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Some Similarity Measures of Spherical Fuzzy Sets Based on the Euclidean Distance and Their Application in Medical Diagnosis

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ABSTRACT

Similarity measure is an important tool in multiple criteria decision-making problems, which can be used to measure the difference between the alternatives. In this paper, some new similarity measures of Spherical Fuzzy Sets (SFS) are defined based on the Euclidean distance measure and the proposed similarity measures satisfy the axiom of the similarity measure. Furthermore, we apply the proposed similarity measures to medical diagnosis decision making problem; the numerical example is used to illustrate the feasibility and effectiveness of the proposed similarity measures of SFS, which are then compared to other existing similarity measures.

1. Introduction

The concept of Fuzzy Set (FS) $A = \{< x_i, \mu_{A_{x_i}} > | x_i \in X\}$ in $X = \{x_1, x_2, \dots, x_n\}$ was proposed by Zadeh [1], where the membership degree $\mu_{A_{x_i}}$ is a single value between zero and one. The FS has been widely applied in many fields, such as medical diagnosis, image processing, supply decision-making [2-4], and so on. In some uncertain decision-making problems, the degree of membership is not exactly as a numerical value but as an interval. Hence, Zadeh [5] proposed the Interval-Valued Fuzzy Sets (IVFS). However, the FS and the IVFS only have the membership degree, and they cannot describe the non-membership degree of the element belonging to the set. Then, Atanassov [6] proposed the Intuitionistic Fuzzy Set (IFS) $E =$

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$\{< x_i, \mu_E(x_i), \vartheta_E(x_i) > | x_i \in X\}$, where $\mu_E(x_i)$ $(0 \leq \mu_E(x_i) \leq 1)$ and $\vartheta_E(x_i)$ $(0 \leq \vartheta_E(x_i) \leq 1)$ represent the membership and the non-membership degree, respectively, and the indeterminacy- membership degree $\pi_E(x_i) = 1 - \mu_E(x_i) - \vartheta_E(x_i)$. The IFS is more effective to deal with the vague information more than the FS and IVFS.

Yang and Chiclana [7] proposed a spherical representation, which allowed us to define a distance function between intuitionistic fuzzy sets. In the spherical representation, hesitancy can be calculated based on the given membership and non-membership values since they only consider the surface of the sphere. Besides, they measure the spherical arc distance between two IFSs. Furthermore, Gong et al. [8] introduced an approach generalizing Yang and Chiclana's work.

The Spherical Fuzzy Sets (SFSs) are based on the fact that the hesitancy of a decision maker can be defined independently from membership and non-membership degrees, satisfying the following condition:

$$0 \leq \mu_{\tilde{A}}{}^2(u) + \vartheta_{\tilde{A}}{}^2(u) + \pi_{\tilde{A}}{}^2(u) \leq 1. \qquad \forall\, u \in U. \tag{1}$$

On the surface of the sphere, ***Eq. (1)*** becomes

$$\mu_{\tilde{A}}{}^2(u) + \vartheta_{\tilde{A}}{}^2(u) + \pi_{\tilde{A}}{}^2(u) = 1. \qquad \forall\, u \in U. \tag{2}$$

On the other hand, similarity measure is an important tool in multiple-criteria decision making problems, which can be used to measure the difference between the alternatives. Many studies about the similarity measures have been obtained. For example, Beg and Ashraf [9] proposed a similarity measure of fuzzy sets based on the concept of $\epsilon -$ fuzzy transitivity and discussed the degree of transitivity of different similarity measures. Song et al. [4] considered the similarity measure and proposed corresponding distance measure between intuitionistic fuzzy belief functions. In addition, cosine similarity measure is also an important similarity measure, and it can be defined as the inner product of two vectors divided by the product of their lengths. There are some scholars who studied the cosine similarity measures [10-15]. Various forms of Spherical fuzzy sets which are applied in Multi–attribute decision making problems are developed in [16-18].

In this paper, we propose a new method to construct the similarity measure of SFSs. They play an important role in practical application, especially in pattern recognition, medical diagnosis, and so on. Furthermore, the proposed similarity measure can be applied more widely in the field of decision–making problems.

2. Preliminaries

Definition 1. [19]. A SFS $\tilde{A}_s$ of the universe of discourse U is given by,

$\tilde{A}_s = \{\langle \mu_{\tilde{A}_s}(u), \vartheta_{\tilde{A}_s}(u), \pi_{\tilde{A}_s}(u) | u \in U \rangle\}$, where $\mu_{\tilde{A}_s}: U \to [0,1]$, $\vartheta_{\tilde{A}_s}: U \to [0,1]$, $\pi_{\tilde{A}_s}: U \to [0,1]$ and $0 \leq \mu_{\tilde{A}_s}{}^2(u) + \vartheta_{\tilde{A}_s}{}^2(u) + \pi_{\tilde{A}_s}{}^2(u) \leq 1. \quad \forall\, u \in U.$

For each u, the numbers $\mu_{\tilde{A}_s}(u), \vartheta_{\tilde{A}_s}(u)$ and $\pi_{\tilde{A}_s}(u)$ are the degree of membership, non-membership and hesitancy of u to $\tilde{A}_s$, respectively.

Definition 2. [9]. Basic operators of spherical fuzzy sets:

Union. $\tilde{A}_s \cup \tilde{B}_s = \{max\{\mu_{\tilde{A}_s}, \mu_{\tilde{B}_s}\}, min\{\vartheta_{\tilde{A}_s}, \vartheta_{\tilde{B}_s}\}, min\{\pi_{\tilde{A}_s}, \pi_{\tilde{B}_s}\}\}$.

Intersection. $\tilde{A}_s \cap \tilde{B}_s = \{min\{\mu_{\tilde{A}_s}, \mu_{\tilde{B}_s}\}, max\{\vartheta_{\tilde{A}_s}, \vartheta_{\tilde{B}_s}\}, max\{\pi_{\tilde{A}_s}, \pi_{\tilde{B}_s}\}\}$.

Addition. $\tilde{A}_s \oplus \tilde{B}_s = \left\{\left({\mu_{\tilde{A}_s}}^2 + {\mu_{\tilde{B}_s}}^2 - {\mu_{\tilde{A}_s}}^2{\mu_{\tilde{B}_s}}^2\right)^{1/2}, \vartheta_{\tilde{A}_s}\vartheta_{\tilde{B}_s}, \ \pi_{\tilde{A}_s}\pi_{\tilde{B}_s}\right\}$.

Multiplication. $\tilde{A}_s \otimes \tilde{B}_s = \left\{\mu_{\tilde{A}_s}\mu_{\tilde{B}_s}, \ \left({\vartheta_{\tilde{A}_s}}^2 + {\vartheta_{\tilde{B}_s}}^2 - {\vartheta_{\tilde{A}_s}}^2{\vartheta_{\tilde{B}_s}}^2\right)^{1/2}, \pi_{\tilde{A}_s}\pi_{\tilde{B}_s}\right\}$.

Multiplication by a scalar, $\lambda > 0$. $\lambda.\tilde{A}_s = \left\{\left(1 - \left(1 - {\mu_{\tilde{A}_s}}^2\right)^{\lambda}\right)^{1/2}, \ {\vartheta_{\tilde{A}_s}}^{\lambda}, \ {\pi_{\tilde{A}_s}}^{\lambda}\right\}$.

Power of $\tilde{A}_s$, $\lambda > 0$. ${\tilde{A}_s}^{\lambda} = \left\{{\mu_{\tilde{A}_s}}^{\lambda}, \left(1 - \left(1 - {\vartheta_{\tilde{A}_s}}^2\right)^{\lambda}\right)^{1/2}, \ {\pi_{\tilde{A}_s}}^{\lambda}\right\}$.

3. Several New Similarity Measures

The similarity measure is a most widely used tool to evaluate the relationship between two sets. The following axiom about the similarity measure of IVSFSs should be satisfied:

Lemma 1. Let $X = \{x_1, x_2, \dots, x_n\}$ be the universal set [12] if the similarity measure S (A, B) between SFSs A and B satisfies the following properties:

- $0 \leq S(A, B) \leq 1$;
- $S(A, B) = 1$ if and only if $A = B$;
- $S(A, B) = S(B, A)$.

Then, the similarity measure $S(A, B)$ is a genuine similarity measure.

3.1. The New Similarity Measures between SFSs

Definition 3: Let $X = \{x_1, x_2, \dots, x_n\}$ be the universal set for any two SFSs $A = \{< x_i, \mu_{A_{x_i}}, \vartheta_{A_{x_i}}, \pi_{A_{x_i}} > | x_i \in X\}$ and $B = \{< x_i, \mu_{B_{x_i}}, \vartheta_{B_{x_i}}, \pi_{B_{x_i}} > | x_i \in X\}$; then the Euclidean distance between SFSs A and B is defined as follows:

$$D_{SFSs}(A, B) = \sqrt{\frac{\sum_{i=1}^{n}\left[\left(\mu_A{}^2(x_i) - \mu_B{}^2(x_i)\right)^2 + \left(\vartheta_A{}^2(x_i) - \vartheta_B{}^2(x_i)\right)^2 + \left(\pi_A{}^2(x_i) - \pi_B{}^2(x_i)\right)^2\right]}{3n}}. \quad (3)$$

Now, we construct new similarity measures of SFSs based on the Euclidean distance measures.

Definition 4. Let $X = \{x_1, x_2, \dots, x_n\}$ be the universal set for any two SFSs $A = \{< x_i, \mu_{A_{x_i}}, \vartheta_{A_{x_i}}, \pi_{A_{x_i}} > | x_i \in X\}$ and $B = \{< x_i, \mu_{B_{x_i}}, \vartheta_{B_{x_i}}, \pi_{B_{x_i}} > | x_i \in X\}$; the similarity measure of SFSs between A and B is defined as follows:

$$S_{1SFSs}(A,B)=\frac{\sum_{i=1}^{n}\left(\min\left(\mu_A{}^2(x_i),\mu_B{}^2(x_i)\right)+\min\left(\vartheta_A{}^2(x_i),\vartheta_B{}^2(x_i)\right)+\min\left(\pi_A{}^2(x_i),\pi_B{}^2(x_i)\right)\right)}{\sum_{i=1}^{n}\left(\max\left(\mu_A{}^2(x_i),\mu_B{}^2(x_i)\right)+\max\left(\vartheta_A{}^2(x_i),\vartheta_B{}^2(x_i)\right)+\max\left(\pi_A{}^2(x_i),\pi_B{}^2(x_i)\right)\right)}. \quad (4)$$

The similarity measure S_{1SFSs} satisfies the properties in ***Lemma 1***.

Next, we propose a new method to construct a new similarity measure of SFSs, and the Euclidean distance, it can be defined as follows:

Definition 5. Let $X=\{x_1,x_2,\dots,x_n\}$ be the universal set for any two SFSs $A=\{<x_i,\mu_{A_{x_i}},\vartheta_{A_{x_i}},\pi_{A_{x_i}}>|x_i\in X\}$ and$B=\{<x_i,\mu_{B_{x_i}},\vartheta_{B_{x_i}},\pi_{B_{x_i}}>|x_i\in X\}$; a new similarity measure $S^*{}_{1SFSs}(A,B)$ is defined as follows:

$$S^*{}_{1SFSs}(A,B)=\frac{1}{2}\left(S_{1SFSs}(A,B)+1-D_{SFSs}(A,B)\right). \quad (5)$$

The proposed similarity measure of SFSs satisfies the ***Theorem 1***.

Theorem 1. The similarity measure $S^*{}_{1SFSs}(A,B)$ between $A=\{<x_i,\mu_{A_{x_i}},\vartheta_{A_{x_i}},\pi_{A_{x_i}}>|x_i\in X\}$ and$B=\{<x_i,\mu_{B_{x_i}},\vartheta_{B_{x_i}},\pi_{B_{x_i}}>|x_i\in X\}$ satisfies the following properties:

- $0\le S^*{}_{1SFSs}(A,B)\le 1$
- $S^*{}_{1SFSs}(A,B)=1$ if and only if $A=B$
- $S^*{}_{1SFSs}(A,B)=S^*{}_{1SFSs}(B,A)$.

Proof. Because $D_{SFSs}(A,B)$ is an Euclidean distance measure, obviously, $0\le D_{SFSs}(A,B)\le 1$. Furthermore, according to lemma 1, we know that $0\le S_{1SFSs}(A,B)\le 1$. Then, $0\le\frac{1}{2}\left(S_{1SFSs}(A,B)+1-D_{SFSs}(A,B)\right)\le 1$, i.e., $0\le S^*{}_{1SFSs}(A,B)\le 1$.

If $S^*{}_{1SFSs}(A,B)=1$, we have $S_{1SFSs}(A,B)+1-D_{SFSs}(A,B)=2$, that is $S_{1SFSs}(A,B)=1+D_{SFSs}(A,B)$. Because $D_{SFSs}(A,B)$ is the Euclidean distance measure $0\le D_{SFSs}(A,B)\le 1$. Furthermore, $0\le S_{1SFSs}(A,B)\le 1$, then $S_{1SFSs}(A,B)=1$ and $D_{SFSs}(A,B)=0$ should be established at the same time. If the Euclidean distance measure $D_{SFSs}(A,B)=0, A=B$ is obvious. According to lemma 1, when $S_{1SFSs}(A,B)=1, A=B$; so if $S^*{}_{1SFSs}(A,B)=1, A=B$ is obtained.

On the other hand, when $A=B$, according to ***Eqs. (3)*** and *(4)* $D_{SFSs}(A,B)=0$ and $S_{1SFSs}(A,B)=1$ are obtained respectively. Furthermore, we can get $S^*{}_{1SFSs}(A,B)=1$. $S^*{}_{1SFSs}(A,B)=S^*{}_{1SFSs}(B,A)$ is straightforward.

From ***Theorem 1***, we know that the proposed new similarity measure $S^*{}_{1SFSs}(A,B)$ is a genuine similarity measure. On the other hand, cosine similarity measure is also an important similarity measure. The cosine similarity measure between SFSs is as follows:

Definition 6. Let $X=\{x_1,x_2,\dots,x_n\}$ be the universal set for any two SFSs $A=\{<x_i,\mu_{A_{x_i}},\vartheta_{A_{x_i}},\pi_{A_{x_i}}>|x_i\in X\}$ and $B=\{<x_i,\mu_{B_{x_i}},\vartheta_{B_{x_i}},\pi_{B_{x_i}}>|x_i\in X\}$; the cosine similarity measure of SFSs between A and B is defined as follows:

$$S_{2SFSs}(A,B) = \frac{1}{n}\sum_{i=1}^{n} \frac{\left(\left(\mu_A{}^2(x_i)\mu_B{}^2(x_i)\right) + \left(\vartheta_A{}^2(x_i)\,\vartheta_B{}^2(x_i)\right) + \left(\pi_A{}^2(x_i)\,\pi_B{}^2(x_i)\right)\right)}{\sqrt{(\mu_A{}^2(x_i))^2 + (\vartheta_A{}^2(x_i))^2 + (\pi_A{}^2(x_i))^2}\sqrt{(\mu_B{}^2(x_i))^2 + (\vartheta_B{}^2(x_i))^2 + (\pi_B{}^2(x_i))^2}}. \quad (6)$$

Now, we are going to propose another similarity measure of SFSs based on the cosine similarity measure and the Euclidean distance D_{SFSs}. It considers the similarity measure not only from the point of view of algebra but also from the point of view of geometry, which can be defined as:

Definition 7. Let $X = \{x_1, x_2, \ldots, x_n\}$ be the universal set for any two SFSs

$A = \{< x_i, \mu_{A_{x_i}}, \vartheta_{A_{x_i}}, \pi_{A_{x_i}} > | x_i \in X\}$ and $B = \{< x_i, \mu_{B_{x_i}}, \vartheta_{B_{x_i}}, \pi_{B_{x_i}} > | x_i \in X\}$; a new similarity measure $S^*{}_{2SFSs}(A,B)$ is defined as follows:

$$S^*{}_{2SFSs}(A,B) = \frac{1}{2}\left(S_{2SFSs}(A,B) + 1 - D_{SFSs}(A,B)\right). \quad (7)$$

Theorem 2. The similarity measure $S^*{}_{2SFSs}(A,B)$ between $A = \{< x_i, \mu_{A_{x_i}}, \vartheta_{A_{x_i}}, \pi_{A_{x_i}} > | x_i \in X\}$ and

$B = \{< x_i, \mu_{B_{x_i}}, \vartheta_{B_{x_i}}, \pi_{B_{x_i}} > | x_i \in X\}$ satisfies the following properties:

- $0 \le S^*{}_{2SFSs}(A,B) \le 1$;
- $S^*{}_{2SFSs}(A,B) = 1$ if and only if $A = B$;
- $S^*{}_{2SFSs}(A,B) = S^*{}_{2SFSs}(B,A)$.

Proof. Because $D_{SFSs}(A,B)$ is an Euclidean distance measure, obviously, $0 \le D_{SFSs}(A,B) \le 1$. Furthermore, according to lemma 1, we know that $0 \le S_{SFSs}(A,B) \le 1$. Then, $0 \le \frac{1}{2}\left(S_{2SFSs}(A,B) + 1 - D_{SFSs}(A,B)\right) \le 1$, i.e., $0 \le S^*{}_{1SFSs}(A,B) \le 1$.

If $S^*{}_{2SFSs}(A,B) = 1$, we have $S_{2SFSs}(A,B) + 1 - D_{SFSs}(A,B) = 2$, that is $S_{2SFSs}(A,B) = 1 + D_{SFSs}(A,B)$. Because $D_{SFSs}(A,B)$ is the Euclidean distance measure $0 \le D_{SFSs}(A,B) \le 1$. Furthermore, $0 \le S_{2SFSs}(A,B) \le 1$, then $S_{2SFSs}(A,B) = 1$ and $D_{SFSs}(A,B) = 0$ should be established at the same time. When $S_{2SFSs}(A,B) = 1$, we have $\mu_A(x_i) = k\mu_B(x_i)$, $\vartheta_A(x_i) = k\vartheta_B(x_i), and\ \pi_A(x_i) = k\pi_B(x_i)$ (k is a constant). When the Euclidean distance measure $D_{SFSs}(A,B) = 0, A = B$. Then $A = B$ is obtained.

On the other hand, when $A = B$, according to ***Eqs. (3)*** and ***(6)*** if $A = B$, $D_{SFSs}(A,B) = 0$ and $S_{2SFSs}(A,B) = 1$ are obtained respectively. Furthermore, we can get $S^*{}_{2SFSs}(A,B) = 1$. $S^*{}_{2SFSs}(A,B) = S^*{}_{2SFSs}(B,A)$ is straightforward.

Thus $S^*{}_{2SFSs}(A,B)$ satisfies all the properties of the ***Theorem 2***.

In the next section, we will apply the proposed new similarity measures to medical diagnosis decision problem; numerical examples are also given to illustrate the application and effectiveness of the proposed new similarity measures.

4. Applications of the Proposed Similarity Measures

4.1. The Proposed Similarity Measures between SFSs for Medical Diagnosis

We first give a numerical example medical diagnosis to illustrate the feasibility of the proposed new similarity measur $S^*_{1SFSs}(A,B)$e and $S^*_{2SFSs}(A,B)$ between SFSs.

Example 1. Consider a medical diagnosis decision problem; Suppose a set of diagnosis $Q = \{Q_1$(viral fever), Q_2(malaria), Q_3(typhoid), Q_4(Gastritis), Q_5(stenocardia)} and a set of symptoms S ={S_1(fever), S_2(headache), S_3(stomach), S_4(cough), S_5(chestpain)}. Assume a patient P_1 has all the symptoms in the process of diagnosis, the SFS evaluate information about P_1 is

P_1(Patient= {$< S_1, 0.8, 0.2, 0.1 >, < S_2, 0.6, 0.3, 0.1 >, < S_3, 0.2, 0.1, 0.8 >) < S_4, 0.6, 0.5, 0.1 >$,
$<S_5, 0.1, 0.4, 0.6 >$}.

The diagnosis information $Q_i(i = 1,2,\ldots,5)$ with respect to symptoms $S_i(i = 1,2,\ldots,5)$ also can be represented by the SFSs, which is shown in ***Table 1***.

Table 1. Diagnosis information.

	S_1	S_2	S_3	S_4	S_5
Q_1	[0.4, 0.6,0.0]	[0.3, 0.2, 0.5]	[0.1, 0.3, 0.7]	[0.4, 0.3, 0.3]	[0.1, 0.2, 0.7]
Q_2	[0.7, 0.3, 0.0]	[0.2, 0.2, 0.6]	[0.0, 0.1, 0.9]	[0.7, 0.3, 0.0]	[0.1, 0.1, 0.8]
Q_3	[0.3, 0.4, 0.3]	[0.6, 0.3, 0.1]	[0.2, 0.1, 0.7]	[0.2, 0.2, 0.6]	[0.1, 0.0, 0.9]
Q_4	[0.1, 0.2, 0.7]	[0.2, 0.2, 0.4]	[0.8, 0.2, 0.0]	[0.2, 0.1, 0.7]	[0.2, 0.1, 0.7]
Q_5	[0.1, 0.1, 0.8]	[0.0, 0.2,0.8]	[0.2, 0.0, 0.8]	[0.3, 0.1, 0.8]	[0.8, 0.1, 0.1]

By applying ***Eqs. (5)*** and *(7)* we can obtain the similarity measure values$S^*_{1SFSs}(P_1, Q_i)$ and $S^*_{2SFSs}(P_1, Q_i)$; the results are shown in ***Table 2***.

Table 2. Similarity measures.

	Q_1	Q_2	Q_3	Q_4	Q_5
$S^*_{1SFSs}(P_1, Q_i)$	0.5980	0.6801	0.5729	0.3919	0.3820
$S^*_{2SFSs}(P_1, Q_i)$	0.4277	0.4581	0.4024	0.3514	0.3155

From the above two similarity measures S^*_{1SFSs} and S^*_{2SFSs} , we can conclude that the diagnoses of the patient P_1 are all malaria (Q_2). The proposed two similarity measures are feasible and effective.

4.2. Comparative Analysis of Existing Similarity Measures

To illustrative the effectiveness of the proposed similarity measures for medical diagnosis, we change the existing similarity measures for SFS and thus will apply the existing similarity measures for comparative analyses.

At first, we introduce the existing similarity measures between SFSs as follows:

Let $A = \{< x_i, \mu_{A_{x_i}}, \vartheta_{A_{x_i}}, \pi_{A_{x_i}} > | x_i \in X\}$ an $B = \{< x_i, \mu_{B_{x_i}}, \vartheta_{B_{x_i}}, \pi_{B_{x_i}} > | x_i \in X\}$d be two SFSs in $X = \{x_1, x_2, \dots, x_n\}$, the existing measures between A and B are defined as follows:

Broumi et al. [20] proposed the similarity measure SM_{SFS}:

$$SM_{SFS}(A, B) = 1 - D_{SFS}(A, B). \quad (8)$$

Sahin and Küçük [21] proposed the similarity measure SD_{SFS}:

$$SD_{SFS} = \frac{1}{1 + D_{SFS}(A, B)}. \quad (9)$$

Ye [22] proposed the improved cosine similarity measure SC_{1SFS} and SC_{2SFS}:

$$SC_{1SFS}(A, B) = \frac{1}{n}\sum_{i=1}^{n} \cos\left[\frac{\pi.\max\left(|\mu_A{}^2(x_i) - \mu_B{}^2(x_i)|, |\vartheta_A{}^2(x_i) - \vartheta_B{}^2(x_i)|, |\pi_A{}^2(x_i) - \pi_B{}^2(x_i)|\right)}{2}\right]. \quad (10)$$

$$SC_{2SFS}(A, B) = \frac{1}{n}\sum_{i=1}^{n} \cos\left[\frac{\pi.\left(|\mu_A{}^2(x_i) - \mu_B{}^2(x_i)| + |\vartheta_A{}^2(x_i) - \vartheta_B{}^2(x_i)| + |\pi_A{}^2(x_i) - \pi_B{}^2(x_i)|\right)}{6}\right]. \quad (11)$$

Yong-Wei et al. [23] proposed the similarity measure $SY_{SFS}(A, B)$:

$$SY_{SFS}(A, B) = \frac{SC_{SFS}(A, B)}{SC_{SFS}(A, B) + D_{SFS}(A, B)}. \quad (12)$$

Example 2. We apply ***Eqs. (4)***, ***(6)*** and ***(8) – (12)*** to calculate ***Example 1*** again; the similarity measure values between P_1 and Q_i $(i = 1,2,..,5)$ are shown on ***Table 3***.

As we can see from ***Table 3***, the patient P_1 is still assigned to malaria (Q_2), and the results are same as the proposed similarity measures in this paper, which means the proposed similarity measures are feasible and effective.

Table 3. Similarity values between patient and symptoms.

	Q_1	Q_2	Q_3	Q_4	Q_5
SM_{SFS}	0.8003	0.8314	0.7449	0.6388	0.6007
SD_{SFS}	0.8335	0.8557	0.7967	0.7346	0.7146
SC_{1SFS}	0.8555	0.9325	0.6469	0.7324	0.6391
SC_{2SFS}	0.9648	0.9759	0.7531	0.885	0.8585
SY_{SFS}	0.8107	0.8468	0.7171	0.6697	0.6154
S_{1SFS}	0.3958	0.5289	0.4010	0.1451	0.1633
S_{2SFS}	0.0551	0.0849	0.0600	0.0191	0.0304

The proposed similarity measures in the paper have some advantages in solving multiple criteria decision making problems. They are constructed based on the existing similarity measures and Euclidean distance, which not only satisfy the axiom of the similarity measure but also consider the similarity measure from the points of view of algebra and geometry. Furthermore, they can be applied more widely in the field of decision making problems.

5. Conclusion

The similarity measure is widely used in multiple criteria decision making problems. This paper proposed a new method to construct the similarity measures combining the existing cosine similarity measure and the Euclidean distance measure. And, the similarity measures are proposed not only from the points of view of algebra and geometry but also satisfy the axiom of the similarity measure. Furthermore, we apply the proposed similarities measures to the medical diagnosis decision problems, and the numerical example is used to illustrate the feasibility and effectiveness of the proposed similarity measure, which are then compared to other existing similarity measures.

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