J. Fuzzy. Ext. Appl. Vol. 1, No. 4 (2020) 291–299

J. Fuzzy. Ext. Appl

Journal of Fuzzy Extension & Applications

www.journal-fea.com

An Overview of Data Envelopment Analysis Models in Fuzzy Stochastic Environments

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PAPER INFO

Chronicle:

Received: 05 July 2020

Reviewed: 29 July 2020

Revised: 11 September 2020

Accepted: 23 October 2020

Keywords:

Decision-Making.

Efficiency.

Stochastic Fuzzy DEA.

ABSTRACT

One of the appropriate and efficient tools in the field of productivity measurement and evaluation is data envelopment analysis, which is used as a non-parametric method to calculate the efficiency of decision-making units. Today, the use of data envelopment analysis technique is expanding rapidly and is used in the evaluation of various organizations and industries such as banks, postal service, hospitals, training centers, power plants, refineries, etc. In real-world problems, the values observed from input and output data are often ambiguous and random. To solve this problem, data envelopment analysis in stochastic fuzzy environment was proposed. Although the DEA has many advantages, one of the disadvantages of this method is that the classic DEA does not actually give us a definitive conclusion and does not allow random changes in input and output. In this paper, we review some of the proposed models in data envelopment analysis with fuzzy and random inputs and outputs.

1. Introduction

Data envelopment analysis is a linear programming method whose basic purpose is to compare and evaluate the performance of a number of identical decision-making units that have different amounts of inputs used and outputs produced. Data Envelopment Analysis (DEA) models used in evaluating the performance of the unit under study can use two separate approaches: reducing the amount of inputs without decreasing the amount of outputs, increasing the outputs without increasing the amount of inputs.

In real world problems, inputs and outputs are considered vague and random. In fact, decision makers may face a specific hybrid environment where there is fuzziness and randomness in the problem. Hatami-Marbini et al. classified the fuzzy DEA methods in the literature into five general groups [1], the tolerance approach [2, 3], the α-level based approach, the fuzzy ranking approach [4, 5], the possibility approach [6], and the fuzzy arithmetic approach [7]. Among these approaches, the α-level based approach is probably the

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DOI: 10.22105/jfea.2020.258330.1030

most popular fuzzy DEA model in the literature. This approach generally tries to transform the FDEA model into a pair of parametric programs for each ⍰-level. Kao and Liu, one of the most cited studies in the ⍰-level approach's category, used Chen and Klein [8] method for ranking fuzzy numbers to convert the FDEA model to a pair of parametric mathematical programs for the given level of ⍰ [9]. Saati et al. proposed a fuzzy CCR model as a possibilistic programming problem and changed it into an interval programming problem by means of the ⍰-level based approach [10]. Parameshwaran et al. proposed an integrated fuzzy analytic hierarchy process and DEA approach for the service performance measurement [11]. Puri and Yadav [12] applied the suggested methodology by Saati et al. [10] to solve fuzzy DEA model with undesirable outputs. Khanjani et al. [13] proposed fuzzy free disposal hull models under possibility and credibility measures. Momeni et al. used fuzzy DEA models to address the impreciseness and ambiguity associated with input and output data in supply chain performance evaluation problems [14]. Payan evaluated the performance of DMUs with fuzzy data by using the common set of weights based on a linear program [15]. Aghayi et al. formulated a model to measure the efficiency of DMUs with interval inputs and outputs based on common sets weights [16].

In recent years, several scholars work on DEA with fuzzy set extension. For example, Edalatpanah et al. [17] for the first time established triangular single-valued neutrosophic data envelopment analysis with application to hospital performance. He also presented data envelopment analysis based on triangular neutrosophic numbers [18]; see also [19-22].

In this research, some models of data envelopment analysis with fuzzy and random data will be mentioned.

2. Existing Models

In this section, we review the proposed models in a random fuzzy environment.

Tavana et al. [23] developed an imprecise DEA-based formulation for dealing with the randomness of fuzzy parameters on a possibility space $(\theta, P(\theta), Pos)$ through efficiency measurement. They considered n DMUs, indexed by $j=1,\dots,n$, where each of them consumes m different random fuzzy inputs, indexed by $\tilde{\tilde{x}}_{ij}\,(i=1,\dots,m)$, to secure s different random fuzzy outputs indexed by $\tilde{\tilde{y}}_{rj}\,(r=1,\dots,s)$. Finally, the final model is as follows:

For $\delta > 0.5$:

$$
\begin{aligned}
&\max \quad \varphi \\
&\text{s.t.} \quad \varphi + \theta_o^O \phi^{-1}(\delta) \le \sum_{r=1}^{s} u_r (y_{ro}^{m_2} + R^{-1}(\gamma) y_{ro}^{\beta}), \\
&\quad \sum_{i=1}^{m} v_i (x_{io}^{m_2} + R^{-1}(\gamma) x_{io}^{\beta}) - \theta_o^I \phi^{-1}(\delta) \ge 1, \\
&\quad \sum_{i=1}^{m} v_i (x_{io}^{m_1} - L^{-1}(\gamma) x_{io}^{\alpha}) + \theta_o^I \phi^{-1}(\delta) \le 1, \\
&\quad \sum_{r=1}^{s} u_r (y_{rj}^{m_1} - L^{-1}(\gamma) y_{rj}^{\alpha}) - \sum_{i=1}^{m} v_i (x_{ij}^{m_2} + R^{-1}(\gamma) x_{ij}^{\beta}) + \phi^{-1}(\delta) \lambda_j \le 0, \quad j = 1,\dots,n, \\
&\quad (\theta_o^O)^2 = \sum_{r=1}^{s} u_r^2 (\hat{y}_{ro}^{m_1} - L^{-1}(\gamma) \hat{y}_{ro}^{\alpha}), \\
&\quad (\theta_o^I)^2 = \sum_{i=1}^{m} v_i^2 (\hat{x}_{io}^{m_1} - L^{-1}(\gamma) \hat{x}_{io}^{\alpha}), \\
&\quad (\lambda_j)^2 = \sum_{r=1}^{s} u_r^2 \hat{y}_{rj}^{m_2} + \sum_{i=1}^{m} v_i^2 \hat{x}_{ij}^{m_2} - L^{-1}(\gamma) (\sum_{r=1}^{s} u_r^2 \hat{y}_{rj}^{\beta} + \sum_{i=1}^{m} v_i^2 \hat{x}_{ij}^{\beta}), \quad j = 1,\dots,n, \\
&\quad u_r, v_i, \theta_p^O, \theta_p^I, \bar{\theta}_p^I, \lambda_j \ge 0, r = 1,\dots,s, i = 1,\dots,m, j = 1,\dots,n.
\end{aligned} \tag{1}
$$

And for $\delta \le 0.5$:

$$
\begin{aligned}
&\max \quad \varphi \\
&\text{s.t.} \quad \varphi - \sum_{r=1}^{s} u_r (y_{ro}^{m_2} + R^{-1}(\gamma) \tilde{\tilde{y}}_{ro}^{\beta}) + \bar{\theta}_o^O \phi^{-1}(\delta) \le 0, \\
&\quad \sum_{i=1}^{m} v_i (x_{io}^{m_2} + R^{-1}(\gamma) x_{io}^{\beta}) - \bar{\theta}_o^I \phi^{-1}(\delta) \ge 1, \\
&\quad \sum_{i=1}^{m} v_i (x_{io}^{m_1} - L^{-1}(\gamma) x_{io}^{\alpha}) + \bar{\theta}_o^I \phi^{-1}(\delta) \le 1, \\
&\quad \sum_{r=1}^{s} u_r (y_{rj}^{m_1} - L^{-1}(\gamma) \sum_{r=1}^{s} u_r y_{rj}^{\alpha}) - \sum_{i=1}^{m} v_i (\tilde{\tilde{x}}_{ip}^{m_2} + R^{-1}(\gamma) \sum_{i=1}^{m} v_i \tilde{\tilde{x}}_{ip}^{\beta}) + \phi^{-1}(\delta) \bar{\lambda}_j \le 0, \quad j = 1,\dots,n, \\
&\quad (\bar{\theta}_o^O)^2 = \sum_{r=1}^{s} u_r^2 (\hat{y}_{ro}^{m_2} + R^{-1}(\gamma) \hat{y}_{ro}^{\beta}), \\
&\quad (\bar{\theta}_o^I)^2 = \sum_{i=1}^{m} v_i^2 (\hat{x}_{io}^{m_2} + R^{-1}(\gamma) \hat{x}_{io}^{\beta}), \\
&\quad (\bar{\lambda}_j)^2 = \sum_{r=1}^{s} u_r^2 \hat{y}_{rj}^{m_2} + \sum_{i=1}^{m} v_i^2 \hat{x}_{ij}^{m_2} + R^{-1}(\gamma) (\sum_{r=1}^{s} u_r^2 \hat{y}_{rj}^{\beta} + \sum_{i=1}^{m} v_i^2 \hat{x}_{ij}^{\beta}), \quad j = 1,\dots,n, \\
&\quad u_r, v_i, \bar{\theta}_p^O, \bar{\theta}_p^I, \lambda_j \ge 0, r = 1,\dots,s, i = 1,\dots,m, j = 1,\dots,n.
\end{aligned} \tag{2}
$$

Furthermore, they presented a Necessity-Probability constrained programming model under fuzzy probability necessity constraints as follow:

For $\delta > 0.5$:

$$
\begin{aligned}
&\max \quad \bar{\varphi} \\
&\text{s.t.} \quad \bar{\varphi}-\sum_{r=1}^{s} u_r y_{ro}^{m_1} + L^{-1}(1-\gamma)\sum_{r=1}^{s} u_r y_{ro}^{\alpha} + \tilde{\theta}_o^{O}\phi^{-1}(\delta) \le 0, \\
&\quad \sum_{i=1}^{m} v_i (x_{io}^{m_1} - L^{-1}(1-\gamma)x_{io}^{\alpha}) - \tilde{\theta}_o^{I}\phi^{-1}(\delta) \ge 1, \\
&\quad \sum_{i=1}^{m} v_i x_{io}^{m_2} + R^{-1}(\gamma)\sum_{i=1}^{m} v_i x_{ip}^{\beta} + \tilde{\theta}_o^{I}\phi^{-1}(\delta) \le 1, \\
&\quad \sum_{r=1}^{s} u_r (y_{rj}^{m_2} + R^{-1}(\gamma)y_{rj}^{\beta}) - \sum_{i=1}^{m} v_i (x_{ij}^{m_1} - L^{-1}(1-\gamma)x_{ij}^{\alpha}) + \tilde{\lambda}_j\phi^{-1}(\delta) \le 0, \quad j=1,\dots,n, \\
&\quad (\tilde{\theta}_o^{O})^2 = \sum_{r=1}^{s} u_r^2 (\hat{y}_{ro}^{m_1} + R^{-1}(\gamma)\hat{y}_{ro}^{\alpha}), \\
&\quad (\tilde{\theta}_o^{I})^2 = \sum_{i=1}^{m} v_i^2 (\hat{x}_{io}^{m_2} + R^{-1}(\gamma)\hat{x}_{io}^{\beta}), \\
&\quad (\tilde{\lambda}_j)^2 = \sum_{r=1}^{s} u_r^2 \hat{y}_{rj}^{m_1} + \sum_{i=1}^{m} v_i^2 \hat{x}_{ij}^{m_1}, \quad j=1,\dots,n, \\
&\quad u_r, v_i, \tilde{\theta}_o^{O}, \tilde{\theta}_o^{I}, \tilde{\lambda}_j \ge 0, r=1,\dots,s, i=1,\dots,m, j=1,\dots,n.
\end{aligned} \tag{3}
$$

For $\delta \le 0/5$:

$$
\begin{aligned}
&\max \quad \bar{\varphi} \\
&\text{s.t.} \quad \bar{\varphi}-\sum_{r=1}^{s} u_r y_{ro}^{m_1} + L^{-1}(1-\gamma)\sum_{r=1}^{s} u_r y_{ro}^{\alpha} + \hat{\theta}_o^{O}\phi^{-1}(\delta) \le 0, \\
&\quad \sum_{i=1}^{m} v_i x_{io}^{m_1} - L^{-1}(1-\gamma)\sum_{i=1}^{m} v_i x_{io}^{\alpha} - \hat{\theta}_o^{I}\phi^{-1}(\delta) \ge 1, \\
&\quad \sum_{i=1}^{m} v_i x_{io}^{m_2} + R^{-1}(\gamma)\sum_{i=1}^{m} v_i x_{ip}^{\beta} + \hat{\theta}_o^{I}\phi^{-1}(\delta) \le 1, \\
&\quad \sum_{r=1}^{s} u_r (y_{rj}^{m_2} + R^{-1}(\gamma)y_{rj}^{\beta}) - \sum_{i=1}^{m} v_i (x_{ij}^{m_1} - L^{-1}(1-\gamma)x_{ij}^{\alpha}) + \hat{\lambda}_j\phi^{-1}(\delta) \le 0, \quad j=1,\dots,n, \\
&\quad (\hat{\theta}_o^{O})^2 = \sum_{r=1}^{s} u_r^2 (\hat{y}_{ro}^{m_1} - L^{-1}(1-\gamma)\hat{y}_{ro}^{\alpha}), \\
&\quad (\hat{\theta}_o^{I})^2 = \sum_{i=1}^{m} v_i^2 (\hat{x}_{io}^{m_1} - L^{-1}(\gamma)\hat{x}_{io}^{\beta}), \\
&\quad (\hat{\lambda}_j)^2 = \sum_{r=1}^{s} u_r^2 \hat{y}_{rj}^{m_1} + \sum_{i=1}^{m} v_i^2 \hat{x}_{ij}^{m_1} - L^{-1}(1-\gamma)(\sum_{i=1}^{m} v_i^2 \hat{x}_{ij}^{\alpha} + \sum_{r=1}^{s} u_r^2 \hat{y}_{rj}^{\alpha}), \quad j=1,\dots,n, \\
&\quad u_r, v_i, \hat{\theta}_o^{O}, \hat{\theta}_o^{I}, \hat{\lambda}_j \ge 0, r=1,\dots,s, i=1,\dots,m, j=1,\dots,n.
\end{aligned} \tag{4}
$$

In 2016, Nasseri et al. [24] proposed a new model of fuzzy stochastic DEA with input-oriented primal data. In this model, the properties and characteristics of the extended normal distribution are used. They considered n DMUs, each unit consumes m fuzzy stochastic inputs, denoted by $\tilde{\tilde{x}}_{ij} = (x_{ij}^{m}, x_{ij}^{\alpha}, x_{ij}^{\beta})_{LR}$, $i=1,\dots,m$, $j=1,\dots,n$, and produces s fuzzy stochastic outputs, denoted by $\tilde{\tilde{y}}_{rj} = (y_{rj}^{m}, y_{rj}^{\alpha}, y_{rj}^{\beta})_{LR}$, $r=1,\dots,s$, $j=1,\dots,n$. Also, they considered x_{ij}^{m} and y_{rj}^{m}, denoted by $x_{ij}^{m} \sim N(x_{ij}, \sigma_{ij}^2)$ and $y_{rj}^{m} \sim N(y_{rj}, \sigma_{rj}^2)$ be normally distributed. Therefore, $x_{ij}(y_{rj})$ and $\sigma_{ij}^2(\sigma_{rj}^2)$ are the mean and the variance of $x_{ij}^{m}(y_{rj}^{m})$ for DMU_j, respectively. Each unit has an extended normal distribution as $\tilde{\bar{x}}_{ij} \sim \overline{N}(\bar{x}_{ij}, \sigma_{ij})$ with $\bar{x}_{ij} = (x_{ij}, x_{ij}^{\alpha}, x_{ij}^{\beta})$ and $\tilde{\bar{y}}_{rj} \sim \overline{N}(\bar{y}_{rj}, \sigma_{rj})$ with $\bar{y}_{rj} = (y_{rj}, y_{rj}^{\alpha}, y_{rj}^{\beta})$. Finally, the final model is as follows:

$$
\begin{aligned}
& E_K^T(\delta,\gamma) = \max \ \varphi \\
& \text{s.t.} \\
& \quad \varphi \le \sum_{r=1}^{s} \hat{y}_{rk}, \\
& \quad \sum_{i=1}^{m} \hat{x}_{ik} = 1, \\
& \quad \sum_{r=1}^{s} \hat{y}_{rj} - \sum_{i=1}^{m} \hat{x}_{ij} \le 0 \quad \forall j, \\
& \quad u_r(y_{rj} - L^{-1}(\delta) y_{rj}^{\alpha} - \sigma_{rj}\phi^{-1}_{1-\frac{\gamma}{2}}) \le \hat{y}_{rj} \le u_r(y_{rj} + R^{-1}(\delta) y_{rj}^{\beta} + \sigma_{rj}\phi^{-1}_{1-\frac{\gamma}{2}}), \forall r, j \\
& \quad v_i(x_{rj} - L^{-1}(\delta) x_{rj}^{\alpha} - \sigma_{ij}\phi^{-1}_{1-\frac{\gamma}{2}}) \le \hat{x}_{ij} \le v_i(x_{ij} + R^{-1}(\delta) x_{ij}^{\beta} + \sigma_{ij}\phi^{-1}_{1-\frac{\gamma}{2}}), \forall i, j \\
& \quad u_r, v_i \ge 0.
\end{aligned} \tag{5}
$$

In 2017, Nasseri et al. [25] introduced a new model of fuzzy stochastic data envelopment analysis with undesirable outputs. The application of this model to the banking industry was demonstrated. They solved the proposed model by using the probability-possibility, probability-necessity and probability-credibility measures in CCP approach. The final models will be as follows:

Probability-possibility approach:

$$
\begin{aligned}
& E_k^{Pos}(\gamma,\delta) = \max \ \varphi \\
& \text{s.t.} \\
& \quad \varphi - \sum_{r=1}^{s_1} u_r^g(\tilde{y}_{rk}^g + R^{-1}(\delta) y_{rk}^{g,\beta}) + \sum_{p=1}^{s_2} u_p^b(\tilde{y}_{pk}^b - L^{-1}(\delta) y_{pk}^{b,\alpha}) \le \sigma_k^y \phi^{-1}_{1-\gamma}, \\
& \quad \sum_{i=1}^{m} v_i(\tilde{x}_{ik} + R^{-1}(\delta) x_{ik}^{\beta}) + \sigma_k^x \phi^{-1}_{1-\gamma} \ge 1, \\
& \quad \sum_{i=1}^{m} v_i(\tilde{x}_{ik} - L^{-1}(\delta) x_{ik}^{\alpha}) - \sigma_k^x \phi^{-1}_{1-\gamma} \le 1, \\
& \quad \sum_{r=1}^{s_1} u_r^g(\tilde{y}_{rj}^g + R^{-1}(\delta_j) y_{rj}^{g,\beta}) - \sum_{p=1}^{s_2} u_p^b(\tilde{y}_{pj}^b - L^{-1}(\delta_j) y_{pj}^{b,\alpha}) - \sigma_j^y \phi^{-1}_{1-\gamma} \ge 0, \\
& \quad \sum_{r=1}^{s_1} u_r^g(\tilde{y}_{rj}^g - L^{-1}(\delta) y_{rj}^{g,\alpha}) - \sum_{p=1}^{s_2} u_p^b(\tilde{y}_{pj}^b + R^{-1}(\delta) y_{pj}^{b,\beta}) - \sum_{i=1}^{m} v_i(\tilde{x}_{ik} + R^{-1}(\delta) x_{ik}^{\beta}) - \sigma_j^A \phi^{-1}_{1-\gamma} \le 0, \\
& \quad u_r^g, u_p^b, v_i \ge 0.
\end{aligned} \tag{6}
$$

Probability-Necessity model:

$$
\begin{aligned}
& E_k^{Nec}(\gamma,\delta)=\max\ \varphi \\
& \text{s.t.} \\
& \quad \varphi \le 1, \\
& \quad \varphi \le \sum_{r=1}^{s_1} u_r^g(\tilde{y}_{rk}^g - L^{-1}(1-\delta)y_{rk}^{g,\alpha}) - \sum_{p=1}^{s_2} u_p^b(\tilde{y}_{pk}^b + R^{-1}(1-\delta)y_{pk}^{b,\beta}) + \sigma_k^y \phi_{1-\gamma}^{-1}, \\
& \quad \sum_{i=1}^{m} v_i(\tilde{x}_{ik} - L^{-1}(1-\delta)x_{ik}^{\alpha}) + \sigma_k^x \phi_{1-\gamma}^{-1} \ge 1, \\
& \quad \sum_{J=1}^{n} v_i(\tilde{x}_{ik} + R^{-1}(1-\delta)x_{ik}^{\beta}) - \sigma_k^x \phi_{1-\gamma}^{-1} \le 1, \\
& \quad \sum_{r=1}^{s_1} u_r^g(\tilde{y}_{rj}^g - L^{-1}(1-\delta)y_{rj}^{g,\alpha}) - \sum_{p=1}^{s_2} u_p^b(\tilde{y}_{pj}^b + R^{-1}(1-\delta)y_{pj}^{b,\beta}) + \sigma_j^y \phi_{1-\gamma}^{-1} \ge 0, \\
& \quad \sum_{r=1}^{s_1} u_r^g(\tilde{y}_{rj}^g + R^{-1}(1-\delta)y_{rj}^{g,\beta}) - \sum_{p=1}^{s_2} u_p^b(\tilde{y}_{pj}^b - L^{-1}(1-\delta)y_{pj}^{b,\alpha}) - \sum_{i=1}^{m} v_i(\tilde{x}_{ij} - L^{-1}(1-\delta)x_{ij}^{\alpha}) - \sigma_j^A \phi_{1-\gamma}^{-1} \le 0, \\
& \quad \sigma_k^x = (\sum_{i=1}^{m} v_i^2 \sigma_{ik}^2)^{\frac{1}{2}}, \\
& \quad \sigma_j^y = (\sum_{r=1}^{s_1} (u_r^g)^2(\sigma_{rj}^g)^2 + \sum_{p=1}^{s_2} (u_p^b)^2(\sigma_{pj}^b)^2)^{\frac{1}{2}}), \\
& \quad \sigma_j^A = (\sum_{r=1}^{s_1} (u_r^g)^2(\sigma_{rj}^g)^2 + \sum_{p=1}^{s_2} (u_p^b)^2(\sigma_{pj}^b)^2) + \sum_{i=1}^{m} v_i^2 \sigma_{ij}^2)^{\frac{1}{2}}, \\
& \quad u_r^g, u_p^b, v_i \ge 0.
\end{aligned} \tag{7}
$$

Probability-Credibility model:

For $\delta \le 0/5$:

$$
\begin{aligned}
& E_k^{Cr}(\gamma,\delta)=\max\ \varphi \\
& \text{s.t.} \\
& \quad \varphi \le 1, \\
& \quad \varphi \le \sum_{r=1}^{s_1} u_r^g(\tilde{y}_{rk}^g + R^{-1}(2\delta)y_{rk}^{g,\beta}) - \sum_{p=1}^{s_2} u_p^b(\tilde{y}_{pk}^b - L^{-1}(2\delta)y_{pk}^{b,\alpha}) + \sigma_k^y \phi_{1-\gamma}^{-1}, \\
& \quad \sum_{i=1}^{m} v_i(\tilde{x}_{ik} + R^{-1}(2\delta)x_{ik}^{\beta}) + \sigma_k^x \phi_{1-\gamma}^{-1} \ge 1, \\
& \quad \sum_{j=1}^{n} v_i(\tilde{x}_{ik} - L^{-1}(2\delta)x_{ik}^{\alpha}) - \sigma_k^x \phi_{1-\gamma}^{-1} \le 1, \\
& \quad \sum_{r=1}^{s_1} u_r^g(\tilde{y}_{rj}^g + R^{-1}(2\delta)y_{rj}^{g,\beta}) - \sum_{p=1}^{s_2} u_p^b(\tilde{y}_{pj}^b - L^{-1}(2\delta)y_{pj}^{b,\alpha}) + \sigma_j^y \phi_{1-\gamma}^{-1} \ge 0, \\
& \quad \sum_{r=1}^{s_1} u_r^g(\tilde{y}_{rj}^g - L^{-1}(2\delta)y_{rj}^{g,\alpha}) - \sum_{p=1}^{s_2} u_p^b(\tilde{y}_{pj}^b + R^{-1}(2\delta)y_{pj}^{b,\beta}) - \sum_{i=1}^{m} v_i(\tilde{x}_{ij} + R^{-1}(2\delta)x_{ij}^{\beta}) - \sigma_j^A \phi_{1-\gamma}^{-1} \le 0, \\
& \quad \sigma_k^x = (\sum_{i=1}^{m} v_i^2 \sigma_{ik}^2)^{\frac{1}{2}}, \\
& \quad \sigma_j^y = (\sum_{r=1}^{s_1} (u_r^g)^2(\sigma_{rk}^g)^2 + \sum_{p=1}^{s_2} (u_p^b)^2(\sigma_{pk}^b)^2)^{\frac{1}{2}}), \\
& \quad \sigma_j^A = (\sum_{r=1}^{s_1} (u_r^g)^2(\sigma_{rj}^g)^2 + \sum_{p=1}^{s_2} (u_p^b)^2(\sigma_{pj}^b)^2) + \sum_{i=1}^{m} v_i^2 \sigma_{ij}^2)^{\frac{1}{2}}, \\
& \quad u_r^g, u_p^b, v_i \ge 0.
\end{aligned} \tag{8}
$$

For $\delta > 0.5$:

$$
\begin{aligned}
& E_k^{Nec}(\gamma,\delta) = \max\ \varphi \\
& s.t. \\
& \quad \varphi \le 1, \\
& \quad \varphi \le \sum_{r=1}^{s_1} u_r^g (\tilde{y}_{rk}^g - L^{-1}(2(1-\delta)) y_{rk}^{g,\alpha}) - \sum_{p=1}^{s_2} u_p^b (\tilde{y}_{pk}^b + R^{-1}(2(1-\delta)) y_{pk}^{b,\beta}) + \sigma_k^y \phi_{1-\gamma}^{-1}, \\
& \sum_{i=1}^{m} v_i (\tilde{x}_{ik} - L^{-1}(2(1-\delta)) x_{ik}^{\alpha}) + \sigma_k^x \phi_{1-\gamma}^{-1} \ge 1, \\
& \sum_{J=1}^{n} v_i (\tilde{x}_{ik} + R^{-1}(2(1-\delta)) x_{ik}^{\beta}) - \sigma_k^x \phi_{1-\gamma}^{-1} \le 1, \\
& \sum_{r=1}^{s_1} u_r^g (\tilde{y}_{rj}^g - L^{-1}(2(1-\delta)) y_{rj}^{g,\alpha}) - \sum_{p=1}^{s_2} u_p^b (\tilde{y}_{pj}^b + R^{-1}(2(1-\delta)) y_{pj}^{b,\beta}) + \sigma_j^y \phi_{1-\gamma}^{-1} \ge 0, \\
& \sum_{r=1}^{s_1} u_r^g (\tilde{y}_{rj}^g + R^{-1}(2(1-\delta)) y_{rj}^{g,\beta}) - \sum_{p=1}^{s_2} u_p^b (\tilde{y}_{pj}^b - L^{-1}(2(1-\delta)) y_{pj}^{b,\alpha}) - \\
& \sum_{i=1}^{m} v_i (\tilde{x}_{ij} - L^{-1}(2(1-\delta)) x_{ij}^{\alpha}) - \sigma_j^A \phi_{1-\gamma}^{-1} \le 0, \\
& \sigma_k^x = (\sum_{i=1}^{m} v_i^2 \sigma_{ik}^2)^{\frac{1}{2}}, \\
& \sigma_j^y = (\sum_{r=1}^{s_1} (u_r^g)^2 (\sigma_{rj}^g)^2 + \sum_{p=1}^{s_2} (u_p^b)^2 (\sigma_{pj}^b)^2)^{\frac{1}{2}}), \\
& \sigma_j^A = (\sum_{r=1}^{s_1} (u_r^g)^2 (\sigma_{rj}^g)^2 + \sum_{p=1}^{s_2} (u_p^b)^2 (\sigma_{pj}^b)^2) + \sum_{i=1}^{m} v_i^2 \sigma_{ij}^2)^{\frac{1}{2}}, \\
& u_r^g, u_p^b, v_i \ge 0.
\end{aligned}
\tag{9}
$$

3. Conclusion

A DEA model basically draws three critical elements: the model specification, the reference set itself, and the definition of the production possibility set. Starting from the latter, the production possibility set can either be defined as complete and known (like in conventional DEA) or as potentially extending beyond or excluding the reference set (like in stochastic DEA). The reference set, the very observations that form the engine of the non-parametric approach, can be either precise (as in conventional DEA), outcomes of stochastic processes (as in stochastic frontier analysis), or imprecise (as in the fuzzy DEA models).

Classic DEA models were originally formulated for optimal inputs and outputs, although undesirable outputs may also appear during production, which should be minimized. In addition, in the real world, there are dimensions and uncertainties in the data. Although DEA has many advantages, one of the disadvantages of this method is that in fact the classic DEA does not lead us to a definite conclusion and does not allow random changes in input and output.

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