J. Fuzzy. Ext. Appl. Vol. 1, No. 3 (2020) 169–186

Journal of Fuzzy Extension & Applications

www.journal-fea.com

Some Remarks on Neutro-Fine Topology

Veerappan Chinnadurai[1,*], Mayandi Pandaram Sindhu[2]

[1]Department of Mathematics, Annamalai University, Chidambaram, Tamilnadu, India.

[2]Department of Mathematics, Faculty of Karpagam College of Engineering, Coimbatore, Tamilnadu, India.

PAPER INFO

Chronicle:

Received: 07 March 2020
Revised: 12 July 2020
Accepted: 14 August 2020

Keywords:

Neutro-Fine-Generalized Open Sets.
Neutro-Fine-Semi Open Sets.
Neutro-Fine-Semi Interior.
Neutro-Fine-Semi Closure.
Neutro-Fine-Generalized Semi Open Sets.
Neutro-Fine Minimal Open Set.
Neutro-Fine Maximal Open Sets.

ABSTRACT

The neutro-fine topological space is a space that contains a combination of neutrosophic and fine sets. In this study, the various types of open sets such as generalized open and semi-open sets are defined in such space. The concept of interior and closure on semi-open sets are defined and some of their basic properties are stated. These definitions extend the concept to generalized semi-open sets. Moreover, the minimal and maximal open sets are defined and some of their properties are studied in this space. As well as, discussed the complement of all these sets as its closed sets. The basic properties of the union and intersection of these open sets are stated in some theorems. Only a few sets satisfy this postulates, and others are disproved as shown in the counterexamples. The converse of some theorems is proved in probable examples.

1. Introduction

The classical set theory developed by Zadeh [40] was termed as a Fuzzy Set (FS), whose elements amuse ambiguous features of true and false membership functions. The FS theory applied in the boundless area of a domain, while Atanassov [39] extended this theory as an Intuitionistic Fuzzy Set (IFS) theory. Later, Smarandache [21] explored a set that contains one more membership function called indeterminacy along with truth and falsity degrees as elements of the Neutrosophic Set (NS). Also, he generalized the NS on IFS [22] and recently proposed his work on attributes valued set, Plithogenic Set (PS) [23]. Nowadays, this set

* Corresponding author
E-mail address: chinnaduraiau@gmail.com
DOI: 10.22105/jfea.2020.251783.1020

made an outstanding impact on many applications [1-4, 11-15, 16, 18, 19] and play a vital role in Decision Making (DM) problems [17, 20, 10] and Multi-Criteria DM (MCDM) problems [5, 9].

Topology is a study of flexible objects under frequent damages without splitting. In recent times, Topological Space (TS) is performing a lead character in the enormous branch of applied sciences and numerous categories of mathematics. The topological structure developed on NS as a generalization of IFTS which was originated by Salama & Alblowi [33, 34], named as Neutrosophic Topological Space (NTS). Few typical sets, open sets, and other TS explored [24, 7, 27, 28, 29, 31], and extended to bi-topological space [6] on such TS.

The most general class of sets which contains few open sets termed as Fine-Open Sets (FOSs), by Powar & Rajak [35], and investigated the special case of generalized TS, called Fine- Topological Space (FTS). Many researchers studied this concept on some sets like FS [26, 30], and others [25, 32]. Recently, this concept extends as Neutro-Fine Topological Space (NFTS) [8], which was introduced by Chinnadurai and Sindhu. The concept of minimal open (closed) and maximal open (closed) sets were exhibited by few researchers [36- 38].

The aspiration of this paper is to instigate the collection of open sets such as generalized open and semi-open sets defined on NFTS. The concept of interior and closure on neutro-fine-semi open sets are defined and some of their basic properties are stated. These definitions extend the concept to generalized semi-open sets. Moreover, the minimal and maximal open sets are defined and some of their properties are studied in this space. Simultaneously, discussed the complement of all these sets as its closed sets. The basic properties of the union and intersection of these sets are stated in some theorems. Only a few sets satisfy this postulates, and others are disproved as shown in the counterexamples. The converse of some theorems is proved in probable examples.

The layout of this proposal is as follows. In Portion 2, essential definitions of NFTS are recollected. In Portion 3, some type of generalized open sets are defined on NFTS and investigated its properties with illustrative examples. In Portion 4, some more open sets like neutro-fine minimal open sets and neutro-fine maximal open sets are explored via perfect examples. In the end, Portion 6 conveyed the conclusions with some future works.

2. Preliminaries

In this portion, we remind a few major descriptions connected to NFTS.

Definition 1. [8]. Let W be a set of universe and $w_i \in W$ where $i \in I$. Let R be a NS over W. Then the subset of NS R with respect to w_i (sub-NS R_{w_i}) and w_i, w_j (sub-NS R_{w_i, w_j}) are denoted as $\varsigma_R(w_i)$ and $\varsigma_R(w_i, w_j)$, and defined as

$$\varsigma_R(w_i)=\Big\{\big\langle w_i, T_R(w_i), I_R(w_i), F_R(w_i)\big\rangle, \big\langle w_{i,j}, \max\big(T_R(w_i), T_R(w_j)\big), \max\big(I_R(w_i), I_R(w_j)\big), \min\big(F_R(w_i), F_R(w_j)\big)\big\rangle,$$
$$\big\langle w_k, T_R(0_n), I_R(0_n), F_R(0_n)\big\rangle, \big\langle w_{k,l}, T_R(0_n), I_R(0_n), F_R(0_n)\big\rangle\Big\}$$

where $i \in I$, $j \in I-\{i\}$, $k,l \in I-\{i,j\}$ and $k \neq l$ and

$$\varsigma_R(w_i,w_j)=\Big\{\big\langle w_i, T_R(w_i), I_R(w_i), F_R(w_i)\big\rangle, \big\langle w_j, T_R(w_j), I_R(w_j), F_R(w_j)\big\rangle, \big\langle w_k, T_R(0_n), I_R(0_n), F_R(0_n)\big\rangle,$$
$$\big\langle w_{i,j}, \max\big(T_R(w_i), T_R(w_j)\big), \max\big(I_R(w_i), I_R(w_j)\big), \min\big(F_R(w_i), F_R(w_j)\big)\big\rangle,$$
$$\big\langle w_{i,k}, \max\big(T_R(w_i), T_R(w_k)\big), \max\big(I_R(w_i), I_R(w_k)\big), \min\big(F_R(w_i), F_R(w_k)\big)\big\rangle,$$
$$\big\langle w_{j,k}, \max\big(T_R(w_j), T_R(w_k)\big), \max\big(I_R(w_j), I_R(w_k)\big), \min\big(F_R(w_j), F_R(w_k)\big)\big\rangle\Big\}$$

where $i,j,k \in I$ and $i \neq j \neq k$, respectively.

Definition 2. [8]. Let W be a set of universe and $w \in W$. Let R be a NS over W and V be any proper non-empty subset of W. Then $\varsigma_R(V)$ is said to be neutro-fine set (NFS) over W.

Definition 3. [8]. Let NFS(W) be the family of all NFSs over W. Then the fine collection of $\varsigma_R(V)$ is denoted as ${}^f\varsigma_W$ and defined over the NT (W, τ_n) as ${}^f\varsigma_W = \{0_{nf}, 1_{nf}, \bigcup \varsigma_R(V)\}$.

Thus the triplet $(W, \tau_n, {}^f\varsigma_W)$ is said to be a NFTS over (W, τ_n). The elements belong to ${}^f\varsigma_W$ are said to be neutro-fine open sets (NFOSs) over (W, τ_n) and the complement of NFOSs are said to be neutro-fine closed sets (NFCSs) over (W, τ_n) and denote the collection by ${}^F\varsigma_W$.

Definition 4. [8]. Let $(W, \tau_n, {}^f\varsigma_W)$ be a NFTS over (W, τ_n). Let $\varsigma_R(V)$ be a NFS over W. Then the neutro-fine interior of $\varsigma_R(V)$ is denoted as $Int_{nf}(\varsigma_R(V))$ and is defined as the union of all NFOSs contained in $\varsigma_R(V)$.

Clearly, $Int_{nf}(\varsigma_R(V))$ is the largest NFOS contained in $\varsigma_R(V)$.

Definition 5. [8]. Let $(W, \tau_n, {}^f\varsigma_W)$ be a NFTS over (W, τ_n). Let $\varsigma_R(V)$ be a NFS over W. Then the neutro-fine closure of $\varsigma_R(V)$ is denoted as $Cl_{nf}(\varsigma_R(V))$ and is defined as the intersection of all NFCSs containing $\varsigma_R(V)$.

Clearly, $Cl_{nf}(\varsigma_R(V))$ is the smallest NFCS containing $\varsigma_R(V)$.

Definition 6. [8]. Let NF(W) be the family of all NFs over the universe W and $w \in W$. Then NFS $w^{\langle \alpha,\beta,\chi \rangle}$ is said to be a neutro-fine point (NFP), for $0 \leq \alpha, \beta, \gamma \leq 1$ and is defined as follows:

$$w^{\langle \alpha,\beta,\gamma \rangle}(v)=\begin{cases}(\alpha,\beta,\gamma), & \text{if } w=v\\ (0,0,1), & \text{if } w\neq v\end{cases}.$$

Every NFS is the union of its NFPs.

Definition 7. [8]. Let $(W, \tau_n, {}^f\varsigma_W)$ be a NFTS over (W, τ_n). Let $\varsigma_R(V)$ be a NFS over W. Then $\varsigma_R(V)$ is said to be a neutro-fine neighborhood of the NFP $w^{\langle\alpha,\beta,\chi\rangle} \in \varsigma_R(V)$, if there exists a NFOS $\varsigma_R(U)$ such that $w^{\langle\alpha,\beta,\chi\rangle} \in \varsigma_R(U) \subseteq \varsigma_R(V)$.

Proposition 1. [8]. Let $(W, \tau_n, {}^f\varsigma_W)$ be a NFTS. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be two NFSs over W. Then,

$\mathrm{Int}_{nf}(0_{nf}) = 0_{nf}$ and $\mathrm{Int}_{nf}(1_{nf}) = 1_{nf}$:

$\varsigma_R(V_1)$ is NFOS $\Rightarrow \mathrm{Int}_{nf}(\varsigma_R(V_1)) = \varsigma_R(V_1)$;

$\mathrm{Int}_{nf}(\varsigma_R(V_1)) \subseteq \varsigma_R(V_1)$;

$\varsigma_R(V_1) \subseteq \varsigma_R(V_2) \Rightarrow \mathrm{Int}_{nf}(\varsigma_R(V_1)) \subseteq \mathrm{Int}_{nf}(\varsigma_R(V_2))$;

$\mathrm{Int}_{nf}(\mathrm{Int}_{nf}(\varsigma_R(V_1))) = \mathrm{Int}_{nf}(\varsigma_R(V_1))$;

$\mathrm{Int}_{nf}(\varsigma_R(V_1) \cap \varsigma_R(V_2)) = \mathrm{Int}_{nf}(\varsigma_R(V_1)) \cap \mathrm{Int}_{nf}(\varsigma_R(V_2))$;

$\mathrm{Int}_{nf}(\varsigma_R(V_1) \cup \varsigma_R(V_2)) \subseteq \mathrm{Int}_{nf}(\varsigma_R(V_1)) \cup \mathrm{Int}_{nf}(\varsigma_R(V_2))$;

$\mathrm{Int}_{nf}(\varsigma_R(V_1)') = [\mathrm{Cl}_{nf}(\varsigma_R(V_1))]'$.

Proof. Straightforward.

Proposition 2. [8]. Let $(W, \tau_n, {}^f\varsigma_W)$ be a NFTS. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be two NFSs over W. Then,

$\mathrm{Cl}_{nf}(0_{nf}) = 0_{nf}$ and $\mathrm{Cl}_{nf}(1_{nf}) = 1_{nf}$;

$\varsigma_R(V_1)$ is NFCS $\Rightarrow \mathrm{Cl}_{nf}(\varsigma_R(V_1)) = \varsigma_R(V_1)$;

$\mathrm{Cl}_{nf}(\varsigma_R(V_1)) \supseteq \varsigma_R(V_1)$;

$\varsigma_R(V_1) \subseteq \varsigma_R(V_2) \Rightarrow \mathrm{Cl}_{nf}(\varsigma_R(V_1)) \subseteq \mathrm{Cl}_{nf}(\varsigma_R(V_2))$;

$\mathrm{Cl}_{nf}(\mathrm{Cl}_{nf}(\varsigma_R(V_1))) = \mathrm{Cl}_{nf}(\varsigma_R(V_1))$;

$\mathrm{Cl}_{nf}(\varsigma_R(V_1) \cup \varsigma_R(V_2)) = \mathrm{Cl}_{nf}(\varsigma_R(V_1)) \cup \mathrm{Cl}_{nf}(\varsigma_R(V_2))$;

$\mathrm{Cl}_{nf}(\varsigma_R(V_1) \cap \varsigma_R(V_2)) \subseteq \mathrm{Cl}_{nf}(\varsigma_R(V_1)) \cap \mathrm{Cl}_{nf}(\varsigma_R(V_2))$;

$Cl_{nf}\left(\varsigma_R(V_1)'\right)=\left[Int_{nf}\left(\varsigma_R(V_1)\right)\right]'$.

Proof. Straightforward.

3. Some Form of Generalized Open Sets in NFTS

In this portion, some open types of generalized open sets on NFTS are defined and probable results are carried by some major expressive examples. This portion is splitted into 3 sub-portions which states neutro-fine-generalized, neutro-fine-semi, and neutro-fine-generalized semi-open sets on NFTS.

3.1. Neutro-Fine-Generalized Open Sets

Let $\varsigma_R(V)$ be a NFS over W of a NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$. Then $\varsigma_R(V_1)$ is said to be a neutro-fine-generalized closed set (nf-GCS) if $Cl_{nf}\left(\varsigma_R(V)\right)\subseteq\varsigma_R(U)$ whenever $\varsigma_R(V)\subseteq\varsigma_R(U)$ and $\varsigma_R(U)$ is NFOS. The complement of nf-GCS is said to be neutro-fine-generalized open set (nf-GOS).

Theorem 1. Every NFCS is a nf-GCS in NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Proof. Let $\varsigma_R(V)$ be a NFCS on $\left(W,\tau_n,{}^f\varsigma_W\right)$. Let $\varsigma_R(V)\subseteq\varsigma_R(U)$, where $\varsigma_R(U)$ is NFOS in $\left(W,\tau_n,{}^f\varsigma_W\right)$. Since $\varsigma_R(V)$ is a NFCS, $\varsigma_R(V)=Cl_{nf}\left(\varsigma_R(V)\right)$ $\Rightarrow Cl_{nf}\left(\varsigma_R(V)\right)\subseteq\varsigma_R(V)$. Thus $Cl_{nf}\left(\varsigma_R(V)\right)\subseteq\varsigma_R(V)\subseteq\varsigma_R(U)$. Hence $\varsigma_R(V)$ is a nf-GCS in NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Remark 1. The converse of the above theorem is not true as shown in the following example.

Example 1. Let $W=\{w_1,w_2,w_3\}$ and $\tau_n=\{0_n,1_n,R,S,T,U\}$ where R, S, T and U are NSs over W and are defined as follows

$$R=\left\{\langle w_1,.2,.4,.7\rangle,\langle w_2,.6,.3,.1\rangle,\langle w_3,.4,.5,.6\rangle\right\},$$

$$S=\left\{\langle w_1,.9,.3,.6\rangle,\langle w_2,.6,.5,.4\rangle,\langle w_3,.7,.8,.1\rangle\right\},$$

$$T=\left\{\langle w_1,.9,.4,.6\rangle,\langle w_2,.6,.5,.1\rangle,\langle w_3,.7,.8,.1\rangle\right\} \text{ and}$$

$$U=\left\{\langle w_1,.2,.3,.7\rangle,\langle w_2,.6,.3,.4\rangle,\langle w_3,.4,.5,.6\rangle\right\}.$$

Thus (W,τ_n) is a NTS over W.

Then NFOSs over (W,τ_n) are ${}^f\varsigma_W=\{0_n,1_n,\varsigma_R(w_1),\varsigma_R(w_3),\varsigma_S(w_2,w_3)\}$, where

$$\varsigma_R(w_1)=\left\{\langle w_1,.2,.4,.7\rangle,\langle w_2,0,0,1\rangle,\langle w_3,0,0,1\rangle,\langle w_{1,2},.6,.4,.1\rangle,\langle w_{1,3},.4,.5,.6\rangle,\langle w_{2,3},0,0,1\rangle\right\},$$

$\varsigma_R(w_3)=\{\langle w_1,0,0,1\rangle,\langle w_2,0,0,1\rangle,\langle w_3,.4,.5,.6\rangle,\langle w_{1,2},0,0,1\rangle,\langle w_{1,3},.4,.5,.6\rangle,\langle w_{2,3},.6,.5,.1\rangle\}$,

$\varsigma_S(w_2,w_3)=\{\langle w_1,0,0,1\rangle,\langle w_2,.6,.5,.4\rangle,\langle w_3,.7,.8,.1\rangle,\langle w_{1,2},.9,.5,.4\rangle,\langle w_{1,3},.9,.8,.1\rangle,\langle w_{2,3},.7,.8,.1\rangle\}$

and NFCSs over (W,τ_n) are ${}^F\varsigma_W=\{0_n,1_n,\varsigma_R(w_1)',\varsigma_R(w_3)'\,\varsigma_S(w_2,w_3)'\}$, where

$\varsigma_R(w_1)'=\{\langle w_1,.7,.6,.2\rangle,\langle w_2,1,1,0\rangle,\langle w_3,1,1,0\rangle,\langle w_{1,2},.1,.6,.6\rangle,\langle w_{1,3},.6,.5,.4\rangle,\langle w_{2,3},1,1,0\rangle\}$,

$\varsigma_R(w_3)'=\{\langle w_1,1,1,0\rangle,\langle w_2,1,1,0\rangle,\langle w_3,.6,.5,.4\rangle,\langle w_{1,2},1,1,0\rangle,\langle w_{1,3},.6,.5,.4\rangle,\langle w_{2,3},.1,.5,.6\rangle\}$,

$\varsigma_S(w_2,w_3)'=\{\langle w_1,1,1,0\rangle,\langle w_2,.4,.5,.6\rangle,\langle w_3,.1,.2,.7\rangle,\langle w_{1,2},.4,.5,.9\rangle,\langle w_{1,3},.1,.2,.9\rangle,\langle w_{2,3},.1,.2,.7\rangle\}$.

Thus $(W,\tau_n,{}^f\varsigma_W)$ is a NFTS over (W,τ_n). Here nf-GCS $=\{\varsigma_S(w_2),\varsigma_S(w_3),\varsigma_S(w_{2,3})\}$. Thus $\varsigma_S(w_2)$ is nf-GCS but not NFCS.

Theorem 2. If $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ are nf-GCSs over $(W,\tau_n,{}^f\varsigma_W)$, then $\varsigma_R(V_1)\cup\varsigma_R(V_2)$ is also a nf-GCS over $(W,\tau_n,{}^f\varsigma_W)$.

Proof. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be nf-GCSs over $(W,\tau_n,{}^f\varsigma_W)$. Then $Cl_{nf}(\varsigma_R(V_1))\subseteq\varsigma_R(U)$ whenever $\varsigma_R(V_1)\subseteq\varsigma_R(U)$ and $\varsigma_R(U)$ is NFOS and $Cl_{nf}(\varsigma_R(V))\subseteq\varsigma_R(U)$ whenever $\varsigma_R(V)\subseteq\varsigma_R(U)$ and $\varsigma_R(U)$ is NFOS. Since $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ are subsets of $\varsigma_R(U)$, $\varsigma_R(V_1)\cup\varsigma_R(V_2)$ are subsets of $\varsigma_R(U)$ and $\varsigma_R(U)$ is NFOS. Then by ***Proposition 2***, $Cl_{nf}(\varsigma_R(V_1)\cup\varsigma_R(V_2))=Cl_{nf}(\varsigma_R(V_1))\cup Cl_{nf}(\varsigma_R(V_2))$. Thus $Cl_{nf}(\varsigma_R(V_1)\cup\varsigma_R(V_2))\subseteq\varsigma_R(U)$. Hence $\varsigma_R(V_1)\cup\varsigma_R(V_2)$ is a nf-GCSs over $(W,\tau_n,{}^f\varsigma_W)$.

Remark 2. The intersection of two nf-GCSs need not be a nf-GCS as shown in the following example.

Example 2. Consider the ***Example 1***. Here nf-GCS $=\{\varsigma_S(w_2),\varsigma_S(w_3),\varsigma_S(w_{2,3})\}$. Then

$\varsigma_S(w_2)\cap\varsigma_S(w_3)=\{\langle w_1,0,0,1\rangle,\langle w_2,0,0,1\rangle,\langle w_3,0,0,1\rangle,\langle w_{1,2},0,0,1\rangle,\langle w_{1,3},0,0,1\rangle,\langle w_{2,3},.7,.8,.1\rangle\}$, is not a nf-GCS.

Theorem 3. If $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ are nf-GCSs over $(W,\tau_n,{}^f\varsigma_W)$, then

$$Cl_{nf}(\varsigma_R(V_1)\cap\varsigma_R(V_2))\subseteq Cl_{nf}(\varsigma_R(V_1))\cap Cl_{nf}(\varsigma_R(V_2)).$$

Proof. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be nf-GCSs over $(W,\tau_n,{}^f\varsigma_W)$. Then $Cl_{nf}(\varsigma_R(V_1))\subseteq\varsigma_R(U)$ whenever $\varsigma_R(V_1)\subseteq\varsigma_R(U)$ and $\varsigma_R(U)$ is NFOS and $Cl_{nf}(\varsigma_R(V))\subseteq\varsigma_R(U)$ whenever $\varsigma_R(V)\subseteq\varsigma_R(U)$ and $\varsigma_R(U)$ is NFOS.

Since $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ are subsets of $\varsigma_R(U)$, $\varsigma_R(V_1)\cap\varsigma_R(V_2)$ are subsets of $\varsigma_R(U)$ and $\varsigma_R(U)$ is NFOS.

Since $\varsigma_R(V_1)\cap\varsigma_R(V_2)\subseteq\varsigma_R(V_1)$ and $\varsigma_R(V_1)\cap\varsigma_R(V_2)\subseteq\varsigma_R(V_1)$, $Cl_{nf}(\varsigma_R(V_1)\cap\varsigma_R(V_2))\subseteq Cl_{nf}(\varsigma_R(V_1))$ and $Cl_{nf}(\varsigma_R(V_1)\cap\varsigma_R(V_2))\subseteq Cl_{nf}(\varsigma_R(V_2))$, by ***Proposition 2***. Thus $Cl_{nf}(\varsigma_R(V_1)\cap\varsigma_R(V_2))\subseteq Cl_{nf}(\varsigma_R(V_1))\cap Cl_{nf}(\varsigma_R(V_2))$.

3.2. Neutro-Fine-Semi Open Sets

Definition 8. Let $\varsigma_R(V)$ be a NFS over W of a NFTS $(W,\tau_n,{}^f\varsigma_W)$. Then $\varsigma_R(V)$ is said to be a neutro-fine-semi closed set (nf-SCS) if $Int_{nf}(Cl_{nf}(\varsigma_R(V)))\subseteq\varsigma_R(V)$.

The complement of nf -SCS is said to be neutro-fine-semi open set (nf -SOS), i.e., $\varsigma_R(V)\subseteq Cl_{nf}(Int_{nf}(\varsigma_R(V)))$.

Theorem 4. Let $(W,\tau_n,{}^f\varsigma_W)$ be a NFTS and $\varsigma_R(V)$ be a NFS over W. Then $\varsigma_R(V)$ is nf-SCS if and only if $\varsigma_R(V)'$ is nf-SOS.

Proof. Let $\varsigma_R(V)$ be a nf-SCS. Then $Int_{nf}(Cl_{nf}(\varsigma_R(V)))\subseteq\varsigma_R(V)$.

Taking complement on both sides,

$$\varsigma_R(V)'\subseteq\left[Int_{nf}(Cl_{nf}(\varsigma_R(V)))\right]' = Cl_{nf}(Cl_{nf}(\varsigma_R(V)))'.$$

By using ***Proposition 1***, $\varsigma_R(V)'=Cl_{nf}(Int_{nf}(\varsigma_R(V)'))$. Thus $\varsigma_R(V)'$ is a nf-SOS. Conversely, assume that $\varsigma_R(V)'$ is a nf-SOS. Then $\varsigma_R(V)'=Cl_{nf}(Int_{nf}(\varsigma_R(V)'))$.
Taking complement on both sides,

$$\varsigma_R(V)\supseteq\left[Cl_{nf}(Int_{nf}(\varsigma_R(V)'))\right]' = Int_{nf}(Int_{nf}(\varsigma_R(V)'))',$$ by ***Proposition 1***.

By **Proposition 2**, $\varsigma_R(V)\supseteq Int_{nf}(Cl_{nf}(\varsigma_R(V)))$. Thus $\varsigma_R(V)$ is a nf-SCS.

Theorem 5. If $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ are nf-SCSs over NFTS $(W,\tau_n,{}^f\varsigma_W)$, then $\varsigma_R(V_1)\cap\varsigma_R(V_2)$ is also a nf -SCS in $(W,\tau_n,{}^f\varsigma_W)$.

Proof. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be nf-SCSs over $(W,\tau_n,{}^f\varsigma_W)$. Then $Int_{nf}(Cl_{nf}(\varsigma_R(V_1)))\subseteq\varsigma_R(V_1)$ and $Int_{nf}(Cl_{nf}(\varsigma_R(V_1)))\subseteq\varsigma_R(V_1)$. Thus $\varsigma_R(V_1)\cap\varsigma_R(V_2)\supseteq Int_{nf}(Cl_{nf}(\varsigma_R(V_1)))\cap Int_{nf}(Cl_{nf}(\varsigma_R(V_2)))$

$$= Int_{nf}(Cl_{nf}(\varsigma_R(V_1))\cap Cl_{nf}(\varsigma_R(V_2))) \supseteq Int_{nf}(Cl_{nf}(\varsigma_R(V_1)\cap\varsigma_R(V_2))),$$ by ***Propositions 1*** and ***2.***

Hence $\varsigma_R(V_1)\cap\varsigma_R(V_2)$ is a nf-SCSs in $(W,\tau_n,{}^f\varsigma_W)$.

Theorem 6. If $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ are nf-SOSs over NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$, then $\varsigma_R(V_1)\cup\varsigma_R(V_2)$ is also a nf-SOS in $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Proof. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be nf-SCSs over $\left(W,\tau_n,{}^f\varsigma_W\right)$. Then $\varsigma_R(V_1)\subseteq Cl_{nf}\left(Int_{nf}\left(\varsigma_R(V_1)\right)\right)$ and $\varsigma_R(V_2)\subseteq Cl_{nf}\left(Int_{nf}\left(\varsigma_R(V_2)\right)\right)$. Thus $\varsigma_R(V_1)\cup\varsigma_R(V_2)\subseteq Cl_{nf}\left(Int_{nf}\left(\varsigma_R(V_1)\right)\right)\cup Cl_{nf}\left(Int_{nf}\left(\varsigma_R(V_2)\right)\right)$

$= Cl_{nf}\left(Int_{nf}\left(\varsigma_R(V_1)\right)\cup Int_{nf}\left(\varsigma_R(V_2)\right)\right) \supseteq Cl_{nf}\left(Int_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)\right)$, by ***Propositions 1*** and ***2***.

Theorem 7. Every NFCS is a nf-SCS in NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Proof. Let $\varsigma_R(V)$ be a NFCS on $\left(W,\tau_n,{}^f\varsigma_W\right)$. Then $\varsigma_R(V)=Cl_{nf}\left(\varsigma_R(V)\right)$. Thus

$Int_{nf}\left(Cl_{nf}\left(\varsigma_R(V)\right)\right)\subseteq Cl_{nf}\left(\varsigma_R(V)\right) \Rightarrow Int_{nf}\left(Cl_{nf}\left(\varsigma_R(V)\right)\right)\subseteq\varsigma_R(V)$. Hence $\varsigma_R(V)$ is a nf-SCS in $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Definition 9. Let $\left(W,\tau_n,{}^f\varsigma_W\right)$ be a NFTS over (W,τ_n). Let $\varsigma_R(V)$ be a NFS over *W*. Then the neutro-fine-semi interior of $\varsigma_R(V)$ is denoted as $S^*Int_{nf}\left(\varsigma_R(V)\right)$ and is defined as the union of all nf-SOSs contained in $\varsigma_R(V)$. Clearly, $S^*Int_{nf}\left(\varsigma_R(V)\right)$ is the largest nf-SOS contained in $\varsigma_R(V)$.

Definition 10. Let $\left(W,\tau_n,{}^f\varsigma_W\right)$ be a NFTS over (W,τ_n). Let $\varsigma_R(V)$ be a NFS over *W*. Then the neutro-fine-semi closure of $\varsigma_R(V)$ is denoted as $S^*Cl_{nf}\left(\varsigma_R(V)\right)$ and is defined as the intersection of all nf-SCSs containing $\varsigma_R(V)$. Clearly, $S^*Cl_{nf}\left(\varsigma_R(V)\right)$ is the smallest nf-SCS containing $\varsigma_R(V)$.

Proposition 3. Let $\left(W,\tau_n,{}^f\varsigma_W\right)$ be a NFTS. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be two NFSs over *W*. Then,

$S^*Int_{nf}\left(\varsigma_R(V_1)\right)\subseteq\varsigma_R(V_1)$;

$\varsigma_R(V_1)$ is nf-SOS $\Rightarrow S^*Int_{nf}\left(\varsigma_R(V_1)\right)=\varsigma_R(V_1)$;

$S^*Int_{nf}\left(S^*Int_{nf}\left(\varsigma_R(V_1)\right)\right)=S^*Int_{nf}\left(\varsigma_R(V_1)\right)$;

$\varsigma_R(V_1)\subseteq\varsigma_R(V_2)\Rightarrow S^*Int_{nf}\left(\varsigma_R(V_1)\right)\subseteq S^*Int_{nf}\left(\varsigma_R(V_2)\right)$.

Proof. Straightforward.

Proposition 4. Let $\left(W,\tau_n,{}^f\varsigma_W\right)$ be a NFTS. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be two NFSs over *W*. Then,

$\varsigma_R(V_1)\subseteq S^*Cl_{nf}\left(\varsigma_R(V_1)\right)$;

$\varsigma_R(V_1)$ is nf-SCS $\Rightarrow S^*Cl_{nf}(\varsigma_R(V_1)) = \varsigma_R(V_1)$;

$S^*Cl_{nf}(S^*Cl_{nf}(\varsigma_R(V_1))) = S^*Cl_{nf}(\varsigma_R(V_1))$;

$\varsigma_R(V_1) \subseteq \varsigma_R(V_2) \Rightarrow S^*Cl_{nf}(\varsigma_R(V_1)) \subseteq S^*Cl_{nf}(\varsigma_R(V_2))$.

Proof. Straightforward.

Proposition 5. Let $(W, \tau_n, {}^f\varsigma_W)$ be a NFTS. Let $\varsigma_R(V)$ be any NFS over W. Then,

$Int_{nf}(\varsigma_R(V)) \subseteq S^*Int_{nf}(\varsigma_R(V)) \subseteq \varsigma_R(V)$;

$\varsigma_R(V) \subseteq S^*Cl_{nf}(\varsigma_R(V)) \subseteq Cl_{nf}(\varsigma_R(V))$;

$S^*Cl_{nf}(\varsigma_R(V)') = [S^*Int_{nf}(\varsigma_R(V))]'$;

$S^*Int_{nf}(\varsigma_R(V)') = [S^*Cl_{nf}(\varsigma_R(V))]'$.

Proof. Straightforward.

Proposition 6. Let $(W, \tau_n, {}^f\varsigma_W)$ be a NFTS. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be two NFSs over W. Then,

$S^*Int_{nf}(\varsigma_R(V_1) \cap \varsigma_R(V_2)) = S^*Int_{nf}(\varsigma_R(V_1)) \cap S^*Int_{nf}(\varsigma_R(V_2))$;

$S^*Int_{nf}(\varsigma_R(V_1) \cup \varsigma_R(V_2)) \supseteq S^*Int_{nf}(\varsigma_R(V_1)) \cup S^*Int_{nf}(\varsigma_R(V_2))$.

Proof. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be two NFSs over W.

Since $\varsigma_R(V_1) \cap \varsigma_R(V_2) \subseteq \varsigma_R(V_1)$ and $\varsigma_R(V_1) \cap \varsigma_R(V_2) \subseteq \varsigma_R(V_2)$, by using ***Proposition 3***, $S^*Int_{nf}(\varsigma_R(V_1) \cap \varsigma_R(V_2)) \subseteq S^*Int_{nf}(\varsigma_R(V_1))$ and $S^*Int_{nf}(\varsigma_R(V_1) \cap \varsigma_R(V_2)) \subseteq S^*Int_{nf}(\varsigma_R(V_2))$.

This implies that,

$$S^*Int_{nf}(\varsigma_R(V_1) \cap \varsigma_R(V_2)) \subseteq S^*Int_{nf}(\varsigma_R(V_1)) \cap S^*Int_{nf}(\varsigma_R(V_2)), \tag{1}$$

By using ***Proposition 3***,

$S^*Int_{nf}(\varsigma_R(V_1)) \subseteq \varsigma_R(V_1)$ and $S^*Int_{nf}(\varsigma_R(V_2)) \subseteq \varsigma_R(V_2)$ $\Rightarrow S^*Int_{nf}(\varsigma_R(V_1)) \cap S^*Int_{nf}(\varsigma_R(V_2)) \subseteq \varsigma_R(V_1) \cap \varsigma_R(V_2)$.

By using ***Proposition 3***,

$$S^*Int_{nf}\left[S^*Int_{nf}\left(\varsigma_R(V_1)\right)\cap S^*Int_{nf}\left(\varsigma_R(V_2)\right)\right]\subseteq S^*Int_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right).$$

By ***Eq. (1)***,

$$S^*Int_{nf}\left(S^*Int_{nf}\left(\varsigma_R(V_1)\right)\right)\cap S^*Int_{nf}\left(S^*Int_{nf}\left(\varsigma_R(V_2)\right)\right)\subseteq S^*Int_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right).$$

By using ***Proposition 3***,

$$S^*Int_{nf}\left(\varsigma_R(V_1)\right)\cap S^*Int_{nf}\left(\varsigma_R(V_2)\right)\subseteq S^*Int_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right), \tag{2}$$

Hence from ***Eqs. (1)-(2)***,

$$S^*Int_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right)=S^*Int_{nf}\left(\varsigma_R(V_1)\right)\cap S^*Int_{nf}\left(\varsigma_R(V_2)\right).$$

Since $\varsigma_R(V_1)\subseteq\varsigma_R(V_1)\cup\varsigma_R(V_2)$ and $\varsigma_R(V_1)\subseteq\varsigma_R(V_1)\cup\varsigma_R(V_2)$, by using ***Proposition 3***,

$S^*Int_{nf}\left(\varsigma_R(V_1)\right)\subseteq S^*Int_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)$ and $S^*Int_{nf}\left(\varsigma_R(V_2)\right)\subseteq S^*Int_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)$. Hence $S^*Int_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)\supseteq S^*Int_{nf}\left(\varsigma_R(V_1)\right)\cup S^*Int_{nf}\left(\varsigma_R(V_2)\right)$.

Proposition 7. Let $\left(W,\tau_n,{}^f\varsigma_W\right)$ be a NFTS. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be two NFSs over *W*. Then,

$$S^*Cl_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)=S^*Cl_{nf}\left(\varsigma_R(V_1)\right)\cup S^*Cl_{nf}\left(\varsigma_R(V_2)\right);$$

$$S^*Cl_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right)\subseteq S^*Cl_{nf}\left(\varsigma_R(V_1)\right)\cap S^*Cl_{nf}\left(\varsigma_R(V_2)\right).$$

Proof. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be two NFSs over *W*.

Since $S^*Cl_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)=S^*Cl_{nf}\left(\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)'\right)'$, by using ***Proposition 5***,

$$S^*Cl_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)=\left[S^*Int_{nf}\left(\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)'\right)\right]'=\left[S^*Int_{nf}\left(\left(\varsigma_R(V_1)'\right)\cap\left(\varsigma_R(V_2)'\right)\right)\right]'.$$

Again by using ***Proposition 5***, $S^*Cl_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)=\left[S^*Int_{nf}\left(\varsigma_R(V_1)'\right)\cap S^*Int_{nf}\left(\varsigma_R(V_2)'\right)\right]'$

$$=\left(S^*Int_{nf}\left(\varsigma_R(V_1)'\right)\right)'\cup\left(S^*Int_{nf}\left(\varsigma_R(V_2)'\right)\right)'.$$

By using ***Proposition 5***, $S^*Cl_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)=S^*Cl_{nf}\left(\left(\varsigma_R(V_1)'\right)\right)'\cup S^*Cl_{nf}\left(\left(\varsigma_R(V_2)'\right)\right)'$

$$=S^*Cl_{nf}\left(\varsigma_R(V_1)\right)\cup S^*Cl_{nf}\left(\varsigma_R(V_2)\right).$$

Hence $S^*Cl_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)=S^*Cl_{nf}\left(\varsigma_R(V_1)\right)\cup S^*Cl_{nf}\left(\varsigma_R(V_2)\right)$.

(ii) Since $\varsigma_R(V_1)\cap\varsigma_R(V_2)\subseteq\varsigma_R(V_1)$ and $\varsigma_R(V_1)\cap\varsigma_R(V_2)\subseteq\varsigma_R(V_2)$, by using ***Proposition 4***, $S^*Cl_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right)\subseteq S^*Cl_{nf}\left(\varsigma_R(V_1)\right)$ and $S^*Cl_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right)\subseteq S^*Cl_{nf}\left(\varsigma_R(V_2)\right)$.

Hence $S^*Cl_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right)\subseteq S^*Cl_{nf}\left(\varsigma_R(V_1)\right)\cap S^*Cl_{nf}\left(\varsigma_R(V_2)\right)$.

3.3. Neutro-Fine-Generalized Semi Open Sets

Definition 11. Let $\varsigma_R(V)$ be a NFS over W of a NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$. Then $\varsigma_R(V)$ is said to be a neutro-fine-generalized semi closed set (nf -GSCS) if $S^*Cl_{nf}\left(\varsigma_R(V)\right)\subseteq\varsigma_R(U)$ whenever $\varsigma_R(V)\subseteq\varsigma_R(U)$ and $\varsigma_R(U)$ is NFOS.

The complement of *nf* -GSCS is said to be neutro-fine-generalized semi open set (*nf* -GSOS), i.e., $\varsigma_R(U)\subseteq S^*Int_{nf}\left(\varsigma_R(V)\right)$ whenever $\varsigma_R(U)\subseteq\varsigma_R(V)$ and $\varsigma_R(U)$ is NFCS.

Example 3. Consider ***Example 1***. Thus *nf* -SCS $=\left\{\varsigma_R(w_1,w_3),\varsigma_S(w_1)\right\}$, *nf* -SOS $=\left\{\varsigma_R(w_1,w_3)',\varsigma_S(w_1)'\right\}$ where $\varsigma_R(w_1,w_3)'=\left\{\langle w_1,.7,.6,.2\rangle,\langle w_2,1,1,0\rangle,\langle w_3,.6,.5,.4\rangle,\langle w_{1,2},.1,.6,.6\rangle,\langle w_{1,3},.6,.5,.4\rangle,\langle w_{2,3},.1,.5,.6\rangle\right\}$, $\varsigma_S(w_1)'=\left\{\langle w_1,.6,.7,.9\rangle,\langle w_2,.1,1,0\rangle,\langle w_3,1,1,0\rangle,\langle w_{1,2},.4,.5,.9\rangle,\langle w_{1,3},.1,.2,.9\rangle,\langle w_{2,3},1,1,0\rangle\right\}$ and *nf* -GSCS $=\left\{\varsigma_S(w_2)\right\}$.

Theorem 8. Every NFCS is a *nf*-GSCS in NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Proof. Let $\varsigma_R(V)$ be a NFCS in NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Let $\varsigma_R(V)\subseteq\varsigma_R(U)$, where $\varsigma_R(U)$ is NFOS in $\left(W,\tau_n,{}^f\varsigma_W\right)$. Since $\varsigma_R(V)$ is a NFCS, $\varsigma_R(V)=Cl_{nf}\left(\varsigma_R(V)\right)$, by ***Proposition 2***. Also, by ***Proposition 5***, $S^*Cl_{nf}\left(\varsigma_R(V)\right)\subseteq Cl_{nf}\left(\varsigma_R(V)\right)$. Thus $S^*Cl_{nf}\left(\varsigma_R(V)\right)\subseteq Cl_{nf}\left(\varsigma_R(V)\right)=\varsigma_R(V)\subseteq\varsigma_R(U)$. Hence $\varsigma_R(V)$ is a *nf*-GSCS in NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Theorem 9. If $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ are *nf*-GSCSs over NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$, then $\varsigma_R(V_1)\cap\varsigma_R(V_2)$ is also a *nf*-GSCS in $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Proof. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be *nf*-GSCSs over $\left(W,\tau_n,{}^f\varsigma_W\right)$.

If $\varsigma_R(V_1)\cap\varsigma_R(V_2)\subseteq\varsigma_R(U)$ and $\varsigma_R(U)$ is a NFOS, then $\varsigma_R(V_1)\subseteq\varsigma_R(U)$ and $\varsigma_R(V_1)\subseteq\varsigma_R(U)$.

Since $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ are *nf*–GSCSs, $S^*Cl_{nf}\left(\varsigma_R(V_1)\right)\subseteq\varsigma_R(U)$ and $S^*Cl_{nf}\left(\varsigma_R(V_2)\right)\subseteq\varsigma_R(U)$.

Thus $S^*Cl_{nf}\left(\varsigma_R(V_1)\right)\cap S^*Cl_{nf}\left(\varsigma_R(V_2)\right)\subseteq\varsigma_R(U)$.

By ***Proposition 7***, $S^*Cl_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right)\subseteq S^*Cl_{nf}\left(\varsigma_R(V_1)\right)\cap S^*Cl_{nf}\left(\varsigma_R(V_2)\right)\subseteq\varsigma_R(U)$. This implies that,

$S^{*}Cl_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right)\subseteq\varsigma_R(U)$. Thus $S^{*}Cl_{nf}\left(\varsigma_R(V_1)\cap\varsigma_R(V_2)\right)\subseteq\varsigma_R(U)$, $\varsigma_R(V_1)\cap\varsigma_R(V_2)\subseteq\varsigma_R(U)$ and $\varsigma_R(U)$ is a NFOS.

Hence $\varsigma_R(V_1)\cap\varsigma_R(V_2)$ is a nf-GSCS over $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Theorem 10. Every NFOS is a nf-GSOS in NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Proof. Let $\varsigma_R(V)$ be a NFOS in NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$. Let $\varsigma_R(U)\subseteq\varsigma_R(V)$, where $\varsigma_R(U)$ is NFCS in $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Since $\varsigma_R(V)$ is a NFOS, $\varsigma_R(V)=Int_{nf}\left(\varsigma_R(V)\right)$, by ***Proposition 1***.

Also, by ***Proposition 5,*** $Int_{nf}\left(\varsigma_R(V)\right)\subseteq S^{*}Int_{nf}\left(\varsigma_R(V)\right)\subseteq\varsigma_R(V)$. Thus $\varsigma_R(V)=S^{*}Int_{nf}\left(\varsigma_R(V)\right)$ $\Rightarrow\varsigma_R(U)\subseteq\varsigma_R(V)=S^{*}Int_{nf}\left(\varsigma_R(V)\right)$.

Hence $\varsigma_R(V)$ is a nf-GSCS in NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Theorem 11. If $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ are nf-GSOSs over NFTS $\left(W,\tau_n,{}^f\varsigma_W\right)$, then $\varsigma_R(V_1)\cup\varsigma_R(V_2)$ is also a nf-GSOS in $\left(W,\tau_n,{}^f\varsigma_W\right)$.

Proof. Let $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ be nf-GSOSs over $\left(W,\tau_n,{}^f\varsigma_W\right)$.

If $\varsigma_R(U)\subseteq\varsigma_R(V_1)\cup\varsigma_R(V_2)$ and $\varsigma_R(U)$ is a NFCS, then $\varsigma_R(U)\subseteq\varsigma_R(V_1)$ and $\varsigma_R(U)\subseteq\varsigma_R(V_2)$.

Since $\varsigma_R(V_1)$ and $\varsigma_R(V_2)$ are nf–GSOSs, $\varsigma_R(U)\subseteq S^{*}Int_{nf}\left(\varsigma_R(V_1)\right)$ and $\varsigma_R(U)\subseteq S^{*}Int_{nf}\left(\varsigma_R(V_2)\right)$.

Thus $\varsigma_R(U)\subseteq S^{*}Int_{nf}\left(\varsigma_R(V_1)\right)\cup S^{*}Int_{nf}\left(\varsigma_R(V_1)\right)$.

By ***Proposition 6***, $\varsigma_R(U)\subseteq S^{*}Int_{nf}\left(\varsigma_R(V_1)\right)\cup S^{*}Int_{nf}\left(\varsigma_R(V_1)\right)\subseteq S^{*}Int_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)$.

This implies that, $\varsigma_R(U)\subseteq S^{*}Int_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)$.

Thus $\varsigma_R(U)\subseteq S^{*}Int_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)$, $\varsigma_R(U)\subseteq S^{*}Int_{nf}\left(\varsigma_R(V_1)\cup\varsigma_R(V_2)\right)$ and $\varsigma_R(U)$ is a NFCS.

Hence $\varsigma_R(V_1)\cup\varsigma_R(V_2)$ is a nf-GSOS in $\left(W,\tau_n,{}^f\varsigma_W\right)$.

4. Neutro-Fine Minimal and Maximal Open Sets

In this portion, the minimal and maximal open sets on NFTS are defined and probable results are carried by some major expressive examples.

Definition 12. Let $\varsigma_R(V)$ be a proper non-empty NFOS of a NFTS $(W,\tau_n,{}^f\varsigma_W)$. Then $\varsigma_R(V)$ is said to be a neutro-fine minimal open set ($\min_{nf}$-OS) if any NFOS which is contained in $\varsigma_R(V)$ is 0_{nf} or $\varsigma_R(V)$. The complement of $\min_{nf}$-OS is said to be neutro-fine minimal closed set ($\min_{nf}$-CS).

Definition 13. Let $\varsigma_R(V)$ be a proper non-empty NFOS of a NFTS $(W,\tau_n,{}^f\varsigma_W)$. Then $\varsigma_R(V)$ is said to be a neutro-fine maximal open set ($\max_{nf}$-OS) if any NFOS which is contained in $\varsigma_R(V)$ is 1_{nf} or $\varsigma_R(V)$. The complement of $\max_{nf}$-OS is said to be neutro-fine maximal closed set ($\max_{nf}$-CS).

Example 4. Let $W=\{w_1,w_2,w_3\}$ and $\tau_n=\{0_n,1_n,R,S\}$ where *R* and *S* are NSs over *W* and are defined as follows

$R=\{\langle w_1,.1,.2,.8\rangle,\langle w_2,.4,.7,.3\rangle,\langle w_3,.6,.5,.2\rangle\}$ and $S=\{\langle w_1,.6,.5,.3\rangle,\langle w_2,.9,.8,.1\rangle,\langle w_3,.7,.6,.1\rangle\}$.

Thus (W,τ_n) is a NTS over *W*. Then ${}^f\varsigma_W=\{0_n,1_n,\varsigma_R(w_1),\varsigma_R(w_2,w_3),\varsigma_S(w_2)\}$, where

$\varsigma_R(w_1)=\{\langle w_1,.1,.2,.8\rangle,\langle w_2,0,0,1\rangle,\langle w_3,0,0,1\rangle,\langle w_{1,2},.4,.7,.3\rangle,\langle w_{1,3},.6,.5,.2\rangle,\langle w_{2,3},0,0,1\rangle\}$,

$\varsigma_R(w_2,w_3)=\{\langle w_1,0,0,1\rangle,\langle w_2,.4,.7,.3\rangle,\langle w_3,.6,.5,.2\rangle,\langle w_{1,2},.4,.7,.3\rangle,\langle w_{1,3},.6,.5,.2\rangle,\langle w_{2,3},.6,.7,.2\rangle\}$,

$\varsigma_S(w_2)=\{\langle w_1,0,0,1\rangle,\langle w_2,.9,.8,.1\rangle,\langle w_3,0,0,1\rangle,\langle w_{1,2},.9,.8,.1\rangle,\langle w_{1,3},0,0,1\rangle,\langle w_{2,3},.9,.8,.1\rangle\}$ are NFOSs over (W,τ_n).

Hence $(W,\tau_n,{}^f\varsigma_W)$ is a NFTS over (W,τ_n). Thus min_{nf}-OS $=\{0_n,\varsigma_R(w_1),\varsigma_S(w_2)\}$, min_{nf}-CS $=\{1_n,\varsigma_R(w_1)',\varsigma_S(w_2)'\}$, $\max_{nf}$-OS $=\{0_n,\varsigma_R(w_2,w_3)\}$ and max_{nf}-CS $=\{1_n,\varsigma_R(w_2,w_3)'\}$.

Example 5. Consider ***Example 1***. Here min_{nf}-OS $=\{0_n,\varsigma_R(w_1),\varsigma_R(w_3)\}$, min_{nf}-CS $=\{1_n,\varsigma_R(w_1)',\varsigma_R(w_3)'\}$,

$\max_{nf}$-OS $=\{0_n,\varsigma_S(w_2,w_3)\}$ and max_{nf}-CS $=\{1_n,\varsigma_S(w_2,w_3)'\}$.

Lemma 1. Let $(W,\tau_n,{}^f\varsigma_W)$ be a NFTS over (W,τ_n).

If $\varsigma_R(U)$ is a $\min_{nf}$-OS and $\varsigma_R(W)$ is NFOS, then $\varsigma_R(U)\cap\varsigma_R(W)=0_{nf}$ or $\varsigma_R(U)\subseteq\varsigma_R(W)$.

If $\varsigma_R(U)$ and $\varsigma_R(V)$ are $\min_{nf}$-OSs, then $\varsigma_R(U)\cap\varsigma_R(V)=0_{nf}$ or $\varsigma_R(U)=\varsigma_R(V)$.

Proof. Let $\varsigma_R(W)$ be a NFOS such that $\varsigma_R(U)\cap\varsigma_R(W)\neq 0_{nf}$.

Since $\varsigma_R(U)$ is a $\min_{nf}$-OS and $\varsigma_R(U)\cap\varsigma_R(W)\subseteq\varsigma_R(U)$, then $\varsigma_R(U)\cap\varsigma_R(W)=\varsigma_R(U)$. Hence $\varsigma_R(U)\subseteq\varsigma_R(W)$.

If $\varsigma_R(U)\cap\varsigma_R(W)\neq 0_{nf}$, then $\varsigma_R(U)\subseteq\varsigma_R(V)$ and $\varsigma_R(V)\subseteq\varsigma_R(U)$, by (i). Hence $\varsigma_R(U)=\varsigma_R(V)$.

Proposition 7. Let $\left(W,\tau_n,{}^f\varsigma_W\right)$ be a NFTS over (W,τ_n). Let $\varsigma_R(U)$ be a $\min_{nf}$-OS. If $w^{\langle\alpha,\beta,\gamma\rangle}$ is a NFP of $\varsigma_R(U)$, then $\varsigma_R(U)\subseteq\varsigma_R(W)$ for any neutro-fine neighborhood $\varsigma_R(W)$ of $w^{\langle\alpha,\beta,\gamma\rangle}$.

Proof. Let $\left(W,\tau_n,{}^f\varsigma_W\right)$ be a NFTS over (W,τ_n).

Let $\varsigma_R(W)$ be a neutro-fine neighborhood of $w^{\langle\alpha,\beta,\gamma\rangle}$ such that $\varsigma_R(U)\not\subset\varsigma_R(W)$. Then $\varsigma_R(U)\cap\varsigma_R(W)$ is a NFOS such that $\varsigma_R(U)\cap\varsigma_R(W)\not\subset\varsigma_R(U)$ and $\varsigma_R(U)\cap\varsigma_R(W)\neq 0_{nf}$.

This contradicts our assumption that $\varsigma_R(U)$ is a $\min_{nf}$-OS. Hence proved.

Proposition 8. Let $\left(W,\tau_n,{}^f\varsigma_W\right)$ be a NFTS over (W,τ_n). Let $\varsigma_R(U)$ be a $\min_{nf}$-OS. Then

$\varsigma_R(U)=\bigcap\left\{\varsigma_R(W):\varsigma_R(W)\text{ is a neutro-fine neighborhood of }w^{\langle\alpha,\beta,\gamma\rangle}\right\}$, for any NFP $w^{\langle\alpha,\beta,\gamma\rangle}$ of $\varsigma_R(U)$.

Proof. Let $\left(W,\tau_n,{}^f\varsigma_W\right)$ be a NFTS over (W,τ_n). Let $\varsigma_R(U)$ be a $\min_{nf}$-OS.

Since $\varsigma_R(U)$ is a neutro-fine neighborhood of $w^{\langle\alpha,\beta,\gamma\rangle}$, by ***Proposition 7***, then

$\varsigma_R(U)\subseteq\bigcap\left\{\varsigma_R(W):\varsigma_R(W)\text{ is a neutro-fine neighborhood of }w^{\langle\alpha,\beta,\gamma\rangle}\right\}\subseteq\varsigma_R(U)$. Thus
$\varsigma_R(U)=\bigcap\left\{\varsigma_R(W):\varsigma_R(W)\text{ is a neutro-fine neighborhood of }w^{\langle\alpha,\beta,\gamma\rangle}\right\}$.

Proposition 9. Let $\left(W,\tau_n,{}^f\varsigma_W\right)$ be a NFTS over (W,τ_n). Let $\varsigma_R(U)$ be a non-empty NFOS. Then the following conditions are equivalent:

- $\varsigma_R(U)$ is a min_{nf}-OS.
- $\varsigma_R(U)\subseteq Cl_{nf}\left(\varsigma_R(V)\right)$ for any NFS $\varsigma_R(V)$ of $\varsigma_R(U)$.
- $Cl_{nf}\left(\varsigma_R(U)\right)\subseteq Cl_{nf}\left(\varsigma_R(V)\right)$ for any NFS $\varsigma_R(V)$ of $\varsigma_R(U)$.

Proof. $(1)\Rightarrow(2)$. Let $\varsigma_R(V)$ be any NFS of $\varsigma_R(U)$.

By ***Proposition 7***, for any NFP $w^{\langle\alpha,\beta,\gamma\rangle}$ of $\varsigma_R(U)$ and any neutro-fine neighborhood $\varsigma_R(W)$ of $w^{\langle\alpha,\beta,\gamma\rangle}$, then $\varsigma_R(V)=\left(\varsigma_R(U)\cap\varsigma_R(V)\right)\subseteq\left(\varsigma_R(W)\cap\varsigma_R(V)\right)$. Thus $\varsigma_R(W)\cap\varsigma_R(V)\neq 0_{nf}$, and hence $\varsigma_R(U)\cap\varsigma_R(W)\neq 0_{nf}$ is a NFP of $Cl_{nf}\left(\varsigma_R(V)\right)$. Therefore $\varsigma_R(U)\subseteq Cl_{nf}\left(\varsigma_R(V)\right)$.

$(2) \Rightarrow (3)$. Since $\varsigma_R(V)$ is any NFS of $\varsigma_R(U)$, then $\varsigma_R(U) \subseteq Cl_{nf}(\varsigma_R(V))$.

Thus by (2), $Cl_{nf}(\varsigma_R(U)) \subseteq Cl_{nf}(Cl_{nf}(\varsigma_R(V))) = Cl_{nf}(\varsigma_R(V))$. Hence $Cl_{nf}(\varsigma_R(U)) \subseteq Cl_{nf}(\varsigma_R(V))$ for any NFS $\varsigma_R(V)$ of $\varsigma_R(U)$.

$(3) \Rightarrow (1)$. Suppose that $\varsigma_R(U)$ is not a $\min_{nf}$-OS.

Then there exists a NFS $\varsigma_R(V)$ such that $\varsigma_R(V) \not\subset \varsigma_R(U)$. Then there exists a NFP $w^{\langle\alpha,\beta,\gamma\rangle} \in \varsigma_R(U)$ such that $w^{\langle\alpha,\beta,\gamma\rangle} \notin \varsigma_R(V)$. This implies that, $w^{\langle\alpha,\beta,\gamma\rangle}$ is a NFS. Then it is clear that $Cl_{nf}(w^{\langle\alpha,\beta,\gamma\rangle}) \subseteq \varsigma_R(V)'$ $\Rightarrow Cl_{nf}(w^{\langle\alpha,\beta,\gamma\rangle}) \neq Cl_{nf}(\varsigma_R(U))$.

Hence the proof.

Lemma 2. Let $(W, \tau_n, {}^f\varsigma_W)$ be a NFTS over (W, τ_n).

If $\varsigma_R(U)$ is a $\max_{nf}$-OS and $\varsigma_R(W)$ is NFOS, then $\varsigma_R(U) \cup \varsigma_R(W) = 1_{nf}$ or $\varsigma_R(W) \subseteq \varsigma_R(U)$.

If $\varsigma_R(U)$ and $\varsigma_R(V)$ are $\max_{nf}$-OSs, then $\varsigma_R(U) \cup \varsigma_R(V) = 1_{nf}$ or $\varsigma_R(U) = \varsigma_R(V)$.

Proof. (i) Let $\varsigma_R(W)$ be a NFOS such that $\varsigma_R(U) \cup \varsigma_R(W) \neq 1_{nf}$.

Since $\varsigma_R(U)$ is a $\max_{nf}$-OS and $\varsigma_R(U) \subseteq \varsigma_R(U) \cup \varsigma_R(W)$, then $\varsigma_R(U) \cup \varsigma_R(W) = \varsigma_R(U)$. Hence $\varsigma_R(W) \subseteq \varsigma_R(U)$.

If $\varsigma_R(U) \cup \varsigma_R(W) \neq 1_{nf}$, then $\varsigma_R(U) \subseteq \varsigma_R(V)$ and $\varsigma_R(V) \subseteq \varsigma_R(U)$, by (i). Hence $\varsigma_R(U) = \varsigma_R(V)$.

Proposition 10. Let $(W, \tau_n, {}^f\varsigma_W)$ be a NFTS over (W, τ_n). Let $\varsigma_R(U)$ be a $\max_{nf}$-OS. If $w^{\langle\alpha,\beta,\gamma\rangle}$ is a NFP of $\varsigma_R(U)$, then for any neutro-fine neighborhood $\varsigma_R(W)$ of $w^{\langle\alpha,\beta,\gamma\rangle}$, $\varsigma_R(U) \cup \varsigma_R(W) = 1_{nf}$ or $\varsigma_R(W) \subseteq \varsigma_R(U)$.

Proof. Follows from the ***Lemma 2***.

Proposition 11. Let $(W, \tau_n, {}^f\varsigma_W)$ be a NFTS over (W, τ_n). Let $\varsigma_R(U)$ be a $\max_{nf}$-OS. Then $\varsigma_R(U) = \bigcup\{\varsigma_R(W) : \varsigma_R(W) \text{ is a neutro-fine neighborhood of } w^{\langle\alpha,\beta,\gamma\rangle} \text{ such that } \varsigma_R(U) \cup \varsigma_R(W) \neq 1_{nf}\}$.

Proof. Let $(W, \tau_n, {}^f\varsigma_W)$ be a NFTS over (W, τ_n). Let $\varsigma_R(U)$ be a $\max_{nf}$-OS.

Since $\varsigma_R(U)$ is a neutro-fine neighborhood of $w^{\langle\alpha,\beta,\gamma\rangle}$, by ***Proposition 10***, then $\varsigma_R(U)\subseteq\bigcup\{\varsigma_R(W):\varsigma_R(W)$ is a neutro-fine neighborhood of $w^{\langle\alpha,\beta,\gamma\rangle}$ such that $\varsigma_R(U)\bigcup\varsigma_R(W)\neq 1_{nf}\}\subseteq\varsigma_R(U)$. Hence the result.

Theorem 12. Let $(W,\tau_n,{}^f\varsigma_W)$ be a NFTS over (W,τ_n). Let $\varsigma_R(U_1)$, $\varsigma_R(U_2)$ and $\varsigma_R(U_3)$ be $\max_{nf}$-OSs such that $\varsigma_R(U_1)\neq\varsigma_R(U_2)$. If $(\varsigma_R(U_1)\bigcap\varsigma_R(U_2))\subseteq\varsigma_R(U_3)$, then $\varsigma_R(U_1)=\varsigma_R(U_3)$ or $\varsigma_R(U_2)=\varsigma_R(U_3)$.

Proof. Let $\varsigma_R(U_1)$, $\varsigma_R(U_2)$ and $\varsigma_R(U_3)$ be $\max_{nf}$-OSs such that $\varsigma_R(U_1)\neq\varsigma_R(U_2)$. Then

$(\varsigma_R(U_1)\bigcap\varsigma_R(U_3))=\varsigma_R(U_1)\bigcap(\varsigma_R(U_3)\bigcap 1_{nf})$

$=\varsigma_R(U_1)\bigcap(\varsigma_R(U_3)\bigcap(\varsigma_R(U_1)\bigcup\varsigma_R(U_2)))$ (by ***Lemma 2***)

$=\varsigma_R(U_1)\bigcap((\varsigma_R(U_3)\bigcap\varsigma_R(U_1))\bigcup(\varsigma_R(U_3)\bigcap\varsigma_R(U_2)))$

$=(\varsigma_R(U_1)\bigcap\varsigma_R(U_3))\bigcup(\varsigma_R(U_3)\bigcap\varsigma_R(U_1)\bigcap\varsigma_R(U_2))$

$=(\varsigma_R(U_1)\bigcap\varsigma_R(U_3))\bigcup(\varsigma_R(U_1)\bigcap\varsigma_R(U_2))$ (since $(\varsigma_R(U_1)\bigcap\varsigma_R(U_2))\subseteq\varsigma_R(U_3)$)

$=\varsigma_R(U_1)\bigcap(\varsigma_R(U_3)\bigcup\varsigma_R(U_2))$.

If $\varsigma_R(U_3)\neq\varsigma_R(U_2)$, then $(\varsigma_R(U_3)\bigcup\varsigma_R(U_2))=1_{nf}$.

Thus $(\varsigma_R(U_1)\bigcap\varsigma_R(U_3))=\varsigma_R(U_1)$ implies $\varsigma_R(U_1)\subseteq\varsigma_R(U_3)$. Since $\varsigma_R(U_1)$ and $\varsigma_R(U_3)$ are $\max_{nf}$-OSs, then hence $\varsigma_R(U_1)=\varsigma_R(U_3)$.

Theorem 13. Let $(W,\tau_n,{}^f\varsigma_W)$ be a NFTS over (W,τ_n). Let $\varsigma_R(U_1)$, $\varsigma_R(U_2)$ and $\varsigma_R(U_3)$ be , $\max_{nf}$-OSs, which are different from each other. Then $(\varsigma_R(U_1)\bigcap\varsigma_R(U_2))\not\subset(\varsigma_R(U_1)\bigcap\varsigma_R(U_3))$.

Proof. Let $\varsigma_R(U_1)$, $\varsigma_R(U_2)$ and $\varsigma_R(U_3)$ be , $\max_{nf}$-OSs.

Suppose assume that $(\varsigma_R(U_1)\bigcap\varsigma_R(U_2))\subseteq(\varsigma_R(U_1)\bigcap\varsigma_R(U_3))$. Then

$(\varsigma_R(U_1)\bigcap\varsigma_R(U_2))\bigcup(\varsigma_R(U_2)\bigcap\varsigma_R(U_3))\subseteq(\varsigma_R(U_1)\bigcap\varsigma_R(U_3))\bigcup(\varsigma_R(U_2)\bigcap\varsigma_R(U_3))$.

Thus $\varsigma_R(U_2)\bigcap(\varsigma_R(U_1)\bigcup\varsigma_R(U_3))\subseteq(\varsigma_R(U_1)\bigcup\varsigma_R(U_2))\bigcap\varsigma_R(U_3)$.

Since $\varsigma_R(U_1)\bigcup\varsigma_R(U_3)=1_{nf}=\varsigma_R(U_1)\bigcup\varsigma_R(U_2)$, then $\varsigma_R(U_2)\subseteq\varsigma_R(U_3)$.

This implies that $\varsigma_R(U_2)=\varsigma_R(U_3)$, which contradicts our assumption. Hence proved.

5. Conclusion

The main objective of this paper is to define some collection of open sets such as neutro-fine-generalized open and neutro-fine-semi open sets on NFTS and analyzed its basic properties with perfect examples. The notion of interior and closure on semi-open sets are described and specified certain properties. These definitions provide the idea of generalized semi-open sets on NFTS. Also, the neutro-fine-minimal and neutro-fine-maximal open sets are defined and some of their properties are studied in this space. Likewise, discussed the complement of all these sets as its closed sets. The basic properties of the union and intersection of these sets are stated in some theorems. Only a few sets satisfy this postulates, and others are disproved as shown in the counterexamples. The converse of some theorems is proved in probable examples. Consequently, the future researchers can extend this NFTS to some special types of sets, whereas soft sets, rough sets, crisp sets, cubic sets, etc., Also, the application part can widen on MCDM problems.

References

[1] Smarandache, F. (2020). Generalizations and alternatives of classical algebraic Structures to NeutroAlgebraic structures and antialgebraic structures. *Journal of fuzzy extension & applications*, *1*(2), 85-87.

[2] Kumar Das, S. (2020). Application of transportation problem under pentagonal Neutrosophic environment. *Journal of fuzzy extension and applications*, *1*(1), 27-41.

[3] Abdel-Basset, M., Mohamed, M., & Smarandache, F. (2020). Comment on" a novel method for solving the fully neutrosophic linear programming problems: suggested modifications”. *Neutrosophic sets and systems*, *31*(1), 305-309.

[4] Smarandache, F. (2020). Extension of HyperGraph to n-SuperHyperGraph and to Plithogenic n-SuperHyperGraph, and Extension of HyperAlgebra to n-ary (Classical-/Neutro-/Anti-) HyperAlgebra. *Neutrosophic sets and systems*, 33, 290-296.

[5] Chinnadurai, V., Smarandache, F., & Bobin, A. (2020). Multi-aspect decision-making process in equity investment using neutrosophic soft matrices. *Neutrosophic Sets and Systems*, *31*, 224-241.

[6] Chinnadurai, V., & Sindhu, M. P. (2020). A Novel Approach for Pairwise Separation Axioms on Bi-Soft Topology Using Neutrosophic Sets and An Output Validation in Real Life Application. *Neutrosophic Sets and Systems*, *35*, 435-463.

[7] Chinnadurai, V., & Sindhu, M. P. (2020). A novel approach: neutro-spot topology and its supra topology with separation axioms and computing the impact on COVID-19. *Neutrosophic sets and systems* (Submitted).

[8] Chinnadurai, V., & Sindhu, M. P. (2020). An introduction to neutro-fine topology with separation axioms and decision making. *International journal of neutrosophic science*, *12*(1), 13-28.

[9] Riaz, M., Naeem, K., Zareef, i., & afzal, d. (2020). Neutrosophic n-soft sets with topsis method for multiple Attribute Decision Making. *Neutrosophic sets and systems*, *32*(1), 146-170.

[10] Guleria, A., Srivastava, S., & Bajaj, R. K. (2019). On parametric divergence measure of neutrosophic sets with its application in decision-making models. *Neutrosophic sets and systems*, *29*(1), 101-120.

[11] Yasser, I., Twakol, A., Abd El-Khalek, A. A., Samrah, A., & Salama, A. A. (2020). COVID-X: novel health-fog framework based on neutrosophic classifier for confrontation Covid-19. *Neutrosophic Sets and Systems*, *35*(1), 1-21.

[12] Nabeeh, N. A., Abdel-Monem, A., & Abdelmouty, A. (2019). A Hybrid Approach of Neutrosophic with MULTIMOORA in Application of Personnel Selection. *Neutrosophic Sets and Systems*, *30*(1), 1-21.

[13] Nogueira, Y. E. M., Ojeda, Y. E. A., Rivera, D. N., León, A. M., & Nogueira, D. M. (2019). Design and application of a questionnaire for the development of the knowledge management audit using neutrosophic iadov technique. *Neutrosophic sets and systems*, *30*(1), 70-87.

[14] Altinirmak, S., Gul, Y., Okoth, B. O., & Karamasa, C. (2018). Performance evaluation of mutual funds via single valued neutrosophic set (svns) perspective: a case study in turkey. *Neutrosophic sets and systems*, *23*(1), 110-125.

[15] Smarandache, F. (2018). Plithogenic Set, an extension of crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets-revisited. *Infinite study*, *21*(1), 153-166.

[16] Abdel-Basset, M., El-hoseny, M., Gamal, A., & Smarandache, F. (2019). A novel model for evaluation Hospital medical care systems based on plithogenic sets. *Artificial intelligence in medicine*, *100*, 101710.

[17] Abdel-Basset, M., Mohamed, R., Zaied, A. E. N. H., & Smarandache, F. (2019). A hybrid plithogenic decision-making approach with quality function deployment for selecting supply chain sustainability metrics. *Symmetry*, *11*(7), 903.

[18] Abdel-Basset, M., Nabeeh, N. A., El-Ghareeb, H. A., & Aboelfetouh, A. (2019). Utilising neutrosophic theory to solve transition difficulties of IoT-based enterprises. *Enterprise information systems*, 1-21.

[19] Abdel-Baset, M., Chang, V., & Gamal, A. (2019). Evaluation of the green supply chain management practices: A novel neutrosophic approach. *Computers in industry*, *108*, 210-220.

[20] Abdel-Basset, M., Mohamed, M., & Smarandache, F. (2018). An extension of neutrosophic AHP–SWOT analysis for strategic planning and decision-making. *Symmetry*, *10*(4), 116.

[21] Smarandache, F. (1998). Neutrosophy: neutrosophic probability, set, and logic: analytic synthesis & synthetic analysis. https://philpapers.org/rec/SMANNP American Research Press, Rehoboth, USA, 105.

[22] Smarandache, F. (2005). Neutrosophic set-a generalization of the intuitionistic fuzzy set. *International journal of pure and applied mathematics*, *24*(3), 287-297

[23] Smarandache, F. (2017). Plithogeny, plithogenic set, logic, probability, and statistics. *Infinity study*, 141.

[24] Chinnadurai, V., & Sindhu, M. P. (2020). Generalization of level sets in neutrosophic soft sets and points: a new approach. *Tathapi*, *19*(50), 54-81.

[25] Kalaiselvi, S., & Sindhu, M. P. (2016). $\tau_1\tau_2$ -fb-open sets in fine-bitopological spaces. *International journal of multidisciplinary research and modern education*, *2*(2), 435-441.

[26] Nandhini, R., & Amsaveni, D. (2020). Fine fuzzy sp closed sets in fine fuzzy topological space. *International journal of engineering and advanced technology*, *9*(3), 1306-1313.

[27] Pushpalatha, A., & Nandhini, T. (2019). *Generalized closed sets via neutrosophic topological spaces*. Malaya journal of mathermatik, *7*(1), 50-54.

[28] Iswarya, P., & Bageerathi, K. (2019). A study on neutrosophic generalized semi-closed sets in neutrosophic topological spaces. *Journal of emerging technologies and innovative research*, *6*(2), 452-457.

[29] Ozturk, T. Y., Aras, C. G., & Bayramov, S. (2019). *A new approach to operations on neutrosophic soft sets and to neutrosophic soft topological spaces.* Communications in mathematics and applications, *10*(3), 481-493.

[30] Rowthri, M., & Amudhambigai, B. (2017). A view on fuzzy fine topological group structures spaces. *International journal of computational and applied mathematics*, *12*(1), 412-422.

[31] Iswarya, P., & Bageerathi, K. (2016). On neutrosophic semi-open sets in neutrosophic topological spaces. Infinite Study. *International journal of mathematics trends and technology*, *37*(3), 214-223.

[32] Rajak, K. (2015). $f\tau^*g^*$ semi-closed sets in fine-topological spaces. *International journal of mathamatical sciences and applica*tions, *5*(2), 329-331.

[33] Salama, A. A., & Alblowi, S. A. (2012). Neutrosophic set and neutrosophic topological spaces. *IOSR journal of mathematics*, *3*(4), 31-35.

[34] Salama, A. A., & Alblowi, S. A. (2012). Generalized neutrosophic set and generalized neutrosophic topological spaces. Infinite Study. *Journal on computer science and engineering, 2*(7), 12-23.

[35] Powar, P. L., & Rajak, K. (2012). Fine-irresolute mappings. *Journal of advance studies in topology*, *3*(4), 125-139.

[36] Nakaoko, F., & Oda, N. (2003). On minimal closed sets. *Proceeding of topological Spaces Theory and its Applications*, 5, 19-21.

[37] Nakaoka, F., & Oda, N. (2003). Some properties of maximal open sets. *International journal of mathematics and mathematical sciences*, *21*, 1331-1340.

[38] Nakaoka, F., & Oda, N. (2001). Some applications of minimal open sets. *International journal of mathematics and mathematical sciences*, 27(8), 471-476.

[39] Atanassov, K. T. (1986). Intuitionistic fuzzy sets. F*uzzy sets and systems,* 20, 87-96.

[40] Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, 8, 338-353.