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Interval Valued Pythagorean Fuzzy Ideals in Semigroups

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ABSTRACT

In this paper, we define the new notion of interval-valued Pythagorean fuzzy ideals in semigroups and established the properties of its with suitable examples. Also, we introduce the concept of interval valued Pythagorean fuzzy sub-semigroup, interval valued Pythagorean fuzzy left (resp. right) ideal, interval valued Pythagorean fuzzy bi-ideal, interval valued Pythagorean fuzzy interior ideal and homomorphism of an interval valued Pythagorean fuzzy ideal in semigroups with suitable illustration. We show that every interval valued Pythagorean fuzzy left (resp. right) ideal is an interval valued Pythagorean fuzzy bi-ideal.

1. Introduction

In 1965, Zadeh [18, 19] introduced the concept of a fuzzy set. He also developed the notion of interval-valued fuzzy set in 1975, which extends the fuzzy set. A semigroup is an algebraic structure comprising a non-empty set together with an associative binary operation. Atanassov [2] introduced the intuitionistic fuzzy set with some properties. Atanassov [3] developed the concept of interval-valued intuitionistic fuzzy set. Thillaigovindan and Chinnadurai [15, 16] discussed interval-valued fuzzy ideals in algebraic structures. In 2018, Chen [4, 5] introduced the concept of interval-valued Pythagorean fuzzy outranking of various methods in the application. Garg [8, 9] presented the notion of interval-valued Pythagorean fuzzy sets of multi-criteria decision-making methods. In 2013, Yager [17] started the notion of Pythagorean fuzzy set, the sum of the squares of membership and non-membership belongs to the unit interval [0, 1]. Peng [13]

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developed the new operations for an interval-valued Pythagorean fuzzy set. Peng and Yang [14] presented the notion of interval-valued Pythagorean fuzzy set. In 2019, Hussain et al. [10] started the notions of rough Pythagorean fuzzy ideals in the semigroups. Akram[1] established the properties of fuzzy lie algebras. Kumar et al. [11] approached transportation decision making problems using Pythagorean fuzzy set. Das and Edalatpanah [6] studied the concept of fuzzy linear fractional progress with trapezoidal fuzzy numbers. Edalatpanah [7] used triangular intuitionistic fuzzy numbers to deal with data envelopment analysis model. Najafi and Edalatpanah [12] used iterative methods to study linear complementarily problems. In this paper, we discuss some of the properties of interval-valued Pythagorean fuzzy ideals in the semigroups.

2. Preliminaries

Definition 1. [12]. Let X be a universe of discourse, A Pythagorean fuzzy set (PFS) $P = \{w, \phi_p(w), \psi_p(w)/w \in X\}$ where $\phi: X \to [0,1]$ and $\psi: X \to [0,1]$ represent the degree of membership and non-membership of the object $w \in X$ to the set P subset to the condition $0 \leq (\phi_p(w))^2 + (\psi_p(w))^2 \leq 1$ for all $w \in X$. For the sake of simplicity a PFS is denoted as $P = (\phi_p(w), \psi_p(w))$.

3. Interval-Valued Pythagorean Fuzzy Ideals in Semigroups

Definition 2. An Interval-Valued Pythagorean Fuzzy Set (IVPFS) $\tilde{P} = [\widetilde{\phi_p}, \widetilde{\psi_p}]$ on S is known to be an interval-valued Pythagorean fuzzy sub-semigroup of S. If for all $w_1, w_2 \in S$, it holds.

$$\widetilde{\phi_p}(w_1w_2) \geq \min\{\widetilde{\phi_p}(w_1), \widetilde{\phi_p}(w_2)\},$$

$$\widetilde{\psi_p}(w_1w_2) \leq \max\{\widetilde{\psi_p}(w_1), \widetilde{\psi_p}(w_2)\}.$$

Example 1. Consider a semigroup $S = \{u, v, w, x, y\}$ with the Cayley Table.

Table 1. Cayley table.

•	u	v	w	x	y
u	u	u	u	u	u
v	u	v	u	x	u
w	u	y	w	w	y
x	u	v	x	x	v
y	u	y	u	w	u

Define an interval-valued Pythagorean fuzzy set(IVPFS) $\tilde{P} = [\widetilde{\phi_p}, \widetilde{\psi_p}]$ in S as follows.

S	$[\widetilde{\phi_p}(w_1), \widetilde{\psi_p}(w_1)]$
u	[0.7,0.8], [0.1,0.2]
v	[0.4,0.6], [0.4,0.5]
w	[0.3,0.5], [0.5,0.6]
x	[0.1,0.2], [0.3,0.5]
y	[0.3,0.5], [0.5,0.6]

$\widetilde{\phi_p}(uv) \geq \min\{\widetilde{\phi_p}(u), \widetilde{\phi_p}(v)\}$
([0.7,0.8],[0.1,0.2]) ≥[0.4,0.6],[0.1,0.2].
$\widetilde{\psi_p}(uv) \leq \max\{\widetilde{\psi_p}(u), \widetilde{\psi_p}(v)\}$.
([0.7,0.8],[0.1,0.2])≤[0.7,0.8],[0.4,0.5].

Thus $\tilde{P} = [\widetilde{\phi_p}, \widetilde{\psi_p}]$ is an Interval-Valued Pythagorean Fuzzy Sub-Semigroup (IVPFSS) of S.

Definition 3. An IVPFS $\tilde{P} = (\widetilde{\phi_p}, \widetilde{\psi_p})$ on semigroup S, is said to be an interval-valued Pythagorean fuzzy left $(\tilde{P}_{LI})$(resp.right$(\tilde{P}_{RI})$) ideal of S. If for all $w_1, w_2 \in S$, it holds.

$\widetilde{\phi_p}(w_1w_2) \geq \widetilde{\phi_p}(w_2)$;
$\widetilde{\psi_p}(w_1w_2) \leq \widetilde{\psi_p}(w_2)$ (resp.right$(\tilde{P}_{RI})$);
$\widetilde{\phi_p}(w_1w_2) \geq \widetilde{\phi_p}(w_1)$;
$\widetilde{\psi_p}(w_1w_2) \leq \widetilde{\psi_p}(w_1)$.

Definition 4. An IVPFS $\tilde{P} = [\widetilde{\phi_p}, \widetilde{\psi_p}]$ on S is called IVPFI $(\tilde{P}_I)$ of S. If for all $w_1, w_2 \in S$, it $\widetilde{P}$ is both a left and right IVPFI of S.

$(w_1w_2) \geq \max\{\widetilde{\phi_p}(w_1), \widetilde{\phi_p}(w_2)\}$;

$\widetilde{\psi_p}(w_1w_2) \leq \min\{\widetilde{\psi_p}(w_1), \widetilde{\psi_p}(w_2)\}$.

Definition 5. An IVPFS $\tilde{P} = [\widetilde{\phi_p}, \widetilde{\psi_p}]$ on S is known to be an interval-Valued Pythagorean Fuzzy Bi-Ideal (IVPFBI) $(\tilde{P}_{BI})$ of S. If for all $a, w_1, w_2 \in S$ and satisfy.

$\widetilde{\phi_p}(w_1aw_2) \geq \min\{\widetilde{\phi_p}(w_1), \widetilde{\phi_p}(w_2)\}$;

$\widetilde{\psi_p}(w_1aw_2) \leq max\{\widetilde{\psi_p}(w_1), \widetilde{\psi_p}(w_2)\}$.

Example 2. Consider a semigroup $S = \{u, v, w, x, y\}$ with the Cayley Table.

Define an interval-valued Pythagorean fuzzy set $\tilde{P} = [\widetilde{\phi_p}, \widetilde{\psi_p}]$ in S as follows.

S	$[\widetilde{\phi_p}(w_1), \widetilde{\psi_p}(w_1)]$
u	[0.8,0.9], [0.1,0.3]
v	[0.3,0.5], [0.7,0.9]
w	[0.4,0.6], [0.6,0.7]
x	[0.3,0.5], [0.7,0.9]
y	[0.7,0.8], [0.4,0.5]

Thus $\tilde{P} = [\widetilde{\phi_p}, \widetilde{\psi_p}]$ is an interval valued Pythagorean fuzzy bi-ideal of S.

Definition 7. An IVPFS $\tilde{P} = \langle[\widetilde{\phi_p}, \widetilde{\psi_p}]\rangle$ on S is known to be an interval-valued Pythagorean fuzzy interior ideal (IVPFII) $(\tilde{P}_{II})$ of S. If for all $a, w_1, w_2 \in S$ and satisfy.

$\widetilde{\phi_p}(w_1aw_2) \geq \widetilde{\phi_p}(a)$;

$\widetilde{\psi_p}(w_1 a w_2) \leq \widetilde{\psi_p}(a)$.

Definition 8. For any non-empty subset N of a semigroup S is defined to be a structure $\chi_N = \{w_1, [\tilde{\phi}_{\chi_N}(w_1), \tilde{\psi}_{\chi_N}(w_1)] | w_1 \in S\}$ which is briefly denoted by $\chi_N = [\tilde{\phi}_{\chi_N}, \tilde{\psi}_{\chi_N}]$

where, $\tilde{\phi}_{\chi_N}(w_1) = \begin{cases} \tilde{1}\ if\ x \in N \\ \tilde{0}\ otherwise \end{cases}$ $\widetilde{\psi}_{\chi_N}(w_1) = \begin{cases} \tilde{0}\ if\ x \in N \\ \tilde{1}\ otherwise \end{cases}$.

Theorem 1. Let S be a semigroup. Then the following are equivalent.

- The intersection of two interval-valued Pythagorean fuzzy sub-semigroup of S, is an interval-valued Pythagorean fuzzy sub-semigroup of S.
- The intersection of two interval-valued Pythagorean fuzzy left (resp. right) ideal of S, is IVPFLI (resp. IVPFRI) of S.

Proof. Let $\widetilde{P_1} = [\tilde{\phi}_{p_1}, \tilde{\psi}_{p_1}]$ and $\widetilde{P_2} = [\tilde{\phi}_{p_2}, \tilde{\psi}_{p_2}]$ be two interval-valued Pythagorean fuzzy sub-semigroup of S. Let $w_1, w_2 \in S$.

Then,

$$(\widetilde{\phi}_{p_1} \cap \widetilde{\phi}_{p_2})(w_1, w_2) = \min\{\widetilde{\phi}_{p_1}(w_1, w_2), \widetilde{\phi}_{p_2}(w_1, w_2)\}$$
$$\geq \min\left\{\min\{\widetilde{\phi}_{p_1}(w_1), \widetilde{\phi}_{p_1}(w_2)\}, \min\{\widetilde{\phi}_{p_2}(w_1), \widetilde{\phi}_{p_2}(w_2)\}\right\}$$
$$= \min\left\{\min\{\widetilde{\phi}_{p_1}(w_1), \widetilde{\phi}_{p_2}(w_1)\}, \min\{\widetilde{\phi}_{p_1}(w_2), \widetilde{\phi}_{p_2}(w_2)\}\right\}$$
$$= \min\{\widetilde{\phi}_{p_1} \cap \widetilde{\phi}_{p_2}(w_1), \widetilde{\phi}_{p_1} \cap \widetilde{\phi}_{p_2}(w_2)\};$$

$$(\widetilde{\psi}_{p_1} \cup \widetilde{\psi}_{p_2})(w_1, w_2) = \max\{\widetilde{\psi}_{p_1}(w_1, w_2), \widetilde{\psi}_{p_2}(w_1, w_2)\}$$
$$\leq \max\left\{\max\{\widetilde{\psi}_{p_1}(w_1), \widetilde{\psi}_{p_1}(w_2)\}, \max\{\widetilde{\psi}_{p_2}(w_1), \widetilde{\psi}_{p_2}(w_2)\}\right\}$$
$$= \max\left\{\max\{\widetilde{\psi}_{p_1}(w_1), \widetilde{\psi}_{p_2}(w_1)\}, \max\{\widetilde{\psi}_{p_1}(w_2), \widetilde{\psi}_{p_2}(w_2)\}\right\}$$
$$= \max\{\widetilde{\psi}_{p_1} \cup \widetilde{\psi}_{p_2}(w_1), \widetilde{\psi}_{p_1} \cup \widetilde{\psi}_{p_2}(w_2)\}.$$

Therefore, $\tilde{P}_1 \cap \tilde{P}_2 = \{\langle(\widetilde{\phi}_{p_1} \cap \widetilde{\phi}_{p_2}), (\widetilde{\psi}_{p_1} \cup \widetilde{\psi}_{p_2})\rangle\}$.

Interval-valued Pythagorean fuzzy sub-semigroup of S.

$$(\widetilde{\phi}_{p_1} \cap \widetilde{\phi}_{p_2})(w_1, w_2) = \min\{\widetilde{\phi}_{p_1}(w_1, w_2), \widetilde{\phi}_{p_2}(w_1, w_2)\}$$
$$\geq \min\{\widetilde{\phi}_{p_1}(w_2), \widetilde{\phi}_{p_2}(w_2)\}$$
$$= (\widetilde{\phi}_{p_1} \cap \widetilde{\phi}_{p_2})(w_2);$$

$$(\widetilde{\psi}_{p_1} \cup \widetilde{\psi}_{p_2})(w_1, w_2) = \max\{\widetilde{\psi}_{p_1}(w_1, w_2), \widetilde{\psi}_{p_2}(w_1, w_2)\}$$
$$\leq \max\{\widetilde{\psi}_{p_1}(w_2), \widetilde{\psi}_{p_2}(w_2)\}$$
$$= (\widetilde{\psi}_{p_1} \cup \widetilde{\psi}_{p_2})(w_2).$$

Therefore, $\tilde{P}_1 \cap \tilde{P}_2 = \{\langle(\tilde{\phi}_{p_1} \cap \tilde{\phi}_{p_2}), (\tilde{\psi}_{p_1} \cup \tilde{\psi}_{p_2})\rangle\}$ is an interval-valued Pythagorean fuzzy left (resp. right) ideal of S.

Theorem 2. An IVPFS $\tilde{P} = [\tilde{\phi}_p, \tilde{\psi}_p]$ of a semigroup S is an IVPFBI of S, if and only if $\langle(\phi_p^L, \phi_p^U), (\psi_p^L, \psi_p^U)\rangle$ of S.

Proof. Let $\tilde{P} = [\tilde{\phi}_p, \tilde{\psi}_p]$ be an interval-valued Pythagorean fuzzy bi-ideal of S, for any $w_1, w_2 \in S$. Then, we have membership

$$[\phi_p^L(w_1w_2), \phi_p^U(w_1w_2)] = \tilde{\phi}_p(w_1w_2)$$
$$\geq \min\{\tilde{\phi}_p(w_1), \tilde{\phi}_p(w_2)\}$$
$$= \min\{[\phi_p^L(w_1), \phi_p^U(w_1)], [\phi_p^L(w_2), \phi_p^U(w_2)]\}$$
$$= \min\{[\phi_p^L(w_1), \phi_p^L(w_2)], [\phi_p^U(w_1), \phi_p^U(w_2)]\}.$$

It follows that $\phi_p^L(w_1w_2) \geq min\{\phi_p^L(w_1), \phi_p^L(w_2)\}$ and $\phi_p^U(w_1w_2) \geq min\{\phi_p^U(w_1), \phi_p^U(w_2)\}$and non-membership

$$[\psi_p^L(w_1w_2), \psi_p^U(w_1w_2)] = \tilde{\psi}_p(w_1w_2)$$
$$\leq \max\{\tilde{\psi}_p(w_1), \tilde{\psi}_p(w_2)\}$$
$$= \max\{[\psi_p^L(w_1), \psi_p^U(w_1)], [\psi_p^L(w_2), \psi_p^U(w_2)]\}$$
$$= \max\{[\psi_p^L(w_1), \psi_p^L(w_2)], [\psi_p^U(w_1), \psi_p^U(w_2)]\}.$$

It follows that $\psi_p^L(w_1w_2) \leq max\{\psi_p^L(w_1), \psi_p^L(w_2)\}$ and $\psi_p^U(w_1w_2) \leq max\{\psi_p^U(w_1), \phi_p^U(w_2)\}$

Therefore, $\tilde{P} = \langle(\phi_p^L, \phi_p^U), (\psi_p^L, \psi_p^U)\rangle$ are Pythagorean fuzzy ideal of S.

Conversely, suppose that $([\phi_p^L, \phi_p^U], [\psi_p^L, \psi_p^U])$ are Pythagorean fuzzy ideal of S, le$w_1, w_2 \in S$ t.

$$\tilde{\phi}_p(w_1w_2) = [\phi_p^L(w_1w_2), \phi_p^U(w_1w_2)]$$
$$\geq [\min\{\phi_p^L(w_1), \phi_p^L(w_2)\}, \min\{\phi_p^U(w_1), \phi_p^U(w_2)\}]$$
$$= \min\{[\phi_p^L(w_1), \phi_p^U(w_1)], [\phi_p^L(w_2), \phi_p^U(w_2)]\}$$
$$= \min\{\tilde{\phi}_p(w_1), \tilde{\phi}_p(w_2)\};$$

$$\tilde{\psi}_p(w_1w_2) = [\psi_p^L(w_1w_2), \psi_p^U(w_1w_2)]$$
$$\leq [\max\{\psi_p^L(w_1), \psi_p^L(w_2)\}, \max\{\psi_p^U(w_1), \psi_p^U(w_2)\}]$$
$$= \max\{[\psi_p^L(w_1), \psi_p^U(w_1)], [\psi_p^L(w_2), \psi_p^U(w_2)]\}$$
$$= \max\{\tilde{\psi}_p(w_1), \tilde{\psi}_p(w_2)\}.$$

$\tilde{P} = [\tilde{\phi}_p, \tilde{\psi}_p]$ is an interval-valued Pythagorean fuzzy sub-semigroup of S.

$$\tilde{\phi}_p(w_1aw_2) = [\phi_p^L(w_1aw_2), \phi_p^U(w_1aw_2)]$$
$$\geq [\min\{\phi_p^L(w_1), \phi_p^L(w_2)\}, \min\{\phi_p^U(w_1), \phi_p^U(w_2)\}]$$
$$= \min\{[\phi_p^L(w_1), \phi_p^U(w_1)], [\phi_p^L(w_2), \phi_p^U(w_2)]\}$$
$$= \min\{\tilde{\phi}_p(w_1), \tilde{\phi}_p(w_2)\};$$

$$\tilde{\psi}_p(w_1aw_2) = [\psi_p^L(w_1aw_2), \psi_p^U(w_1aw_2)]$$
$$\leq [\max\{\psi_p^L(w_1), \psi_p^L(w_2)\}, \max\{\psi_p^U(w_1), \psi_p^U(w_2)\}]$$
$$= \max\{[\psi_p^L(w_1), \psi_p^U(w_1)], [\psi_p^L(w_2), \psi_p^U(w_2)]\}$$

$= \max\{\tilde{\psi}_p(w_1), \tilde{\psi}_p(w_2)\}$.

$\tilde{P} = [\tilde{\phi}_p, \tilde{\psi}_p]$ is an interval-valued Pythagorean fuzzy bi-ideal of S.

Theorem 3. If $\{P_i\}_{i\in I}$ is a family of interval-valued Pythagorean fuzzy bi-ideal of a semigroup S. Then $\cap P_i$ is an interval-valued Pythagorean fuzzy bi-ideal of S. Where $\cap P_i = (\cap \tilde{\phi}_{p_i}, \cup \tilde{\psi}_{p_i})$.

$\cap (\tilde{\phi}_{p_i}) = inf\{(\tilde{\phi}_{p_i})(w_1)/i \in I, w_1 \in S\}$, $\cup (\tilde{\psi}_{p_i}) = sup\{(\tilde{\psi}_{p_i})(w_1)/i \in I, w_1 \in S\}$ and $i \in I$ is any index set.

Proof. Since $\tilde{P}_i = \langle[\tilde{\phi}_{p_i}, \tilde{\psi}_{p_i}] | i \in I\rangle$ is a family of interval-valued Pythagorean fuzzy bi-ideal of S.

Let $a, w_1, w_2 \in S$.

$$\cap \tilde{\phi}_{p_i}(w_1, w_2) = \inf\{\tilde{\phi}_{p_i}(w_1, w_2)/i \in I, w_1, w_2 \in S\}$$
$$\geq \inf\left\{\min\{\tilde{\phi}_{p_i}(w_1), \tilde{\phi}_{p_i}(w_2)\}\right\}$$
$$= \min\left\{\inf\left(\tilde{\phi}_{p_i}(w_1)\right), \inf\left(\tilde{\phi}_{p_i}(w_2)\right)\right\}$$
$$= \min\{\cap \tilde{\phi}_{p_i}(w_1), \cap \tilde{\phi}_{p_i}(w_2)\};$$

$$\cup \tilde{\psi}_{p_i}(w_1 w_2) = \sup\{\tilde{\psi}_{p_i}(w_1 w_2)/i \in I, w_1, w_2 \in S\}$$
$$\leq \sup\left\{\max\{\tilde{\psi}_{p_i}(w_1), \tilde{\psi}_{p_i}(w_2)\}\right\}$$
$$= \max\left\{\sup\left(\tilde{\psi}_{p_i}(w_1)\right), \sup\left(\tilde{\psi}_{p_i}(w_2)\right)\right\}$$
$$= \max\{\cup \tilde{\psi}_{p_i}(w_1), \cup \tilde{\psi}_{p_i}(w_2)\}.$$

Hence, $\cap \tilde{P}_i = (\cap \tilde{\phi}_{p_i}, \cup \tilde{\psi}_{p_i})$ is an interval-valued Pythagorean fuzzy sub-semigorup of S.

$$\cap \tilde{\phi}_{p_i}(w_1 a w_2) = \inf\{\tilde{\phi}_{p_i}(w_1 a w_2)/i \in I, a, w_1, w_2 \in S\}$$
$$\geq \inf\left\{\min\{\tilde{\phi}_{p_i}(w_1), \tilde{\phi}_{p_i}(w_2)\}\right\}$$
$$= \min\left\{\inf\left(\tilde{\phi}_{p_i}(w_1)\right), \inf\left(\tilde{\phi}_{p_i}(w_2)\right)\right\}$$
$$= \min\{\cap \tilde{\phi}_{p_i}(w_1), \cap \tilde{\phi}_{p_i}(w_2)\}.$$

$$\cup \tilde{\psi}_{p_i}(w_1 a w_2) = \sup\{\tilde{\psi}_{p_i}(w_1 a w_2)/i \in I, a, w_1, w_2 \in S\}$$
$$\leq \sup\left\{\max\{\tilde{\psi}_{p_i}(w_1), \tilde{\psi}_{p_i}(w_2)\}\right\}$$
$$= \max\left\{\sup\left(\tilde{\psi}_{p_i}(w_1)\right), \sup\left(\tilde{\psi}_{p_i}(w_2)\right)\right\}$$
$$= \max\{\cup \tilde{\psi}_{p_i}(w_1), \cup \tilde{\psi}_{p_i}(w_2)\}.$$

Hence, $\cap P_i = (\cap \tilde{\phi}_{p_i}, \cup \tilde{\psi}_{p_i})$ is an interval-valued Pythagorean fuzzy bi-ideals of S.

Theorem 4. Let N be any non-empty subset of a semigroup S. Then N is a bi-ideal of S, if and only if the characteristic interval-valued Pythagorean fuzzy set $\chi_N = [\tilde{\phi}_{p\chi_N}, \tilde{\psi}_{p\chi_N}]$ is IVPFBI of S.

Proof. Assume that N is a bi-ideal of S. Let $a, w_1, w_2 \in S$.

Suppose that $\tilde{\phi}_{p\chi_N}(w_1w_2) < min\{\tilde{\phi}_{p\chi_N}(w_1), \tilde{\phi}_{p\chi_N}(w_2)\}$ and $\tilde{\psi}_{p\chi_N}(w_1w_2) > max\{\tilde{\psi}_{p\chi_N}(w_1), \tilde{\psi}_{p\chi_N}(w_2)\}$ it follows that $\tilde{\phi}_{p\chi_N}(w_1w_2) = 0,\ min\{\tilde{\phi}_{p\chi_N}(w_1), \tilde{\phi}_{p\chi_N}(w_2)\} = 1$

,$\tilde{\psi}_{p\chi_N}(w_1w_2) = 1,\ max\{\tilde{\psi}_{p\chi_N}(w_1), \tilde{\psi}_{p\chi_N}(w_2)\} = 0$.

This implies that $w_1, w_2 \in N$ by $w_1, w_2 \notin N$ a contradiction to N.

So $\tilde{\phi}_{p\chi_N}(w_1w_2) \geq min\{\tilde{\phi}_{p\chi_N}(w_1), \tilde{\phi}_{p\chi_N}(w_2)\}$, $\tilde{\psi}_{p\chi_N}(w_1w_2) \leq max\{\tilde{\psi}_{p\chi_N}(w_1), \tilde{\psi}_{p\chi_N}(w_2)\}$.

Suppose that $\tilde{\phi}_{p\chi_N}(w_1aw_2) < min\{\tilde{\phi}_{p\chi_N}(w_1), \tilde{\phi}_{p\chi_N}(w_2)\}$ and $\tilde{\psi}_{p\chi_N}(w_1aw_2) > max\{\tilde{\psi}_{p\chi_N}(w_1), \tilde{\psi}_{p\chi_N}(w_2)\}$ it follows that $\tilde{\phi}_{p\chi_N}(w_1aw_2) = 0,\ min\{\tilde{\phi}_{p\chi_N}(w_1), \tilde{\phi}_{p\chi_N}(w_2)\} = 1$, $\tilde{\psi}_{p\chi_N}(w_1w_2) = 1$, $max\{\tilde{\psi}_{p\chi_N}(w_1), \tilde{\psi}_{p\chi_N}(w_2)\} = 0$.

This implies that $a, w_1, w_2 \in N$ by $a, w_1, w_2 \notin N$ a contradiction to N.

So $\tilde{\phi}_{p\chi_N}(w_1aw_2) \geq min\{\tilde{\phi}_{p\chi_N}(w_1), \tilde{\phi}_{p\chi_N}(w_2)\}$, $\tilde{\psi}_{p\chi_N}(w_1aw_2) \leq max\{\tilde{\psi}_{p\chi_N}(w_1), \tilde{\psi}_{p\chi_N}(w_2)\}$.

This shows that χ_N is an interval-valued Pythagorean fuzzy bi-ideal of S.

Conversely, $\chi_N = [\tilde{\phi}_{p\chi_N}, \tilde{\psi}_{p\chi_N}]$ is an IVPFBI of S for any subset N of S.

Let $w_1, w_2 \in N$ then $\tilde{\phi}_{p\chi_N}(w_1) = \tilde{\phi}_{p\chi_N}(w_2) = \tilde{1}$, $\tilde{\psi}_{p\chi_N}(w_1) = \tilde{\psi}_{p\chi_N}(w_2) = \tilde{0}$, since χ_N is an IVPFBI of S.

$\tilde{\phi}_{p\chi_N}(w_1w_2) \geq min\{\tilde{\phi}_{p\chi_N}(w_1), \tilde{\phi}_{p\chi_N}(w_2)\} \geq min\{\tilde{1}, \tilde{1}\} = \tilde{1}$, $\tilde{\psi}_{p\chi_N}(w_1w_2) \leq max\{\tilde{\psi}_{p\chi_N}(w_1), \tilde{\psi}_{p\chi_N}(w_2)\} \leq max\{\tilde{0}, \tilde{0}\} = \tilde{0}$.

This implies that $w_1, w_2 \in N$.

Let $a, w_1, w_2 \in N$ then $\tilde{\phi}_{p\chi_N}(w_1) = \tilde{\phi}_{p\chi_N}(a) = \tilde{\phi}_{p\chi_N}(w_2) = \tilde{1}$,

$\tilde{\psi}_{p\chi_N}(w_1) = \tilde{\psi}_{p\chi_N}(a) = \tilde{\psi}_{p\chi_N}(w_2) = \tilde{0}$, since χ_N is an IVPFBI of S.

$\tilde{\phi}_{p\chi_N}(w_1aw_2) \geq min\{\tilde{\phi}_{p\chi_N}(w_1), \tilde{\phi}_{p\chi_N}(w_2)\} \geq min\{\tilde{1}, \tilde{1}\} = \tilde{1}$, $\tilde{\psi}_{p\chi_N}(w_1aw_2) \leq max\{\tilde{\psi}_{p\chi_N}(w_1), \tilde{\psi}_{p\chi_N}(w_2)\} \leq max\{\tilde{0}, \tilde{0}\} = \tilde{0}$.

Which implies that $w_1, w_2 \in N$. Hence N is a bi- ideal of S.

Theorem 5. If $\{\tilde{P}_i\}_{i \in I}$ is a family of interval-valued Pythagorean fuzzy interior ideal of a semigroup S. Then $\cap\ \tilde{P}_i$ is an interval-valued Pythagorean fuzzy interior ideal (IVPFII) of S.

Where $\cap\ \tilde{P}_i = (\cap\, \tilde{\phi}_{p_i}, \cup\, \tilde{\psi}_{p_i})$;

$\cap\,(\tilde{\phi}_{p_i}) = inf\{(\tilde{\phi}_{p_i})(w_1)/i \in I, w_1 \in S\}$, $\cup\,(\tilde{\psi}_{p_i}) = sup\{(\tilde{\psi}_{p_i})(w_1)/i \in I, w_1 \in S\}$ and $i \in I$ is any index set.

Theorem 6. Let N be any non-empty subset of a semigroup S. Then N is a interior ideal of S, if and only if the characteristic interval-valued Pythagorean fuzzy set $\chi_N = [\tilde{\phi}_{p\chi_N}, \tilde{\psi}_{p\chi_N}]$ is IVPFII of S.

4. Homomorphism of Interval-Valued Pythagorean Fuzzy Ideals in Semigroups

Let R and T be two non-empty sets of semigroup S. A mapping $f: R \to T$ is called a homomorphism if $(rt) = f(r)f(t)\ \forall r, t \in R$.

Definition 9. Let f be a mapping from a set R to a set T and $\tilde{P} = [\tilde{\phi}_p, \tilde{\psi}_p]$ be an interval-valued Pythagorean fuzzy set R the image of R (i.e.) $f(\tilde{P}) = (f(\tilde{\phi}_p), f(\tilde{\psi}_p))$ is an interval-valued Pythagorean fuzzy set of T is defined by

$$f(\tilde{P})(r) = \begin{cases} f(\widetilde{\phi_P})(r) = \begin{cases} \sup\limits_{t \in f'(r)} (\widetilde{\phi_P})(t), & \text{if} f^{-1}(r) = 0 \\ [0,0] & \text{otherwise} \end{cases} \\ f(\widetilde{\psi_P})(r) = \begin{cases} \inf\limits_{t \in f'(r)} (\widetilde{\psi_P})(t), & \text{if} f^{-1}(r) = 0 \\ [1,1] & \text{otherwise} \end{cases} \end{cases}$$

Let f be a mapping from a set R to T and $\tilde{P} = [\tilde{\phi}_p, \tilde{\psi}_p]$ be an interval-valued Pythagorean fuzzy set of T then the preimage of T (i.e.) $f^{-1}(\tilde{P}) = \{(f^{-1}(\tilde{\phi}_p), f^{-1}(\tilde{\psi}_p))\}$ is an interval-valued Pythagorean fuzzy set of R is defined as

$$f^{-1}(\tilde{P})(r) = \begin{cases} f^{-1}(\widetilde{\phi_p})(r) = \widetilde{\phi_p}(f(r)) \\ f^{-1}(\widetilde{\psi_p})(r) = \widetilde{\psi_p}(f(r)) \end{cases}.$$

Theorem 7. Let R, T be a semigroups, $f: R \to T$ be a homomorphism of semigroups.

If $\tilde{P} = [\tilde{\phi}_p, \tilde{\psi}_p]$ is an interval-valued Pythagorean fuzzy sub-semigroup of T the the preimage $f^{-1}(\tilde{P}) = (f^{-1}(\tilde{\phi}_p), f^{-1}(\tilde{\psi}_p))$ is an interval-valued Pythagorean fuzzy sub-semigroup of R.

If $\tilde{P} = [\tilde{\phi}_p, \tilde{\psi}_p]$ is an interval-valued Pythagorean fuzzy left (resp.right) ideal of T the the preimage $f^{-1}(\tilde{P}) = (f^{-1}(\tilde{\phi}_p), f^{-1}(\tilde{\psi}_p))$ is an interval-valued Pythagorean fuzzy left ideal (resp. right ideal) of R.

Proof. Assume that $\tilde{P} = [\tilde{\phi}_p, \tilde{\psi}_p]$ is an interval-valued Pythagorean fuzzy sub-semigroup of T and $r, t \in R$. Then

$$\begin{aligned} & f^{-1}(\widetilde{\phi}_p)(rt) = \widetilde{\phi}_p(f(rt)) \\ & = \widetilde{\phi}_p(f(r)f(t)) \\ & \geq \min\{\widetilde{\phi}_p(f(r)), \widetilde{\phi}_p(f(t))\} \\ & = \min\{f^{-1}(\widetilde{\phi}_p)(r), f^{-1}(\widetilde{\phi}_p)(f(t))\}; \end{aligned}$$

$$f^{-1}(\widetilde{\psi}_p)(rt) = \widetilde{\psi}_p(f(rt))$$

$= \widetilde{\psi}_p(f(r)f(t))$
$\leq \max\{\widetilde{\psi}_p(f(r)), \widetilde{\psi}_p(f(t))\}$
$= \max\{f^{-1}(\widetilde{\psi}_p)(r), f^{-1}(\widetilde{\psi}_p)(f(t))\}.$

Hence, $f^{-1}(\tilde{P}) = (f^{-1}(\tilde{\phi}_p), f^{-1}(\widetilde{\psi}_p))$ is an interval-valued Pythagorean fuzzy sub-semigroup of R.

$f^{-1}(\widetilde{\phi}_p)(rt) = \widetilde{\phi}_p(f(rt))$
$= \widetilde{\phi}_p(f(r)f(t))$
$\geq \widetilde{\phi}_p(f(t))$
$= f^{-1}(\widetilde{\phi}_p)(f(t));$

$f^{-1}(\widetilde{\psi}_p)(rt) = \widetilde{\psi}_p(f(rt))$
$= \widetilde{\psi}_p(f(r)f(t))$
$\leq \widetilde{\psi}_p(f(t))$
$= f^{-1}(\widetilde{\psi}_p)(f(t)).$

Hence, $f^{-1}(\tilde{P}) = (f^{-1}(\tilde{\phi}_p), f^{-1}(\tilde{\psi}_p))$ is an interval-valued Pythagorean fuzzy left (resp.right) ideal of R.

Theorem 8. Let R, T be a semigroups, $f: R \to T$ be a homomorphism of semigroups. If $\tilde{P} = [\tilde{\phi}_p, \tilde{\psi}_p]$ is an interval-valued Pythagorean fuzzy bi-ideal of T the the preimage $f^{-1}(\tilde{P}) = (f^{-1}(\tilde{\phi}_p), f^{-1}(\tilde{\psi}_p))$ is an interval-valued Pythagorean fuzzy bi-ideal of R.

Proof. Assume that $\tilde{P} = [\tilde{\phi}_p, \tilde{\psi}_p]$ is an interval-valued Pythagorean fuzzy sub-semigroup of T and $a, r, t \in R$. Then

$f^{-1}(\widetilde{\phi}_p)(rat) = \widetilde{\phi}_p(f(rat))$
$= \widetilde{\phi}_p(f(r)f(a)f(t))$
$\geq \min\{\widetilde{\phi}_p(f(r)), \widetilde{\phi}_p(f(t))\}$
$= \min\{f^{-1}(\widetilde{\phi}_p)(r), f^{-1}(\widetilde{\phi}_p)(f(t))\};$

$f^{-1}(\widetilde{\psi}_p)(rat) = \widetilde{\psi}_p(f(rat))$
$= \widetilde{\psi}_p(f(r)f(a)f(t))$
$\leq \max\{\widetilde{\psi}_p(f(r)), \widetilde{\psi}_p(f(t))\}$
$= \max\{f^{-1}(\widetilde{\psi}_p)(r), f^{-1}(\widetilde{\psi}_p)(f(t))\}.$

Hence $f^{-1}(\tilde{P}) = (f^{-1}(\tilde{\phi}_p), f^{-1}(\tilde{\psi}_p))$ is an interval-valued Pythagorean fuzzy bi-ideal of R.

Theorem 9. Let R, T be a semigroups, $f: R \to T$ be a homomorphism of semigroups. If $\tilde{P} = [\tilde{\phi}_p, \tilde{\psi}_p]$ is an interval-valued Pythagorean fuzzy interior ideal of T the preimage $f^{-1}(P) = (f^{-1}(\tilde{\phi}_p), f^{-1}(\tilde{\psi}_p))$ is an interval-valued Pythagorean fuzzy interior ideal of R.

5. Conclusion

In this paper interval valued Pythagorean fuzzy sub-semigroup, interval valued Pythagorean fuzzy left (resp. right) ideal, interval valued Pythagorean fuzzy ideal, interval valued Pythagorean fuzzy bi-ideal, interval valued Pythagorean fuzzy interior ideal and Homomorphism of interval valued Pythagorean fuzzy ideal in semigroups are studied and investigated some properties with suitable examples.

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