Neutrosophic Soft Matrices and Its Application in Medical Diagnosis

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Abstract

In real life situations, there are many issues in which we face uncertainties, vagueness, complexities, and unpredictability. Neutrosophic sets are a mathematical tool to address some issues which cannot be met using the existing methods. Neutrosophic soft matrices play a crucial role in handling indeterminant and inconsistent information during decision making process. The main focus of this article is to discuss the concept of neutrosophic sets, neutrosophic soft sets, and neutrosophic soft matrices theory which are very useful and applicable in various situations involving uncertainties and imprecisions. Thereafter our intention is to find a new method for constructing a decision matrix using neutrosophic soft matrices as an application of the theory. A neutrosophic soft matrix based algorithm is considered to solve some problems in the diagnosis of a disease from the occurrence of various symptoms in patients. This article deals with patient-symptoms and symptoms-disease neutrosophic soft matrices. To come to a decision, a score matrix is defined where multiplication based on max-min operation and complementation of neutrosophic soft matrices are taken into considerations.

Keywords: Fuzzy sets, Soft sets, Soft matrix, Neutrosophic soft sets, Neutrosophic soft matrix.

1 | Introduction

The theory of fuzzy sets introduced by Zadeh [1] showed many applications in many areas of research. The idea of fuzzy sets was needed because it deals with those uncertainties and vauueness which cannot be handled using crisp sets. Fuzzy sets has membership function which assigns to each element of the Universe of discourse, a number from the unit interval [0,1], to indicate the degree of belongingness of the set under consideration. In fuzzy set theory, the membership of an element to a fuzzy set is a single value between zero and one. But in reality it may not always happen that the degree of non membership of an element in a fuzzy set is equal to one minus the membership degree because there may exist some hesitation degree as well.
Therefore, a generalization of fuzzy set was realized by Atanassov [2] who introduced a new set and named it intuitionistic fuzzy set. Intuitionistic fuzzy set incorporated the degree of hesitation known as hesitation margin.

Intuitionistic fuzzy sets which is generalization of fuzzy sets is useful in some situations when the problem involving linguistic variable are considered. For example, in decision making problems particularly in the cases of medical diagnosis, sales analysis, new product marketing, financial services, etc. there is a fair chance of a non null hesitation part in each moment of evaluation of an unknown project.

In real life situations, most of the problems in economics, social sciences, environment, etc. have various uncertainties. However, most of the existing mathematical tools for formal modeling, reasoning and computation are crisp deterministic and precise in character. There are theories namely theory of probability, evidence, fuzzy set, intuitionistic fuzzy set, rough set, etc. for dealing with uncertainties. These theories have their own difficulties as pointed out by Molodsov [3], and as such the novel concept of soft set theory was initiated. Soft set theory has rich potential for application in solving practical problems in economics, social science, medical sciences, etc. Maji et al. ([4] and [5]) have studied the theory of fuzzy soft set. In work [6], it can be seen that the theory of fuzzy soft sets have been extended to intuitionistic fuzzy soft sets. Smarandache [7] generalized soft set to Hypersoft sets and further used it in decision making processes. Vellapandi and Gunasekaran [9], studied a new decision making approach using multi soft set logic.

Intuitionistic fuzzy sets can only handle the incomplete information considering both the truth membership and falsity membership values. It does not handle the interminant and inconsistent information that exists in belief system. Smarandache [9], introduced the concept of neutrosophic sets as a mathematical tool to deal with some situations which involves impreciseness, inconsistencies and interminancy. It is expected that neutrosophic sets will produce more accurate results than those obtained using fuzzy sets or intuitionistic fuzzy sets. Many researchers as can be found in ([10]-[14]) who have worked on the applications of neutrosophic sets in decision making processes. Applications of neutrosophic soft sets now catching momentum.

2 | Definitions and Preliminaries

Some basic definitions which are useful in the subsequent sections of the article are discussed in this section.

**Definition 1. Soft set.** Let $U$ be the initial Universe of discourse and $E$ is the set of parameter. Let $P(U)$ denote the power set of $U$. A pair $(E,F)$ is called a soft set over $U$ where $F$ is a mapping given by $F: E \rightarrow P(U)$. Clearly soft set is a mapping from parameters to $P(U)$.

**Example 1.** Let $U = \{u_1, u_2, u_3, u_4\}$ be a set of four types of ornaments and $E = \{costly(e_1), medium(e_2), cheap(e_3)\}$ be the set of parameters. If $A = \{e_1, e_2\} \subseteq E$. Let $F(e_1) = \{u_1, u_4\}$ and $F(e_2) = \{u_2, u_3\}$. Then the soft set can be described as $(F,E) = \{(e_1, \{u_1, u_4\}), (e_2, \{u_2, u_3\})\}$ over $U$ which describe the “quality of furnitures” which Mr. X is going to buy. This soft set can be represented in the following form.
Definition 2. **Fuzzy soft set.** Let $U$ be the Universe of discourse and $E$ is the set of parameters. Let $P(U)$ denotes the collections of all fuzzy subsets of $U$. A pair $(F_A, E)$ is called fuzzy soft set over $U$ where $F_A$ is a mapping given by $F_A : E \to P(U)$.

Example 2. Let $U = \{u_1, u_2, u_3, u_4\}$ be a set of four types of ornaments and $E = \{\text{costly}(e_1), \text{medium}(e_2), \text{cheap}(e_3), \text{very cheap}(e_4)\}$ be the set of parameters. Let us consider the following case

\[
F_A(e_1) = [(u_1, 0.7), (u_2, 0.8), (u_3, 0.0), (u_4, 0.5)],
\]

\[
F_A(e_3) = [(u_1, 0.3), (u_2, 0.4), (u_3, 0.6), (u_4, 0.5)].
\]

<table>
<thead>
<tr>
<th>U</th>
<th>Costly(e_1)</th>
<th>Medium(e_2)</th>
<th>Cheap(e_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U_1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>U_2</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>U_3</td>
<td>0</td>
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<td>1</td>
</tr>
<tr>
<td>U_4</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Definition 3. **Intuitionistic fuzzy sets.** Let $U$ be the Universe of discourse. Then the intuitionistic fuzzy set $A$ is an object having of the form $A = \langle x, \mu_A(x), \nu_A(x) : x \in U \rangle$, where the function $\mu_A(x), \nu_A(x) : U \to [0, 1]$ define the degree of membership and degree of non membership of the element $x \in X$ to the set $A$ with the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 4. **Intuitionistic fuzzy soft sets.** Let $U$ be the Universe of discourse and $E$ is the set of parameters. Let $P(U)$ denotes the collections of all intuitionistic fuzzy subsets of $U$. A pair $(F_A, E)$ is called an intuitionistic fuzzy soft set over $U$ where $F_A$ is a mapping given by $F_A : E \to P(U)$.

Definition 5. **Neutrosophic sets.** Let $U$ be the Universe of discourse. The neutrosophic set $A$ on the Universe of discourse $U$ is defined as $A = \langle x, T_A(x), I_A(x), F_A(x) : x \in U \rangle$, where the characteristic functions $T, I, F : U \to [0, 1]$ and $-0 \leq T + I + F \leq 3$; $T, I, F$ are neutrosophic components which defines the degree of membership, the degree of indeterminacy and the degree of non membership, respectively.

Definition 6. **Neutrosophic soft sets.** Let $U$ be the Universe of discourse and $E$ is the set of parameters. Let $P(U)$ denotes the collections of all neutrosophic subsets of $U$. A pair $(F_A, E)$ is called a neutrosophic soft set over $U$ where $F_A$ is a mapping given by $F_A : E \to P(U)$. The following example will illustrate the concept.

Let $U$ be the set of houses under consideration and $E$ be the set of parameters where each parameter includes neutrosophic words.

Example 3. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ be a set of six types of ornaments and $E = \{\text{costly}(e_1), \text{medium}(e_2), \text{cheap}(e_3), \text{very cheap}(e_4)\}$ be the set of parameters. Let us consider the following case
The tabular representation of the NSS is

| \(u_1\) | \(0.3,0.4,0.2\) | \(0.4,0.5,0.1\) | \(0.5,0.2,0.2\) |
| \(u_2\) | \(0.5,0.4,0.1,0.8\) | \(0.4,0.2,0.3,0\) | \(0.4,0.5,0.1\) |
| \(u_5\) | \(0.5,0.2,0.2,0\) | \(0.3,0.4,0.2\) | \(0.4,0.4,0.2\) |
| \(u_6\) | \(0.5,0.2,0.2,0\) | \(0.5,0.3,0.1\) | \(0.5,0.3,0.1\) |

**Definition 7. Neutrosophic soft matrix.** Let \((\mathcal{F}_A,E)\) is a neutrosophic soft set over \(U\) where \(\mathcal{F}_A\) is a mapping given by \(\mathcal{F}_A : E \rightarrow \mathcal{P}(U)\) and \(\mathcal{P}(U)\) is the collection of all neutrosophic subsets of \(U\). Then the subsets of \(U \times E\) is uniquely defined by \(R_A = \{(u,e) : e \in A, u \in \mathcal{F}_A(e)\}\) and this is called a relation form of \((\mathcal{F}_A,E)\). Now the relation \(R_A\) is characterized by truth membership function \(T_A : U \times E \rightarrow [0,1]\), interminancy membership function \(I_A : U \times E \rightarrow [0,1]\) and falsity membership function \(F_A : U \times E \rightarrow [0,1]\) where \(T_A(u,e)\) is the truth membership value, \(I_A(u,e)\) is the interminancy membership value \(F_A(u,e)\) is the falsity membership value of the object \(u\) associated with the parameter \(e\).

Let \(U = \{u_1,u_2,u_3,\ldots,u_m\}\) be the Universe set and \(E = \{x_1,x_2,x_3,\ldots,x_n\}\) be the set of parameters. Then \(R_A\) can be represented by tabular form as follows.

\[
\begin{array}{cccc}
\text{e_1} & \text{e_2} & \cdots & \text{e_n} \\
\hline
\text{U_1} & (T_{A_{11}}, I_{A_{11}}, F_{A_{11}}) & \cdots & (T_{A_{1n}}, I_{A_{1n}}, F_{A_{1n}}) \\
\text{U_2} & (T_{A_{21}}, I_{A_{21}}, F_{A_{21}}) & \cdots & (T_{A_{2n}}, I_{A_{2n}}, F_{A_{2n}}) \\
\vdots & \vdots & \vdots & \vdots \\
\text{U_m} & (T_{A_{m1}}, I_{A_{m1}}, F_{A_{m1}}) & \cdots & (T_{A_{mn}}, I_{A_{mn}}, F_{A_{mn}}) \\
\end{array}
\]

where \((T_{A_{mn}}, I_{A_{mn}}, F_{A_{mn}}) = (T_A(u_m, e_n), I_A(u_m, e_n), F_A(u_m, e_n))\), If \(a_{ij} = (T_{A_{ij}}, I_{A_{ij}}, F_{A_{ij}})\) , a matrix can be defined as

\[
\mathbf{a}_{ij} = \begin{bmatrix}
a_{i1} & a_{i2} & \cdots & a_{in} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{m1} & a_{m2} & \cdots & a_{mn}
\end{bmatrix}
\]

This is called neutrosophic soft matrix corresponding to the neutrosophic soft set \((\mathcal{F}_A,E)\) over \(U\).
Example 4. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ be the Universal set and $E = \{e_1, e_2, e_3, e_4\}$ be the set of parameters, $A = \{e_1, e_2, e_3\}$.

Let us consider the following case

$F_A(e_1) = \{(u_1, 0.3, 0.4, 0.2), (u_2, 0.5, 0.4, 0.31), (u_3, 0.4, 0.5, 0.4), (u_4, 0.6, 0.3, 0.2), (u_5, 0.8, 0.1, 0.3), (u_6, 0.7, 0.2, 0.1)\}$,

$F_A(e_2) = \{(u_1, 0.4, 0.5, 0.2), (u_2, 0.6, 0.2, 0.3), (u_3, 1, 0.4), (u_4, 0.6, 0.2, 0.5), (u_5, 0.3, 0.4, 0.3), (u_6, 0.5, 0.4, 0.4)\}$,

$F_A(e_3) = \{(u_1, 0.6, 0.2, 0.3), (u_2, 0.4, 0.3, 0.3), (u_3, 0.5, 0.1, 0.4), (u_4, 0.4, 0.2, 0.3), (u_5, 0.6, 0.4, 0.2), (u_6, 0.7, 0.3, 0.2)\}$.

Then the NSS$(F_A, E)$ is a parameterized family $\{(F_A(e_1), F_A(e_2), F_A(e_3))\}$ of all NSS over $U$ and gives an approximate description of the object. Hence neutrosophic soft matrix can be represented by

$$ A = \begin{bmatrix}
(0.3, 0.4, 0.2) & (0.4, 0.5, 0.2) & (0.6, 0.2, 0.3) \\
(0.5, 0.4, 0.3) & (0.6, 0.2, 0.3) & (0.4, 0.3, 0.3) \\
(0.4, 0.5, 0.4) & (0.1, 0.0, 0.4) & (0.5, 0.1, 0.4) \\
(0.6, 0.3, 0.2) & (0.6, 0.2, 0.5) & (0.4, 0.2, 0.3) \\
(0.8, 0.1, 0.3) & (0.3, 0.4, 0.3) & (0.6, 0.4, 0.2) \\
(0.7, 0.2, 0.1) & (0.5, 0.4, 0.4) & (0.7, 0.3, 0.2)
\end{bmatrix}. $$

**Definition 8. Complement of neutrosophic soft matrices.** Let $A = [T_{ij}^A, I_{ij}^A, F_{ij}^A] \in \text{NSM}_{m \times n}$, then the complement of the neutrosophic soft matrix $A$ is denoted by $A^c$ and is defined as $A^c = [F_{ij}^A, I_{ij}^A, T_{ij}^A] \in \text{NSM}_{m \times n}$ for all $i$ and $j$. If the above mentioned neutrosophic fuzzy matrix is considered then the complement of the said matrix will be

$$ A^c = \begin{bmatrix}
(0.2, 0.4, 0.3) & (0.2, 0.5, 0.4) & (0.3, 0.2, 0.6) \\
(0.3, 0.4, 0.5) & (0.3, 0.2, 0.6) & (0.3, 0.3, 0.4) \\
(0.4, 0.5, 0.4) & (0.4, 0.0, 0.1) & (0.4, 0.1, 0.5) \\
(0.2, 0.3, 0.6) & (0.5, 0.2, 0.6) & (0.3, 0.2, 0.4) \\
(0.3, 0.1, 0.8) & (0.3, 0.4, 0.3) & (0.2, 0.4, 0.6) \\
(0.1, 0.2, 0.7) & (0.4, 0.4, 0.5) & (0.2, 0.3, 0.7)
\end{bmatrix}. $$

**Definition 9. Max-min product of neutrosophic soft matrices.** Let $A = [T_{ij}^A, I_{ij}^A, F_{ij}^A]$ and $B = [T_{ij}^B, I_{ij}^B, F_{ij}^B]$ be two neutrosophic soft matrices. Then the max-min product of the two neutrosophic soft matrices $A$ and $B$ is denoted as $A \ast B$ is defined as

$$ A \ast B = \{\max \min (T_{ij}^A, T_{ij}^B), \min \max (I_{ij}^A, I_{ij}^B), \min \max (F_{ij}^A, F_{ij}^B)\} \text{ for all } i \text{ and } j. $$

**Definition 10. Score matrix.** The score matrix $A$ and $b$ is defined as $S/(A, B) = [V - W]$, where $V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$, $V = T_{ij}^A + I_{ij}^A - F_{ij}^A$ and $W = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$, $W = T_{ij}^B + I_{ij}^B - F_{ij}^B$ are called membership value matrices.
In this article, it is intended to find $S((A \ast B),(A \ast B^c))$. Suppose $S((A \ast B),(A \ast B^c)) = V' - W'$. Then $V' = [v'_q], v'_q = T_q + I_q - F_q$ and $W' = [w'_q], w'_q = T_q + I_q - F_q$.

In this article, with the help of the above mentioned definitions, it is intended to get an approximate conception of the diseases revealing from some symptoms which the patients may convey to the doctors. The following sections deals with the algorithm followed by a case study to make the concept clear.

3 | Algorithm

Step 1. Input neutrosophic soft sets $(F,E)$ and $(G,E)$ and obtain neutrosophic soft matrices $A$ and $B$.

Step 2. Write the neutrosophic soft complement set $(G,E)^c$ and obtain neutrosophic soft complement matrix $B^c$.

Step 3. Compute patient symptom disease matrix $A \ast B$.

Step 4. Compute patient symptom non disease matrix $A \ast B^c$.

Step 5. Compute $V', W'$.

Step 6. Compute score matrix $S(A \ast B, A \ast B^c)$.

Step 7. Identify maximum score for the patient $P_i$ and conclude that the patient $P_i$ is suffering from the disease $D_i$.

4 | Case Studies

Suppose the test results of four patients $P = \{P_1, P_2, P_3, P_4\}$ as the Universal set where $P_1, P_2, P_3$ and $P_4$ represents patients Ram, Shyam, Jadu, and Madhu with systems $S = \{s_1, s_2, s_3, s_4, s_5\}$ where $s_1, s_2, s_3, s_4, s_5$ represents symptoms temperature, headaches, coughs, stomach pain, and body pain, respectively. Let the possible diseases relating to the above symptoms $D = \{D_1, D_2, D_3\}$ be viral fever, typhoid, and malaria.

Again let the set $S = \{s_1, s_2, s_3, s_4, s_5\}$ be a Universal set where $s_1, s_2, s_3, s_4, s_5$ represents symptoms temperature, headaches, coughs, stomach pain, and body pain, respectively. Let the possible diseases relating to the above symptoms $D = \{D_1, D_2, D_3\}$ be viral fever, typhoid, and malaria. Suppose that $\text{NSSF}(F,S)$ over $P$, where $F$ is a mapping $F : S \rightarrow \mathcal{F}^P$ gives a collection of an approximate description of patient symptoms in the hospital. Let

$$(F(S))=[F(s)]=((p_1,0.7,0.1,0.2),(p_2,0.1,0.8,0.1),(p_3,0.6,0.1,0.4),(p_4,0.5,0.2,0.4)),$$

$$(F(s))=[F(s)]=(p_1,0.6,0.1,0.3),(p_2,0.4,0.4,0.5),(p_3,0.8,0.1,0.2),(p_4,0.5,0.4,0.1),$$

$$(F(s))=[F(s)]=(p_1,0.2,0.8,0.2),(p_2,0.6,0.1,0.3),(p_3,0.0,0.6,0.4),(p_4,0.3,0.4,0.5).$$
The neutrosophic soft set is represented by the following neutrosophic soft matrix to describe the patient symptoms relationship.

\[
\begin{bmatrix}
    s_1 & s_2 & s_3 & s_4 & s_5 \\
    0.1,0.4,0.5 & 0.2,0.8,0.2 & 0.1,0.6,0.3 \\
    0.1,0.8,0.2 & 0.1,0.7,0.2 & 0.1,0.8,0.2 \\
    0.6,0.1,0.4 & 0.6,0.0,0.1 & 0.6,0.5,0.1 \\
    0.5,0.4,0.1 & 0.3,0.4,0.5 & 0.3,0.4,0.4 \\
\end{bmatrix}
\]

Again let the set \( S = \{s_1, s_2, s_3, s_4, s_5\} \) be a Universal set where \( s_1, s_2, s_3, s_4 \) and \( s_5 \) represents symptoms temperature, headaches, coughs, stomach pain, and body pain, respectively. Let the possible diseases relating to the above symptoms \( D = \{D_1, D_2, D_3\} \) be viral fever, typhoid, and malaria. Suppose that \( NSS(G,D) \) over \( S \), where \( G \) is a mapping \( G:D \rightarrow F^S \) gives a collection of an approximate description of medical knowledge of the three diseases and their symptoms. Let

\[
\begin{align*}
G(D_1) &= \{(s_1,0.6,0.2,0.3), (s_2,0.3,0.5,0.4), (s_3,0.1,0.8,0.1), (s_4,0.4,0.5,0.3), (s_5,0.7,0.4,0.2)\}, \\
G(D_2) &= \{(s_1,0.6,0.2,0.2), (s_2,0.2,0.6,0.3), (s_3,0.5,0.4,0.3), (s_4,0.7,0.2,0.1), (s_5,0.1,0.8,0.2)\}, \\
G(D_3) &= \{(s_1,0.3,0.4,0.3), (s_2,0.7,0.2,0.4), (s_3,0.7,0.2,0.3), (s_4,0.3,0.4,0.4), (s_5,0.2,0.7,0.3)\}.
\end{align*}
\]

Neutrosophic soft set can be represented by the following neutrosophic soft matrix

\[
\begin{bmatrix}
    D_1 & D_2 & D_3 \\
    s_1 & (0.6,0.2,0.3) & (0.6,0.2,0.2) & (0.3,0,4,0.3) \\
    s_2 & (0.3,0,5,0.4) & (0.2,0,6,0.3) & (0.7,0,2,0.4) \\
    s_3 & (0.1,0,8,0.1) & (0.5,0,4,0.3) & (0.7,0,2,0.3) \\
    s_4 & (0.4,0,5,0.3) & (0.7,0,2,0.1) & (0.3,0,4,0.4) \\
    s_5 & (0.7,0,4,0.2) & (0.1,0,8,0.2) & (0.2,0,7,0.3) \\
\end{bmatrix}
\]

Neutrosophic soft complement matrix

\[
\begin{bmatrix}
    D_1 & D_2 & D_3 \\
    s_1 & (0.3,0,2,0.6) & (0.2,0,2,0.6) & (0.3,0,4,0.3) \\
    s_2 & (0.4,0,5,0.3) & (0.3,0,6,0.2) & (0.4,0,2,0.7) \\
    s_3 & (0.1,0,8,0.1) & (0.3,0,4,0.5) & (0.3,0,2,0.7) \\
    s_4 & (0.3,0,5,0.4) & (0.1,0,2,0.7) & (0.4,0,4,0.3) \\
    s_5 & (0.2,0,4,0.7) & (0.2,0,8,0.1) & (0.3,0,7,0.2) \\
\end{bmatrix}
\]
Max-min compositions of two neutrosophic soft matrices will produce the following results:

Let us suppose $A \ast B = C_{ij} \ast_{mn}$, where

$$
C_{11} = \left[ \begin{array}{c}
\max(0.6,0.3,0.1,0.4,0.1), \min(0.2,0.5,0.8,0.5,0.6), \\
\min(0.2, 0.4, 0.2, 0.3, 0.3) = (0.6, 0.2, 0.2),
\end{array} \right]
$$

$$
C_{12} = \left[ \begin{array}{c}
\max(0.6,0.2,0.2,0.6,0.1), \min(0.2,0.6,0.8,0.2,0.8), \\
\min(0.2, 0.3, 0.3, 0.2, 0.3) = (0.6, 0.2, 0.2),
\end{array} \right]
$$

$$
C_{13} = \left[ \begin{array}{c}
\max(0.3,0.6,0.2,0.3,0.1), \min(0.4,0.2,0.8,0.4,0.7), \\
\min(0.3, 0.4, 0.3, 0.4, 0.3) = (0.6, 0.2, 0.3),
\end{array} \right]
$$

$$
C_{21} = \left[ \begin{array}{c}
\max(0.6,0.3,0.1,0.1,0.1), \min(0.8,0.5,0.8,0.7,0.8), \\
\min(0.3, 0.5, 0.3, 0.3, 0.2) = (0.6, 0.5,0.2),
\end{array} \right]
$$

$$
C_{22} = \left[ \begin{array}{c}
\max(0.1,0.2,0.5,0.1,0.1), \min(0.8,0.6,0.4,0.7,0.8), \\
\min(0.2, 0.5, 0.3, 0.2, 0.2) = (0.5, 0.4,0.2),
\end{array} \right]
$$

$$
C_{23} = \left[ \begin{array}{c}
\max(0.1,0.4,0.6,0.1,0.1), \min(0.8,0.4,0.2,0.7,0.8), \\
\min(0.3, 0.5, 0.3, 0.4, 0.2) = (0.6, 0.2,0.3),
\end{array} \right]
$$

$$
C_{31} = \left[ \begin{array}{c}
\max(0.6,0.3,0.0,0.4,0.6), \min(0.2,0.5,0.8,0.5,0.5), \\
\min(0.4, 0.4, 0.4, 0.2, 0.2) = (0.6, 0.2,0.2),
\end{array} \right]
$$

$$
C_{32} = \left[ \begin{array}{c}
\max(0.6,0.2,0.0,0.5,0.1), \min(0.2,0.6,0.6,0.3,0.8), \\
\min(0.4, 0.3, 0.4, 0.3, 0.2) = (0.7, 0.2,0.2),
\end{array} \right]
$$

$$
C_{33} = \left[ \begin{array}{c}
\max(0.3,0.7,0.0,0.3,0.2), \min(0.4,0.2,0.6,0.4,0.7), \\
\min(0.4, 0.4, 0.4, 0.4, 0.3) = (0.7, 0.2,0.3),
\end{array} \right]
$$

$$
C_{41} = \left[ \begin{array}{c}
\max(0.5,0.3,0.1,0.4,0.3), \min(0.2,0.5,0.8,0.5,0.4), \\
\min(0.4, 0.4, 0.5, 0.3, 0.4) = (0.5, 0.2,0.3),
\end{array} \right]
$$

$$
C_{42} = \left[ \begin{array}{c}
\max(0.5,0.2,0.3,0.7,0.1), \min(0.2,0.6,0.4,0.3,0.8), \\
\min(0.4, 0.3, 0.5, 0.1, 0.4) = (0.7, 0.2,0.1),
\end{array} \right]
$$

$$
C_{43} = \left[ \begin{array}{c}
\max(0.3,0.5,0.3,0.3,0.2), \min(0.4,0.4,0.4,0.4,0.7), \\
\min(0.4, 0.4, 0.5, 0.4, 0.4) = (0.5, 0.4,0.4).
\end{array} \right]
$$

Then

$$
A \ast B = \begin{bmatrix}
\begin{array}{c}
D_1 \\
D_2 \\
D_3
\end{array}
\end{bmatrix}
$$

Hence

$$
\begin{bmatrix}
P_1 \\
P_2 \\
P_3 \\
P_4
\end{bmatrix}
\begin{bmatrix}
(0.6,0.2,0.2) & (0.6,0.2,0.2) & (0.6,0.2,0.3) \\
(0.6,0.5,0.2) & (0.5,0.4,0.2) & (0.6,0.2,0.3) \\
(0.6,0.2,0.2) & (0.6,0.2,0.2) & (0.7,0.2,0.3) \\
(0.5,0.2,0.3) & (0.7,0.2,0.1) & (0.5,0.4,0.4)
\end{bmatrix}
$$
Similarly if $A \times B^c$, is calculated then it is obtained that

$$
A \times B^c = \begin{bmatrix}
     D_1 & D_2 & D_3 \\
     P_1 & (0.4,0.2,0.2) & (0.3,0.2,0.3) & (0.4,0.2,0.3) \\
     P_2 & (0.4,0.2,0.3) & (0.3,0.4,0.2) & (0.4,0.2,0.2) \\
     P_3 & (0.4,0.2,0.3) & (0.6,0.2,0.1) & (0.4,0.2,0.2) \\
     P_4 & (0.4,0.2,0.3) & (0.3,0.2,0.2) & (0.4,0.4,0.3)
\end{bmatrix}
$$

Hence

$$
W = \begin{bmatrix}
     D_1 & D_2 & D_3 \\
     P_1 & 0.4 & 0.2 & 0.3 \\
     P_2 & 0.3 & 0.5 & 0.4 \\
     P_3 & 0.3 & 0.7 & 0.4 \\
     P_4 & 0.3 & 0.3 & 0.5
\end{bmatrix}
$$

and finally it is observed that

$$
V - W = \begin{bmatrix}
     D_1 & D_2 & D_3 \\
     P_1 & 0.2 & 0.4 & 0.2 \\
     P_2 & 0.6 & 0.2 & 0.1 \\
     P_3 & 0.3 & -0.1 & 0.2 \\
     P_4 & 0.1 & 0.5 & 0.0
\end{bmatrix}
$$

It is clear from the above matrix that the patients $\{p_1, p_4\}$ suffering from disease $\{D_2\}$ and patients $\{p_2, p_3\}$ are suffering from disease $\{D_1\}$.

5 | Conclusions

It is seen that this method of finding the patients suffering from various diseases from studying the occurrence of systems with the help of one simple application. In future our effort will be to apply this method to various other situations involving some of the uncertain parameters.
References