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Using Interval Type-2 Fuzzy Logic to Analyze Igbo Emotion Words

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ABSTRACT

Several attempts had been made to analyze emotion words in the fields of linguistics, psychology and sociology; with the advent of computers, the analyses of these words have taken a different dimension. Unfortunately, limited attempts have so far been made to using Interval Type-2 Fuzzy Logic (IT2FL) to analyze these words in native languages. This study used IT2FL to analyze Igbo emotion words. IT2F sets are computed using the interval approach method which is divided into two parts: the data part and the fuzzy set part. The data part preprocessed data and its statistics computed for the interval that survived the preprocessing stages while the fuzzy set part determined the nature of the footprint of uncertainty; the IT2F set mathematical models for each emotion characteristics of each emotion word is also computed. The data used in this work was collected from fifteen subjects who were asked to enter an interval for each of the emotion characteristics: Valence, Activation and Dominance on an interval survey of the thirty Igbo emotion words. With this, the words are being analyzed and can be used for the purposes of translation between vocabularies in consideration to context.

1. Introduction

Words are vital in our description and understanding of emotions and means different things to different people based on different instances and some are uncertain [1]. Hence, there is a need for a model that can capture the uncertainties of these words. Emotions are feelings or involuntary physiological response of a person to a situation, words or things. Emotion is defined using two approaches: the classical approach and the use of multidimensional space. The classical approach uses fixed number of emotion classes such as {positive, non-positive}, {negative, non-negative}, and {angry, non-angry} in describing emotion-related states. However, this approach lacks the ability to handle many types of real-life emotions while the second approach represents emotion using each point in the multidimensional space and the dimensions that are mostly used are valence, activation, and dominance [2, 3]. Valence represents negative

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to positive axis, activation represents calm to excited axis while dominance represents weak to strong axis in the three dimensional space [4]. However, the two approaches have failed to represent the uncertainty a person has about the emotion words as reflected in real-world and are somewhat vague and not precisely defined sets.

Therefore, application of fuzzy set model is well suitable because of its ability to cope with uncertainties [5, 6] and can represent each emotion dimension using intervals rather than fixed points [7].

The Type -1 Fuzzy Set (T1-FS) is a generalization of traditional classical sets in which a concept can possess a certain degree of truth, where the truth value may range between completely true and completely false. However, T1-FS lacks the ability to adequately represent or directly handle data uncertainty because its membership function is crisp in nature [8]. The optimal design of fuzzy systems enables making decisions based on a structure built from the knowledge of experts and guided by membership functions and fuzzy rules [28]. "The membership functions of type-2 fuzzy sets are (n+2)-dimensional, while membership functions of type-1 fuzzy sets are only (n+1)-dimensional (assuming that the universe of discourse has n dimensions). Thus, type-2 fuzzy sets allow more degrees of freedom in representing uncertainty" [32]. Type-2 Fuzzy Set (T2-FS) which is an extension of T1-FS was used in [9] to address the inabilities of T1-FLS. Interval Type-2 Fuzzy Logic (IT2FL) is a simplified version of the general T2FLS that uses intervals to handle uncertainty in the membership function. The structure of T2-FIS is similar to T1-FLS but with additional component called the type reducer modified to accommodate T2F set. Type Reducer is used to reduce the output of the T2 inference engine to type-1 before defuzzification. An IT2FL can better model intrapersonal and interpersonal uncertainties, which are intrinsic to natural language, because the membership grade of an IT2 Fuzzy set is an interval instead of a crisp number as in a T1 FS [31].

Though, many works have been done in the field of emotion words, [10], [8], [11], [12], [13], [14], [15], [16], [17], [18], however, attempts to analyze emotion words in Igbo non-English languages have not been impacted in any way. This study uses IT2FL to analyze Igbo Emotion Words. The term "Igbo" in this context is used to describe the language spoken primarily by Igbo ethnic group in Nigeria. The Igbo belong to the Sudanic linguistic group of the Kwa division according to [19, 20]. This wide presence of the Igbos is the basis for selecting the emotions words in Igbo language for analysis using IT2FL.

This study used IT2FL to analyze Igbo emotion words because of its ability to adequately handle emotion word uncertainties described by its Footprint Of Uncertainty (FOU), which is the uncertainty about the union of all the primary MFs. Uncertainty is a characteristic of information, which may be incomplete, inaccurate, undefined, inconsistent. Primary Membership Functions with where both the standard deviation and the uncertain are popular FOUs for a Gaussian because of their parsimony and differentiability [29]. IT2F sets are computed using the interval approach method which is divided into two parts: the data part and the fuzzy set part. The data part preprocessed data and its statistics computed for the interval that survived the preprocessing stages while the fuzzy set part determined the nature of the footprint of uncertainty; the IT2F set mathematical models for each emotion characteristics of each emotion word is also computed. The data used in this work was collected from 15 (fifteen) subjects who were asked to enter an interval for each of the emotion characteristics: Valence, Activation and Dominance on an interval survey of the 30 (thirty) Igbo emotion words.

IT2F sets are computed using the interval approach method which is divided into two parts: the data part and the fuzzy set part. The data part preprocessed data and its statistics computed for the interval that survived the preprocessing stages while the fuzzy set part determined the nature of the footprint of uncertainty; the IT2F set mathematical models for each emotion characteristics of each emotion word is also computed. The data used in this work was collected from 15 (fifteen) subjects who were asked to enter an interval for each of the emotion characteristics: Valence, Activation and Dominance on an interval survey of the 30 (thirty) Igbo emotion words. This paper is organized as follows: In the following section, the Emotion Space, Emotion Vocabularies and Variables, the Igbos and Emotions, T2FLS and Sets, and the IT2FLS are described. The research methodology is presented, the results and discussion are described and the conclusions are drawn.

1.1. Emotion Space, Emotion Vocabularies and Variables

The emotion space E can be considered as a set of all possible emotions and it is represented using variables on the Cartesian product space of valence, activation and dominance scales. An emotional variable $\mathcal{E}$ represents an arbitrary region in the emotion space i.e. $\varepsilon \subset E$. An emotional vocabulary in ***Eq. (1)***,

$$V = (W_V, eval_V) \qquad (1)$$

is a set of words W_V and a function $eval_V$ that maps words of W_V to their corresponding region in the emotional space, $eval: W_V \to E$. Thus, an emotional vocabulary can be seen as a dictionary for looking up the meaning of an emotion word. Words in an emotional vocabulary can be seen as constant emotional variables.

1.2. Igbos and Emotions

Emotions affect every human including the Igbo's and the Igbo emotions have become even more complex over time hence when the Igbos speak of their emotions, the analysis, interpretation and translation to other languages is highly needed. For instance, an Igbo businessman may say "iwe ne enwe m" which means "I am angry". The emotion word in his statement is "iwe" translated as "anger" in English could either be with regards to a business situation or a person. Assuming it is a person that is an individual who annoyed him, it would not change his disposition about his business hence at almost the same time the same businessman could be heard saying "oba go" meaning "it has entered" which insinuates happiness of some sought.

1.3. Type-2 Fuzzy Logic System and Sets

T2FLS is the generalized standard type-1 in order to accommodate and handle more uncertainties in the MF [21]. According to [22] "words mean different things to different people". T2FLS is based on the T2F sets where the MF has multiple values for a crisp input of x, making the need for the creation of a 3-dimensional MF for all x Ɛ X. A characteristic feature of T2FS is the FOU, which is the union of all primary memberships and upper MFs and a lower MFs that are the bounds for the FOU of a T2Fs.

Uncertainty in relations and uncertainty in values of the variables are majorly the types of uncertainty considered in developing systems since the high overlapping of the MFS, defining precise values for linguistic variables is not possible [30].

Given a T2Fs $\tilde{A}$, the representation of $\tilde{A}$ is as shown ***Eq. (2)***.

$$\tilde{A} = \{((x, u), \mu_{\tilde{A}}(x, u)) | x \in X, u \in J_x \in [0,1]\}. \quad (2)$$

Where $\mu_{\tilde{A}}(x, u)$ is the type-2 fuzzy MF in which $0 \le \mu_{\tilde{A}}(x, u) \le 1$

1.4. Interval Type-2 Fuzzy Logic System (IT2FLS)

As a result of the computational complexity of using a general T2FS, IT2Fs is mostly used as a special case an express as, when all $\mu_{\tilde{A}}(x, u) = 1$, then $\tilde{A}$ can rightly be described as an IT2FL as seen in ***Eq. (3)*** and in ***Fig. 1***. The interval type-2 membership function is always equal to 1.

$$\tilde{A} = \int_{x\, \varepsilon\, X} \int_{u\, \varepsilon\, Jx} \frac{1}{(x,u)}, \int_x \varepsilon\ [0, 1]. \quad (3)$$

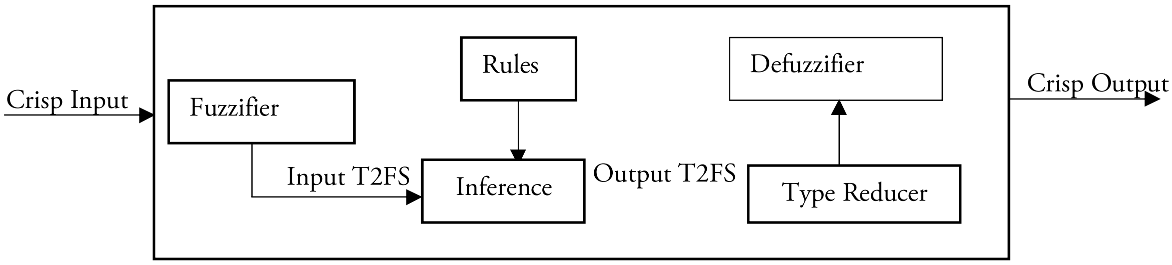

Fig. 1. Typical structure of interval type-2 fuzzy logic system [23].

IT2FLS consists of the fuzzifier, rule base, Inference Engine (IE), Type Reducer (TR). Fuzzification module maps the crisp input to a T2Fs using Gaussian MF. Inference Engine module evaluates the rules in the knowledge base against T2Fs from Fuzzification, to produce another T2Fs. TR uses Karnik-Mendel algorithm to reduce an IT2Fs to T1Fs. Defuzzification module maps the fuzzy set produced by TR to a crisp output using center of gravity defuzzification method. Fuzzy knowledge base stores rules generated from experts' knowledge used by the IE.

2. Research Methodology

The paper employs interval approach where data and fuzzy set parts are considered.

2.1. Interval Approach (IA)

The IA is a method for estimating MFs where the subject does not need to be knowledgeable about fuzzy sets and it has a simple and unambiguous mapping from data to FoU. Feilong and Mendel [24] used the

IA to capture the strong points of two previous methods: the Person-MF Approach and the End-point Approach. Using the IA, the data collected from different subjects is subjected to probability distribution. The mean and standard deviation of the distribution are then mapped into the parameters of a T1 MF which are then transformed into T2 MF from which the IT2 MF is derived. The IA is divided into two parts namely the Data Part and the Fuzzy Set Part (FSP) as seen in ***Figs. (2) & (3)***, respectively.

2.1.1. The Data Part (DP)

The DP of the IA consists of data collection, data preprocessing and probability distribution assignment parts as shown in ***Fig. 2***. The DP highlights the valence layer and this is repeated for each word in the vocabulary. In ***Fig. 2***, the following steps are carried out to achieve an input to the FSP.

- Data Collection. Here, interval survey is performed to collect human intuition about fuzzy predicate.
- Data Preprocessing. Pre-processing for n interval data $[a^{(i)}, b^{(i)}]$, i=1,…,n, are performed consisting of 3 stages:
- Bad Data Processing: At this stage, results from the survey that do not fall within the given range are removed. If interval end-points given by the respondents satisfy,

$$\left.\begin{array}{l} 0 \leq \mathrm{a}^{\mathrm{i}} \leq 10 \\ 0 \leq \mathrm{b}^{\mathrm{i}} \leq 10 \\ \mathrm{b}_{\mathrm{i}} \geq \mathrm{a}_{\mathrm{i}} \end{array}\right\} \forall\, \mathrm{i} = 1, \cdots, \mathrm{n}, \tag{4}$$

then accept the interval; else reject it. After this stage what remains is $n' \leq n$ intervals

- Outlier Processing. At this stage, a Box and Whisker test is used to eliminate outliers. Outliers are points which do not satisfy

$$\begin{array}{l} a^{(i)} \in [Q_a(0.25) - 1.5IQR_a, Q_a(0.75) + 1.5IQR_a] \\ b^{(i)} \in [Q_b(0.25) - 1.5IQR_b, Q_b(0.75) + 1.5IQR_b] \end{array}. \tag{5}$$

$Q_a(p)$ and $IQR_a = Q_a(0.75) - Q_a(0.25)$ are the p quartile and inter-quartile range for the left end-points and $Q_b(p)$ and $IQR_b = Q_b(0.75) - Q_b(0.25)$ are the p quartile and inter-quartile range for the right end-points, respectively.

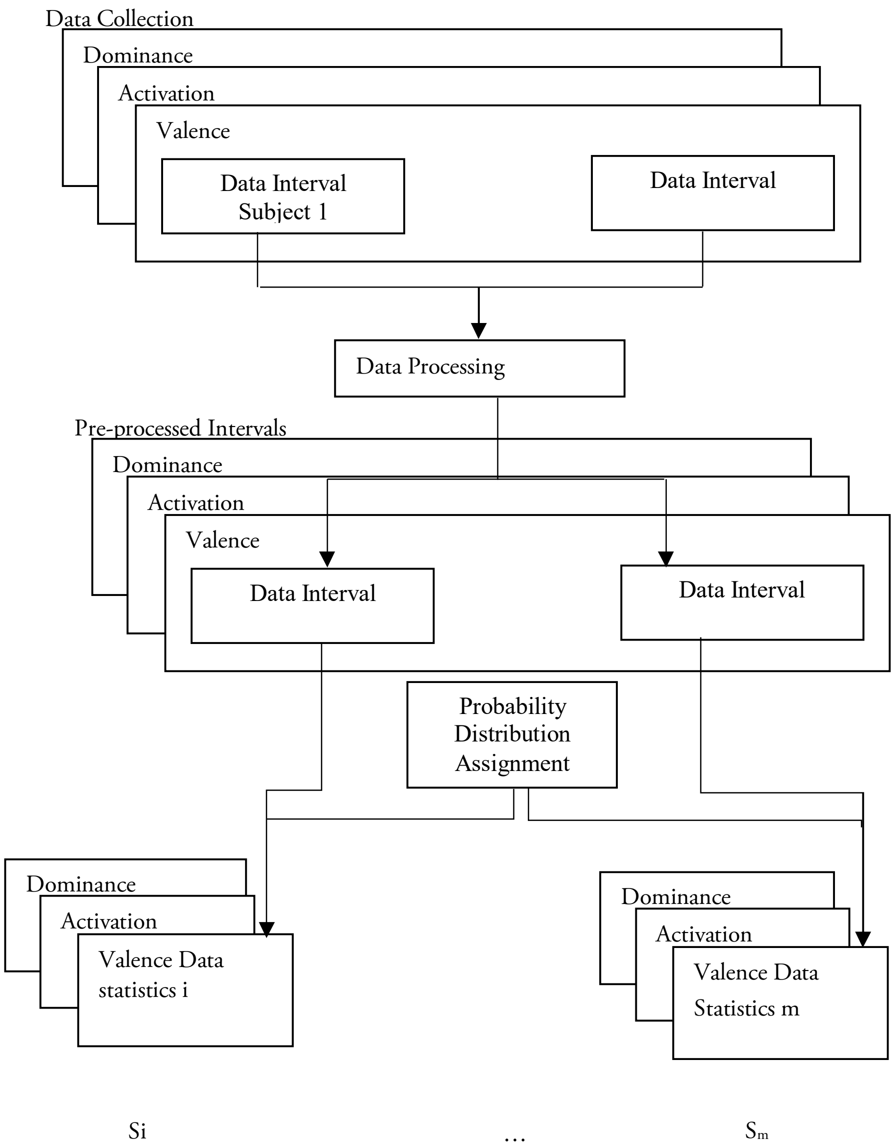

Fig. 2. The data part of the interval approach [25].

- Tolerance-Limit Processing. Here, tolerance-limit test is processed using ***Eqs. (6) & (7)*** and if the interval passes, it is accepted else, it is rejected. Then the data intervals are reduced to $m'' \leq m'$.

$$a^{(l)} \in [m_l - k\sigma_l, m_l + k\sigma_l,]. \tag{6}$$

$$b^{(r)} \in [m_r - k\sigma_r, m_r + k\sigma_r,]. \tag{7}$$

k is determined by confidence analysis, such that with a $100(1-\gamma)\%$ confidence the given limits contain at least the proportion$1-\alpha$ measurements.

After the data preprocessing, we are left with m data intervals such that $1 \leq m \leq n$.

- Assign Probability Distribution to the Interval Data. Here, a probability distribution is assigned to each subject's data interval. The distribution can be uniform, triangular, normal, etc. but for the purpose of this work, the uniform distribution is used.
- Compute Data Statistics for the Interval Data. The data statistics $m_l, \sigma_l, m_r and \sigma_r$, which are the sample means and standard deviation of the left- and right-end points respectively, are computed based on interval data that remains. The last two steps are often merged since they work hand in hand.

The probability distribution $S_i = (m_i, \sigma_i)$ is assigned to the remaining intervals, then the statistics are calculated using the formula for random variables with random distribution stated as

$$m = (a + b)/2 \text{ and} \tag{8}$$

$$\sigma = (b - a)/\sqrt{12}. \tag{9}$$

Then the probability distribution, S_i, is evaluated in ***Eq. (10)*** and forms the input to the fuzzy set part

$$S_i = \left(m_Y^i, \sigma_Y^i\right) \forall\, i = 1, \dots, m. \tag{10}$$

The S_i is then input to the next stage, the fuzzy set part.

2.1.2. The Fuzzy Set Part (FSP)

The fuzzy set part constructs the interval type-2 fuzzy sets as shown in ***Fig. 3.*** It takes the result of Data part, i.e. the S_i as input from which it creates the IT2Fs. The Layers denote individual fuzzy sets for valence, activation and dominance. This framework is repeated for each word in the vocabulary.

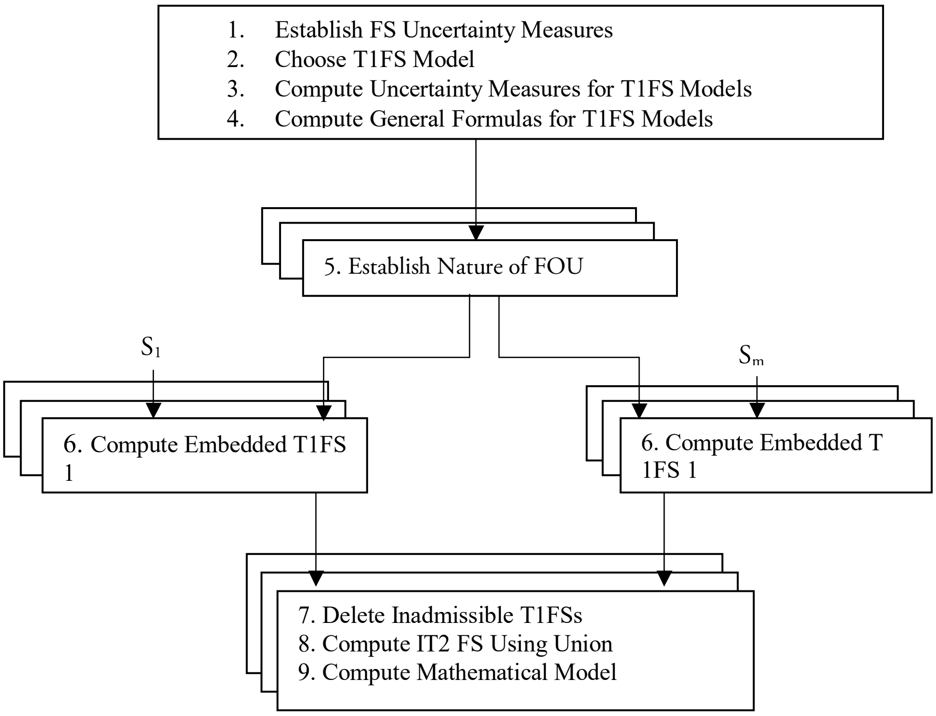

Fig. 3. Fuzzy set part of the interval approach [25].

In ***Fig. 3***, the fuzzy set part of IT2FL process is divided into nine (9) steps. Step 1, establishes FS uncertainty measures while Step 2 selects the T1FS model. In Step 3, the uncertainty measures for the selected models are computed. The uncertainty measures on the symmetrical interior triangle compute the mean and Standard Deviation (SD) derived from the mean and SD of the triangular and uniform distributions. In step 4, the S_i from the data part is recollected. The mean and standard SD from the interval model is equated with the corresponding parameters from the previous step. In Step 5, the nature of the FoU is established using the parameters of the models associated with each interval to classify whether an interval should be mapped to an interior MF, or a left or right shoulder MF. The input interval is mapped to a shoulder MF if the parameters show that the distribution is out of the range of the scales. In Step 6, the T1Fs is computed with the intervals and the decision is made in the previous step. Since the MFs derived in this step are based on statistics and not the raw intervals themselves, another preprocessing stage is carried out to delete inadmissible T1FS is with range outside of the limit of the scale of the variable of interest. In Step 7, the fuzzy sets derived are the subject-specific T1FSs. The aggregate is taken to compute the IT2FSs that contain all the subject-specific T1FS in its FOU. This aggregation can be said to be a type-2 union of T1FSs, where the embedded T1FSs describe the FOU of IT2FSs. This includes Steps 8 and 9. After these steps, the IT2FSs word model is derived. The study analyzes the 30 emotion words collected from a wide range of psychological domains based on their MF values of the three characteristics: Valence, Activation, and Dominance.

3. System Design

The architecture for analyzing emotion words from Igbo using IT2FL presented in ***Fig. 4***. The architecture is made up of the Knowledge Engine, the Knowledge Base and the User Interface. The Knowledge base further comprises the Database Model, the Interval Surveys and the IT2FL Model. The IT2FL model is composed of the Interval Approach Data Part and the Interval Approach Fuzzy Set Part. The knowledge engine stores all the variables required for the system, the knowledge base stores values for the variables defined in the knowledge engine. The Database stores an organized data to be used in the system as collected from the subjects. Interval survey is where the data are collected in a range of intervals which defines a subject's view of each emotion word given with regards to the emotion characteristics defined in the knowledge engine. The IT2FL model is used to represent words using IT2FSs. The IT2FL comprises Interval Approach which is made up of data and fuzzy set parts. The data part processes the data and makes it ready for use in estimating the MF of each emotion characteristics in each emotion word. The fuzzy set part evaluates the MF representing each emotion characteristics of each word.

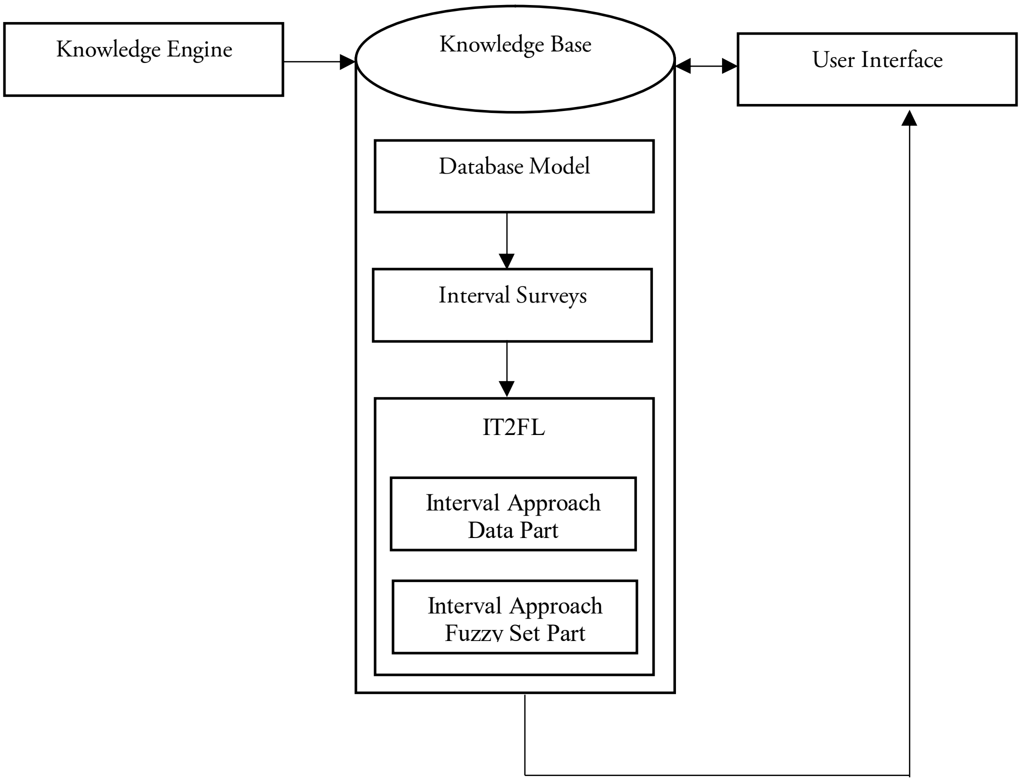

Fig. 4. Architecture for analyzing Igbo emotion words using interval IT2FL.

3.1. The Knowledge Engine for Analyzing Igbo Emotion Words

For the purpose of this paper, the variables used include the emotional characteristics (Valence, Activation and Dominance). Valence defines an emotion word on the basis of its positivity or negativity. Activation defines an emotion word on the basis of its calmness or excitement. Dominance defines an emotion word on the basis of its submissiveness or aggressiveness. The Igbo emotion vocabulary used in this work is contained in the knowledge engine. The emotion vocabulary used in this research work is the Igbo Emotions vocabulary of 30 words which include: Iwe, Obi Uto, Onuma, Ujo, Ntukwasi Obi, Obi Ojo, Onu, Anuri, Egwu, Ihunanya, Mgbagwoju Anya, Mwute, Ikpe Mara, Enyo, Ihere, Iwe Oku, Ekworo, Anyaufu, Anya Ukwu, Obi Ike, Nrugide, Akwa Uta, Kpebisiri Ike, Kenchekwube, Keechiche, Mkpako, Nwayoo, Obi Abuo, Obi Mgbawa, Ara.

Table 1. The Igbo emotion words and the english equivalence.

Igbo	English
Iwe	Anger, Annoyance
Obi Uto	Happiness, Delighted
Onuma	Wrath, Great Anger
Ujo	Fear, Shock
Ntukwasi Obi	Trust
Obi Ojo	Wickedness, Bitter
Onu	Joy
Anuri	Gladness
Egwu	Great Fear, Dread
Ihunaya	Love
Mgbagwoju Anya	Confused
Mwute	Regret
Ikpe Mara	Guilty
Enyo	Suspicious
Ihere	Shame
Iwe Oku	Hot-Temper
Ekworo	Jealousy
Anyaufu	Envy
Anya Ukwu	Greed, Discontentment
Obi Ike	Confidence, Courage
Nrugide	Persuasion
AkwaUta	Regret, Condemned
Kpebisiri Ike	Determination, Strong-willed
Kenchekwube	Hopeful
Keechiche	Worry
Mpako	Arrogance, Pride
Nwayoo	Gentle, Calm
Obi Abuo	Doubt, Unsteady
Obi Mgbawa	Heartbroken
Ara	Mad, Disturbed

3.2. The Database Model

The IT2F inference system uses the data as organized in the database. The database schema is created to hold the collected data, data statistics and the membership function values, respectively.

3.3. The Interval Survey

An interval survey is carried out using questionnaires which are given to 15 native speakers of Igbo. The questionnaire began by giving the users instructions then sequentially providing the words to the users. The reason for the interval survey is to collect human intuition about fuzzy predicate which is emotion in this case and provision is given for the subject to give an interval within the range 0-10 that defines the emotion characteristics.

3.4. The IT2FL Model to Analyze Igbo Emotion Words

The IT2FL model for analyzing Igbo emotion words using Interval Approach. The interval approach is used because it takes on the strength of the interval endpoints and the person membership function approaches. Based on ***Figs.*** ***(2)*** & (***3)***, the data and fuzzy set parts of Interval Approach for analyzing Igbo emotion words using IT2FL are evaluated.

3.4.1. Interval approach data part for analyzing Igbo emotion words using IT2FL

Using ***Fig. 2***, in the data collection part, data are collected using the interval surveys from 10 subjects who are native speakers of the Igbo Language. The 30 emotion words are randomly ordered and presented to the respondents. Each was asked to provide the end-points on an interval for each word on the scale of 0-10. Data processing is performed in estimating the MF of each emotion characteristic in each emotion word and then calculates the statistics of the data intervals. Validation of the data intervals is done starting at 'n' intervals at the data preprocessing part. Bad data processing is performed on the collected dataset and the intervals that do not satisfy the condition in ***Eq. (4)*** is removed. After bad processing, outlier processing is carried out on the remaining dataset and the intervals that do not satisfy the box and whisker test in (5) are eliminated. Tolerance –limit processing is performed on the remaining intervals and the data intervals are accepted if they satisfy ***Eqs. (6)*** & ***(7)*** otherwise, they are rejected.

The constant 'k' is used as estimated by Mendel and Liu [26, 27] as shown in ***Table 2*** for 10 intervals at 0.95 confidence limit since we have an average of ten intervals per characteristic.

Table 2. Tolerance factor k for a number of collected data (m'), a proportion of the data (1 - α), and a confidence level 1- γ.

m'	1-γ = 0.95		γ = 0.99	
	1 – α		1 – α	
	0.9	0.95	0.9	0.95
10	2.839	3.379	3.582	4.265
15	2.48	2.954	2.945	3.507
20	2.31	2.752	2.659	3.168
30	2.14	2.549	2.358	2.841
50	1.996	2.379	2.162	2.576
100	1.874	2.233	1.977	2.355
1000	1.709	2.036	1.736	2.718
∞	1.645	1.96	1.645	1.96

Reasonable data intervals are evaluated for the overlapped data intervals and only reasonable data intervals are kept. At this point, the intervals remain the same, i.e. $m = m''$. After the data has been validated, the mean and SD of the data intervals are computed in ***Eqs. (8)*** & ***(9)*** on the assumption that the data intervals are uniformly distributed and the probability distribution, S_i, computed in ***Eq. (10)*** and then becomes an input to the fuzzy set part.

3.4.2. Interval approach fuzzy set part for analyzing Igbo emotion words using

The Interval Approach Fuzzy Set Part for Analyzing Igbo emotion words using IT2FL is used to evaluate the MF based on ***Fig. 3***. It consists of nine steps. Step 1 selects a T1FS and computes the mean and SD of the data intervals using symmetrical triangle interior T1FS, left-shoulder T1FS and the right-shoulder T1FS only. In Step 2, the mean and standard deviation are calculated to establish FS uncertainty measures using ***Eqs. (11)*** & ***(12)***.

$$m_A = \frac{\int_{aMF}^{bMF} x\mu_A(x)dx}{\int_{aMF}^{bMF} \mu_A(x)dx}. \tag{11}$$

$$\sigma_A = \left[\frac{\int_{aMF}^{bMF} (x - m_A)^2\mu_A(x)dx}{\int_{aMF}^{bMF} \mu_A(x)dx}\right]^{1/2}. \tag{12}$$

Obviously, $\mu_A(x)/\int_{aMF}^{bMF}\mu_A(x)dx$ is the probability distribution of x, where$x \in \left[a_{MF}, b_{MF}\right]$, then m_A and σ_A are the same as the mean and standard deviation used in probability. In Step 3, the Uncertainty Measures are computed for T1FS by calculating the mean and SD for symmetric Triangle Interior (TI) and the Left-Shoulder (LS) and Right-Shoulder (RS) T1MFs using ***Eqs. (13) – (15)***, respectively.

$$TI\text{: } m_{MF} = (a_{MF} + b_{MF})/2; \sigma_{MF} = (b_{MF} - a_{MF})/2\sqrt{6}. \tag{13}$$

$$LS\text{: } m_{MF} = (2a_{MF} + b_{MF})/3; \sigma_{MF} = \left[\tfrac{1}{6}[(a_{MF} + b_{MF})^2 + 2a_{MF}^2] - m_{MF}^2\right]^{1/2}. \tag{14}$$

$$RS: m_{MF} = (2a_{MF} + b_{MF})/3; \sigma_{MF} = \left[\frac{1}{6}\left[(a'_{MF} + b'_{MF})^2 + 2a'^2_{MF}\right] - m'^2_{MF}\right]^{1/2}. \quad (15)$$

Where, $a'_{MF} = M - b_{MF}$; $b'_{MF} = M - a_{MF}$; $m'_{MF} = M - m_{MF}$. In Step 4, we compute for the parameters of T1FS models by equating the mean and SD of a T1FS to the mean and SD of the data intervals i.e. $m^i_{MF} = m^i_Y$ and $\sigma^i_{MF} = \sigma^i_Y$ to have ***Eqs. (16) – (18)***.

$$\text{IT: } a_{MF} = (a+b)/2 - \sqrt{2}(b-a)/2; b_{MF} = (a+b)/2 + \sqrt{2}(b-a)/2. \quad (16)$$

$$\text{LS: } a_{MF} = (a+b)/2 - (b-a)/\sqrt{6}; b_{MF} = (a+b)/2 + \sqrt{6}(b-a)/3. \quad (17)$$

$$\text{RS: } a_{MF} = M - (a'+b')/2 - (b'-a')/\sqrt{6}; \ b_{MF} = M - (a'+b')/2 + \sqrt{6}(b'-a')/3. \quad (18)$$

Where, $a' = M - b$ and $b' = M - a$. In Step 5, the nature of the FOU is established by mapping the 'm' data intervals into an IT, LS or a RS FOUs using a scale of [0, 10] if and only if (i), (ii) or (iii).

$$\underset{\text{(i)}}{\left.\begin{matrix} a^i_{MF} \geq 0 \\ b^i_{MF} \leq 10 \\ b^i_{MF} \geq a^i_{MF} \end{matrix}\right\}} \quad \underset{\text{(ii)}}{\left.\begin{matrix} a^i_{MF} \geq M \\ b^i_{MF} \leq 10 \\ b^i_{MF} \geq a^i_{MF} \end{matrix}\right\}} \quad \underset{\text{(iii)}}{\left.\begin{matrix} a^i_{MF} \geq 0 \\ b^i_{MF} \leq M \\ b^i_{MF} \geq a^i_{MF} \end{matrix}\right\}} \ \forall \ i = 1, \dots, m \quad (19)$$

From the mapping, we achieve, 12 TI FOUs, 35 LS FOUs and 23 RS FOUs respectively. In Step 6, embedded T1FSs is computed where the actual mapping to a T1FS is applied to the remaining 'm' data intervals for each word using ***Eq. (20)***.

$$A^i = (a^i, b^i) \rightarrow (a^i_{MF}, b^i_{MF}), i = 1, \dots, m. \quad (20)$$

In Step 7, we delete inadmissible T1FSs for all data intervals for any T1FS that do not satisfy (19(i)) – (19(iii)) and m reduces to m*. Step 8 computes an IT2FS using the representation theorem for an IT2FS $\tilde{A}$ in ***Eq. (21)***,

$$\tilde{A} = \bigcup_{i=1}^{m*} A^i. \quad (21)$$

Where, A^i is just the computed ith embedded T1FS. In Step 9, the mathematical model for FOU ($\tilde{A}$) is computed by first approximating the parameters of UMF ($\tilde{A}$) and the LMF($\tilde{A}$) for each of the FOU models as shown in ***Eqs. (22) – (27)***, respectively.

$$\left.\begin{matrix} \underline{a}_{MF} \equiv \min\limits_{i=1,\dots m*} \{a^i_{MF}\} \\ \bar{a}_{MF} \equiv \max\limits_{i=1,\dots m*} \{a^i_{MF}\} \end{matrix}\right\}. \quad (22)$$

$$\left.\begin{matrix} \underline{b}_{MF} \equiv \min\limits_{i=1,\dots m*} \{b^i_{MF}\} \\ \bar{b}_{MF} \equiv \max\limits_{i=1,\dots m*} \{b^i_{MF}\} \end{matrix}\right\}. \quad (23)$$

$$C^i_{MF} = \frac{a^i_{MF} + b^i_{MF}}{2}. \quad (24)$$

$$\left.\begin{matrix} \underline{C}_{MF} \equiv \min\limits_{i=1,\dots m*} \{C^i_{MF}\} \\ \bar{C}_{MF} \equiv \max\limits_{i=1,\dots m*} \{C^i_{MF}\} \end{matrix}\right\}. \quad (25)$$

$$p = \frac{\underline{b}_{MF}(\overline{C}_{MF}-\overline{a}_{MF})+\overline{a}_{MF}(\underline{b}_{MF}-\underline{C}_{MF})}{(\overline{C}_{MF}-\overline{a}_{MF})+(\underline{b}_{MF}-\underline{C}_{MF})}. \quad (26)$$

$$\mu_p = \frac{\underline{b}_{MF} - p}{(\underline{b}_{MF} - \underline{C}_{MF})}. \quad (27)$$

The mathematical model of the UMF and the LMF are shown in ***Table 3*** for TI, L-S and R-S FOU.

Table 3. The IT2FS UMF and the LMF of IT, L-S and R-S FOUs.

	Upper MF	Lower MF
Triangle (Interior) FOU	$(\underline{a}_{MF}, 0)$, $(\underline{C}_{MF}, 1)$, $(\overline{C}_{MF}, 1)$, $(\overline{b}_{MF}, 0)$	$(\underline{a}_{MF}, 0)$, $(\overline{a}_{MF}, 0)$, (p, μ_p), $(\underline{b}_{MF}, 0)$, $(\overline{b}_{MF}, 0)$
Left-Shoulder FOU	$(0, 1)$, $(\overline{a}_{MF}, 1)$, $(\overline{b}_{MF}, 0)$	$(0, 1)$, $(\underline{a}_{MF}, 1)$, $(\underline{b}_{MF}, 0)$, $(\overline{b}_{MF}, 0)$
Right-Shoulder FOU	$(\underline{a}_{MF}, 0)$, $(\underline{b}_{MF}, 1)$, $(M, 1)$	$(\underline{a}_{MF}, 0)$, $(\overline{a}_{MF}, 0)$, $(\overline{b}_{MF}, 1)$, $(M, 1)$

4. Model Experiment

This paper uses IT2FL to analyze Igbo emotion words. The IT2F sets are computed using the interval approach method which comprises data part and fuzzy set part. In order to illustrate the methodology proposed in this paper, we conduct some experiments for Igbo emotion words described in this work.

4.1. The Data Part

Table 4. shows the data intervals collected from the subjects. Bad data are processed and sample presented in ***Table 5***. Outlier processing is performed in ***Table 5*** and the sample result is presented in ***Table 6***. The tolerance limit processing in ***Table 6*** is shown in ***Table 7***. Sample result of reasonable interval processing is seen in ***Table 8***. The data statistics is computed for all the surviving *m* data intervals and sample are shown in ***Table 9***.

Table 4. Parts of the data intervals collected from the subjects.

		S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15
	Val	8-10	0-8	3-6	7-9	5-8	5-7	0-5	7-10	6-8	6-9	5-8	8-10	7-10	4-5	4-8
	Act	3-5	1-6	4-7	5-8	7-9	6-7	7-10	0-4	6-10	7-10	7-10	8-10	0-5	9-10	5-9
Iwe	Dom	7-10	2-10	8-10	|||	7-10	8-10	2-9	6-8	8-10	8-10	8-10	8-10	5-8	8-10	6-9
	Val	9-10	5-6	4-7	7-10	5-8	6-8	5-10	1-4	0-5	0-4	3-5	1-3	5-8	4-5	3-8
	Act	9-10	7-9	5-8	5-8	7-10	8-10	1-10	9-10	8-10	5-9	8-10	8-10	6-9	7-10	5-9
Obi Uto	Dom	0-3	2-6	6-9	6-9	8-10	4-6	8-10	3-5	0-5	4-8	5-8	1-3	8-10	3-5	4-10
	Val	7-9	3-9	8-10	9-10	5-8	4-7	2-7	7-10	9-10	8-10	8-10	8-10	7-10	5-8	4-8
	Act	0-3	2-6	5-8	4-9	9-10	4-6	7-9	5-8	9-10	6-10	3-5	8-10	0-6	7-10	3-7
Onuma	Dom	8-10	3-9	7-10	8-9	5-8	6-9	2-6	10	9-10	7-10	4-6	8-10	5-8	7-10	5-9
	Val	0-3	4-10	2-4	1-3	5-8	6-9	8-10	1-3	8-10	5-8	7-9	8-10	8-10	4-6	2-10
	Act	0-4	2-6	3-5	5-8	7-10	3-6	4-5	0-2	0-5	2-7	0-5	1-3	5-7	7-8	3-9
Ujo	Dom	4-5	3-9	6-8	5-9	7-10	0-4	8-9	0-2	0-5	1-4	3-6	1-3	0-5	8-10	5-10
	Val	7-10	5-7	1-3	6-8	6-10	0-4	3-10	8-10	0-5	1-5	0-3	1-3	7-10	4-7	1-9
	Act	2-3	4-8	4-6	6-10	6-8	4-7	1-9	7-9	0-5	3-6	3-5	1-3	7-10	7-8	2-8
Ntukwasi Obi	Dom	1-3	5-6	8-10	4-8	6-9	4-7	4-9	6-7	0-5	5-8	3-5	1-3	8-10	8-10	5-7
	Val	5-8	0-10	2-5	6-10	5-10	6-10	2-4	5-8	8-10	8-10	8-10	8-10	6-10	4-5	2-8
Obi Ojo	Act	0-2	2-8	1-4	5-10	6-10	5-7	4-8	7-9	0-2	6-10	3-5	5-6	5-9	7-8	3-6

Table 5. Sample result of bad data processing.

		Valence	Activation	Dominance
1	IWE	15	15	14
2	OBI UTO	15	15	15
3	ONUMA	15	15	14
4	UJO	15	15	15
5	NTUKWASI OBI	15	15	15

Table 6. Sample result of outlier processing.

		Valence	Activation	Dominance
1	IWE	9	9	2
2	OBI UTO	12	5	11
3	ONUMA	5	10	5
4	UJO	15	13	12
5	NTUKWASI OBI	15	11	12

Table 7. Sample result of tolerance-limit processing.

		Valence	Activation	Dominance
1	IWE	9	9	2
2	OBI UTO	12	5	11
3	ONUMA	5	10	5
4	UJO	15	13	12
5	NTUKWASI OBI	15	11	12

Table 8. Sample result of reasonable interval processing.

		Valence	Activation	Dominance
1	IWE	9	9	2
2	OBI UTO	12	5	11
3	ONUMA	5	10	5
4	UJO	15	13	12
5	NTUKWASI OBI	15	11	12

4.2. The Fuzzy Set Part

In the fuzzy set part, only the symmetrical triangle interior T1FS, left-shoulder T1FS and the right-shoulder T1FS are used. The mean and SD are the same as in the data part as shown ***Table 9.*** The uncertainty measure for the chosen T1FS models are computed as is shown in parts in ***Table 10.*** The nature of FOU for each emotion characteristic for each word is determined as shown in ***Table 11.*** The embedded T1FSs, a_{MF} and b_{MF} of each interval are calculated as shown in ***Table 12*** where M=5. The LMF and the UMF of the FOU ($\tilde{A}$) for the five experimental intervals are calculated and summarized in ***Table 13.***

Table 10. Sample result of the uncertainty measures for T1FS models.

	m_{MF}	σ_{MF}
Interior	5	2.041241
Left	3.333333	3.535887
Right	3.333333	2.022657

Table 9. The sample mean and SD computed for all the surviving m data intervals.

			Mean						Standard Deviation					
			S1	S2	S3	S4	…	S15	S1	S2	S3	S4	…	S15
1	Iwe	Val	0	4	4.5	0	…	6	0	2.309401	0.866025	0	…	1.154701
		Act	4	3.5	5.5	6.5	…	7	0.57735	1.443376	0.866025	0.866025	…	1.154701
		Dom	0	0	0	0	…	0	0	0	0	0	…	0
							…						…	
							…						…	
2	Obi Uto	Val	0	5.5	5.5	0	…	5.5	0	0.288675	0.866025	0	…	1.443376
		Act	0	0	6.5	6.5	…	7	0	0	0.866025	0.866025	…	1.154701
		Dom	1.5	4	7.5	7.5	…	0	0.866025	1.154701	0.866025	0.866025	…	0
							…						…	
							…						…	
3	Onuma	Val	0	0	0	0	…	6	0	0	0	0	…	1.154701
		Act	1.5	4	6.5	6.5	…	5	0.866025	1.154701	0.866025	1.443376	…	1.154701
		Dom	0	0	0	0	…	7	0	0	0	0	…	1.154701
							…						…	
							…						…	
4	Ujo	Val	1.5	7	3	2	…	6	0.866025	1.732051	0.57735	0.57735	…	2.309401
		Act	2	4	4	6.5	…	0	1.154701	1.154701	0.57735	0.866025	…	0
		Dom	4.5	6	7	7	…	0	0.288675	1.732051	0.57735	1.154701	…	0
							…						…	
							…						…	
5	Ntukwasi Obi	Val	8.5	6	2	7	…	5	0.866025	0.57735	0.57735	0.57735	…	2.309401
		Act	2.5	6	5	0	…	5	0.288675	1.154701	0.57735	0	…	1.732051
		Dom	2	5.5	0	6	…	6	0.57735	0.288675	0	1.154701	…	0.57735

Table 11. Sample result of the nature of the FOU for each emotion characteristics.

			Interior	Left-Shoulder	Right-Shoulder
1	Iwe	Val		1	
		Act		1	
		Dom	1		
2	Obi Uto	Val		1	
		Act	1		
		Dom			1
3	Onuma	Val	1		
		Act		1	
		Dom	1		
4	Ujo	Val	1		
		Act	1		
		Dom	1		
5	Ntukwasi Obi	Val	1		
		Act	1		
		Dom	1		

Table 12. Embedded T1FS for each emotion characteristics.

			amf	Bmf
1	Iwe	Val	6.183503	8.632993
		Act	7.183503	9.632993
		Dom	5.585786	8.414214
2	Obi Uto	Val	6.183503	8.632993
		Act	5.37868	9.62132
		Dom	1.275255	3.724745
3	Onuma	Val	4.37868	8.62132
		Act	7.183503	9.632993
		Dom	4.171573	9.828427
4	Ujo	Val	7.585786	10.41421
		Act	6.792893	8.207107
		Dom	7.792893	9.207107
5	Ntukwasi Obi	Val	7.585786	10.41421
		Act	6.792893	8.207107
		Dom	5.37868	9.62132

Table 13. Computed LMF and the UMF of the FOU ($\tilde{A}$).

S/No	Word		UMF	LMF
1	Iwe	Val	(6,8)	(0,5,8)
		Act	(7,9)	(0,4,9)
		Dom	(5,6.5,7,8)	(5,6,6.8,8)
2	Obi Uto	Val	(6,8)	(0,3,8)
		Act	(5,6.5,7.5,9)	(5,6,7,8)
		Dom	(0,3,10)	(0,6,9,10)
3	Onma	Val	(2,4.5,6.5,8)	(2,5,5.7,7)
		Act	(7,9)	(0,3,9)
		Dom	(2,4,7,9)	(2,5,5.5,6)
4	Ujo	Val	(8,10)	(0,3,10)
		Act	(7,8)	(0,2,8)
		Dom	(8,9)	(0,2,9)
5	Ntukwasi Obi	Val	(8,10)	(0,3,10)
		Act	(7,8)	(0,3,8)
		Dom	(0,3,10)	(0,6,9,10)

5. Result and Discussion

5.1. The Fuzzy Set Part

The IT2FL model used to analyze the Igbo emotion words are simulated using Matlab, Microsoft excel and Netbeans. The data as collected are input into a spreadsheet, preprocessed as discussed in the paper.

Then, the model is run on the data yielding the MFs and the FOUs of each of the first 5 emotion words. Parts of the results of simulation for dimensions Valence, Activation and Dominance are shown in *Figs.* (*4*)-(*16*).

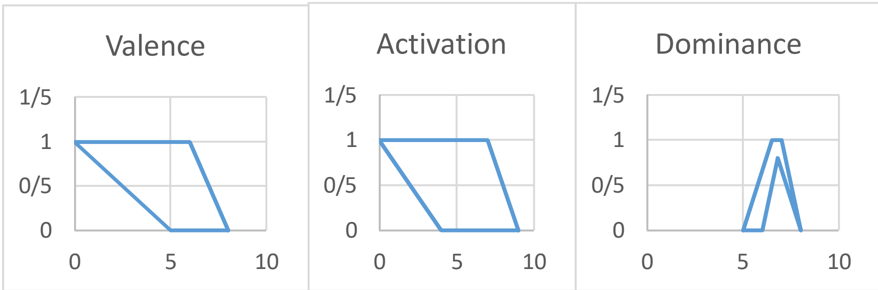

Fig. 4. The FOU of Iwe (Anger) for dimensions valence, activation and dominance.

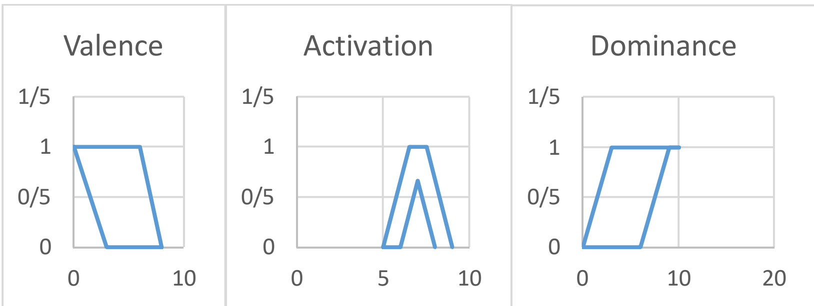

Fig. 5. The FOU of Obi Uto (Happiness) for dimensions valence, activation and dominance.

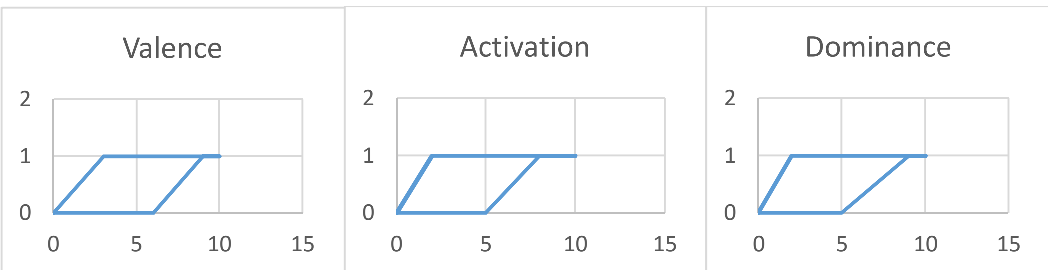

Fig. 6. The FOU of Ihere (Shame) for dimensions valence, activation and dominance.

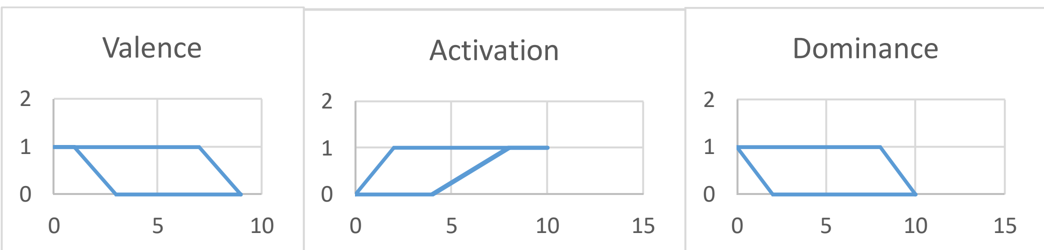

Fig. 7. The FOU of Akwa Uta (Remorse) for dimensions valence, activation and dominance.

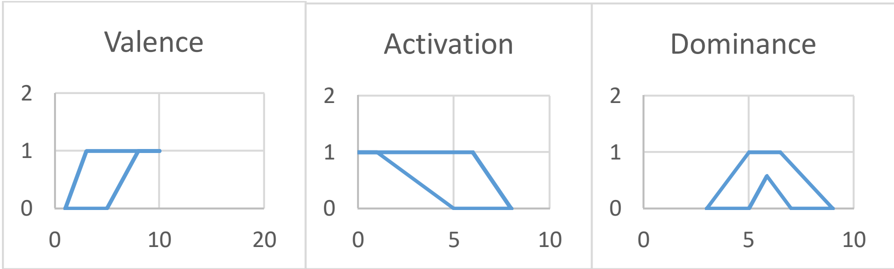

Fig. 8. The FOU of Ara (Mad) for dimensions valence, activation and dominance.

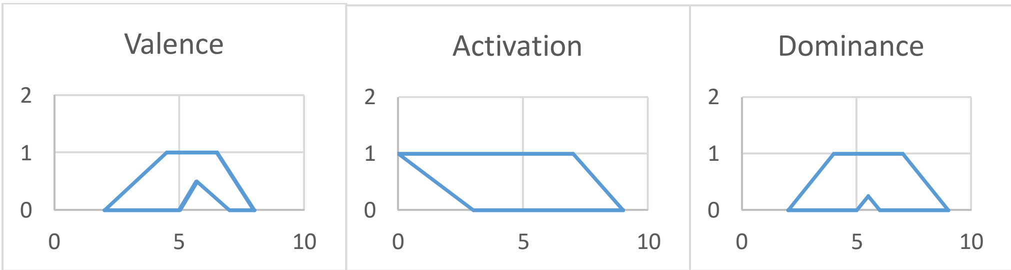

Fig. 9. The FOU of Onuma (Wrath) for dimensions valence, activation and dominance.

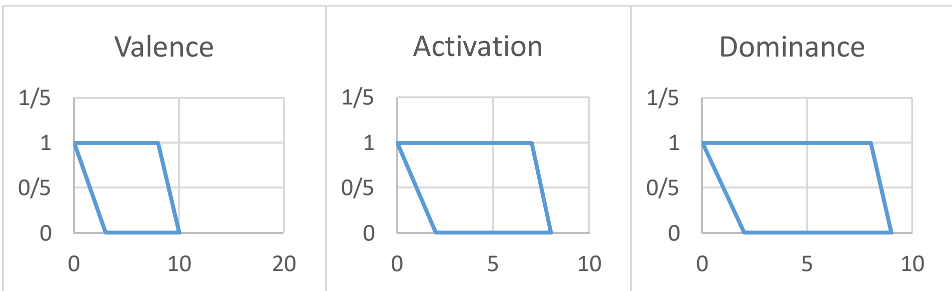

Fig. 10. The FOU of Ujo (Fear) for dimensions valence, activation and dominance.

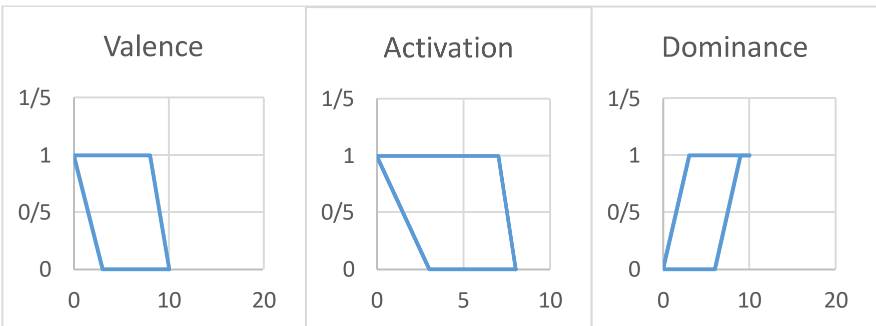

Fig. 11. The FOU of Ntukwasi (Trust) for dimensions valence, activation and dominance.

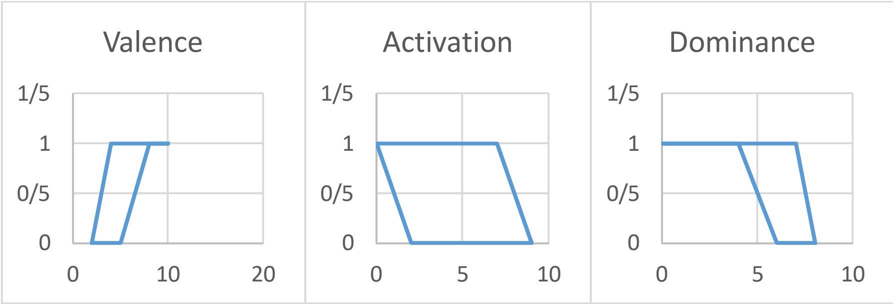

Fig. 12. The FOU of Ojo (Wicked) for dimensions valence, activation and dominance.

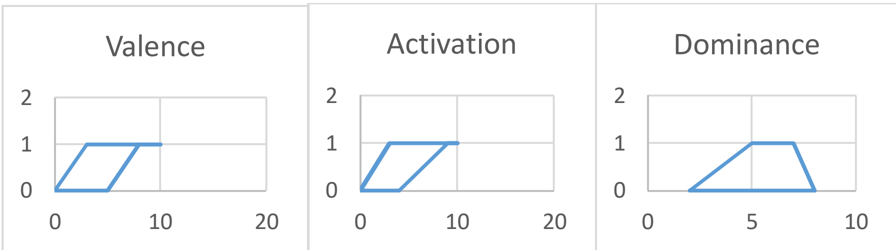

Fig. 13. The FOU of Onu (Joy) for dimensions valence, activation and dominance.

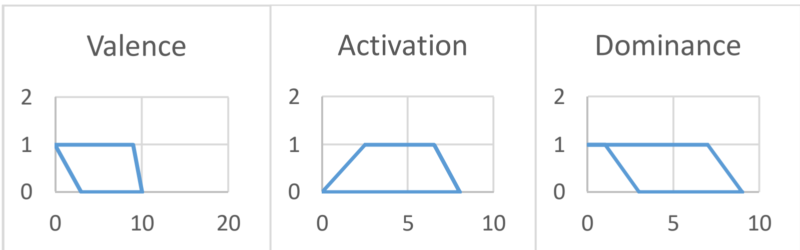

Fig. 14. The FOU of Anuri (Glad) for dimensions valence, activation and dominance.

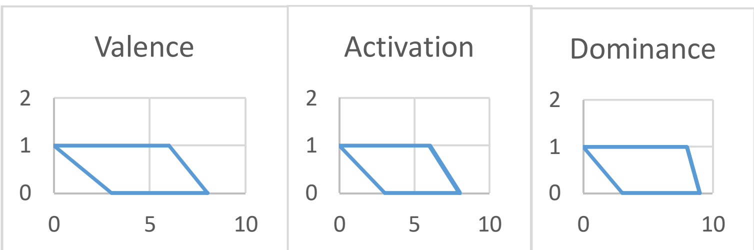

Fig. 15. The FOU of Egwu (Dread) for dimensions valence, activation and dominance.

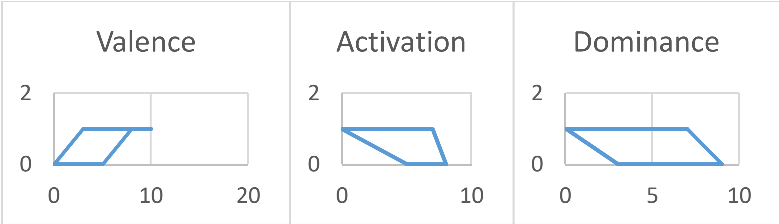

Fig. 16. The FOU of Ihunaya (Love) for dimensions valence, activation and dominance.

5.2. Discussion

Figs. (4) – (16) show parts of the MFs that were calculated from the survey data. These Figures indicate 3 general tendencies of graph: the MF could be either steeper and more peripheral if the emotion words are in accordance with their real meaning, or less steep and more central if the emotion words have ambiguous and undetermined meaning, or shifted to the opposite side of at least one of the scales if the emotion words do not conform with the real meaning. The valence dimension for the word "Iwe (Anger)" is not so broad, indicating a value a bit low. The activation dimension indicates a high value while the dominance dimension is narrow and has a small footprint of uncertainty. This means the word "Iwe (Anger)" carries a meaning that is well determined by the valence and dominance dimensions but is not well determined by the activation dimension. The word "Obi Uto (Happiness)" has a high value in its valence dimension, low value in its activation dimension and high value in its dominance dimension. This implies that the meaning of the word "Obi Uto (Happiness)" is well determined by its activation dimension. For the word "Ihere (Shame)", the valence dimension has a high value. The activation and dominance dimensions have low values. This indicates that the meaning of the word "Ihere (Shame)" is well determined by its activation and dominance dimensions and not the valence dimension. The word "AkwaUta (Remorse)" has a high value in its valence dimension, a low value in its activation dimension and a high value in its dominance dimension. This indicates that the meaning of the word "AkwaUta (Remorse)" is well determined by all the three dimensions. For the word "Ara (Mad)", the valence dimension has a low value while the activation dimension has a high value. Its dominance dimension though not so broad has a small footprint of uncertainty. This shows that the meaning of the word "Ara (Mad)" is well determined by all the three characteristics.

5. Conclusion

This paper involves the implementation of the IT2FL model for analyzing thirty (30) Igbo emotion words. Interval survey is conducted using Igbo native speakers to collect human intuition about fuzzy predicate which is emotion. Using an interval approach, user data are associated with each emotion word with intervals on the three scales of emotional characteristics-valence, activation, and dominance which are collected and used to estimate IT2F MFs for each scale. Results indicate that the study is able to demonstrate that the use of the proposed system will be of immense benefit to every aspect of Natural Language Processing (NLP) and affective computing and that the IT2FL model for words is more suitable for any purpose in which emotion words may be computed. This is because of the interval approach method used to analyze the words to yield IT2FSs which captures most uncertainty that are contained in an emotion word. Also, the study will help the users in selecting specific Igbo emotion words for easy communication and understanding.

In the future, more emotion words can be added to the system and IT2FL tool can be employed in the translation of Igbo emotion words in English language.

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