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## Evaluation and Selection of Supplier in Supply Chain with Fuzzy Analytical Network Process Approach

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### Abstract

One of the most important issues concerning the designing a supply chain is selecting the supplier. Selecting proper suppliers is one of the most crucial activities of an organization towards the gradual improvement and a promotion in performance. This intricacy is because suppliers fulfil a part of customer's expectancy and selecting among them is multi-criteria decision, which needs a systematic and organized approach without which this decision may lead to failure. The purpose of this research is proposing a new method for assessment and rating the suppliers. We have identified several evaluation criteria and attributes; the selection among them was by the Simple Multi-Attribute Rating Technique (SMART) method, then we have specified the connection and the influence of the criteria on each other by DEMATEL method. After that, suppliers were graded by using the Fuzzy Analytical Network Process (FANP) approach and the most efficient one was selected. The innovation of this research is combining the SMART method, DEMATEL method, and Analytical Network Process in Fuzzy state which lead to more exact and efficient results which is proposed for the first time by the researchers of this study.

**Keywords:** Supply Chain Management (SCM), Supplier selection, Multi Criteria Decision Making (MCDM), DEMATEL, Fuzzy Analytical Network Process (FANP).

## 1 | Introduction

The objective of supplier selection is to identify suppliers with the highest potential for meeting an organization's needs undeviatingly and at an acceptable price. Selection is considered a comparison of suppliers using a set of criteria and measures. However, the level of detail used for examining potential suppliers may vary depending on an organization's needs. The ultimate goal of selection is to identify suppliers with the highest potential. A supply chain is composed of all links from suppliers to customers: suppliers, manufacturing plants, warehouses, distribution centers, and retailers.



Supplier selection and evaluation are recognized as a strategic and crucial component of supply chain strategy. A good coordination between a manufacturer and suppliers is necessary because the failure of coordination contributes to excessive delays, products with poor quality and ultimately results in poor customer services. When companies turn to outside suppliers and manufacturers and become more dependent on outsourcers, the consequences of poor decision-making become more severe. As a result, it is too important for an outsourced-type manufacturer to assess, manage and select its suppliers. The objective of supplier selection process is to identify high potential suppliers for meeting a manufacturer's needs undeviatingly and at an acceptable overall performance. Selecting suppliers among a large number of possible ones with various levels of capabilities and potential is an extremely difficult task and a Supplier Selection. Multi Criteria Decision Making (MCDM) problem. supplier selection decisions are intricate owing to the fact that various criteria must be taken into consideration in the decision-making process. To select the prospective suppliers, the organization judges each supplier's ability to meet consistently and cost-effectively its needs using selection criteria and proper measures.

Criteria and measures are developed to be applicable to all the suppliers being considered and to indicate the organization's needs and its supply and technology strategy. It may be demanding to convert its needs into useful criteria, due to the fact that needs are often expressed as general qualitative concepts while criteria should be specific requirements that can be quantitatively assessed. As more companies become interested in developing and executing strategic partnership with their suppliers, an effective tool is needed to help these organizations in prequalifying their suppliers based on their overall performances, in selecting the best suppliers and in developing and managing the strategic partnership. A number of alternative approaches have been proposed to consider these criteria, called mathematical programming models, multiple attribute decision aid methods, cost-based methods, statistical and probabilistic methods, combined methodologies, and other methods. Consequently, significant amounts of researches have been conducted for supplier assessment and selection problem since the 60s.

Increasing in the number of commercial competition and expanse of global markets cause the organizations to pay more attention to improving the quality of their activities and processes taking into account the measures and competitive criteria, which entails the suppliers selection. Harland et al. [1] argued about the main aim of Supply Chain Management (SCM) is to improve the competitive advantage by focusing on productivity of suppliers' processes, technology, and their abilities. Waters [2] studied the reason of supply chain existence is to meet customers' expectations taking into consideration bringing benefits for the various segments of the chain and ultimate aim of a chain is to maximize the benefits and whole values of segments of the chain. The main reason that organizations focus on their supply chains is due to the short life of products and the variable customers' expectations as a threat, and information technology improvement as an opportunity. Karpak et al. [3] surveyed the main aim of SCM is considered as improving the operational effectiveness, profitability and competitive state of an agency and its supply chain which includes all segments. Pang & Bai [4] argued about evaluation and selection of suppliers are known as a strategic and important part of choosing a long-term approach regarding the supply chain. A thorough cooperation between the factory and the supplier is needed owing to the fact that the failure of corporation leads to excessive delays, poor quality product, and superfluous costs and eventually results in poor servicing; hence, substandard decision-making makes more problems. Therefore, the issue of the best supplier selection and evaluation for a producer is considered a crucial case develops a supplier evaluation approach based on the Analytic Network Process (ANP) and fuzzy synthetic evaluation under a fuzzy environment. The importance weights of various criteria are considered as linguistic variables.

These linguistic ratings can be expressed in triangular fuzzy numbers by using the fuzzy extent analysis. Fuzzy synthetic evaluation is used to select a supplier alternative and the Fuzzy ANP (FANP) method is applied to calculate the importance of the criteria weights. Then an integrated FANP and fuzzy synthetic evaluation methodology is proposed for evaluating and selecting the most suitable suppliers. De Boer et al. [5] stated the most important subjects concerning the purchase management are: supplier selection, the commercial partner, and the issue of determining the optimal amount of order. Weber et al. [6], [7] found the weak point in most of the supplier selection methods is not considering the criteria as variable components and in a span. Although the criteria are quantified using methods such as Analytical Hierarchy Process (AHP), they considered price, delivery time, and quality as criteria to select the suppliers Carrera and Mayorga [8] provided a fuzzy set application in supplier selection for new product development. The model quantifies these four multiple criteria in terms of Taguchi quality loss and then uses an AHP to combine them into one global variable for decision-making.

Kubat and Yuce [9] proposed a hybrid intelligent approach for supply chain management system, which combines AHP, Fuzzy AHP and Genetic Algorithm (GA). Some researchers have applied the FANP based approach to solve complex decision-making problems [10]-[19]; some scholars also proposed decision-making models in generalized fuzzy sets [20]-[29]. Narasimhan et al. [30] proposed an AHP-based methodology for supplier selection and performance evaluation. Jeong and Lee [31] proposed a Multi-Criteria Supplier Selection (MCSS) model to deal with the supplier selection problems in the SCM, where a fuzzy-based methodology is used to assess the ratings for the qualitative factors, such as profitability and quality. Xia and Wu [32] proposed an integrated AHP approach based on rough sets theory with multiple criteria and with supplier's capacity constraints. Among the available multi-attribute decision-making methods, only the ANP can be used to evaluate the most suitable suppliers systematically due to the dependencies and feedbacks caused by the mutual effects of the criteria. Kumar et al. [33] studied a fuzzy multi-objective integer programming problem incorporating three important goals: cost-minimization, quality-maximization and maximization of on-time delivery with the realistic constraints such as meeting the buyers' demand, vendors' capacity, vendors' quota flexibility, etc. In the proposed model, various input parameters have been treated as vague with a linear membership function of fuzzy type. The model acts as a decision tool facilitating the vendor selection and their quota allocation under different degrees of information vagueness in the decision parameters of a supply chain modeling.

Recent information and communication developments caused that global organizations spread out their markets throughout the world. In this environment, local exclusive markets have been replaced with global competitive ones. Therefore, organizations must concentrate on their main operations to survive in such an environment. To do so, managers have intended to cooperate with some financial partners in long-term relations. In this paper, the aim is to develop a FANP model to evaluate the potential suppliers and select the best one(s) with respect to the vendor important factors. Additionally, ANP is developed by fuzzy sets theory to cover the indeterminacy of decisions made in this field. The authors have augmented the model with a non-linear programming model to elicit eigenvectors from fuzzy comparison matrices. Hybridization of these two concepts can model supplier selection problem in all circumstances and reaches the optimal choice.

## 2 | Methodology

### 2.1 | The Steps of Methodology

This study includes the following steps:

**Step1:** Criteria selection.

**Step 2:** Choosing and building a prototype.

**Step 3:** Screening criteria by Simple Multi-Attribute Rating Technique (SMART) method.

**Step 4:** Identifying the effects of the criteria on each other by DEMATEL method.

**Step 5:** Forming matrixes of FANP method.

**Step 6:** Selecting the best supplier.

## 2.2| SMART Method

SMART was introduced by Winterfeldt and Edwards in 1986 [34], [35], in which a limited number of alternatives are examined based on a limited number of attributes. The present method aimed to rank the alternatives by a combination of quantitative and qualitative attributes. This is a convenient technique because of its ease of use, which is used in many cases such as evaluation of nuclear waste disposal sites and ERP system selection [36]. SMART assigns the center of gravity of weights method to the purposes through the following way. Suppose that  $w_1$  is the weight of the most important (the first) purpose,  $w_2$  is the weight of the second important purpose,  $w_3$  is the weight of the third purpose, etc.

$$W_1 = \frac{1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}}{n},$$

$$W_2 = \frac{0 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}}{n},$$

$$W_k = \frac{0 + 0 + 0 + \dots + \frac{1}{n}}{n}.$$

The sum of all weights, in this way, equals to one. The more the number of purposes, the less errors in estimating the weights.

## 2.3| DEMATEL Method

The DEcision-MAking Trial and Evaluation Laboratory (DEMATEL) method was introduced by Fonetla and Gabus in 1971 [37]-[40], mainly used to study very complex global issues. The DEMATEL method is applied to construct a network relation design in order to examine the internal relation among the attributes.

The steps used in DEMATEL technique are as the followings:

**First Step:** Determine the fundamental elements of the system.

**Second Step:** Assign the given elements to the points of a diagram and determine the effects between them. Elements are compared in pairs and the judgments are made just for the direct effects of the elements.

**Third Step:** Ask the experts for the intensity of the final effects of elements on each other. This intensity is in the form of grading (for instance 0-4, 0-10, or 0-100).

**Fourth Step:** Show the numbers resulted from the previous steps in a  $\hat{M}$  matrix.

**Fifth Step:** Add the numbers in each row and find the row with the highest sum ( $\alpha$ ) and then divide each number of  $\hat{M}$  matrix by  $\alpha$  ( $M = \alpha \cdot \hat{M}$ ).

**Sixth Step:** the result of the following equation is the final direct and indirect effect on each other:

$$S = M + M^2 + M^3 + \dots + M^t = \frac{M(I-M)}{(I-M)} ; \lim_{t \rightarrow \infty} M^t = 0 = \frac{M}{(I-M)} = M(I-M)^{-1}.$$

We use the FANP method that has been developed in [41].

## 2.4 | The Analytic Network Process

Saaty [42], proposed ANP to decompose a multi-criteria decision making problem into components. Final decision is gained by a logical determination of components' values and aggregation of them [43]-[45]. A comprehensive research was performed by Taslicali and Ercan [46], comparing the analytic hierarchy process and the analytic network process. The authors concluded that the ANP compensates weaknesses of the AHP. They mentioned some of the drawbacks of AHP as followings. AHP can model linear and strictly hierarchical structure, while ANP can be applied to tackle more general structure including interrelationships between different criteria in different clusters or within the same cluster. Moreover, it is indicated that ANP is more accurate in complex situation due to its capability of modeling complex structure and the way in which comparisons are performed. Hence, the ANP can be considered as a more general form of the AHP in which dependencies and feedbacks between elements of a decision can be modeled.

The following sections describe the steps of the ANP. Although ANP is one of the most complete and comprehensive multi-attribute decision-making methods as it encompass the criteria and alternatives in an integrated manner, a great drawback of this method is the pair-wise comparison section. This section consists of deterministic comparisons, while real world has an indeterminate nature. Therefore, fuzzy sets theory is adopted in this research to cope with this drawback.

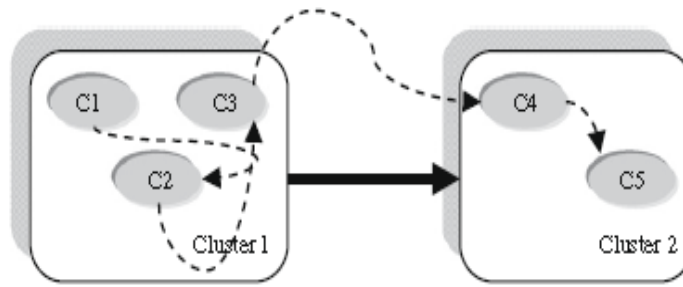
**Criteria Definition.** In this step, criteria which affect the decision being made must be defined. In order to define the criteria, a group of managers who make the decision or consultants (e.g., an expert group) can be an appropriate choice. In order to select suppliers, various criteria have been introduced through years during which supplier selection problem have been challenged by academicians and practitioners.

**Network Formation.** Network formation comprises two steps described as follows:

- *Clustering.* Some clusters are formed with respect to the criteria. Then, the criteria are assigned to the clusters to which are mostly related. Finally, alternatives make a separate cluster.
- *Connecting.* In this step, the related clusters are connected with respect to the dependencies between their corresponding criteria. The connections which reflect interrelationships and feedback structure can be either inner (between two criteria within the same cluster) or outer (between two different clusters). An inner connection is like a loop on the corresponding cluster. Connection between two criteria is signed with an arrow from the affecting criterion to the dependent one. As it is shown in Fig. 1, the dotted arrows show connections between criteria within one cluster or two different clusters, while the connection between two clusters are represented by solid arrows as a result of connections between criterion 3 and criterion 4 from two clusters.

**Pair-Wise Comparisons.** Pairwise comparisons are performed between each pair of criteria with respect to a control criterion. Control criterion is the criterion to which some other criteria are dependent. In other words, the group of criteria connected to a specific (control) criterion is compared

pair-wisely. In addition to the comparisons of criteria, clusters of the network must be compared pair-wisely with respect to the control cluster.



**Fig. 1. A network with clusters and connections.**

The comparisons are performed using a similar scale to the AHP's (Saaty [42]) (shown in *Table 1*). Similar to the AHP, each comparison results in a matrix with an eigenvector showing the final priorities regarding the control criterion. *Eq. (1)* shows the normalization of the comparison matrices, while  $a_{ij}$  are the scores assigned to factors  $i$  being compared with factors  $j$ . Calculation of eigenvector  $w$  is indicated in *Eq. (2)*, where  $w_i$  are the relative importance of factors  $i$ :

$$A=[a_{ij}/\sum_j a_{ij}], \quad (1)$$

$$\Rightarrow \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{\sum_{j=1}^n a_{1j}}{\sum_{j=1}^n \sum_{i=1}^n a_{ij}} \\ \frac{\sum_{j=1}^n a_{2j}}{\sum_{j=1}^n \sum_{i=1}^n a_{ij}} \\ \dots \\ \frac{\sum_{j=1}^n a_{nj}}{\sum_{j=1}^n \sum_{i=1}^n a_{ij}} \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ \dots \\ w_n \end{bmatrix} \quad (2)$$

**Supermatrix Calculation.** Un-weighted supermatrix is constructed by putting the eigenvectors together (please refer to the super matrix in Statement).

**Table 1. Scoring scales pair-wise comparison.**

Score	Definition	
1	Equal importance	Two activities contribute equally to the objective.
2	Weak importance	
3	Moderate importance	Experience and judgment slightly favor one activity over another.
4	Moderate plus	
5	Strong importance	Experience and judgment strongly favor one activity over another.
6	Strong plus	
7	Very strong demonstrated importance	An activity is favored very strongly over another; its dominance demonstrated in practice.
8	Very, very strong	
9	Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation.



This super matrix consists of blocks  $w_{ij}$  which are the matrices of eigenvectors corresponding to the comparison of elements of cluster  $i$  with respect to the control elements of cluster  $j$ .

$$W = \begin{bmatrix} w_{11} & w_{12} & \dots & w_{1n} \\ w_{21} & w_{22} & \dots & w_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ w_{n1} & w_{n2} & \dots & w_{nm} \end{bmatrix} \quad (3)$$

**Weighted Supermatrix.** The un-weighted supermatrix is not necessarily column stochastic, i.e., all columns do not sum to one. To normalize the columns, each eigenvector in a column is multiplied by the clusters' relative priorities obtained using comparison matrix of the clusters. The resulted column stochastic matrix is the weighted supermatrix.

**Limit Supermatrix.** In the network model, the direct dependencies are signed by connections and the indirect ones are neglected. For example in *Fig. 1*,  $C5$  is dependent to  $C4$ ,  $C4$  is dependent to  $C3$ , but  $C5$  is not directly connected to  $C3$ . These indirect dependencies are not taken into account in weighted super matrix as it is formed regarding only direct dependencies and feedbacks in the model. To cope with this problem, weighted super matrix is powered until it converges, i.e., its rows stabilize to a unique value. The resulted matrix is limit super matrix.

**Selection.** Final priorities or weights of alternatives are the values in rows corresponding to the alternatives based upon which final decision is made.

### 3 | Proposed Algorithm

To cope with the complexity of the problem as well as cover different dependent and independent aspects and criteria, the analytic network process is applied. The effective application of the ANP is accomplished providing that decision makers know the objectives, decision environment and decision elements. A suitably modeled decision is the result of this knowledge, as the decision makers need it to define the criteria and their dependencies. Additionally, pairwise comparisons must be realistic as much as possible.

As the complete and accurate information is not always available and the decision making process is indeterminate in nature, the proposed ANP is accompanied with fuzzy sets theory. To model the indeterminacy of a comparison, fuzzy comparison is a solution in which a limited and continuous interval of numbers is utilized. With the assumption of triangular fuzzy numbers, paired numbers  $(l_{ij}, u_{ij})$  representing the lower and upper possible values of  $a_{ij}$ , indicate the preference of  $i$  to  $j$ , resulting a matrix like the matrix shown in *Statement (4)*. The conventional way to elicit the weights in ANP is not applicable.

Therefore, a special optimization model is adopted to elicit the best weights. Calculation of the eigenvectors is described in the following sub-section, as it is dissimilar to the conventional ANP.

$$\begin{bmatrix} 1 & (l_{12}, u_{12}) & \dots & (l_{1n}, u_{1n}) \\ (l_{21}, u_{21}) & 1 & \dots & (l_{2n}, u_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ (l_{n1}, u_{n1}) & (l_{n2}, u_{n2}) & \dots & 1 \end{bmatrix}. \quad (4)$$

As mentioned, the fuzzy comparison matrices are similar to the one demonstrated in *Statement (4)*. It is clear that the reciprocal value of  $(l_{ij}, u_{ij})$  is  $(1 / u_{ij}, 1 / l_{ij})$  (Chang [47]).

If  $W = (w_1, w_2, \dots, w_n)$  is the eigenvector of the matrix shown in *Eq. (4)*, in which  $w_i$  are the relative weights of  $i$  from the comparison matrix, following conditions are met

$$w_1 + w_2 + \dots + w_n = 1, \quad (6)$$

$$\forall i, j \quad (7)$$

$$\text{s.t. } i \in \{1, 2, 3, \dots, n-1\}; j \in \{2, 3, \dots, n\} \rightarrow l_{ij} \leq w_i / w_j \leq u_{ij}, \quad (8)$$

$$\forall i \in \{1, 2, \dots, n\} \quad w_i > 0. \quad (8)$$

Therefore, vector  $W$  must be determined such that it meets the preceding conditions as well as demonstrates the decision-makers preferences.  $\tilde{N}$  is a normalized fuzzy set of triangular fuzzy numbers. This set is defined by three values  $a \leq b \leq c$  for any number of which a membership function is defined. This membership function has the following attributes:

It is a continuous function such that  $R \rightarrow [0, 1]$ .

Following function is confirmed.

$$\mu_N(x) = \begin{cases} (c-x) / (c-b) & b \leq x \leq c, \\ (x-a) / (b-a) & a \leq x \leq b, \\ 0 & \text{otherwise.} \end{cases} \quad (8)$$

For which

$$\int_{-\infty}^{\infty} \mu_N(x) dx = 1. \quad (9)$$

In

this kind of fuzzy numbers,  $b$  is the central value with the highest probability,  $a$  and  $c$  represent the fuzziness. The triangular numbers are selected as the real  $w_i$  is a unique number. It is assumed to apply symmetric triangular fuzzy numbers, hence the best value is obtained when  $w_i / w_j$  closes to  $(l_{ij} + u_{ij}) / 2$  using fuzzy comparison values  $(l_{ij}, u_{ij})$ . By means of this concept, weights of criteria are elicited. Supposing *Eq. (10)*, the membership function of the number  $\varphi_{ij}$  is defined as *Eq. (11)* in interval  $(l_{ij}, u_{ij})$ .

$$\varphi_{ij} = w_i / w_j, \quad m_{ij} = (l_{ij} + u_{ij}) / 2, \quad (10)$$

$$M(\varphi_{ij}) = \begin{cases} (\varphi_{ij} - m_{ij}) / (u_{ij} - m_{ij}) & ; \quad \varphi_{ij} > m_{ij}, \\ (m_{ij} - \varphi_{ij}) / (m_{ij} - l_{ij}) & ; \quad \varphi_{ij} \leq m_{ij}, \\ 0 & ; \quad \varphi_{ij} > u_{ij} \text{ or } \varphi_{ij} < l_{ij}. \end{cases} \quad (11)$$

It must be noted that the membership function is a triangular function with the base to the top. This function is an index of how distant  $w_i / w_j$  is from average  $m_{ij}$ . Thus,  $W$  is optimum when minimizes *Eq. (12)* and follows *Eqs. (5)–(7)*.



$$\sum_{i < j} M(\varphi_{ij}) \quad ; \quad i \in \{1, 2, 3, \dots, n-1\}, \quad j \in \{2, 3, 4, \dots, n\}. \quad (12)$$

So the resulted mathematical model is as follows

$$\begin{aligned} & \text{Min } \sum_{i < j} M(\varphi_{ij}), \\ & \text{s.t. } -w_i + l_{ij} w_j \leq 0 \quad i \in \{1, 2, 3, \dots, n-1\}, \end{aligned} \quad (13)$$

$$j \in \{2, 3, 4, \dots, n\}, \quad w_i - u_{ij} w_j \leq 0 \quad i \in \{1, 2, 3, \dots, n-1\}, \quad (14)$$

$$j \in \{2, 3, 4, \dots, n\}, \quad (15)$$

$$w_1 + w_2 + \dots + w_n = 1, \quad (16)$$

$$w_i > 0 \quad ; \quad \forall i \in \{1, 2, 3, \dots, n\}. \quad (17)$$

If the mathematical model is infeasible, the judgments are inconsistent and must be revised. As the membership function is multifunctional, optimization of this function is onerous with the conventional methods. To do so, the distance between  $\varphi_{ij}$  and  $m_{ij}$  is minimized using the model described by Eq. (18). It must be noted that minimization of square of the distance is the same as the distance itself.

$$\text{Min } \sum_{i < j} \left( \frac{\varphi_{ij} - m_{ij}}{(u_{ij} - l_{ij})/2} \right)^2, \quad (18)$$

$$\text{s.t. } -w_i + l_{ij} w_j \leq 0 \quad i \in \{1, 2, 3, \dots, n-1\}, \quad j \in \{2, 3, 4, \dots, n\},$$

$$w_i - u_{ij} w_j \leq 0 \quad i \in \{1, 2, 3, \dots, n-1\}, \quad j \in \{2, 3, 4, \dots, n\},$$

$$w_1 + w_2 + \dots + w_n = 1,$$

$$w_i > 0 \quad ; \quad \forall i \in \{1, 2, 3, \dots, n\}.$$

Solving this model using optimizer software packages, the eigenvector of fuzzy comparison matrix is gained which can be utilized in the ANP. It is notable that the model can be solved by the optimizer, because the number of elements being compared pair-wisely is recommended to be less than seven (Saaty, [42]). Pang and bai [4] recommend the numbers in the following tables for making decision in a fuzzy state.

Table 2. Linguistic expression for fuzzy scale of relative weights of criteria.

Definition of Linguistic Variables for Relative Weights of Criteria	Triangular Fuzzy Scale	Triangular Fuzzy Reciprocal Scale
Just equal	(1,1,1)	(1,1,1)
Equally important	M1=(1,1,3)	(1/3,1,1)
Weakly important	M3=(1,3,5)	(1/5,1/3,1)
Essentially important	M5=(3,5,7)	(1/7,1/5,1/3)
Strongly important	M7=(5,7,9)	(1/9,1/7,1/5)
Absolutely important	M9=(7,9,9)	(1/9,1/9,1/7)
Intermediate values between two adjacent M2, M4, M6, M8		

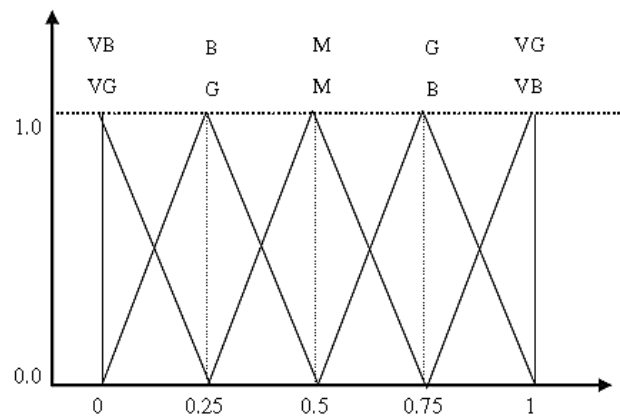


Fig. 2. Membership functions of linguistic values for criteria rating.

Table 3. Linguistic values and mean of fuzzy numbers for alternatives.

Linguistic Values of Negative Criteria for Alternatives	Linguistic Values for Criteria For Alternatives	The Mean of Fuzzy Numbers
Very bad (VB)	Very good (VG)	1
Bad (B)	Good (G)	0.75
Medium (M)	Medium (M)	0.5
Good (G)	Bad (B)	0.25
Very good (VG)	Very bad (VB)	0

## 5 | Numerical Example

Suppose that a firm has three suppliers ( $A_1, A_2, A_3$ ). Considering the introduced model, and the information about each supplier, the best supplier will be selected.

**Step 1. Criteria Selection.** The following criteria are selected by the experts from the criteria in **Table 1**: quality, on time delivery, cost, reliability, services, technical capabilities, location, financial status, partnership, operational control, supplier reputation, packing, background, reciprocity, flexibility, discounts, transport, and risks.

**Step 2. Choosing and Building a Prototype.** All supplier selection criteria can be classified into two categories as following:

Company status.

Performance criteria.

**Step 3. Screening Criteria by SMART.** For this purpose, we have considered the following indicators for screening: 1) applicability, 2) being measurable, 3) the frequency of the criteria in other researches, and 4) being perfect. The weight of these indicators:

$$w_1=0.521, w_2=0.271, w_3=0.145, w_4=0.63.$$

Screening and ranking the criteria by using these indicators and SMART method. In this stage, the criteria which have the highest points are selected as following.

**Table 4. Selected criteria.**

No	Name	Selected Criteria	Points
1	C1	Cost	94.56
2	C2	On time delivery	85.79
3	C3	Quality	83.87
4	C4	Flexibility	63.73
5	C5	Services	64.48
6	C6	Financial status	67.12
7	C7	Supplier reputation	69.65
8	C8	Technical capabilities	76.26
9	C9	Background	67.90
10	C10	Location	57.60

**Step 4. Identify the Effects of the Criteria on Each Other by DEMATEL.** Our experts were asked to rate and grade the effects of criteria on each other from 0 – 4 in some matrixes, the result of which is as the following:

These matrixes are for the C1 to C5 criteria:

$$\hat{M} = \begin{bmatrix} 0 & 3 & 4 & 3 & 2 \\ 3 & 0 & 2 & 2 & 1 \\ 3 & 2 & 0 & 1 & 1 \\ 2 & 1 & 2 & 0 & 1 \\ 2 & 1 & 2 & 1 & 0 \end{bmatrix},$$

$$M = \begin{bmatrix} 0 & 0.25 & 0.33 & 0.25 & 0.166 \\ 0.25 & 0 & 0.166 & 0.166 & 0.083 \\ 0.25 & 0.166 & 0 & 0.083 & 0.083 \\ 0.166 & 0.083 & 0.166 & 0 & 0.083 \\ 0.166 & 0.083 & 0.166 & 0.083 & 0 \end{bmatrix},$$

$$M(I - M)^{-1} = \begin{bmatrix} 0.814 & 0.827 & 1.000 & 0.737 & 0.586 \\ 0.776 & 0.46 & 0.676 & 0.521 & 0.404 \\ 0.727 & 0.563 & 0.492 & 0.428 & 0.377 \\ 0.583 & 0.43 & 0.55 & 0.291 & 0.327 \\ 0.583 & 0.43 & 0.55 & 0.366 & 0.251 \end{bmatrix}.$$

These matrixes are for the C6 to C10 criteria:

$$\widehat{M} = \begin{bmatrix} 0 & 3 & 4 & 2 & 1 \\ 3 & 0 & 1 & 2 & 0 \\ 2 & 3 & 0 & 2 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 2 & 2 & 1 & 2 & 0 \end{bmatrix},$$

$$M = \begin{bmatrix} 0 & 0.3 & 0.4 & 0.2 & 0.1 \\ 0.3 & 0 & 0.1 & 0.2 & 0 \\ 0.2 & 0.3 & 0 & 0.2 & 0 \\ 0.1 & 0.1 & 0 & 0 & 0 \\ 0.2 & 0.2 & 0.1 & 0.2 & 0 \end{bmatrix},$$

$$M(I - M)^{-1} = \begin{bmatrix} 0.435 & 0.716 & 0.66 & 0.591 & 0.144 \\ 0.518 & 0.313 & 0.344 & 0.445 & 0.052 \\ 0.481 & 0.578 & 0.255 & 0.473 & 0.048 \\ 0.195 & 0.203 & 0.1 & 0.104 & 0.019 \\ 0.478 & 0.504 & 0.346 & 0.475 & 0.048 \end{bmatrix}.$$

The final diagram of the pre-mentioned matrixes is as it follows.

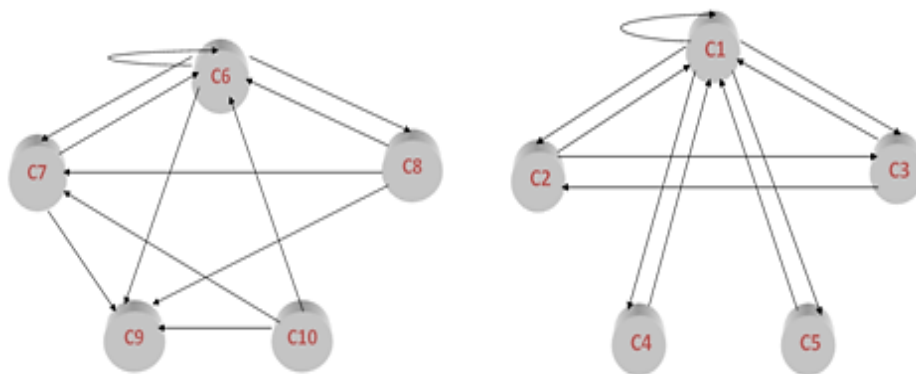


Fig. 3. Effects of criteria on each other.

The final diagram of the whole criteria is as the following.

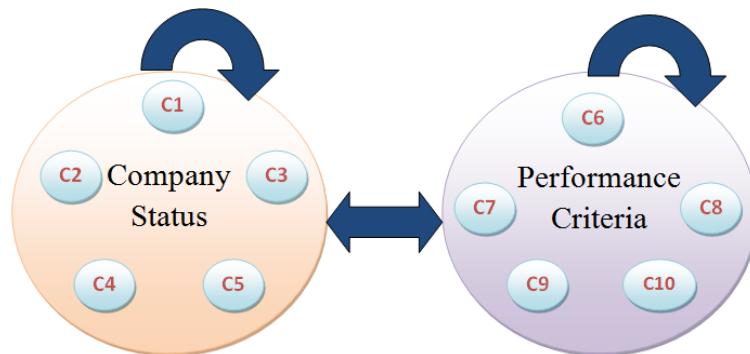


Fig. 4. Final diagram of effects of criteria on each other.

By adding the suppliers, this diagram is resulted:

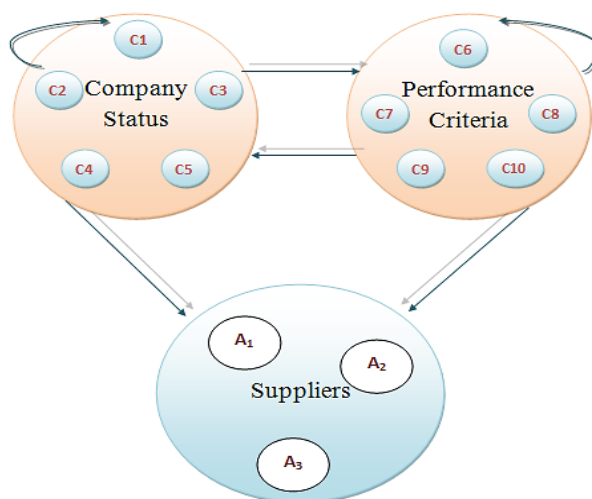


Fig. 5. Final diagram of effects of criteria and suppliers on each other.

**Step 5. Forming matrixes of FANP.** The suppliers' information is categorized in the following tables:

Table 5. Suppliers compared with cost.

Cost	A1	A2	A3
A1	1	1.05	1.15
A2	0.95	1	1.1
A3	0.87	0.91	1

The cost calculation in the following tables is the result of the above-mentioned table:

Table 6. Suppliers compared with cost.

Cost	A1	A2	A3
A1	1	1.05	1.15
A2	0.95	1	1.1
A3	0.87	0.91	1
Sum	2.82	2.96	3.25

The normalized form is

Table 7. Suppliers compared with cost.

Cost	A1	A2	A3
A1	0.3546	0.3547	0.3538
A2	0.3369	0.3378	0.3385
A3	0.3085	0.3075	0.3077

To sum up:

$$W_1 = (0.3546 + 0.3547 + 0.3538) / 3 = 0.3544,$$

$$W_2 = (0.3369 + 0.3378 + 0.3385) / 3 = 0.3377,$$

$$W_3 = (0.3085 + 0.3075 + 0.3077) / 3 = 0.3079.$$

Other information for the other criteria is provided in a fuzzy state, some of which are mentioned in the following tables:

**Table 8. Suppliers compared with on time delivery.**

On Time Delivery	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>
A <sub>1</sub>	(1,1)	(2/5 , 2/3)	(1/3 , 1/2 )
A <sub>2</sub>	(3/2 , 5/2)	(1 , 1)	(2/5 , 2/3)
A <sub>3</sub>	(2 , 3)	(3/2 , 5/2)	(1 , 1)

Therefore:

$$\text{Min } (900w_1^2/w_2^2 - 960w_1/w_2 + 144w_1^2/w_3^2 - 120w_1/w_3 + 900w_2^2/w_3^2 - 960w_2/w_3 + 537)$$

s.t.

$$-w_1 + 2/5 w_2 \leq 0,$$

$$w_1 - 2/3 w_2 \leq 0,$$

$$-w_1 + 1/3 w_3 \leq 0,$$

$$w_1 - 1/2 w_3 \leq 0,$$

$$-w_2 + 2/5 w_3 \leq 0,$$

$$w_2 - 2/3 w_3 \leq 0,$$

$$w_1 + w_2 + w_3 = 1.$$

To sum up:

$$w_1 = 0.1732,$$

$$w_2 = 0.3016,$$

$$w_3 = 0.5251,$$

and the information about the flexibility of suppliers are given in the following table.

**Table 9. Suppliers compared with flexibility.**

Flexibility	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>
A <sub>1</sub>	(1 , 1)	(1/2 , 3/2)	(1 , 2)
A <sub>2</sub>	(2/3 , 2)	(1 , 1)	(1 , 2)
A <sub>3</sub>	(1/2 , 1)	(1/2 , 1)	(1 , 1)

Therefore:

$$\text{Min } (4w_1^2/w_2^2 - 8w_1/w_2 + 4w_1^2/w_3^2 - 12w_1/w_3 + 4w_2^2/w_3^2 - 12w_2/w_3 + 22)$$

s.t.

$$-w_1 + 1/2 w_2 \leq 0,$$

$$w_1 - 3/2 w_2 \leq 0,$$

$$-w_1 + w_3 \leq 0,$$

$$w_1 - 2w_3 \leq 0,$$

$$-w_2 + w_3 \leq 0,$$

$$w_2 - 2w_3 \leq 0,$$

$$w_1 + w_2 + w_3 = 1.$$

To sum up:

$$w_1 = 0.3750,$$

$$w_2 = 0.3750,$$

$$w_3 = 0.2500.$$

Step 6. Selecting the Best Supplier. We assign the resulted weights to un-weighted supermatrix and continue the calculations.



**Table 10. Un-weighted supermatrix respecting the change.**

	<b>A1</b>	<b>A2</b>	<b>A3</b>	<b>C1</b>	<b>C2</b>	<b>C3</b>	<b>C4</b>	<b>C5</b>	<b>C6</b>	<b>C7</b>	<b>C8</b>	<b>C9</b>	<b>C10</b>
<b>A1</b>	0.0000	0.0000	0.0000	0.3544	0.1732	0.1732	0.1732	0.1732	0.3333	0.3333	0.3333	0.3750	0.5062
<b>A2</b>	0.0000	0.0000	0.0000	0.3377	0.3016	0.3016	0.3016	0.3016	0.3333	0.3333	0.3333	0.3750	0.3072
<b>A3</b>	0.0000	0.0000	0.0000	0.3079	0.5251	0.5251	0.5251	0.5251	0.3333	0.3333	0.3333	0.2500	0.1865
<b>C1</b>	0.0000	0.0000	0.0000	0.8140	0.7760	0.7270	0.5830	0.5830	0.5000	0.5000	0.2000	0.3000	0.1000
<b>C2</b>	0.0000	0.0000	0.0000	0.8270	0.0000	0.5630	0.0000	0.0000	0.6000	0.3000	0.7000	0.5000	0.3000
<b>C3</b>	0.0000	0.0000	0.0000	1.0000	0.6760	0.0000	0.0000	0.0000	0.5000	0.3000	0.4000	0.4000	0.2000
<b>C4</b>	0.0000	0.0000	0.0000	0.7370	0.0000	0.0000	0.0000	0.0000	0.5000	0.0000	0.4000	0.4000	0.2000
<b>C5</b>	0.0000	0.0000	0.0000	0.5860	0.0000	0.0000	0.0000	0.0000	0.5000	0.0000	0.5000	0.5000	0.3000
<b>C6</b>	0.0000	0.0000	0.0000	0.8000	0.5000	0.7000	0.1000	0.4000	0.4350	0.5180	0.4810	0.0000	0.4780
<b>C7</b>	0.0000	0.0000	0.0000	0.0000	0.7000	0.7000	0.8000	0.7000	0.7160	0.0000	0.5780	0.0000	0.5040
<b>C8</b>	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2000	0.0000	0.6600	0.0000	0.0000	0.0000	0.0000
<b>C9</b>	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5910	0.4450	0.4730	0.0000	0.4750
<b>C10</b>	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

For calculating the limit supermatrix, weighted supermatrix should be multiplied in itself for several times to reach the stability, as it was mentioned. In this way the limit supermatrix is resulted such as the table above. Taking into consideration the resulted weights, the volunteer suppliers are ranked as the following:

$A_3 > A_2 > A_1$ .

Table 11. Weighted supermatrix respecting the change.

[illegible]

**Table 12. Limit supermatrix respecting the change.**

[illegible]

## 6 | Conclusion

The supplier selection process is a MCDM issue. However, the exact and certain information is not always available for making decision. As it was noted earlier, most of the information is qualitative, therefore, in this study linguistic and fuzzy variables and a form of FANP are used to comprise the uncertain and ambiguous states; taking into account the combination of the pre-mentioned method with Smarter and Dematel methods, a precise process has been designed which can select, evaluate, grade, and determine the connections and effects of elements in the best form, and as a result they lead to selecting the best supplier. This model is able to cover all levels of decision making for supplier selection, suppliers rates, and proportional and final weights of each criterion. These weights signify the importance of each criterion in relation to the purpose, which is supplier selection. Taking into consideration the characteristics of this model, the mutual effects of the decision making elements can be applied to calculation and the decisions can be made in the best form.

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