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Multi-Item Inventory Model Include Lead Time with Demand Dependent Production Cost and Set-Up-Cost in Fuzzy Environment

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Abstract

In this paper, we have developed the multi-item inventory model in the fuzzy environment. Here we considered the demand rate is constant and production cost is dependent on the demand rate. Set-up- cost is dependent on average inventory level as well as demand. Lead time crashing cost is considered the continuous function of leading time. Limitation is considered on storage of space. Due to uncertainty all cost parameters of the proposed model are taken as generalized trapezoidal fuzzy numbers. Therefore this model is very real. The formulated multi objective inventory problem has been solved by various techniques like as Geometric Programming (GP) approach, Fuzzy Programming Technique with Hyperbolic Membership Function (FPTHMF), Fuzzy Nonlinear Programming (FNLP) technique and Fuzzy Additive Goal Programming (FAGP) technique. An example is given to illustrate the model. Sensitivity analysis and graphical representation have been shown to test the parameters of the model.

Keywords: Inventory, Multi-Item, Leading time; Generalized trapezoidal fuzzy number, Fuzzy techniques; GP technique.

1 | Introduction

Inventory models deal with decisions that minimize the total average cost or maximize the total average profit. In that way to construct a real life mathematical inventory model on based on various assumptions and notations and approximations. Multi-item is also an important factor in the inventory control system. The basic well known Economic Order Quantity (EOQ) model was first introduced by Harris in 1913; Abou-el-ata and Kotb studied a multi-item EOQ inventory model with varying holding costs under two restrictions with a geometric programming approach [1]. Chen [7] presented an optimal determination of quality level, selling quantity and purchasing price for intermediate firms. Liang and Zhou [11] discussted two warehouse inventory model for deteriorating items and stock dependent demand under conditionally permissible delay in payment.

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Das et al. [12] developed a multi-item inventory model with quantity dependent inventory costs and demand-dependent unit cost under imprecise objectives and restrictions with a geometric programming approach. Das and Islam [13] considered a multi-objective two echelon supply chain inventory model with customer demand dependent purchase cost and production rate dependent production cost. Shaikh et al. [30] discussed an inventory model for deteriorating items with preservation facility of ramp type demand and trade credit.

The concept of fuzzy set theory was first introduced by Zadeh [27]. Afterward, Zimmermann [28] applied the fuzzy set theory concept with some useful membership functions to solve the linear programming problem with some objective functions. Bit [2] applied fuzzy programming with hyperbolic membership functions for multi objective capacitated transportation problems. Bortolan and Degani [4] discussed a review of some methods for ranking fuzzy subsets. Maiti [19] developed a fuzzy inventory model with two warehouses under possibility measure in fuzzy goals. Mandal et al. [22] presented a multi-objective fuzzy inventory model with three constraints with a geometric programming approach. Shaikh et al. [26] developed a fuzzy inventory model for a deteriorating Item with variable demand, permissible delay in payments and partial backlogging with Shortage Following Inventory (SFI) policy. Garai et al. [29] discussed multi-objective inventory model with both stock-dependent demand rate and holding cost rate under fuzzy random environment.

In the global market system lead time is an important matter. Ben-Daya and Rauf [3] considered an inventory model involving lead-time as a decision variable. Chuang et al. [8] presented a note on periodic review inventory model with controllable setup cost and lead time. Hariga and Ben-Daya [14] discussed some stochastic inventory models with deterministic variable lead time. Ouyang et al. [20] studied mixture inventory models with backorders and lost sales for variable lead time. Ouyang and Wu [21] established a min-max distribution free procedure for mixed inventory models with variable lead time. Sarkar et al. [24] developed an integrated inventory model with variable lead time and defective units and delay in payments. Sarkar et al. [25] studied quality improvement and backorder price discount under controllable lead time in an inventory model.

Geometric Programming (GP) is a powerful optimization technique developed to solve a class of non-linear optimization programming problems especially found in engineering design and manufacturing. Multi objective geometric programming techniques are also interesting in the EOQ model. GP was introduced by Duffin et al. in 1966 [10] and published a famous book in 1967 [9]. Beightler et al. [5] applied GP. Biswal [6] considered fuzzy programming techniques to solve multi-objective geometric programming problems. Islam [16] discussed multi-objective geometric-programming problem and its application. Mandal et al. [22] developed a multi-objective fuzzy inventory model with three constraints with a geometric programming approach. Mandal et al. [23] discussed an inventory model of deteriorating items with a constraint with a geometric Programming approach. Islam [17] studied a multi-objective marketing planning inventory model with a geometric programming approach. Kotb et al. [18] presented a multi-item EOQ model with both demand dependent on unit cost and varying lead time via geometric programming.

In this paper, we have developed an inventory model of multi-item with space constraint in a fuzzy environment. Here we considered the constant demand rate and production cost is dependent on the demand rate. Set-up- cost is dependent on average inventory level as well as demand. Lead time crashing cost is considered the continuous function of leading time. Due to uncertainty all cost parameters are taken as generalized trapezoidal fuzzy numbers. The proposal has been solved by various techniques like GP approach, FPTHMF, FNLP, and FAGP. Numerical example is given to illustrate the model. Finally sensitivity analysis and graphical representation have been shown to test the parameters of the model.

2| Mathematical Model

2.1| Notations

h_i: Holding cost per unit per unit time for i^{th} item.

T_i: The length of cycle time for i^{th}item, $T_i > 0$.

D_i : Demand rate per unit time for the i^{th} item.

L_i : Rate of leading time for the i^{th} item.

SS: Safety stock.

k: Safety factor.

$I_i(t)$: Inventory level of the i^{th} item at time *t*.

C_p^i : Unit production cost of i^{th} item.

$S_c{}^i(Q_i, D_i)$: Set up cost for i^{th} item.

$R^i(L_i)$: Lead time crashing cost for the i^{th} item.

Q_i: The order quantity for the duration of a cycle of length T_i for i^{th} item.

$TAC_i(D_i, Q_i, L_i)$: Total average profit per unit for the i^{th} item.

w_i: Storage space per unit time for the i^{th} item.

W: Total area of space.

$\widetilde{w}_i$: Fuzzy storage space per unit time for the i^{th} item.

$\widetilde{h}_i$: Fuzzy holding cost per unit per unit time for the i^{th} item.

$\widetilde{TAC}_i(D_i, Q_i, L_i)$: Fuzzy total average cost per unit for the i^{th} item.

$\widehat{w}_i$: Defuzzyfication of the fuzzy number $\widetilde{w}_i$.

$\widehat{h}_i$: Defuzzyfication of the fuzzy number $\widetilde{h}_i$.

$\widehat{TAC}_i(D_i, Q_i, L_i)$: Defuzzyfication of the fuzzy number $\widetilde{TAC}_i(D_i, Q_i, L_i)$.

2.2| Assumptions

- *Multi-item is considered.*
- *The replenishment occurs instantaneously at infinite rate.*
- *The lead time is considered.*
- *Shortages are not allowed.*
- *Production cost is inversely related to the demand. Here considered* $C_p^i(D_i) = \alpha_i D_i^{-\beta_i}$, *where* $\alpha_i > 0$ *and* $\beta_i > 1$ *are constant real numbers.*

- *The set up cost is dependent on the demand as well as average inventory level. Here considered* $S_c^{\,i}(Q_i, D_i) = \gamma_i \left(\frac{Q_i}{2}\right)^{\delta_i} D_i^{\sigma_i}$ *where* $0 < \gamma_i$, $0 < \delta_i \ll 1$ *and* $0 < \sigma_i \ll 1$ *are constant real numbers.*
- *Lead time crashing cost is dependent on the lead time by a function of the form* $R^i(L_i) = \rho_i L_i^{-\tau_i}$*, where* $\rho_i > 0$ *and* $0 < \tau_i \leq 0.5$ *are constant real numbers.*
  Li.
- *Deterioration is not allowed.*

2.3 | Formulation of the Model

The inventory level for i^th^ item is illustrated in *Fig. 1*. During the period $[0, T_i]$ the inventory level reduces due to demand rate. In this time period, the governing differential equation is

$$\frac{dI_i(t)}{dt} = -D_i, 0 \leq t \leq T_i. \tag{1}$$

With boundary condition, $I_i(0) = Q_i$, $I_i(T_i) = 0$.

Solving *Eq. (1)* we have,

$$I_i(t) = Q_i - D_i t,\ \ 0 \leq t \leq T_i\ \ . \tag{2}$$

$$T_i = \frac{Q_i}{D_i}. \tag{3}$$

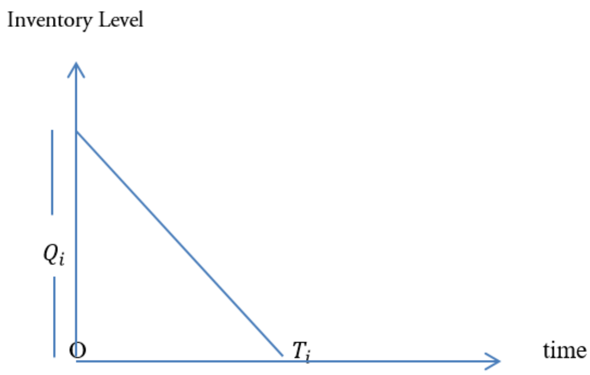

Fig. 1. Inventory level for the ith item.

Now calculating various averages cost for ith item,

Average production cost $(PC_i) = \frac{Q_i C_p^i(D_i)}{T_i} = \alpha_i D_i^{(1-\beta_i)}$;

Average holding cost$(HC_i) = \frac{1}{T_i}\int_0^{T_i} h_i I_i(t)dt + h_i k\omega\sqrt{L_i}\ = h_i\left(\frac{Q_i}{2} + k\omega\sqrt{L_i}\right)$;

Average set-up-cost $(SC_i)\ = \frac{1}{T_i}\left[\gamma_i\left(\frac{Q_i}{2}\right)^{\delta_i} D_i^{\sigma_i}\right] = \frac{\gamma_i Q_i^{\delta_i - 1} D_i^{\sigma_i + 1}}{2^{\delta_i}}$;

Average lead time crashing cost $(CC_i) = \frac{\rho_i L_i^{-\tau_i}}{T_i} = \frac{D_i \rho_i L_i^{-\tau_i}}{Q_i}$.

Total average cost for ith item is

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$$\mathrm{TAC_i(D_i,Q_i,L_i)} = (\mathrm{PC_i+HC_i+SC_i+CC_i}) = \alpha_i D_i^{(1-\beta_i)} + h_i\left(\frac{Q_i}{2}+k\omega\sqrt{L_i}\right) + \frac{\gamma_i Q_i^{\delta_i-1} D_i^{\sigma_i+1}}{2^{\delta_i}} + \frac{D_i \rho_i L_i^{-\tau_i}}{Q_i}. \tag{4}$$

A Multi-Objective Inventory Model (MOIM) can be written as:

$$\mathrm{Min}\ \{\mathrm{TAC_1, TAC_2, TAC_3, \dots \dots \dots \dots \dots, TAC_n}\},$$

$$\mathrm{TAC_i(D_i,Q_i,L_i)} = \alpha_i D_i^{(1-\beta_i)} + h_i\left(\frac{Q_i}{2}+k\omega\sqrt{L_i}\right) + \frac{\gamma_i Q_i^{\delta_i-1} D_i^{\sigma_i+1}}{2^{\delta_i}} + \frac{D_i \rho_i L_i^{-\tau_i}}{Q_i}, \tag{5}$$

Subject to

$$\sum_{i=1}^{n} w_i Q_i \le W,\ D_i > 0, Q_i > 0, L_i > 0, \text{ for } i = 1,2, \dots \dots \dots . . n.$$

2.4 | Fuzzy Model

Due to uncertainty, we consider all the parameters $(\alpha_i, \beta_i, h_i, \rho_i, \gamma_i, \delta_i, \sigma_i, \tau_i)$of the model and storage space w_i as Generalized Trapezoidal Fuzzy Number (GTrFN)$(\widetilde{\alpha_\imath}, \widetilde{\beta_\imath}, \widetilde{h_\imath}, \widetilde{\rho_\imath}, \widetilde{\gamma_\imath}, \widetilde{\delta_\imath}, \widetilde{\sigma_\imath}, \widetilde{w_\imath}, \widetilde{\tau_\imath})$. Here

$$\widetilde{\alpha_\imath} = \left(\alpha_i^1, \alpha_i^2, \alpha_i^3, \alpha_i^4; \varphi_{\alpha_i}\right), 0 < \varphi_{\alpha_i} \le 1; \widetilde{h_\imath} = \left(h_i^1, h_i^2, h_i^3, h_i^4; \varphi_{h_i}\right), 0 < \varphi_{h_i} \le 1;$$

$$\widetilde{\beta_\imath} = \left(\beta_i^1, \beta_i^2, \beta_i^3, \beta_i^4; \varphi_{\beta_i}\right), 0 < \varphi_{\beta_i} \le 1; \widetilde{\rho_\imath} = \left(\rho_i^1, \rho_i^1, \rho_i^1, \rho_i^1; \varphi_{\rho_i}\right), 0 < \varphi_{\rho_i} \le 1;$$

$$\widetilde{\gamma_\imath} = \left(\gamma_i^1, \gamma_i^2, \gamma_i^3, \gamma_i^4; \varphi_{\gamma_i}\right), 0 < \varphi_{\gamma_i} \le 1; \widetilde{w_\imath} = \left(w_i^1, w_i^2, w_i^3, w_i^4; \varphi_{w_i}\right), 0 < \varphi_{w_i} \le 1;$$

$$\widetilde{\delta_\imath} = \left(\delta_i^1, \delta_i^2, \delta_i^3, \delta_i^4; \varphi_{\delta_i}\right), 0 < \varphi_{\delta_i} \le 1; \widetilde{\sigma_\imath} = \left(\sigma_i^1, \sigma_i^2, \sigma_i^3, \sigma_i^4; \varphi_{\sigma_i}\right), 0 < \varphi_{\sigma_i} \le 1;$$

$$\widetilde{\tau_\imath} = \left(\tau_i^1, \tau_i^2, \tau_i^3, \tau_i^4; \varphi_{\tau_i}\right), 0 < \varphi_{\tau_i} \le 1; (\ i = 1,2, \dots \dots \dots, n).$$

Then the above inventory *Model* (5) becomes the fuzzy inventory model as

$$\mathrm{Min}\quad \left\{\widetilde{\mathrm{TAC}_1}, \widetilde{\mathrm{TAC}_2}, \widetilde{\mathrm{TAC}_3}, \dots \dots \dots \dots \dots \dots \dots . , \widetilde{\mathrm{TAC}_n}\right\},$$

Subject to

$$\sum_{i=1}^{n} \widetilde{w_\imath} Q_i \le W, \text{ for } i = 1,2, \dots \dots \dots . . n. \tag{6}$$

Where

$$\mathrm{TAC}_\imath(\widetilde{D_\imath, Q_\imath}, L_\imath) = \widetilde{\alpha_\imath} D_i^{(1-\widetilde{\beta_\imath})} + \widetilde{h_\imath}\left(\frac{Q_i}{2}+k\omega\sqrt{L_i}\right) + \frac{\widetilde{\gamma_\imath} Q_i^{\widetilde{\delta_\imath}-1} D_i^{\widetilde{\sigma_\imath}+1}}{2^{\widetilde{\delta_\imath}}} + \frac{D_i \widetilde{\rho_\imath} L_i^{-\widetilde{\tau_\imath}}}{Q_i}.$$

$\lambda -$Integer method is used to defuzzify the fuzzy number. In this method the defuzzify value of the fuzzy number $\tilde{A} = (a, b, c, d; \varphi)$ *is* $\varphi\left(\frac{a+b+c+d}{4}\right)$. So using the defuzzified values $(\widehat{\alpha_\imath}, \widehat{\beta_\imath}, \widehat{h_\imath}, \widehat{\rho_\imath}, \widehat{\gamma_\imath}, \widehat{\delta_\imath}, \widehat{\sigma_\imath}, \widehat{w_\imath}, \widehat{\tau_\imath})$ of the GTrFN $(\widetilde{\alpha_\imath}, \widetilde{\beta_\imath}, \widetilde{h_\imath}, \widetilde{\rho_\imath}, \widetilde{\gamma_\imath}, \widetilde{\delta_\imath}, \widetilde{\sigma_\imath}, \widetilde{w_\imath}, \widetilde{\tau_\imath})$, the above fuzzy inventory *Model* (6) reduces to

$$\operatorname{Min}\left\{\widehat{TAC_1}, \widehat{TAC_2}, \widehat{TAC_3}, \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots ., \widehat{TAC_n}\right\},$$

Subject to

$$\sum_{i=1}^{n} \widehat{w_i} Q_i \leq W, \tag{7}$$

Where

$$TAC_i\left(\widehat{D_i, Q_i}, L_i\right) = \widehat{\alpha_i} D_i^{\left(1-\widehat{\beta_i}\right)} + \widehat{h_i}\left(\frac{Q_i}{2} + k\omega\sqrt{L_i}\right) + \frac{\widehat{\gamma_i} {Q_i}^{\widehat{\delta_i}-1} D_i^{\widehat{\sigma_i}+1}}{2^{\widehat{\delta_i}}} + \frac{D_i \widehat{\rho_i} {L_i}^{-\widehat{\tau_i}}}{Q_i},$$

$$D_i > 0, Q_i > 0, L_i > 0, \text{ for } i = 1,2, \dots \dots \dots . . n.$$

3| Fuzzy Programming Techniques to Solve MOIM

Solve the MOIM as a single objective NLP using only one objective at a time and ignoring the others. So we get the ideal solutions. Using the ideal solutions the pay-off matrix as follows:

$$\begin{array}{cc}
 & \begin{array}{cccc} TAC_1(D_1,Q_1,L_1) & TAC_2(D_2,Q_2,L_2) \dots & \dots \dots \dots & TAC_n(D_n,Q_n,L_n) \end{array} \\
\begin{array}{c} (D_1^1,Q_1^1,L_1^1) \\ (D_2^2,Q_2^2,L_2^2) \\ \dots \\ \dots \\ (D_n^n,Q_n^n,L_n^n) \end{array} &
\left(\begin{array}{cccc}
TAC_1^*(D_1^1,Q_1^1,L_1^1) & TAC_2(D_1^1,Q_1^1,L_1^1) \dots & \dots \dots \dots & TAC_n(D_1^1,Q_1^1,L_1^1) \\
TAC_1(D_2^2,Q_2^2,L_2^2) & TAC_2^*(D_2^2,Q_2^2,L_2^2) \dots & \dots \dots & TAC_n(D_2^2,Q_2^2,L_2^2) \\
\dots \dots & \dots \dots \dots & \dots \dots \dots & \dots \dots \dots \\
\dots \dots & \dots \dots \dots & \dots \dots \dots & \dots \dots \dots \\
TAC_1(D_n^n,Q_n^n,L_n^n) & TAC_2(D_n^n,Q_n^n,L_n^n) \dots & \dots \dots . & TAC_n^*(D_n^n,Q_n^n,L_n^n)
\end{array}\right)
\end{array} \tag{8}$$

Let $U^k = \max\{TAC_k(D_i^i, Q_i^i, L_i^i), i = 1,2, \dots ., n\}$ for $k = 1,2, \dots ., n$ and

$L^k = TAC_k^*(D_k^k, Q_k^k, L_k^k)$ for $k = 1,2, \dots ., n$.

Hence U^k, L^k are identified, $L^k \leq TAP_k(D_i^i, Q_i^i, L_i^i) \leq U^k$, for $i = 1,2, \dots ., n$; $k = 1,2, \dots ., n$.

3.1| Fuzzy Programming Technique Using Hyperbolic Membership Function (FPTHMF)

Now fuzzy non-linear hyperbolic membership functions $\mu_{TAC_k}^H\left(TAC_k(D_k, Q_k, L_k)\right)$ for the kth objective functions $TAC_k(D_k, Q_k, L_k)$ respectively for $k = 1,2, \dots ., n$ are defined as follows:

$$\mu_{TAC_k}^H\left(TAC_k(D_k, Q_k, L_k)\right) = \frac{1}{2}\tanh\left(\left(\frac{U^k + L^k}{2} - TAC_k(D_k, Q_k, L_k)\right)\sigma_k\right) + \frac{1}{2}.$$

Where α_k is a parameter, $\sigma_k = \frac{3}{(U^k - L^k)/2} = \frac{6}{U^k - L^k}$.

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In this technique the problem is defined as follows:

$$\text{Max } \lambda,$$

Subject to

$$\frac{1}{2}\tanh\left(\left(\frac{U^k+L^k}{2} - TAC_k(D_k, Q_k, L_k)\right)\sigma_k\right) + \frac{1}{2} \geq \lambda,$$

$$\sum_{i=1}^{n} \widehat{w_i} Q_i \leq W, \lambda \geq 0, \quad D_k >, Q_k > 0, L_k > 0, \text{ for } k = 1,2, \ldots \ldots \ldots .. n.$$

After simplification the above problem can be written as

$$\text{Max } y,$$

Subject to

$$y + \sigma_k TAC_k(D_k, Q_k, L_k) \leq \frac{U^k+L^k}{2}\sigma_k,$$

$$\sum_{i=1}^{n} \widehat{w_i} Q_i \leq W, y \geq 0, D_k >, Q_k > 0, L_k > 0 \text{ for } k = 1,2, \ldots \ldots \ldots .. n.$$

Now the above problem can be freely solved by suitable mathematical programming algorithm and then we shall get the appropiet solution of the MOIM.

3.2| Fuzzy Non-Linear Programming (FNLP) Technique based on Max-Min

In this technique fuzzy membership function $\mu_{TAC_k}(TAC_k(Q_k, D_k))$ for the k[th] objective function $TAC_k(D_k, Q_k, L_k)$ respectively for $k = 1,2, \ldots .., n$ are defined as follows:

$$\mu_{TAC_k}(TAC_k(D_k, Q_k, L_k)) = \begin{cases} 1 & \text{for } TAC_k(D_k, Q_k, L_k) < L^k \\ \dfrac{U^k - TAC_k(D_k, Q_k, L_k)}{U^k - L^k} & \text{for } L^k \leq TAC_k(D_k, Q_k, L_k) \leq U^k \\ 0 & \text{for } TAC_k(D_k, Q_k, L_k) > U^k \end{cases}$$

$$\text{for } k = 1,2, \ldots .., n.$$

In this technique the problem is defined as follows:

$$\text{Max } \alpha',$$

Subject to

$$TAC_k(D_k, Q_k, L_k) + \alpha'(U^k - L^k) \leq U^k \quad , \qquad \text{for } k = 1,2, \ldots .., n,$$

$$\sum_{i=1}^{n} \widehat{w_i} Q_i \leq W, 0 \leq \alpha' \leq 1, \ D_k >, Q_k > 0, L_k > 0.$$

Now the above problem can be freely solved by suitable mathematical programming algorithm and then we shall get the required solution of the MOIM.

3.3| Fuzzy Additive Goal Programming (FAGP) Technique Based on Additive Operator

Using the above membership function, fuzzy non-linear programming problem is formulated as

$$\text{Max} \sum_{k=1}^{n} \frac{U^k - TAC_k(D_k, Q_k, L_k)}{U^k - L^k},$$

Subject to

$$U^k - TAC_k(D_k, Q_k, L_k) \leq U^k - L^k,$$

$$\sum_{i=1}^{n} \widehat{w_1} Q_i \leq W, \; D_k >, Q_k > 0, L_k > 0 \text{ for } k = 1,2, \ldots, n.$$

Now the above problem can be solved by suitable mathematical programming algorithm and then we shall get the solution of the MOIM.

4| Geometric Programming Technique

Let us consider a Multi Objective Geometric Programming (MOGP) problem is as follows

$$\text{Minimize } g_{s0}(t) = \sum_{k=1}^{T_{s0}} c_{s0k} \prod_{j=1}^{m} t_j^{\alpha_{s0kj}}, \; s = 1,2,3, \ldots \ldots \ldots, n,$$

Subject to

$$g_r(t) = \sum_{k=1}^{l_r} c_{rk} \prod_{j=1}^{m} t_j^{\alpha_{rkj}} \leq 1, \; r = 1,2,3, \ldots \ldots \ldots, p,$$

$$t_j > 0, j = 1,2, \ldots, m.$$

Where $c_{rk}, c_{s0k} (> 0)$, α_{rkj} and α_{s0kj} $(j = 1,2, \ldots, m; r = 0,1,2, \ldots, p; \; k = 1,2, \ldots . l_r \; ; \; s = 1,2,3, \ldots \ldots \ldots, n)$are all real numbers. T_{s0} is the number of terms in the s^{th} objective function and l_r is the number of terms in the r^{th} constraint.

Now introducing the weights w_i $(i = 1,2,3, \ldots \ldots \ldots, n)$, the above MOGP converted into the single objective geometric programming problem as following

Primal Problem.

$$\text{Minimize } g(t) = \sum_{s=1}^{n} w_s \sum_{k=1}^{T_{s0}} c_{s0k} \prod_{j=1}^{m} t_j^{\alpha_{s0kj}}, \; s = 1,2,3, \ldots \ldots \ldots, n,$$

$$\text{i.e.} = \sum_{s=1}^{n} \sum_{k=1}^{T_{s0}} w_s c_{s0k} \prod_{j=1}^{m} t_j^{\alpha_{s0kj}},$$

Subject to

$$g_r(t) = \sum_{k=1}^{l_r} c_{rk} \prod_{j=1}^{m} t_j^{\alpha_{rkj}} \leq 1, \; r = 1,2,3, \ldots \ldots \ldots, p, \quad (9)$$

$$t_j > 0, j = 1,2, \ldots, m,$$

$$\sum_{i=1}^{n} w_i = 1, w_i > 0, i = 1,2,3, \ldots \ldots \ldots, n.$$

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Let T be the total numbers of terms (including constraints), number of variables is m. Then the degree of the difficulty (DD) is $T-(m+1)$.

Dual Program.

The dual programming of *Eq. (9)* is given as follows:

$$\text{Maximize } v(\theta) = \prod_{s=1}^{n}\prod_{k=1}^{T_{so}}\left(\frac{w_s c_{s0k}}{\theta_{0sk}}\right)^{\theta_{0sk}} \prod_{r=1}^{p}\prod_{k=1}^{l_r}\left(\frac{c_{rk}}{\theta_{rk}}\right)^{\theta_{rk}} \left(\sum_{k=1}^{l_r}\theta_{rk}\right)^{\sum_{k=1}^{l_r}\theta_{rk}},$$

Subject to

$$\sum_{s=1}^{n}\sum_{k=1}^{T_{s0}}\theta_{0sk} = 1, \text{ (Normality condition)}$$

$$\sum_{r=1}^{p}\sum_{k=1}^{l_r}\alpha_{rkj}\theta_{rk} + \sum_{s=1}^{n}\sum_{k=1}^{T_{s0}}\alpha_{s0kj}\theta_{0sk} = 0, \ (j=1,2,\dots,m)$$
(Orthogonality conditions)

$$\theta_{0sk}, \theta_{rk} > 0, \ (r=0,1,2,\dots,p;\ k=1,2,\dots,l_r;\ s=1,2,3,\dots,n).$$
(Positivity conditions)

Now here three cases may arises

Case I. $T_0 = m+1$, (i.e. DD=0). So DP presents a system of linear equations for the dual variables. So we have a unique solution vector of dual variables.

Case II. $T_0 > m+1$, So a system of linear equations is presented for the dual variables, where the number of linear equations is less than the number of dual variables. So it is concluded that dual variables vector have many solutions.

Case III. $T_0 < m+1$, so a system of linear equations is presented for the dual variables, where the number of linear equations is greater than the number of dual variables. It is seen that generally no solution vector exists for the dual variables here.

4.1| Solution Procedure of My Proposed Problem

Primal Problem.

$$\text{Minimize } TAC(D,Q,L) = \sum_{i=1}^{n} w_i'\left(\widehat{\alpha}_i D_i^{(1-\widehat{\beta}_i)} + \widehat{h}_i\left(\frac{Q_i}{2} + k\omega\sqrt{L_i}\right) + \frac{\widehat{\gamma}_i Q_i^{\widehat{\delta}_i - 1} D_i^{\widehat{\sigma}_i + 1}}{2^{\widehat{\delta}_i}} + D_i\widehat{\rho}_i L_i^{-\widehat{\tau}_i} Q_i^{-1}\right), \tag{10}$$

Subject to

$$\sum_{i=1}^{n}\frac{\widehat{w}_i}{W}Q_i \le 1,$$

$$\sum_{i=1}^{n} w_i' = 1, w_i' > 0, i = 1,2,3,\dots,n.$$

Dual Program.

The dual programming of *Eq. (10)* is given as follows:

$$
\begin{aligned}
&\text{Maximize } v(\theta)\\
&= \prod_{i=1}^{n} \left(\frac{w_i'\widehat{\alpha_1}}{\theta_{i1}}\right)^{\theta_{i1}} \left(\frac{w_i'\widehat{h_1}}{2\theta_{i2}}\right)^{\theta_{i2}} \left(\frac{w_i' k\omega}{\theta_{i3}}\right)^{\theta_{i3}} \left(\frac{w_i'\widehat{\gamma_1}}{2^{\widehat{\delta_1}}\theta_{i4}}\right)^{\theta_{i4}} \left(\frac{w_i'\widehat{\rho_1}}{\theta_{i5}}\right)^{\theta_{i5}} \left(\frac{\widehat{w_1}}{W\theta_{i1}'}\right)^{\theta_{i1}'} \left(\sum_{i=1}^{n}\theta_{i1}'\right)^{\sum_{i=1}^{n}\theta_{i1}'},\\
&\text{Subject to}\\
&\theta_{i1}+\theta_{i2}+\theta_{i3}+\theta_{i4}+\theta_{i5}=1,\\
&\left(1-\widehat{\beta_1}\right)\theta_{i1}+(\widehat{\sigma_1}+1)\theta_{i4}+\theta_{i5}=0,\\
&\theta_{i2}+\left(\widehat{\delta_1}-1\right)\theta_{i4}-\theta_{i5}+\theta_{i1}'=0,\\
&\frac{\theta_{i3}}{2}-\widehat{\tau_1}\theta_{i5}=0,\\
&\sum_{i=1}^{n} w_i'=1, w_i'>0,\\
&\theta_{i1},\theta_{i2},\theta_{i3},\theta_{i4},\theta_{i5},\theta_{i1}'\geq 0 \text{ for } i=1,2,3,\ldots\ldots\ldots..,n.
\end{aligned}
\tag{11}
$$

Solving the above linear equations we have

$$\theta_{i1}=\frac{\widehat{\sigma_\iota}+1}{\widehat{\beta_\iota}-1}y_i+\frac{1}{\widehat{\beta_\iota}-1}x_i,\ \theta_{i2}=1-\left\{(1+2\widehat{\tau_\iota})+\frac{1}{\widehat{\beta_\iota}-1}x_i\right\}-\frac{\widehat{\sigma_\iota}+\widehat{\beta_\iota}}{\widehat{\beta_\iota}-1}y_i,\ \theta_{i3}=2\widehat{\tau_\iota}x_i,\ \theta_{i4}=y_i,\ \theta_{i5}=x_i,$$

$$\theta_{i1}'=1-\left(2\widehat{\tau_\iota}+\frac{1}{\widehat{\beta_\iota}-1}\right)x_i-\left(\frac{\widehat{\sigma_\iota}+\widehat{\beta_\iota}}{\widehat{\beta_\iota}-1}+\widehat{\delta_\iota}-1\right)y_i.$$

Putting the above values in *Eq. (11)* we have

$$
\begin{aligned}
&\text{Maximize } v(x,y)\\
&= \prod_{i=1}^{n}\left(\frac{w_i'\left(\widehat{\beta_1}-1\right)\widehat{\alpha_1}}{(\widehat{\sigma_1}+1)y_i+x_i}\right)^{\frac{\widehat{\sigma_1}+1}{\widehat{\beta_1}-1}y_i+\frac{1}{\widehat{\beta_1}-1}x_i}\left(\frac{w_i'\widehat{h_1}}{2\left\{1-\left\{(1+2\widehat{\tau_1})+\frac{1}{\widehat{\beta_1}-1}x_i\right\}-\frac{\widehat{\sigma_1}+\widehat{\beta_1}}{\widehat{\beta_1}-1}y_i\right\}}\right)^{1-\left\{(1+2\widehat{\tau_1})+\frac{1}{\widehat{\beta_1}-1}x_i\right\}-\frac{\widehat{\sigma_1}+\widehat{\beta_1}}{\widehat{\beta_1}-1}y_i}\\
&\quad\left(\frac{w_i'k\omega}{2\widehat{\tau_1}x_i}\right)^{2\widehat{\tau_1}x_i}\left(\frac{w_i'\widehat{\gamma_1}}{2^{\widehat{\delta_1}}y_i}\right)^{y_i}\left(\frac{w_i'\widehat{\rho_1}}{x_i}\right)^{x_i}\left(\frac{\widehat{w_1}}{Wz_i}\right)^{z_i}\left(\sum_{i=1}^{n}z_i\right)^{\sum_{i=1}^{n}z_i},\\
&\sum_{i=1}^{n}w_i'=1,\\
&\text{Where } z_i=1-\left(2\widehat{\tau_1}+\frac{1}{\widehat{\beta_1}-1}\right)x_i-\left(\frac{\widehat{\sigma_1}+\widehat{\beta_1}}{\widehat{\beta_1}-1}+\widehat{\delta_1}-1\right)y_i,\\
&\qquad x_i,y_i>0, w_i'>0 \text{ and } x=(x_1,x_2,x_3,\ldots..,x_n),\ y=(y_1,y_2,y_3,\ldots\ldots\ldots,y_n).
\end{aligned}
$$

Now using the primal-dual relation we have

$$TAC^{*}(D,Q,L) = n\left(v^{*}(x,y)\right)^{1/n};$$

$$w_i'\widehat{\alpha}_1 D_i^{*(1-\widehat{\beta}_1)} = \theta_{i1}^{*}\left(v^{*}(x,y)\right)^{1/n};$$

$$\frac{w_i'\widehat{h}_1 Q_i^{*}}{2} = \theta_{i2}^{*}\left(v^{*}(x,y)\right)^{1/n};$$

wi⁄hikωLi∗=θi3∗v∗x,y1/n, for i=1,2,3,……….,n.

5| Numerical Example

Here we consider an inventory system which consists of two items with following parameter values in proper units. Total storage area $W = 500\ Sq.ft.$ and $k = 3, \omega = 5, w_1' = 0.5, w_1' = 0.5$.

Table 1. Input imprecise data for shape parameters.

Parameters	Items	
	I	II
$\widetilde{\alpha}_1$	(200,205,210,215; 0.9)	(215,220,225,230; 0.8)
$\widetilde{\beta}_1$	(4,5,6,7; 0.8)	(5,6,7,8; 0.8)
$\widetilde{h}_1$	(2,4,5,6; 0.9)	(2,2.5,3,3.5; 0.8)
$\widetilde{\rho}_1$	(2,2.3,2.4,2.5; 0.9)	(3,3.1,3.2,3.3; 0.9)
$\widetilde{\gamma}_1$	(90,95,100,105; 0.7)	(92,95,98,102; 0.8)
$\widetilde{\delta}_1$	(0.02,0.03,0.04,0.05; 0.8)	(0.04,0.05,0.06,0.07; 0.8)
$\widetilde{\sigma}_1$	(0.2,0.3,0.4,0.5; 0.8)	(0.2,0.3,0.4,0.5; 0.9)
$\widetilde{w}_1$	(1.5,1.6,1.7,1.8; 0.7)	(1.7,1.8,1.9,2.0; 0.9)
$\widetilde{\tau}_1$	$\left(\frac{1}{6},\frac{1}{5},\frac{1}{4},\frac{1}{3};0.9\right)$	$\left(\frac{1}{7},\frac{1}{6},\frac{1}{5},\frac{1}{4};0.9\right)$

Approximate value of the above parameter is

Table 2. Defuzzification of the fuzzy numbers.

Defuzzification of the Fuzzy Numbers	Items	
	I	II
$\widehat{\alpha}_1$	186.75	178
$\widehat{\beta}_1$	4.4	5.2
$\widehat{h}_1$	3.825	2.2
$\widehat{\rho}_1$	2.07	2.835
$\widehat{\gamma}_1$	68.25	77.4
$\widehat{\delta}_1$	0.028	0.044
$\widehat{\sigma}_1$	0.28	0.315
$\widehat{w}_1$	1.155	2.115
$\widehat{\tau}_1$	0.21375	0.17089

Table 3. Optimal solutions of MOIM using different methods.

Methods	D_1^*	Q_1^*	L_1^*	TAC_1^*	D_2^*	Q_2^*	L_2^*	TAC_2^*
FPTHMF	2.50	11.51	0.34×10^{-3}	53.95	2.28	15.54	0.30×10^{-3}	41.03
FNLP	2.50	11.51	0.34×10^{-3}	53.95	2.30	15.57	0.33×10^{-3}	41.03
FAGP	2.49	11.35	0.35×10^{-3}	53.96	2.30	15.63	0.37×10^{-3}	41.03
GP	2.58	10.07	0.27×10^{-3}	54.58	2.20	17.58	0.42×10^{-3}	41.52

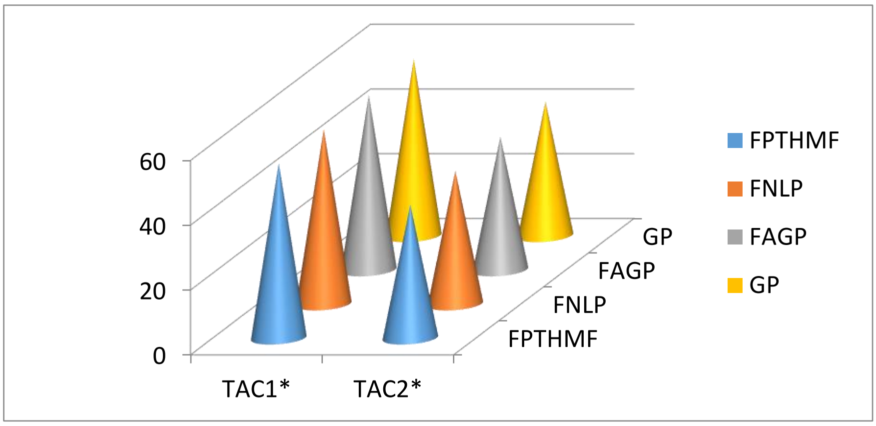

Fig. 2. Minimizing cost of both items using different methods.

From the above figure shows that GP, FPTHMF, FNLP, and FAGP methods almost provide the same results.

6| Sensitivity Analysis

In the sensitivity analysis the optimal solutions have been found buy using FNLP method.

Table 4. Optimal solution of MOIM for different values of α_1, α_2.

Method	α_1, α_2	D_1^*	Q_1^*	L_1^*	TAC_1^*	D_2^*	Q_2^*	L_2^*	TAC_2^*
FNLP	-20%	2.36	11.12	0.33×10^{-3}	52.14	2.19	15.12	0.29×10^{-3}	39.82
	-10%	2.43	11.33	0.34×10^{-3}	53.09	2.24	15.36	0.33×10^{-3}	40.45
	10%	2.56	11.68	0.35×10^{-3}	54.75	2.34	15.78	0.37×10^{-3}	41.55
	20%	2.61	10.84	0.35×10^{-3}	55.49	2.38	17.97	0.42×10^{-3}	42.03

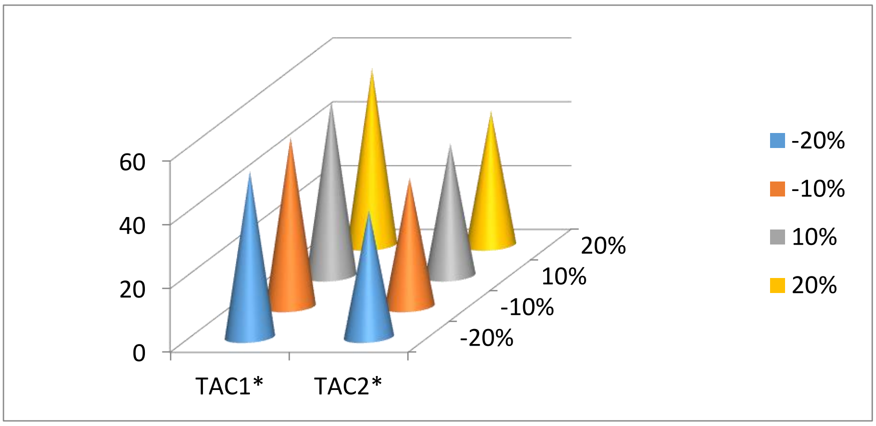

Fig. 3. Minimizing cost of both items for different values of α_1, α_2.

From the *Fig. 3* suggests that the minimum cost of both items is increased when values of α_1, α_2 are increased.

Table 5. Optimal solution of MOIM for different values of β_1, β_2.

Method	β_1, β_2	D_1^*	Q_1^*	L_1^*	TAC_1^*	D_2^*	Q_2^*	L_2^*	TAC_2^*
	-20%	2.93	12.73	0.37×10^{-3}	62.82	2.67	17.19	0.32×10^{-3}	47.24
	-10%	2.69	12.05	0.35×10^{-3}	57.77	2.46	16.30	0.31×10^{-3}	43.71
FNLP	10%	2.35	11.07	0.33×10^{-3}	50.98	2.16	15.00	0.29×10^{-3}	38.92
	20%	2.22	10.70	0.32×10^{-3}	48.59	2.06	14.52	0.28×10^{-3}	37.23

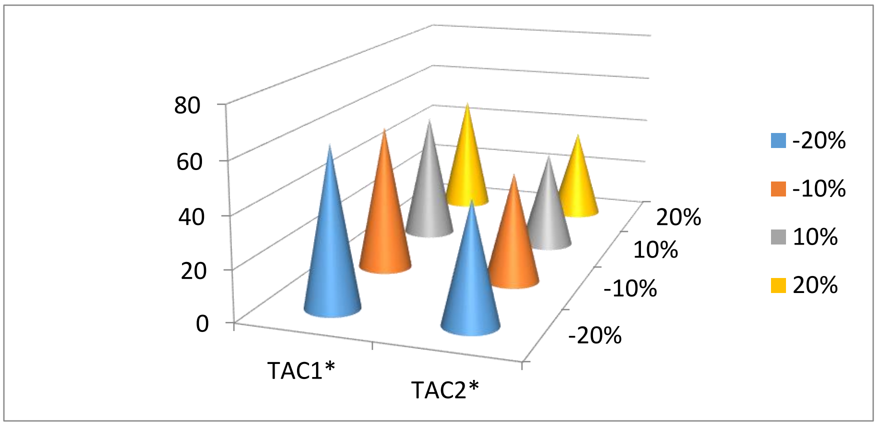

Fig. 4. Minimizing cost of 1st and 2nd items for different values of β_1, β_2.

From the *Fig. 4* suggests that the optimal cost of both items is decreased when values of β_1, β_2 are increased.

Table 6. Optimal solution of MOIM for different values of γ_1, γ_2.

Method	γ_1, γ_2	D_1^*	Q_1^*	L_1^*	TAC_1^*	D_2^*	Q_2^*	L_2^*	TAC_2^*
FNLP	-20%	2.57	10.58	0.40×10^{-3}	49.70	2.34	14.26	0.35×10^{-3}	37.60
	10%-	2.53	11.06	0.37×10^{-3}	51.89	2.32	14.94	0.32×10^{-3}	39.36
	10%	2.47	11.94	0.32×10^{-3}	55.92	2.27	16.19	0.28×10^{-3}	42.60
	20%	2.45	12.35	0.30×10^{-3}	57.78	2.25	16.77	0.26×10^{-3}	44.11

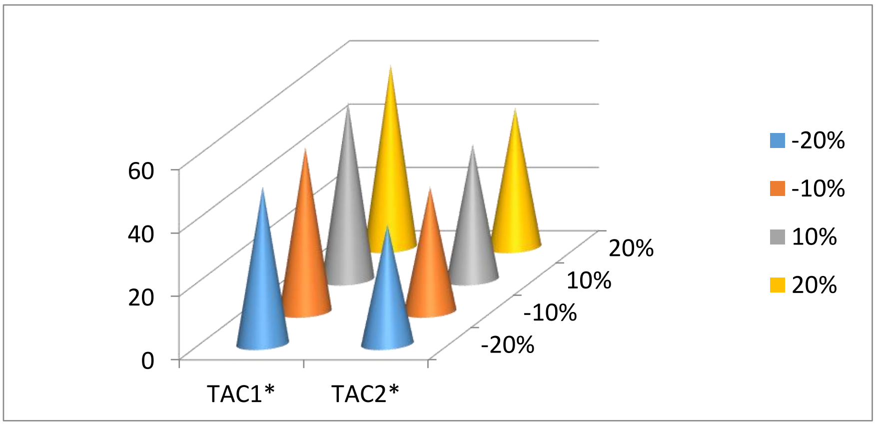

Fig. 5. Minimizing cost of both items for different values of γ_1, γ_2.

From the above *Fig. 5* suggests that the optimal cost of both items is increased when values of γ_1, γ_2 are increased.

Table 7. Optimal solutions of MOIM for different values of σ_1, σ_2.

Method	σ_1, σ_2	D_1^*	Q_1^*	L_1^*	TAC_1^*	D_2^*	Q_2^*	L_2^*	TAC_2^*
	-20%	2.54	11.36	0.35×10^{-3}	52.92	2.33	15.35	0.31×10^{-3}	40.18
	-10%	2.52	11.44	0.35×10^{-3}	53.44	2.31	15.46	0.31×10^{-3}	40.60
FNLP	10%	2.48	11.59	0.33×10^{-3}	54.47	2.28	15.70	0.29×10^{-3}	41.45
	20%	2.46	11.67	0.33×10^{-3}	54.99	2.26	15.81	0.29×10^{-3}	41.87

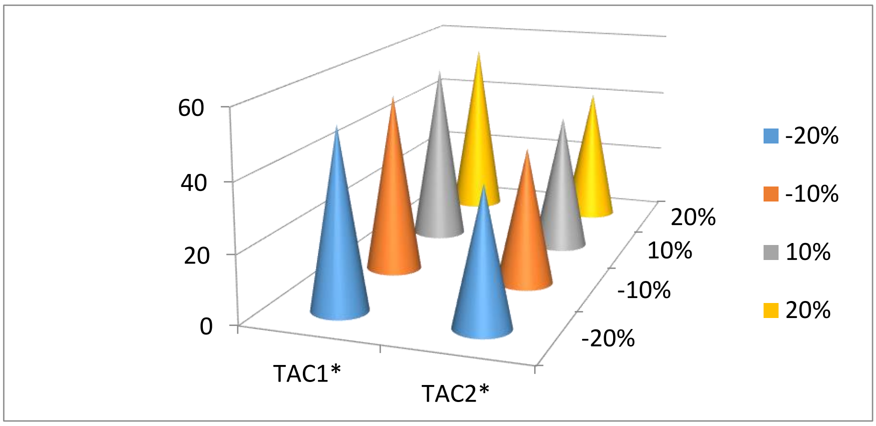

Fig. 6. Minimizing cost of 1st and 2nd items for different values of σ_1, σ_2.

From the **above *Fig. 6*** suggests that the minimum cost of both items is increased when values of σ_1, σ_2 are increased.

Table 8. Optimal solutions of MOIM for different values of ρ_1, ρ_2.

Method	ρ_1, ρ_2	D_1^*	Q_1^*	L_1^*	TAC_1^*	D_2^*	Q_2^*	L_2^*	TAC_2^*
	-20%	2.50	11.42	0.25×10^{-3}	53.44	2.29	15.47	0.23×10^{-3}	40.68
	-10%	2.50	11.47	0.30×10^{-3}	53.70	2.29	15.53	0.26×10^{-3}	40.86
FNLP	10%	2.50	11.55	0.39×10^{-3}	54.20	2.29	15.64	0.34×10^{-3}	41.19
	20%	2.50	11.60	0.43×10^{-3}	54.45	2.29	15.69	0.39×10^{-3}	41.35

Table 9. Optimal solutions of MOIM for different values of τ_1, τ_2.

Method	τ_1, τ_2	D_1^*	Q_1^*	L_1^*	TAC_1^*	D_2^*	Q_2^*	L_2^*	TAC_2^*
	-20%	2.50	11.40	0.15×10^{-3}	53.15	2.29	15.47	0.14×10^{-3}	40.58
	-10%	2.50	11.46	0.23×10^{-3}	53.54	2.29	15.52	0.21×10^{-3}	40.80
FNLP	10%	2.50	11.57	0.48×10^{-3}	54.39	2.29	15.64	0.42×10^{-3}	41.26
	20%	2.50	11.62	0.67×10^{-3}	54.83	2.29	15.69	0.57×10^{-3}	41.50

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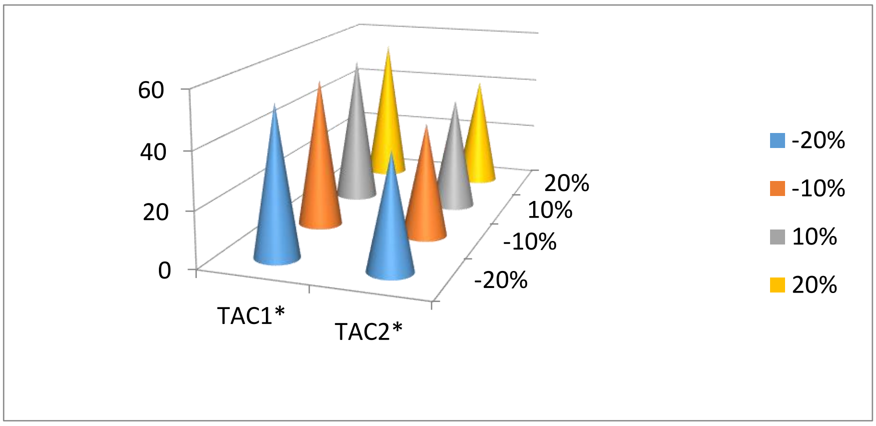

Fig. 7. **Minimizing cost of both items for different values of ρ_1, ρ_2.**

From the above *Fig. 7* suggests that the minimum cost of both items is increased when values of ρ_1, ρ_2 are increased.

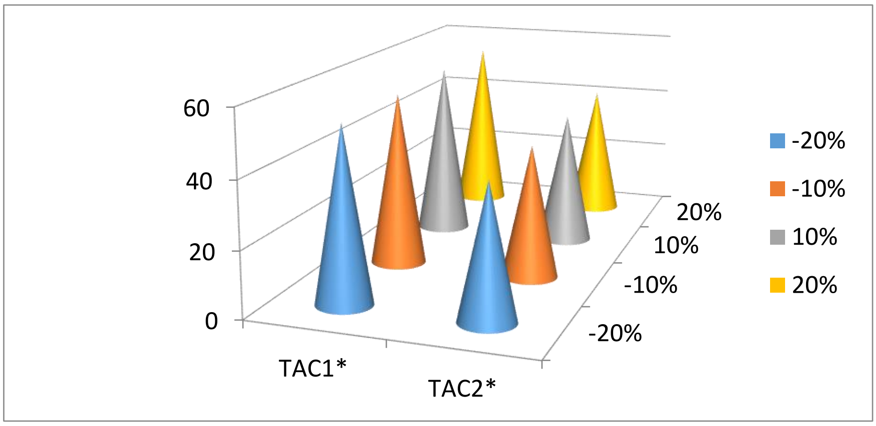

Fig. 8. Minimizing cost of 1st and 2nd items for different values of τ_1, τ_2.

From the above *Fig. 8* suggests that the minimum cost of both items is increased when values of τ_1, τ_2 are increased.

7| Conclusion

In this article, we have developed an inventory model of multi-item with limitations on storage space in a fuzzy environment. Here we considered the constant demand rate and production cost is dependent on the demand rate. Set-up- cost is dependent on average inventory level as well as demand. Lead time crashing cost is considered the continuous function of leading time. Due to uncertainty all cost parameters are taken as a generalized trapezoidal fuzzy number. The formulated problem has been solved by various techniques like GP approach, FPTHMF, FNLP, and FAGP. Numerical example is given under considering two items to illustrate the model. A numerical problem is solved by using LINGO13 software.

This paper will be extended by using linear, quadratic demand, ramp type demand, power demand, and stochastic demand etc., introduce shortages, generalize the model under two-level credit period strategy etc. Inflation plays a crucial position in Inventory Management (IM) but here it is not considered. So inflation can be used in this model for practical. Also other types of fuzzy numbers like triangular fuzzy numbers; PfFN, pFN, etc. may be used for all cost parameters of the model.

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