], [9].

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Topology is a study of flexible objects under frequent damages without splitting. In recent times, Topological Space (TS) is performing a lead character in the enormous branch of applied sciences and numerous categories of mathematics. The topological structure developed on NS as a generalization of IFTS which was originated by Salama & Alblowi [33], [34], named as Neutrosophic Topological Space (NTS). Few typical sets, open sets, and other TS explored [7], [24], [27], [28], [29], [31], and extended to bi-topological space [6] on such TS.

The most general class of sets which contains few open sets termed as Fine-Open Sets (FOSs), by Powar & Rajak [35], and investigated the special case of generalized TS, called Fine- Topological Space (FTS). Many researchers studied this concept on some sets like FS [26], [30], and others [25], [32]. Recently, this concept extends as Neutro-Fine Topological Space (NFTS) [8], which was introduced by Chinnadurai and Sindhu. The concept of minimal open (closed) and maximal open (closed) sets were exhibited by few researchers [36]- [38].

The aspiration of this paper is to instigate the collection of open sets such as generalized open and semi-open sets defined on NFTS. The concept of interior and closure on neutro-fine-semi open sets are defined and some of their basic properties are stated. These definitions extend the concept to generalized semi-open sets. Moreover, the minimal and maximal open sets are defined and some of their properties are studied in this space. Simultaneously, discussed the complement of all these sets as its closed sets. The basic properties of the union and intersection of these sets are stated in some theorems. Only a few sets satisfy this postulates, and others are disproved as shown in the counterexamples. The converse of some theorems is proved in probable examples.

The layout of this proposal is as follows. In Portion 2, essential definitions of NFTS are recollected. In Portion 3, some type of generalized open sets are defined on NFTS and investigated its properties with illustrative examples. In Portion 4, some more open sets like neutro-fine minimal open sets and neutro-fine maximal open sets are explored via perfect examples. In the end, Portion 6 conveyed the conclusions with some future works.

2| Preliminaries

In this portion, we remind a few major descriptions connected to NFTS.

Definition 1. [8]. Let W be a set of universe and $w_i \in W$ where $i \in I$. Let R be a NS over W. Then the subset of NS R with respect to w_i (sub-NS R_{w_i}) and w_i, w_j (sub-NS R_{w_i,w_j}) are denoted as $\varsigma_R(w_i)$ and $\varsigma_R(w_i,w_j)$, and defined as

$$\varsigma_R(w_i)=\left\{\begin{array}{l}\left\langle w_i, T_R(w_i), I_R(w_i), F_R(w_i)\right\rangle, \left\langle \begin{array}{l} w_{i,j}, max\left(T_R(w_i), T_R(w_j)\right), max\left(I_R(w_i), I_R(w_j)\right), \\ min\left(F_R(w_i), F_R(w_j)\right)\end{array}\right\rangle, \\ \left\langle w_k, T_R(0_n), I_R(0_n), F_R(0_n)\right\rangle, \left\langle w_{k,l}, T_R(0_n), I_R(0_n), F_R(0_n)\right\rangle\end{array}\right\}$$

where $i \in I$, $j \in I-\{i\}$, $k,l \in I-\{i,j\}$ and $k \neq l$ and

Chinnadurai and Sindhu |J. Fuzzy. Ext. Appl. 1(3) (2020) 159-179

$$\varsigma_R(w_i,w_j)=\left\{\begin{array}{l}\left\langle w_i,T_R(w_i),I_R(w_i),F_R(w_i)\right\rangle,\left\langle w_j,T_R(w_j),I_R(w_j),F_R(w_j)\right\rangle,\left\langle w_k,T_R(0_n),I_R(0_n),F_R(0_n)\right\rangle,\\ \left\langle w_{i,j},max\left(T_R(w_i),T_R(w_j)\right),max\left(I_R(w_i),I_R(w_j)\right),min\left(F_R(w_i),F_R(w_j)\right)\right\rangle,\\ \left\langle w_{i,k},max\left(T_R(w_i),T_R(w_k)\right),max\left(I_R(w_i),I_R(w_k)\right),min\left(F_R(w_i),F_R(w_k)\right)\right\rangle,\\ \left\langle w_{j,k},max\left(T_R(w_j),T_R(w_k)\right),max\left(I_R(w_j),I_R(w_k)\right),min\left(F_R(w_j),F_R(w_k)\right)\right\rangle\end{array}\right\}$$

where $i,j,k\in I$ and $i\neq j\neq k$, respectively.

Definition 2. [8]. Let W be a set of universe and $w\in W$. Let R be a NS over *W* and *V* be any proper non- empty subset of *W*. Then $\varsigma_R(V)$ is said to be neutro-fine set (NFS) over *W*.

Definition 3. [8]. Let NFS(W) be the family of all NFSs over W. Then the fine collection of $\varsigma_R(V)$ is denoted as ${}^f\varsigma_W$ and defined over the NT (W, τ_n) as ${}^f\varsigma_W=\left\{0_{nf},1_{nf},\bigcup\varsigma_R(V)\right\}$.

Thus the triplet $\left(W,\tau_n,{}^f\varsigma_W\right)$ is said to be a NFTS over (W, τ_n). The elements belong to ${}^f\varsigma_W$ are said to be neutro-fine open sets (NFOSs) over (W, τ_n) and the complement of NFOSs are said to be neutro-fine closed sets (NFCSs) over (W, τ_n) and denote the collection by ${}^F\varsigma_W$.

Definition 4. [8]. Let $\left(W,\tau_n,{}^f\varsigma_W\right)$ be a NFTS over (W, τ_n). Let $\varsigma_R(V)$ be a NFS over W. Then the neutro-fine interior of $\varsigma_R(V)$ is denoted as $Int_{nf}\left(\varsigma_R(V)\right)$ and is defined as the union of all NFOSs contained in $\varsigma_R(V)$.

Clearly, $Int_{nf}\left(\varsigma_R(V)\right)$ is the largest NFOS contained in $\varsigma_R(V)$.

Definition 5. [8]. Let $\left(W,\tau_n,{}^f\varsigma_W\right)$ be a NFTS over (W, τ_n). Let $\varsigma_R(V)$ be a NFS over W. Then the neutro-fine closure of $\varsigma_R(V)$ is denoted as $Cl_{nf}\left(\varsigma_R(V)\right)$ and is defined as the intersection of all NFCSs containing $\varsigma_R(V)$.

Clearly, $Cl_{nf}\left(\varsigma_R(V)\right)$ is the smallest NFCS containing $\varsigma_R(V)$.

Definition 6. [8]. Let NF(W) be the family of all NFs over the universe W and $w\in W$. Then NFS $w^{\langle\alpha,\beta,\chi\rangle}$ is said to be a neutro-fine point (NFP), for $0\leq\alpha,\beta,\gamma\leq 1$ and is defined as follows:

$$w^{\langle\alpha,\beta,\gamma\rangle}(v)=\begin{cases}(\alpha,\beta,\gamma), & \text{if } w=v\\ (0,0,1), & \text{if } w\neq v\end{cases}.$$

Every NFS is the union of its NFPs.

Definition 7. [8]. Let $\left(W,\tau_n,{}^f\varsigma_W\right)$ be a NFTS over (W,τ_n). Let $\varsigma_R(V)$ be a NFS over W. Then $\varsigma_R(V)$ is said to be a neutro-fine neighborhood of the NFP $w^{\langle\alpha,\beta,\chi\rangle}\in\varsigma_R(V)$, if there exists a NFOS $\varsigma_R(U)$ such that $w^{\langle\alpha,\beta,\chi\rangle}\in\varsigma_R(U)\subseteq\varsigma_R(V)$.

Proposition 1. [8]. Let $\left(W,\tau_n,{}^f\varsigma_W\right)$ be a NFTS. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be two NFSs over W. Then,

$Int_{nf}\left(0_{nf}\right)=0_{nf}$ and $Int_{nf}\left(1_{nf}\right)=1_{nf}$:

$\varsigma_R(V_1)$ is NFOS $\Rightarrow Int_{nf}\left(\varsigma_R(V_1)\right)=\varsigma_R(V_1)$;

$Int_{nf}\left(\varsigma_R(V_1)\right)\subseteq\varsigma_R(V_1)$;

$\varsigma_R(V_1)\subseteq\varsigma_R(V_2)\Rightarrow Int_{nf}\left(\varsigma_R(V_1)\right)\subseteq Int_{nf}\left(\varsigma_R(V_2)\right)$;

$Int_{nf}\left(Int_{nf}\left(\varsigma_R(V_1)\right)\right)=Int_{nf}\left(\varsigma_R(V_1)\right)$;

$Int_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right)=Int_{nf}\left(\varsigma_R(V_1)\right)\cap Int_{nf}\left(\varsigma_R(V_2)\right)$;

$Int_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)\subseteq Int_{nf}\left(\varsigma_R(V_1)\right)\cup Int_{nf}\left(\varsigma_R(V_2)\right)$;

$Int_{nf}\left(\varsigma_R(V_1)'\right)=\left[Cl_{nf}\left(\varsigma_R(V_1)\right)\right]'$.

Proof. Straightforward.

Proposition 2. [8]. Let $\left(W,\tau_n,{}^f\varsigma_W\right)$ be a NFTS. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be two NFSs over W. Then,

$Cl_{nf}\left(0_{nf}\right)=0_{nf}$ and $Cl_{nf}\left(1_{nf}\right)=1_{nf}$;

$\varsigma_R(V_1)$ is NFCS $\Rightarrow Cl_{nf}\left(\varsigma_R(V_1)\right)=\varsigma_R(V_1)$;

$Cl_{nf}\left(\varsigma_R(V_1)\right)\supseteq\varsigma_R(V_1)$;

$\varsigma_R(V_1)\subseteq\varsigma_R(V_2)\Rightarrow Cl_{nf}\left(\varsigma_R(V_1)\right)\subseteq Cl_{nf}\left(\varsigma_R(V_2)\right)$;

$Cl_{nf}\left(Cl_{nf}\left(\varsigma_R(V_1)\right)\right)=Cl_{nf}\left(\varsigma_R(V_1)\right)$;

$$Cl_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)=Cl_{nf}\left(\varsigma_R(V_1)\right)\cup Cl_{nf}\left(\varsigma_R(V_2)\right);$$

$$Cl_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right)\subseteq Cl_{nf}\left(\varsigma_R(V_1)\right)\cap Cl_{nf}\left(\varsigma_R(V_2)\right);$$

$$Cl_{nf}\left(\varsigma_R(V_1)'\right)=\left[Int_{nf}\left(\varsigma_R(V_1)\right)\right]'.$$

Proof. Straightforward.

3| Some Form of Generalized Open Sets in NFTS

In this portion, some open types of generalized open sets on NFTS are defined and probable results are carried by some major expressive examples. This portion is splitted into 3 sub-portions which states neutro-fine-generalized, neutro-fine-semi, and neutro-fine-generalized semi-open sets on NFTS.

3.1| Neutro-Fine-Generalized Open Sets

Let $\varsigma_R(V)$ be a NFS over W of a NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$. Then $\varsigma_R(V_1)$ is said to be a neutro-fine-generalized closed set (nf-GCS) if $Cl_{nf}\left(\varsigma_R(V)\right)\subseteq\varsigma_R(U)$ whenever $\varsigma_R(V)\subseteq\varsigma_R(U)$ and $\varsigma_R(U)$ is NFOS. The complement of nf-GCS is said to be neutro-fine-generalized open set (nf-GOS).

Theorem 1. Every NFCS is a nf-GCS in NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Proof. Let $\varsigma_R(V)$ be a NFCS on $\left(W,\tau_n,{}^f\varsigma_W\right)$. Let $\varsigma_R(V)\subseteq\varsigma_R(U)$, where $\varsigma_R(U)$ is NFOS in $\left(W,\tau_n,{}^f\varsigma_W\right)$. Since $\varsigma_R(V)$ is a NFCS, $\varsigma_R(V)=Cl_{nf}\left(\varsigma_R(V)\right)$ $\Rightarrow Cl_{nf}\left(\varsigma_R(V)\right)\subseteq\varsigma_R(V)$. Thus $Cl_{nf}\left(\varsigma_R(V)\right)\subseteq\varsigma_R(V)\subseteq\varsigma_R(U)$. Hence $\varsigma_R(V)$ is a nf-GCS in NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Remark 1. The converse of the above theorem is not true as shown in the following example.

Example 1. Let $W=\left\{w_1,w_2,w_3\right\}$ and $\tau_n=\left\{0_n,1_n,R,S,T,U\right\}$ where R, S, T and U are NSs over W and are defined as follows

$$R=\left\{\left\langle w_1,.2,.4,.7\right\rangle,\left\langle w_2,.6,.3,.1\right\rangle,\left\langle w_3,.4,.5,.6\right\rangle\right\},$$

$$S=\left\{\left\langle w_1,.9,.3,.6\right\rangle,\left\langle w_2,.6,.5,.4\right\rangle,\left\langle w_3,.7,.8,.1\right\rangle\right\},$$

$$T=\left\{\left\langle w_1,.9,.4,.6\right\rangle,\left\langle w_2,.6,.5,.1\right\rangle,\left\langle w_3,.7,.8,.1\right\rangle\right\}$$ and

$$U=\left\{\left\langle w_1,.2,.3,.7\right\rangle,\left\langle w_2,.6,.3,.4\right\rangle,\left\langle w_3,.4,.5,.6\right\rangle\right\}.$$

Thus (W,τ_n) is a NTS over W.

Then NFOSs over (W,τ_n) are ${}^f\varsigma_W=\left\{0_n,1_n,\varsigma_R(w_1),\varsigma_R(w_3),\varsigma_S(w_2,w_3)\right\}$, where

$$\varsigma_R(w_1)=\left\{\left\langle w_1,.2,.4,.7\right\rangle,\left\langle w_2,0,0,1\right\rangle,\left\langle w_3,0,0,1\right\rangle,\left\langle w_{1,2},.6,.4,.1\right\rangle,\left\langle w_{1,3},.4,.5,.6\right\rangle,\left\langle w_{2,3},0,0,1\right\rangle\right\},$$

$$\varsigma_R(w_3)=\left\{\left\langle w_1,0,0,1\right\rangle,\left\langle w_2,0,0,1\right\rangle,\left\langle w_3,.4,.5,.6\right\rangle,\left\langle w_{1,2},0,0,1\right\rangle,\left\langle w_{1,3},.4,.5,.6\right\rangle,\left\langle w_{2,3},.6,.5,.1\right\rangle\right\},$$

$$\varsigma_S(w_2,w_3)=\left\{\left\langle w_1,0,0,1\right\rangle,\left\langle w_2,.6,.5,.4\right\rangle,\left\langle w_3,.7,.8,.1\right\rangle,\left\langle w_{1,2},.9,.5,.4\right\rangle,\left\langle w_{1,3},.9,.8,.1\right\rangle,\left\langle w_{2,3},.7,.8,.1\right\rangle\right\}$$

and NFCSs over (W,τ_n) are ${}^F\varsigma_W=\left\{0_n,1_n,\varsigma_R(w_1)',\varsigma_R(w_3)'\ \varsigma_S(w_2,w_3)'\right\}$, where

$$\varsigma_R(w_1)'=\left\{\left\langle w_1,.7,.6,.2\right\rangle,\left\langle w_2,1,1,0\right\rangle,\left\langle w_3,1,1,0\right\rangle,\left\langle w_{1,2},.1,.6,.6\right\rangle,\left\langle w_{1,3},.6,.5,.4\right\rangle,\left\langle w_{2,3},1,1,0\right\rangle\right\},$$

$$\varsigma_R(w_3)'=\left\{\left\langle w_1,1,1,0\right\rangle,\left\langle w_2,1,1,0\right\rangle,\left\langle w_3,.6,.5,.4\right\rangle,\left\langle w_{1,2},1,1,0\right\rangle,\left\langle w_{1,3},.6,.5,.4\right\rangle,\left\langle w_{2,3},.1,.5,.6\right\rangle\right\},$$

$$\varsigma_S(w_2,w_3)'=\left\{\left\langle w_1,1,1,0\right\rangle,\left\langle w_2,.4,.5,.6\right\rangle,\left\langle w_3,.1,.2,.7\right\rangle,\left\langle w_{1,2},.4,.5,.9\right\rangle,\left\langle w_{1,3},.1,.2,.9\right\rangle,\left\langle w_{2,3},.1,.2,.7\right\rangle\right\}.$$

Thus $\left(W,\tau_n,{}^f\varsigma_W\right)$ is a NFTS over (W,τ_n). Here nf-GCS$=\left\{\varsigma_S(w_2),\varsigma_S(w_3),\varsigma_S(w_{2,3})\right\}$. Thus $\varsigma_S(w_2)$ is nf-GCS but not NFCS.

Theorem 2. If $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ are nf-GCSs over $\left(W,\tau_n,{}^f\varsigma_W\right)$, then $\varsigma_R(V_1)\cup\varsigma_R(V_2)$ is also a nf-GCS over $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Proof. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be nf-GCSs over $\left(W,\tau_n,{}^f\varsigma_W\right)$. Then $Cl_{nf}\left(\varsigma_R(V_1)\right)\subseteq\varsigma_R(U)$ whenever $\varsigma_R(V_1)\subseteq\varsigma_R(U)$ and $\varsigma_R(U)$ is NFOS and $Cl_{nf}\left(\varsigma_R(V)\right)\subseteq\varsigma_R(U)$ whenever

Chinnadurai and Sindhu |J. Fuzzy. Ext. Appl. 1(3) (2020) 159-179

$\varsigma_R(V)\subseteq\varsigma_R(U)$ and $\varsigma_R(U)$ is NFOS. Since $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ are subsets of $\varsigma_R(U)$, $\varsigma_R(V_1)\cup\varsigma_R(V_2)$ are subsets of $\varsigma_R(U)$ and $\varsigma_R(U)$ is NFOS. Then by *Proposition 2*, $Cl_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)=Cl_{nf}\left(\varsigma_R(V_1)\right)\cup Cl_{nf}\left(\varsigma_R(V_2)\right)$. Thus $Cl_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)\subseteq\varsigma_R(U)$. Hence $\varsigma_R(V_1)\cup\varsigma_R(V_2)$ is a nf-GCSs over $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Remark 2. The intersection of two nf-GCSs need not be a tnf-GCS as shown in the following example.

Example 2. Consider the *Example 1*. Here nf-GCS $=\left\{\varsigma_S(w_2),\varsigma_S(w_3),\varsigma_S(w_{2,3})\right\}$. Then

$$\varsigma_S(w_2)\cap\varsigma_S(w_3)=\left\{\left\langle w_1,0,0,1\right\rangle,\left\langle w_2,0,0,1\right\rangle,\left\langle w_3,0,0,1\right\rangle,\left\langle w_{1,2},0,0,1\right\rangle,\left\langle w_{1,3},0,0,1\right\rangle,\left\langle w_{2,3},.7,.8,.1\right\rangle\right\},$$

is not a nf-GCS.

Theorem 3. If $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ are nf-GCSs over $\left(W,\tau_n,{}^f\varsigma_W\right)$, then

$$Cl_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right)\subseteq Cl_{nf}\left(\varsigma_R(V_1)\right)\cap Cl_{nf}\left(\varsigma_R(V_2)\right).$$

Proof. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be nf-GCSs over $\left(W,\tau_n,{}^f\varsigma_W\right)$. Then $Cl_{nf}\left(\varsigma_R(V_1)\right)\subseteq\varsigma_R(U)$ whenever $\varsigma_R(V_1)\subseteq\varsigma_R(U)$ and $\varsigma_R(U)$ is NFOS and $Cl_{nf}\left(\varsigma_R(V)\right)\subseteq\varsigma_R(U)$ whenever $\varsigma_R(V)\subseteq\varsigma_R(U)$ and $\varsigma_R(U)$ is NFOS.

Since $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ are subsets of $\varsigma_R(U)$, $\varsigma_R(V_1)\cap\varsigma_R(V_2)$ are subsets of $\varsigma_R(U)$ and $\varsigma_R(U)$ is NFOS.

Since $\varsigma_R(V_1)\cap\varsigma_R(V_2)\subseteq\varsigma_R(V_1)$ and $\varsigma_R(V_1)\cap\varsigma_R(V_2)\subseteq\varsigma_R(V_1)$, $Cl_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right)\subseteq Cl_{nf}\left(\varsigma_R(V_1)\right)$ and $Cl_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right)\subseteq Cl_{nf}\left(\varsigma_R(V_2)\right)$, by *Proposition 2*. Thus $Cl_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right)\subseteq Cl_{nf}\left(\varsigma_R(V_1)\right)\cap Cl_{nf}\left(\varsigma_R(V_2)\right)$.

3.2 | Neutro-Fine-Semi Open Sets

Definition 8. Let $\varsigma_R(V)$ be a NFS over W of a NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$. Then $\varsigma_R(V)$ is said to be a neutro-fine-semi closed set (nf-SCS) if $Int_{nf}\left(Cl_{nf}\left(\varsigma_R(V)\right)\right)\subseteq\varsigma_R(V)$.

The complement of nf-SCS is said to be neutro-fine-semi open set (nf-SOS), i.e., $\varsigma_R(V)\subseteq Cl_{nf}\left(Int_{nf}\left(\varsigma_R(V)\right)\right)$.

Theorem 4. Let $\left(W, \tau_n, {}^f\varsigma_W\right)$ be a NFTS and $\varsigma_R(V)$ be a NFS over W. Then $\varsigma_R(V)$ is nf-SCS if and only if $\varsigma_R(V)'$ is nf-SOS.

Proof. Let $\varsigma_R(V)$ be a nf-SCS. Then $Int_{nf}\left(Cl_{nf}\left(\varsigma_R(V)\right)\right) \subseteq \varsigma_R(V)$.

Taking complement on both sides,

$$\varsigma_R(V)' \subseteq \left[Int_{nf}\left(Cl_{nf}\left(\varsigma_R(V)\right)\right)\right]' = Cl_{nf}\left(Cl_{nf}\left(\varsigma_R(V)\right)\right)'.$$

By using *Proposition 1*, $\varsigma_R(V)' = Cl_{nf}\left(Int_{nf}\left(\varsigma_R(V)'\right)\right)$. Thus $\varsigma_R(V)'$ is a nf-SOS. Conversely, assume that $\varsigma_R(V)'$ is a nf-SOS. Then $\varsigma_R(V)' = Cl_{nf}\left(Int_{nf}\left(\varsigma_R(V)'\right)\right)$.

Taking complement on both sides,

$$\varsigma_R(V) \supseteq \left[Cl_{nf}\left(Int_{nf}\left(\varsigma_R(V)'\right)\right)\right]' = Int_{nf}\left(Int_{nf}\left(\varsigma_R(V)'\right)\right)', \text{ by } \textit{Proposition 1}.$$

By Proposition 2, $\varsigma_R(V) \supseteq Int_{nf}\left(Cl_{nf}\left(\varsigma_R(V)\right)\right)$. Thus $\varsigma_R(V)$ is a nf-SCS.

Theorem 5. If $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ are nf-SCSs over NFTS $\left(W, \tau_n, {}^f\varsigma_W\right)$, then $\varsigma_R(V_1) \cap \varsigma_R(V_2)$ is also a nf-SCS in $\left(W, \tau_n, {}^f\varsigma_W\right)$.

Proof. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be nf-SCSs over $\left(W, \tau_n, {}^f\varsigma_W\right)$. Then $Int_{nf}\left(Cl_{nf}\left(\varsigma_R(V_1)\right)\right) \subseteq \varsigma_R(V_1)$ and $Int_{nf}\left(Cl_{nf}\left(\varsigma_R(V_1)\right)\right) \subseteq \varsigma_R(V_1)$. Thus $\varsigma_R(V_1) \cap \varsigma_R(V_2) \supseteq Int_{nf}\left(Cl_{nf}\left(\varsigma_R(V_1)\right)\right) \cap Int_{nf}\left(Cl_{nf}\left(\varsigma_R(V_2)\right)\right)$

$$= Int_{nf}\left(Cl_{nf}\left(\varsigma_R(V_1)\right) \cap Cl_{nf}\left(\varsigma_R(V_2)\right)\right) \supseteq Int_{nf}\left(Cl_{nf}\left(\varsigma_R(V_1) \cap \varsigma_R(V_2)\right)\right), \text{ by } \textit{Propositions 1} \text{ and } \textit{2}.$$

Hence $\varsigma_R(V_1) \cap \varsigma_R(V_2)$ is a nf-SCSs in $\left(W, \tau_n, {}^f\varsigma_W\right)$.

Theorem 6. If $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ are nf-SOSs over NFTS $\left(W, \tau_n, {}^f\varsigma_W\right)$, then $\varsigma_R(V_1) \cup \varsigma_R(V_2)$ is also a nf-SOS in $\left(W, \tau_n, {}^f\varsigma_W\right)$.

Chinnadurai and Sindhu |J. Fuzzy. Ext. Appl. 1(3) (2020) 159-179

Proof. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be nf-SCSs over $\left(W,\tau_n,{}^f\varsigma_W\right)$. Then $\varsigma_R(V_1)\subseteq Cl_{nf}\left(Int_{nf}\left(\varsigma_R(V_1)\right)\right)$ and $\varsigma_R(V_2)\subseteq Cl_{nf}\left(Int_{nf}\left(\varsigma_R(V_2)\right)\right)$. Thus $\varsigma_R(V_1)\cup\varsigma_R(V_2)\subseteq Cl_{nf}\left(Int_{nf}\left(\varsigma_R(V_1)\right)\right)\cup Cl_{nf}\left(Int_{nf}\left(\varsigma_R(V_2)\right)\right)$

$=Cl_{nf}\left(Int_{nf}\left(\varsigma_R(V_1)\right)\cup Int_{nf}\left(\varsigma_R(V_2)\right)\right)\ \supseteq Cl_{nf}\left(Int_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)\right)$, by *Propositions 1* and *2*.

Theorem 7. Every NFCS is a nf-SCS in NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Proof. Let $\varsigma_R(V)$ be a NFCS on $\left(W,\tau_n,{}^f\varsigma_W\right)\frac{1}{2}$. Then $\varsigma_R(V)=Cl_{nf}\left(\varsigma_R(V)\right)$. Thus

$Int_{\eta f}\left(Cl_{nf}\left(\varsigma_R(V)\right)\right)\subseteq Cl_{\eta f}\left(\varsigma_R(V)\right)\ \Rightarrow Int_{\eta f}\left(Cl_{nf}\left(\varsigma_R(V)\right)\right)\subseteq\varsigma_R(V)$. Hence $\varsigma_R(V)$ is a nf-SCS in $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Definition 9. Let $\left(W,\tau_n,{}^f\varsigma_W\right)$ be a NFTS over (W,τ_n). Let $\varsigma_R(V)$ be a NFS over W. Then the neutro-fine-semi interior of $\varsigma_R(V)$ is denoted as $S^*Int_{nf}\left(\varsigma_R(V)\right)$ and is defined as the union of all nf-SOSs contained in $\varsigma_R(V)$. Clearly, $S^*Int_{nf}\left(\varsigma_R(V)\right)$ is the largest nf-SOS contained in $\varsigma_R(V)$.

Definition 10. Let $\left(W,\tau_n,{}^f\varsigma_W\right)$ be a NFTS over (W,τ_n). Let $\varsigma_R(V)$ be a NFS over W. Then the neutro-fine-semi closure of $\varsigma_R(V)$ is denoted as $S^*Cl_{nf}\left(\varsigma_R(V)\right)$ and is defined as the intersection of all nf-SCSs containing $\varsigma_R(V)$. Clearly, $S^*Cl_{nf}\left(\varsigma_R(V)\right)$ is the smallest nf-SCS containing $\varsigma_R(V)$.

Proposition 3. Let $\left(W,\tau_n,{}^f\varsigma_W\right)$ be a NFTS. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be two NFSs over W. Then,

$S^*Int_{nf}\left(\varsigma_R(V_1)\right)\subseteq\varsigma_R(V_1)$;

$\varsigma_R(V_1)$ is nf-SOS $\Rightarrow S^*Int_{nf}\left(\varsigma_R(V_1)\right)=\varsigma_R(V_1)$;

$S^*Int_{nf}\left(S^*Int_{nf}\left(\varsigma_R(V_1)\right)\right)=S^*Int_{nf}\left(\varsigma_R(V_1)\right)$;

$\varsigma_R(V_1)\subseteq\varsigma_R(V_2)\Rightarrow S^*Int_{nf}\left(\varsigma_R(V_1)\right)\subseteq S^*Int_{nf}\left(\varsigma_R(V_2)\right)$.

Proof. Straightforward.

Proposition 4. Let $\left(W,\tau_n,{}^f\varsigma_W\right)$ be a NFTS. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be two NFSs over W. Then,

$\varsigma_R(V_1) \subseteq S^* Cl_{nf}\left(\varsigma_R(V_1)\right)$;

$\varsigma_R(V_1)$ is nf-SCS $\Rightarrow S^* Cl_{nf}\left(\varsigma_R(V_1)\right) = \varsigma_R(V_1)$;

$S^* Cl_{nf}\left(S^* Cl_{nf}\left(\varsigma_R(V_1)\right)\right) = S^* Cl_{nf}\left(\varsigma_R(V_1)\right)$;

$\varsigma_R(V_1) \subseteq \varsigma_R(V_2) \Rightarrow S^* Cl_{nf}\left(\varsigma_R(V_1)\right) \subseteq S^* Cl_{nf}\left(\varsigma_R(V_2)\right)$.

Proof. Straightforward.

Proposition 5. Let $\left(W, \tau_n, {}^f\varsigma_W\right)$ be a NFTS. Let $\varsigma_R(V)$ be any NFS over W. Then,

$Int_{nf}\left(\varsigma_R(V)\right) \subseteq S^* Int_{nf}\left(\varsigma_R(V)\right) \subseteq \varsigma_R(V)$;

$\varsigma_R(V) \subseteq S^* Cl_{nf}\left(\varsigma_R(V)\right) \subseteq Cl_{nf}\left(\varsigma_R(V)\right)$;

$S^* Cl_{nf}\left(\varsigma_R(V)'\right) = \left[S^* Int_{nf}\left(\varsigma_R(V)\right)\right]'$;

$S^* Int_{nf}\left(\varsigma_R(V)'\right) = \left[S^* Cl_{nf}\left(\varsigma_R(V)\right)\right]'$.

Proof. Straightforward.

Proposition 6. Let $\left(W, \tau_n, {}^f\varsigma_W\right)$ be a NFTS. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be two NFSs over W. Then,

$S^* Int_{nf}\left(\varsigma_R(V_1) \cap \varsigma_R(V_2)\right) = S^* Int_{nf}\left(\varsigma_R(V_1)\right) \cap S^* Int_{nf}\left(\varsigma_R(V_2)\right)$;

$S^* Int_{nf}\left(\varsigma_R(V_1) \cup \varsigma_R(V_2)\right) \supseteq S^* Int_{nf}\left(\varsigma_R(V_1)\right) \cup S^* Int_{nf}\left(\varsigma_R(V_2)\right)$.

Proof. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be two NFSs over W.

Since $\varsigma_R(V_1) \cap \varsigma_R(V_2) \subseteq \varsigma_R(V_1)$ and $\varsigma_R(V_1) \cap \varsigma_R(V_2) \subseteq \varsigma_R(V_2)$, by using *Proposition 3*, $S^* Int_{nf}\left(\varsigma_R(V_1) \cap \varsigma_R(V_2)\right) \subseteq S^* Int_{nf}\left(\varsigma_R(V_1)\right)$ and $S^* Int_{nf}\left(\varsigma_R(V_1) \cap \varsigma_R(V_2)\right) \subseteq S^* Int_{nf}\left(\varsigma_R(V_2)\right)$.

This implies that,

$$S^* \mathrm{Int}_{nf}\left(\varsigma_R(V_1) \cap \varsigma_R(V_2)\right) \subseteq S^* \mathrm{Int}_{nf}\left(\varsigma_R(V_1)\right) \cap S^* \mathrm{Int}_{nf}\left(\varsigma_R(V_2)\right), \quad (1)$$

By using *Proposition 3*,

$S^* Int_{nf}\left(\varsigma_R(V_1)\right) \subseteq \varsigma_R(V_1)$ and $S^* Int_{nf}\left(\varsigma_R(V_2)\right) \subseteq \varsigma_R(V_2)$

$\Rightarrow S^* Int_{nf}\left(\varsigma_R(V_1)\right) \cap S^* Int_{nf}\left(\varsigma_R(V_2)\right) \subseteq \varsigma_R(V_1) \cap \varsigma_R(V_2)$.

Chinnadurai and Sindhu |J. Fuzzy. Ext. Appl. 1(3) (2020) 159-179

By using *Proposition 3*,

$$S^{*}Int_{nf}\left[S^{*}Int_{nf}\left(\varsigma_R(V_1)\right)\cap S^{*}Int_{nf}\left(\varsigma_R(V_2)\right)\right]\subseteq S^{*}Int_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right).$$

By *Eq. (1)*,

$$S^{*}Int_{nf}\left(S^{*}Int_{nf}\left(\varsigma_R(V_1)\right)\right)\cap S^{*}Int_{nf}\left(S^{*}Int_{nf}\left(\varsigma_R(V_2)\right)\right)\subseteq S^{*}Int_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right).$$

By using *Proposition 3*,

$$S^{*}Int_{nf}\left(\varsigma_R(V_1)\right)\cap S^{*}Int_{nf}\left(\varsigma_R(V_2)\right)\subseteq S^{*}Int_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right), \tag{2}$$

Hence from *Eqs. (1)-(2)*,

$$S^{*}Int_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right)=S^{*}Int_{nf}\left(\varsigma_R(V_1)\right)\cap S^{*}Int_{nf}\left(\varsigma_R(V_2)\right).$$

Since $\varsigma_R(V_1)\subseteq\varsigma_R(V_1)\cup\varsigma_R(V_2)$ and $\varsigma_R(V_1)\subseteq\varsigma_R(V_1)\cup\varsigma_R(V_2)$, by using *Proposition 3*,

$S^{*}Int_{nf}\left(\varsigma_R(V_1)\right)\subseteq S^{*}Int_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)$ and $S^{*}Int_{nf}\left(\varsigma_R(V_2)\right)\subseteq S^{*}Int_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)$. Hence $S^{*}Int_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)\supseteq S^{*}Int_{nf}\left(\varsigma_R(V_1)\right)\cup S^{*}Int_{nf}\left(\varsigma_R(V_2)\right)$.

Proposition 7. Let $\left(W,\tau_n,{}^{f}\varsigma_W\right)$ be a NFTS. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be two NFSs over *W*. Then,

$$S^{*}Cl_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)=S^{*}Cl_{nf}\left(\varsigma_R(V_1)\right)\cup S^{*}Cl_{nf}\left(\varsigma_R(V_2)\right);$$

$$S^{*}Cl_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right)\subseteq S^{*}Cl_{nf}\left(\varsigma_R(V_1)\right)\cap S^{*}Cl_{nf}\left(\varsigma_R(V_2)\right).$$

Proof. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be two NFSs over *W*.

(i) Since $S^{*}Cl_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)=S^{*}Cl_{nf}\left(\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)'\right)'$, by using *Proposition 5*,

$$S^{*}Cl_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)=\left[S^{*}Int_{nf}\left(\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)'\right)\right]'=\left[S^{*}Int_{nf}\left(\left(\varsigma_R(V_1)'\right)\cap\left(\varsigma_R(V_2)'\right)\right)\right]'.$$

Again by using *Proposition 5*, $S^{*}Cl_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)=\left[S^{*}Int_{nf}\left(\varsigma_R(V_1)'\right)\cap S^{*}Int_{nf}\left(\varsigma_R(V_2)'\right)\right]'$

$$=\left(S^{*}Int_{nf}\left(\varsigma_R(V_1)'\right)\right)'\cup\left(S^{*}Int_{nf}\left(\varsigma_R(V_2)'\right)\right)'.$$

By using *Proposition 5*, $S^{*}Cl_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)=S^{*}Cl_{nf}\left(\left(\varsigma_R(V_1)'\right)\right)'\cup S^{*}Cl_{nf}\left(\left(\varsigma_R(V_2)'\right)\right)'$

$$=S^{*}Cl_{nf}\left(\varsigma_R(V_1)\right)\cup S^{*}Cl_{nf}\left(\varsigma_R(V_2)\right).$$

Hence $S^*Cl_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)=S^*Cl_{nf}\left(\varsigma_R(V_1)\right)\cup S^*Cl_{nf}\left(\varsigma_R(V_2)\right)$.

(ii) Since $\varsigma_R(V_1)\cap\varsigma_R(V_2)\subseteq\varsigma_R(V_1)$ and $\varsigma_R(V_1)\cap\varsigma_R(V_2)\subseteq\varsigma_R(V_2)$, by using *Proposition 4*, $S^*Cl_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right)\subseteq S^*Cl_{nf}\left(\varsigma_R(V_1)\right)$ and $S^*Cl_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right)\subseteq S^*Cl_{nf}\left(\varsigma_R(V_2)\right)$.

Hence $S^*Cl_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right)\subseteq S^*Cl_{nf}\left(\varsigma_R(V_1)\right)\cap S^*Cl_{nf}\left(\varsigma_R(V_2)\right)$.

3.3 | Neutro-Fine-Generalized Semi Open Sets

Definition 11. Let $\varsigma_R(V)$ be a NFS over W of a NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$. Then $\varsigma_R(V)$ is said to be a neutro-fine-generalized semi closed set (nf -GSCS) if $S^*Cl_{nf}\left(\varsigma_R(V)\right)\subseteq\varsigma_R(U)$ whenever $\varsigma_R(V)\subseteq\varsigma_R(U)$ and $\varsigma_R(U)$ is NFOS.

The complement of nf-GSCS is said to be neutro-fine-generalized semi open set (nf-GSOS), i.e., $\varsigma_R(U)\subseteq S^*Int_{nf}\left(\varsigma_R(V)\right)$ whenever $\varsigma_R(U)\subseteq\varsigma_R(V)$ and $\varsigma_R(U)$ is NFCS.

Example 3. Consider *Example 1*. Thus nf-SCS $=\left\{\varsigma_R(w_1,w_3),\varsigma_S(w_1)\right\}$, nf-SOS $=\left\{\varsigma_R(w_1,w_3)',\varsigma_S(w_1)'\right\}$ where

$\varsigma_R(w_1,w_3)'=\left\{\left\langle w_1,.7,.6,.2\right\rangle,\left\langle w_2,1,1,0\right\rangle,\left\langle w_3,.6,.5,.4\right\rangle,\left\langle w_{1,2},.1,.6,.6\right\rangle,\left\langle w_{1,3},.6,.5,.4\right\rangle,\left\langle w_{2,3},.1,.5,.6\right\rangle\right\}$,

$\varsigma_S(w_1)'=\left\{\left\langle w_1,.6,.7,.9\right\rangle,\left\langle w_2,.1,1,0\right\rangle,\left\langle w_3,1,1,0\right\rangle,\left\langle w_{1,2},.4,.5,.9\right\rangle,\left\langle w_{1,3},.1,.2,.9\right\rangle,\left\langle w_{2,3},1,1,0\right\rangle\right\}$ and

nf-GSCS $=\left\{\varsigma_S(w_2)\right\}$.

Theorem 8. Every NFCS is a nf-GSCS in NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Proof. Let $\varsigma_R(V)$ be a NFCS in NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Let $\varsigma_R(V)\subseteq\varsigma_R(U)$, where $\varsigma_R(U)$ is NFOS in $\left(W,\tau_n,{}^f\varsigma_W\right)$. Since $\varsigma_R(V)$ is a NFCS, $\varsigma_R(V)=Cl_{nf}\left(\varsigma_R(V)\right)$, by *Proposition 2*. Also, by *Proposition 5*, $S^*Cl_{nf}\left(\varsigma_R(V)\right)\subseteq Cl_{nf}\left(\varsigma_R(V)\right)$. Thus $S^*Cl_{nf}\left(\varsigma_R(V)\right)\subseteq Cl_{nf}\left(\varsigma_R(V)\right)=\varsigma_R(V)\subseteq\varsigma_R(U)$. Hence $\varsigma_R(V)$ is a nf-GSCS in NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Chinnadurai and Sindhu |J. Fuzzy. Ext. Appl. 1(3) (2020) 159-179

Theorem 9. If $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ are nf-GSCSs over NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$, then $\varsigma_R(V_1)\cap\varsigma_R(V_2)$ is also a nf-GSCS in $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Proof. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be nf-GSCSs over $\left(W,\tau_n,{}^f\varsigma_W\right)$.

If $\varsigma_R(V_1)\cap\varsigma_R(V_2)\subseteq\varsigma_R(U)$ and $\varsigma_R(U)$ is a NFOS, then $\varsigma_R(V_1)\subseteq\varsigma_R(U)$ and $\varsigma_R(V_1)\subseteq\varsigma_R(U)$.

Since $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ are nf–GSCSs, $S^*Cl_{nf}\left(\varsigma_R(V_1)\right)\subseteq\varsigma_R(U)$ and $S^*Cl_{nf}\left(\varsigma_R(V_2)\right)\subseteq\varsigma_R(U)$.

Thus $S^*Cl_{nf}\left(\varsigma_R(V_1)\right)\cap S^*Cl_{nf}\left(\varsigma_R(V_2)\right)\subseteq\varsigma_R(U)$.

By *Proposition 7*, $S^*Cl_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right)\subseteq S^*Cl_{nf}\left(\varsigma_R(V_1)\right)\cap S^*Cl_{nf}\left(\varsigma_R(V_2)\right)\subseteq\varsigma_R(U)$. This implies that,

$S^*Cl_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right)\subseteq\varsigma_R(U)$. Thus $S^*Cl_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right)\subseteq\varsigma_R(U)$, $\varsigma_R(V_1)\cap\varsigma_R(V_2)\subseteq\varsigma_R(U)$ and $\varsigma_R(U)$ is a NFOS.

Hence $\varsigma_R(V_1)\cap\varsigma_R(V_2)$ is a nf-GSCS over $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Theorem 10. Every NFOS is a nf-GSOS in NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Proof. Let $\varsigma_R(V)$ be a NFOS in NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$. Let $\varsigma_R(U)\subseteq\varsigma_R(V)$, where $\varsigma_R(U)$ is NFCS in $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Since $\varsigma_R(V)$ is a NFOS, $\varsigma_R(V)=Int_{nf}\left(\varsigma_R(V)\right)$, by *Proposition 1*.

Also, by *Proposition 5*, $Int_{nf}\left(\varsigma_R(V)\right)\subseteq S^*Int_{nf}\left(\varsigma_R(V)\right)\subseteq\varsigma_R(V)$. Thus $\varsigma_R(V)=S^*Int_{nf}\left(\varsigma_R(V)\right)$ $\Rightarrow\varsigma_R(U)\subseteq\varsigma_R(V)=S^*Int_{nf}\left(\varsigma_R(V)\right)$.

Hence $\varsigma_R(V)$ is a nf-GSCS in NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Theorem 11. If $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ are nf-GSOSs over NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$, then $\varsigma_R(V_1)\cup\varsigma_R(V_2)$ is also a nf-GSOS in $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Proof. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be nf-GSOSs over $\left(W,\tau_n,{}^f\varsigma_W\right)$.

If $\varsigma_R(U)\subseteq\varsigma_R(V_1)\cup\varsigma_R(V_2)$ and $\varsigma_R(U)$ is a NFCS, then $\varsigma_R(U)\subseteq\varsigma_R(V_1)$ and $\varsigma_R(U)\subseteq\varsigma_R(V_2)$.

Since $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ are nf–GSOSs, $\varsigma_R(U)\subseteq S^*Int_{nf}\left(\varsigma_R(V_1)\right)$ and $\varsigma_R(U)\subseteq S^*Int_{nf}\left(\varsigma_R(V_2)\right)$.

Thus $\varsigma_R(U)\subseteq S^*Int_{nf}\left(\varsigma_R(V_1)\right)\cup S^*Int_{nf}\left(\varsigma_R(V_1)\right)$.

By *Proposition 6*, $\varsigma_R(U)\subseteq S^*Int_{nf}\left(\varsigma_R(V_1)\right)\cup S^*Int_{nf}\left(\varsigma_R(V_1)\right)\subseteq S^*Int_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)$.

This implies that, $\varsigma_R(U)\subseteq S^*Int_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)$.

Thus $\varsigma_R(U)\subseteq S^*Int_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)$, $\varsigma_R(U)\subseteq S^*Int_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)$ and $\varsigma_R(U)$ is a NFCS.

Hence $\varsigma_R(V_1)\cup\varsigma_R(V_2)$ is a nf-GSOS in $\left(W,\tau_n,{}^f\varsigma_W\right)$.

4| Neutro-Fine Minimal and Maximal Open Sets

In this portion, the minimal and maximal open sets on NFTS are defined and probable results are carried by some major expressive examples.

Definition 12. Let $\varsigma_R(V)$ be a proper non-empty NFOS of a NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$. Then $\varsigma_R(V)$ is said to be a neutro-fine minimal open set (min_{nf}-OS) if any NFOS which is contained in $\varsigma_R(V)$ is 0_{nf} or $\varsigma_R(V)$. The complement of min_{nf}-OS is said to be neutro-fine minimal closed set (min_{nf}-CS).

Definition 13. Let $\varsigma_R(V)$ be a proper non-empty NFOS of a NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$. Then $\varsigma_R(V)$ is said to be a neutro-fine maximal open set (max_{nf}-OS) if any NFOS which is contained in $\varsigma_R(V)$ is 1_{nf} or $\varsigma_R(V)$. The complement of max_{nf}-OS is said to be neutro-fine maximal closed set (max_{nf}-CS).

Example 4. Let $W=\left\{w_1,w_2,w_3\right\}$ and $\tau_n=\left\{0_n,1_n,R,S\right\}$ where R and *S* are NSs over *W* and are defined as follows

$R=\left\{\left\langle w_1,.1,.2,.8\right\rangle,\left\langle w_2,.4,.7,.3\right\rangle,\left\langle w_3,.6,.5,.2\right\rangle\right\}$ and

$S=\left\{\left\langle w_1,.6,.5,.3\right\rangle,\left\langle w_2,.9,.8,.1\right\rangle,\left\langle w_3,.7,.6,.1\right\rangle\right\}$.

Chinnadurai and Sindhu |J. Fuzzy. Ext. Appl. 1(3) (2020) 159-179

Thus (W, τ_n) is a NTS over W. Then ${}^f\varsigma_W = \left\{0_n, 1_n, \varsigma_R(w_1), \varsigma_R(w_2, w_3), \varsigma_S(w_2)\right\}$, where

$\varsigma_R(w_1) = \left\{\left\langle w_1, .1, .2, .8\right\rangle, \left\langle w_2, 0, 0, 1\right\rangle, \left\langle w_3, 0, 0, 1\right\rangle, \left\langle w_{1,2}, .4, .7, .3\right\rangle, \left\langle w_{1,3}, .6, .5, .2\right\rangle, \left\langle w_{2,3}, 0, 0, 1\right\rangle\right\}$,

$\varsigma_R(w_2, w_3) = \left\{\left\langle w_1, 0, 0, 1\right\rangle, \left\langle w_2, .4, .7, .3\right\rangle, \left\langle w_3, .6, .5, .2\right\rangle, \left\langle w_{1,2}, .4, .7, .3\right\rangle, \left\langle w_{1,3}, .6, .5, .2\right\rangle, \left\langle w_{2,3}, .6, .7, .2\right\rangle\right\}$,

$\varsigma_S(w_2) = \left\{\left\langle w_1, 0, 0, 1\right\rangle, \left\langle w_2, .9, .8, .1\right\rangle, \left\langle w_3, 0, 0, 1\right\rangle, \left\langle w_{1,2}, .9, .8, .1\right\rangle, \left\langle w_{1,3}, 0, 0, 1\right\rangle, \left\langle w_{2,3}, .9, .8, .1\right\rangle\right\}$ are NFOSs over (W, τ_n).

Hence $\left(W, \tau_n, {}^f\varsigma_W\right)$ is a NFTS over (W, τ_n). Thus min_{nf}-OS $= \left\{0_n, \varsigma_R(w_1), \varsigma_S(w_2)\right\}$, min_{nf}-CS $= \left\{1_n, \varsigma_R(w_1)', \varsigma_S(w_2)'\right\}$, $\max_{nf}$-OS $= \left\{0_n, \varsigma_R(w_2, w_3)\right\}$ and max_{nf}-CS $= \left\{1_n, \varsigma_R(w_2, w_3)'\right\}$.

Example 5. Consider *Example 1.* Here min_{nf}-OS $= \left\{0_n, \varsigma_R(w_1), \varsigma_R(w_3)\right\}$, min_{nf}-CS $= \left\{1_n, \varsigma_R(w_1)', \varsigma_R(w_3)'\right\}$,

$\max_{nf}$-OS $= \left\{0_n, \varsigma_S(w_2, w_3)\right\}$ and max_{nf}-CS $= \left\{1_n, \varsigma_S(w_2, w_3)'\right\}$.

Lemma 1. Let $\left(W, \tau_n, {}^f\varsigma_W\right)$ be a NFTS over (W, τ_n).

If $\varsigma_R(U)$ is a min_{nf}-OS and $\varsigma_R(W)$ is NFOS, then $\varsigma_R(U) \cap \varsigma_R(W) = 0_{nf}$ or $\varsigma_R(U) \subseteq \varsigma_R(W)$.

If $\varsigma_R(U)$ and $\varsigma_R(V)$ are min_{nf}-OSs, then $\varsigma_R(U) \cap \varsigma_R(V) = 0_{nf}$ or $\varsigma_R(U) = \varsigma_R(V)$.

Proof. Let $\varsigma_R(W)$ be a NFOS such that $\varsigma_R(U) \cap \varsigma_R(W) \neq 0_{nf}$.

Since $\varsigma_R(U)$ is a min_{nf}-OS and $\varsigma_R(U) \cap \varsigma_R(W) \subseteq \varsigma_R(U)$, then $\varsigma_R(U) \cap \varsigma_R(W) = \varsigma_R(U)$. Hence $\varsigma_R(U) \subseteq \varsigma_R(W)$.

If $\varsigma_R(U) \cap \varsigma_R(W) \neq 0_{nf}$, then $\varsigma_R(U) \subseteq \varsigma_R(V)$ and $\varsigma_R(V) \subseteq \varsigma_R(U)$, by (i). Hence $\varsigma_R(U) = \varsigma_R(V)$.

Proposition 7. Let $\left(W, \tau_n, {}^f\varsigma_W\right)$ be a NFTS over (W, τ_n). Let $\varsigma_R(U)$ be a min_{nf}-OS. If $w^{\langle\alpha,\beta,\gamma\rangle}$ is a NFP of $\varsigma_R(U)$, then $\varsigma_R(U) \subseteq \varsigma_R(W)$ for any neutro-fine neighborhood $\varsigma_R(W)$ of $w^{\langle\alpha,\beta,\gamma\rangle}$.

Proof. Let $\left(W, \tau_n, {}^f\varsigma_W\right)$ be a NFTS over (W, τ_n).

Let $\varsigma_R(W)$ be a neutro-fine neighborhood of $w^{\langle\alpha,\beta,\gamma\rangle}$ such that $\varsigma_R(U) \not\subset \varsigma_R(W)$. Then $\varsigma_R(U) \cap \varsigma_R(W)$ is a NFOS such that $\varsigma_R(U) \cap \varsigma_R(W) \not\subset \varsigma_R(U)$ and $\varsigma_R(U) \cap \varsigma_R(W) \neq 0_{nf}$.

This contradicts our assumption that $\varsigma_R(U)$ is a min_{nf}-OS. Hence proved.

Proposition 8. Let $\left(W, \tau_n, {}^f\varsigma_W\right)$ be a NFTS over (W, τ_n). Let $\varsigma_R(U)$ be a min_{nf}-OS. Then

$\varsigma_R(U) = \cap\left\{\varsigma_R(W) : \varsigma_R(W) \text{ is a neutro-fine neighborhood of } w^{\langle\alpha,\beta,\gamma\rangle}\right\}$, for any NFP $w^{\langle\alpha,\beta,\gamma\rangle}$ of $\varsigma_R(U)$.

Proof. Let $\left(W, \tau_n, {}^f\varsigma_W\right)$ be a NFTS over (W, τ_n). Let $\varsigma_R(U)$ be a min_{nf}-OS.

Since $\varsigma_R(U)$ is a neutro-fine neighborhood of $w^{\langle\alpha,\beta,\gamma\rangle}$, by *Proposition 7*, then

$\varsigma_R(U) \subseteq \cap\left\{\varsigma_R(W) : \varsigma_R(W) \text{ is a neutro-fine neighborhood of } w^{\langle\alpha,\beta,\gamma\rangle}\right\} \subseteq \varsigma_R(U)$. Thus $\varsigma_R(U) = \cap\left\{\varsigma_R(W) : \varsigma_R(W) \text{ is a neutro-fine neighborhood of } w^{\langle\alpha,\beta,\gamma\rangle}\right\}$.

Proposition 9. Let $\left(W, \tau_n, {}^f\varsigma_W\right)$ be a NFTS over (W, τ_n). Let $\varsigma_R(U)$ be a non-empty NFOS. Then the following conditions are equivalent:

$\varsigma_R(U)$ is a min_{nf}-OS.

$\varsigma_R(U) \subseteq Cl_{nf}\left(\varsigma_R(V)\right)$ for any NFS $\varsigma_R(V)$ of $\varsigma_R(U)$.

$Cl_{nf}\left(\varsigma_R(U)\right) \subseteq Cl_{nf}\left(\varsigma_R(V)\right)$ for any NFS $\varsigma_R(V)$ of $\varsigma_R(U)$.

Proof. (1) $\Rightarrow$ (2). Let $\varsigma_R(V)$ be any NFS of $\varsigma_R(U)$.

Chinnadurai and Sindhu |J. Fuzzy. Ext. Appl. 1(3) (2020) 159-179

By *Proposition 7*, for any NFP $w^{\langle\alpha,\beta,\gamma\rangle}$ of $\varsigma_R(U)$ and any neutro-fine neighborhood $\varsigma_R(W)$ of $w^{\langle\alpha,\beta,\gamma\rangle}$, then $\varsigma_R(V)=\left(\varsigma_R(U)\cap\varsigma_R(V)\right)\subseteq\left(\varsigma_R(W)\cap\varsigma_R(V)\right)$. Thus $\varsigma_R(W)\cap\varsigma_R(V)\neq 0_{nf}$, and hence $\varsigma_R(U)\cap\varsigma_R(W)\neq 0_{nf}$ is a NFP of $Cl_{nf}\left(\varsigma_R(V)\right)$. Therefore $\varsigma_R(U)\subseteq Cl_{nf}\left(\varsigma_R(V)\right)$.

(2)$\Rightarrow$(3). Since $\varsigma_R(V)$ is any NFS of $\varsigma_R(U)$, then $\varsigma_R(U)\subseteq Cl_{nf}\left(\varsigma_R(V)\right)$.

Thus by (2), $Cl_{nf}\left(\varsigma_R(U)\right)\subseteq Cl_{nf}\left(Cl_{nf}\left(\varsigma_R(V)\right)\right)=Cl_{nf}\left(\varsigma_R(V)\right)$. Hence $Cl_{nf}\left(\varsigma_R(U)\right)\subseteq Cl_{nf}\left(\varsigma_R(V)\right)$ for any NFS $\varsigma_R(V)$ of $\varsigma_R(U)$.

(3)$\Rightarrow$(1). Suppose that $\varsigma_R(U)$ is not a min_{nf}-OS.

Then there exists a NFS $\varsigma_R(V)$ such that $\varsigma_R(V)\not\subset\varsigma_R(U)$. Then there exists a NFP $w^{\langle\alpha,\beta,\gamma\rangle}\in\varsigma_R(U)$ such that $w^{\langle\alpha,\beta,\gamma\rangle}\notin\varsigma_R(V)$. This implies that, $w^{\langle\alpha,\beta,\gamma\rangle}$ is a NFS. Then it is clear that $Cl_{nf}\left(w^{\langle\alpha,\beta,\gamma\rangle}\right)\subseteq\varsigma_R(V)'$

$$\Rightarrow Cl_{nf}\left(w^{\langle\alpha,\beta,\gamma\rangle}\right)\neq Cl_{nf}\left(\varsigma_R(U)\right).$$

Hence the proof.

Lemma 2. Let $\left(W,\tau_n,{}^f\varsigma_W\right)$ be a NFTS over $(\mathbb{W},\tau_n)$.

If $\varsigma_R(U)$ is a max_{nf}-OS and $\varsigma_R(W)$ is NFOS, then $\varsigma_R(U)\cup\varsigma_R(W)=1_{nf}$ or $\varsigma_R(W)\subseteq\varsigma_R(U)$.

If $\varsigma_R(U)$ and $\varsigma_R(V)$ are max_{nf}-OSs, then $\varsigma_R(U)\cup\varsigma_R(V)=1_{nf}$ or $\varsigma_R(U)=\varsigma_R(V)$.

Proof. (i) Let $\varsigma_R(W)$ be a NFOS such that $\varsigma_R(U)\cup\varsigma_R(W)\neq 1_{nf}$.

Since $\varsigma_R(U)$ is a max_{nf}-OS and $\varsigma_R(U)\subseteq\varsigma_R(U)\cup\varsigma_R(W)$, then $\varsigma_R(U)\cup\varsigma_R(W)=\varsigma_R(U)$. Hence $\varsigma_R(W)\subseteq\varsigma_R(U)$.

If $\varsigma_R(U)\cup\varsigma_R(W)\neq 1_{nf}$, then $\varsigma_R(U)\subseteq\varsigma_R(V)$ and $\varsigma_R(V)\subseteq\varsigma_R(U)$, by (i). Hence $\varsigma_R(U)=\varsigma_R(V)$.

Proposition 10. Let $\left(W,\tau_n,{}^f\varsigma_W\right)$ be a NFTS over $(\mathbb{W},\tau_n)$. Let $\varsigma_R(U)$ be a max_{nf}-OS. If $w^{\langle\alpha,\beta,\gamma\rangle}$ is a NFP of $\varsigma_R(U)$, then for any neutro-fine neighborhood $\varsigma_R(W)$ of $w^{\langle\alpha,\beta,\gamma\rangle}$, $\varsigma_R(U)\cup\varsigma_R(W)=1_{nf}$ or $\varsigma_R(W)\subseteq\varsigma_R(U)$.

Proof. Follows from the *Lemma 2*.

Proposition 11. Let $\left(W, \tau_n, {}^f\varsigma_W\right)$ be a NFTS over (W, τ_n). Let $\varsigma_R(U)$ be a max_{nf}-OS. Then $\varsigma_R(U) = \cup\left\{ \varsigma_R(W) : \varsigma_R(W) \text{is a neutro-fine neighborhood of } w^{\langle \alpha,\beta,\gamma \rangle} \text{ such that } \varsigma_R(U) \cup \varsigma_R(W) \neq 1_{nf} \right\}$.

Proof. Let $\left(W, \tau_n, {}^f\varsigma_W\right)$ be a NFTS over (W, τ_n). Let $\varsigma_R(U)$ be a max_{nf}-OS.

Since $\varsigma_R(U)$ is a neutro-fine neighborhood of $w^{\langle \alpha,\beta,\gamma \rangle}$, by *Proposition 10*, then $\varsigma_R(U) \subseteq \cup\left\{ \varsigma_R(W) : \varsigma_R(W) \text{is a neutro-fine neighborhood of } w^{\langle \alpha,\beta,\gamma \rangle} \text{ such that } \varsigma_R(U) \cup \varsigma_R(W) \neq 1_{nf} \right\}$ $\subseteq \varsigma_R(U)$. Hence the result.

Theorem 12. Let $\left(W, \tau_n, {}^f\varsigma_W\right)$ be a NFTS over (W, τ_n). Let $\varsigma_R(U_1)$, $\varsigma_R(U_2)$ and $\varsigma_R(U_3)$ be max_{nf}-OSs such that $\varsigma_R(U_1) \neq \varsigma_R(U_2)$. If $\left(\varsigma_R(U_1) \cap \varsigma_R(U_2)\right) \subseteq \varsigma_R(U_3)$, then $\varsigma_R(U_1) = \varsigma_R(U_3)$ or $\varsigma_R(U_2) = \varsigma_R(U_3)$.

Proof. Let $\varsigma_R(U_1)$, $\varsigma_R(U_2)$ and $\varsigma_R(U_3)$ be max_{nf}-OSs such that $\varsigma_R(U_1) \neq \varsigma_R(U_2)$. Then

$\left(\varsigma_R(U_1) \cap \varsigma_R(U_3)\right) = \varsigma_R(U_1) \cap \left(\varsigma_R(U_3) \cap 1_{nf}\right)$

$= \varsigma_R(U_1) \cap \left(\varsigma_R(U_3) \cap \left(\varsigma_R(U_1) \cup \varsigma_R(U_2)\right)\right)$ (by *Lemma 2*)

$= \varsigma_R(U_1) \cap \left(\left(\varsigma_R(U_3) \cap \varsigma_R(U_1)\right) \cup \left(\varsigma_R(U_3) \cap \varsigma_R(U_2)\right)\right)$

$= \left(\varsigma_R(U_1) \cap \varsigma_R(U_3)\right) \cup \left(\varsigma_R(U_3) \cap \varsigma_R(U_1) \cap \varsigma_R(U_2)\right)$

$= \left(\varsigma_R(U_1) \cap \varsigma_R(U_3)\right) \cup \left(\varsigma_R(U_1) \cap \varsigma_R(U_2)\right)$ (since $\left(\varsigma_R(U_1) \cap \varsigma_R(U_2)\right) \subseteq \varsigma_R(U_3)$)

$= \varsigma_R(U_1) \cap \left(\varsigma_R(U_3) \cup \varsigma_R(U_2)\right)$.

If $\varsigma_R(U_3) \neq \varsigma_R(U_2)$, then $\left(\varsigma_R(U_3) \cup \varsigma_R(U_2)\right) = 1_{nf}$.

Thus $\left(\varsigma_R(U_1) \cap \varsigma_R(U_3)\right) = \varsigma_R(U_1)$ implies $\varsigma_R(U_1) \subseteq \varsigma_R(U_3)$. Since $\varsigma_R(U_1)$ and $\varsigma_R(U_3)$ are max_{nf}-OSs, then hence $\varsigma_R(U_1) = \varsigma_R(U_3)$.

Theorem 13. Let $\left(W, \tau_n, {}^f\varsigma_W\right)$ be a NFTS over (W, τ_n). Let $\varsigma_R(U_1)$, $\varsigma_R(U_2)$ and $\varsigma_R(U_3)$ be , max_{nf}-OSs, which are different from each other. Then $\left(\varsigma_R(U_1) \cap \varsigma_R(U_2)\right) \not\subset \left(\varsigma_R(U_1) \cap \varsigma_R(U_3)\right)$.

Chinnadurai and Sindhu |J. Fuzzy. Ext. Appl. 1(3) (2020) 159-179

Proof. Let $\varsigma_R(U_1)$, $\varsigma_R(U_2)$ and $\varsigma_R(U_3)$ be , max_{nf} -OSs.

Suppose assume that $\left(\varsigma_R(U_1)\cap\varsigma_R(U_2)\right)\subseteq\left(\varsigma_R(U_1)\cap\varsigma_R(U_3)\right)$. Then

$$\left(\varsigma_R(U_1)\cap\varsigma_R(U_2)\right)\cup\left(\varsigma_R(U_2)\cap\varsigma_R(U_3)\right)\subseteq\left(\varsigma_R(U_1)\cap\varsigma_R(U_3)\right)\cup\left(\varsigma_R(U_2)\cap\varsigma_R(U_3)\right).$$

Thus $\varsigma_R(U_2)\cap\left(\varsigma_R(U_1)\cup\varsigma_R(U_3)\right)\subseteq\left(\varsigma_R(U_1)\cup\varsigma_R(U_2)\right)\cap\varsigma_R(U_3)$.

Since $\varsigma_R(U_1)\cup\varsigma_R(U_3)=1_{nf}=\varsigma_R(U_1)\cup\varsigma_R(U_2)$, then $\varsigma_R(U_2)\subseteq\varsigma_R(U_3)$.

This implies that $\varsigma_R(U_2)=\varsigma_R(U_3)$, which contradicts our assumption. Hence proved.

5| Conclusion

The main objective of this paper is to define some collection of open sets such as neutro-fine-generalized open and neutro-fine-semi open sets on NFTS and analyzed its basic properties with perfect examples. The notion of interior and closure on semi-open sets are described and specified certain properties. These definitions provide the idea of generalized semi-open sets on NFTS. Also, the neutro-fine-minimal and neutro-fine-maximal open sets are defined and some of their properties are studied in this space. Likewise, discussed the complement of all these sets as its closed sets. The basic properties of the union and intersection of these sets are stated in some theorems. Only a few sets satisfy this postulates, and others are disproved as shown in the counterexamples. The converse of some theorems is proved in probable examples. Consequently, the future researchers can extend this NFTS to some special types of sets, whereas soft sets, rough sets, crisp sets, cubic sets, etc., Also, the application part can widen on MCDM problems.

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