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Fuzzy Hypersoft Sets and Its Weightage Operator for Decision Making

Somen Debnath *

Department of Mathematics, Umakanta Academy, Agartala-799001, Tripura, India; somen008@rediffmail.com.

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Abstract

Hypersoft set is an extension of the soft set where there is more than one set of attributes occur and it is very much helpful in multi-criteria group decision making problem. In a hypersoft set, the function F is a multi-argument function. In this paper, we have used the notion of Fuzzy Hypersoft Set (FHSS), which is a combination of fuzzy set and hypersoft set. In earlier research works the concept of Fuzzy Soft Set (FSS) was introduced and it was applied successfully in various fields. The FHSS theory gives more flexibility as compared to FSS to tackle the parameterized problems of uncertainty. To overcome the issue where FSS failed to explain uncertainty and incompleteness there is a dire need for another environment which is known as FHSS. It works well when there is more complexity involved in the parametric data i.e the data that involves vague concepts. This work includes some basic set-theoretic operations on FHSSs and for the reliability and the authenticity of these operations, we have shown its application with the help of a suitable example. This example shows that how FHSS theory plays its role to solve real decision-making problems.

Keywords: Fuzzy set, Soft set, Hypersoft set, Decision making.

1 | Introduction

Uncertainty is a part and parcel of our daily life activities and it exists in various forms. The classical set-theoretic approach could not find any way to deal with incomplete information i.e the information which is blurred. For the scientific computation of vague data, we need a powerful tool that gives us a precise idea about objects so that we get insight into the objects and classify them into different groups. Finally, fuzzy set theory was introduced by Zadeh [24] in 1965 for dealing with uncertain, incomplete, indeterministic information in a systematic way. After the introduction of fuzzy set theory, it has been used successfully in various fields such as engineering, social science, computer science, control theory, game theory, pattern recognition, logic, etc. In the fuzzy set, every object has some membership value and it is called the degree of membership and each membership value belongs to the unit closed interval [0, 1]. So, a fuzzy set is an extension of a crisp set where there are only two choices (0 or 1) to denote the membership of an object i.e

Corresponding Author: somen008@rediffmail.com
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we choose 1 for belongingness and 0 for non-belongingness of an object. Compared to classical set, by using fuzzy set theory we are enabled to extend the range of domain under the fuzzy environment where each object is a fuzzy word or fuzzy sentence or fuzzy axiom, etc. It gives a general formula to model vague or uncertain or indeterministic or incomplete concepts lucidly. But we know that every theory has its limitations so does fuzzy theory. Using fuzzy theory, we only determine the degree of membership of an object but, there is no scope of non-membership degree. We have experienced the co-existence of two opposite concepts like agreement-disagreement, truth-falsity, success-failure, yes-no, attraction-repulsion in a real-life scenario to make a balance. Like this, there is a demand for the coexistence of membership value and non-membership value. To realize the importance of non-membership value along with membership value another set-theoretical notion known as Intuitionistic Fuzzy Set (IFS) was introduced by Atanassov [5] in 1986. In IFS, every object has two values i.e membership value and non-membership value, and their sum range from 0 to 1. For the need of the hour, fuzzy set theory has been applied successfully to develop new theories, propositions, axioms, etc. Some of them are given in [6], [10], [11], [12], [25], [26].

Due to the more complex like uncertainty in data the fuzzy set and its variants are not sufficient for mathematical modeling. It is due to the parameters involved in an attribute. To handle parametric data comprehensively we need another tool to solve the issue. This creates the invention of the Soft Set (SS). Soft set theory was introduced by Molodtsov [16] in 1999. It gives more rigidity to model the vague concept in a parametric way. It is the more general framework as compared to the fuzzy set and its variants. The soft set has been progressed more rapidly and applied in different fields with great success. Some of the novel's work on soft set theory given in [3], [4], [7], [8], [14], [15]. In 2001, a new concept known as Fuzzy Soft Set (FSS) was introduced by Maji et al. [13]. FSS is a combination of Fuzzy Set (FS) and SS. Some applications and extensions of FSS are given in [9], [17], [21], [22].

Recently, Smarandache [20] generalize the concept of the soft set to the hypersoft set, where the function F is transformed into a multi-attribute function. The main motivation of using Hypersoft Set (HSS) is that when the attributes are more than one and further bisected, the SS environment cannot be applied to handle such types of cases. So, there is a worth need to define a new approach to solve these. Afterward, Saeed et al. [19] studied the fundamentals of hypersoft set theory, Abbas et al. [1] defined the basic operations on hypersoft sets, Yolcu and Ozturk [23] introduced fuzzy hypersoft set and its application in decision-making, Ajay and Charisma [2] defined neutrosophic hypersoft topological spaces, aggregate operators of neutrosophic hypersoft set studied in [18], Extension of TOPSIS method under intuitionistic fuzzy hypersoft set environment is discussed in [27], some fundamental operations on interval-valued neutrosophic hypersoft set are discussed in [28].

In this work, we have used the notion of the Fuzzy Hypersoft Set (FHSS), which is an amalgamation of the FS and HSS. Afterward, we define different set-theoretic operations on them, and then there is an attempt to use this concept effectively in multi-criteria decision-making problems using weightage aggregate operator.

2 | Preliminaries

This section includes some basic definitions with examples that are relevant for subsequent discussions.

Definition 1. [24] and [26]. Let X be a non-empty set. Then a fuzzy set A, defined on X, is a set having the form $A=\{(x,\mu_A(x)):x\in X\}$, where the function $\mu_A:X\to[0,1]$ is called the membership function and $\mu_A(x)$ is called the degree of membership of each element $x\in X$.

Definition 2. [16]. Let U be an initial universe and E be a set of parameters. Let $P(U)$ denotes the power set of U, and $A \subseteq E$. Then the pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \rightarrow P(U)$.

Definition 3. [13]. Let U be an initial universe and E be a set of parameters. Let I^U be the set of all fuzzy subsets of U, and $A \subseteq E$. Then the pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by $F : A \rightarrow I^U$.

Definition 4. [20]. Let ξ be the set of the universe and $P(\xi)$ denotes the power set of ξ. Consider $l^1, l^2, ..., l^n$, for $n \geq 1$, be n well-defined attributes, whose corresponding values are respectively the set $L^1, L^2, ..., L^n$ with $L^i \cap L^j = \varnothing$, for $i \neq j$ and $i, j \in \{1, 2, ..., n\}$, then the pair $(F, L^1 \times L^2 \times .... \times L^n)$ is said to be hypersoft set over ξ, where $F : L^1 \times L^2 \times .... \times L^n \rightarrow P(\xi)$.

Example 1. Let $U = \{c_1, c_2, c_3, c_4\}$ be the set of the universe of cars under consideration and $A = \{c_1, c_3\} \subset U$. We consider the attributes to be x_1 = size, x_2 = color, x_3 = cost price (in a dollar), x_4 = mileage,and x_5 = model, and their respective values are given by Size= X_1 = {small, medium, big}; color = X_2 = {white, black, red}; cost price (in dollar)= X_3 = {1000,1050,1080}; model= X_4 = {honda amaze, tata tigor, ford figo}.

Let the function be: $F : X^1 \times X^2 \times X^3 \times X^4 \rightarrow P(U)$. In respect of A, one has assumed that F = ({small, white, 1000, Honda amaze}, {small, white, 1000, tata tigor}, {small, white, 1000, ford figo}, {small, white, 1050, Honda amaze}, {small, white, 1050, tata tigor}, {small, white, 1050, ford figo}, {small, white, 1080, Honda amaze}, {small, white, 1080, tata tigor}, {small, white, 1080, ford figo}, {small, blue, 1000, Honda amaze}, {small, blue, 1000, tata tigor}, {small, blue, 1000, ford figo}, {small, blue, 1050, Honda amaze}, {small, blue, 1050, tata tigor}, {small, blue, 1050, ford figo}, {small, blue, 1080, Honda amaze}, {small, blue, 1080, tata tigor}, {small, blue, 1080, ford figo}, {small, red, 1000, Honda amaze}, {small, red, 1000, tata tigor}, {small, red, 1000, ford figo}, {small, red, 1050, Honda amaze}, {small, red, 1050, tata tigor}, {small, red, 1050, ford figo}, {small, red, 1080, Honda amaze}, {small, red, 1080, tata tigor}, {small, red, 1080, ford figo}, {medium, white, 1000, Honda amaze}, {medium, white, 1000, tata tigor}, {medium, white, 1000, ford figo}, {medium, white, 1050, Honda amaze}, {medium, white, 1050, tata tigor}, {medium, white, 1050, ford figo}, {medium, white, 1080, Honda amaze}, {medium, white, 1080, tata tigor}, {medium, white, 1080, ford figo}, {medium, blue, 1000, Honda amaze}, {mediuml, blue, 1000, tata tigor}, {medium, blue, 1000, ford figo}, {medium, blue, 1050, Honda amaze}, {medium, blue, 1050, tata tigor}, {medium, blue, 1050, ford figo}, {medium, blue, 1080, Honda amaze}, {medium, blue, 1080, tata tigor}, {medium, blue, 1080, ford figo}, {medium, red, 1000, Honda amaze}, {medium, red, 1000, tata tigor}, {medium, red, 1000, ford figo}, {medium, red, 1050, Honda amaze}, {medium, red, 1050, tata tigor}, {medium, red, 1050, ford figo}, {medium, red, 1080, Honda amaze}, {medium, red, 1080, tata tigor}, {medium, red, 1080, ford figo}, {big,white, 1000, Honda amaze}, {big, white, 1000, tata tigor}, {big, white, 1000, ford figo}, {big, white, 1050, Honda amaze}, {big, white, 1050, tata tigor}, {big,white, 1050, ford figo}, {big,white, 1080, Honda amaze}, {big, white, 1080, tata tigor}, {big, white, 1080, ford figo}, {big, blue, 1000, Honda amaze}, {big, blue, 1000, tata tigor}, {big, blue, 1000, ford figo}, {big, blue, 1050, Honda amaze}, {big, blue, 1050, tata tigor}, {big, blue, 1050, ford figo}, {big, blue, 1080, Honda amaze}, {big, blue, 1080, tata tigor}, {big, blue, 1080, ford figo}, {big, red, 1000, Honda amaze}, {big, red, 1000, tata tigor}, {big, red, 1000, ford figo}, {big, red, 1050, Honda amaze}, {big, red, 1050, tata tigor}, {big, red, 1050, ford figo}, {big, red, 1080, Honda amaze}, {big, red, 1080, tata tigor}, {big, red, 1080, ford figo})= $\{c_1, c_3\}$.

Thus, there are 81 possible hypersoft sets to describe $\{c_1, c_3\}$.

Definition 5. [23]. Let ς be the universe of discourse and $P(\varsigma)$ be the power set of ς. Suppose $l^1, l^2, \ldots\ldots, l^n$, for $n \geq 1$, be n well-defined attributes, whose corresponding values are respectively the set $L^1, L^2, \ldots\ldots, L^n$ with $L^i \cap L^j = \varnothing$, for $i \neq j$ and $i, j \in \{1, 2, \ldots\ldots, n\}$, and $L^1 \times L^2 \times \ldots\ldots \times L^n = S$, then the pair (F, S) is said to be the FHSS over ς, where $F : L^1 \times L^2 \times \ldots\ldots \times L^n \to P(\varsigma)$ and $\Gamma_S = F\left(L^1 \times L^2 \times \ldots\ldots \times L^n\right) = \left\{\left\langle x, \mu\left(F(S)\right)\right\rangle : x \in \varsigma\right\}$, where μ is the membership function which determines the value of the degree of belongingness and $\mu : \varsigma \to [0,1]$.

Example 2. Let us consider an example where we have proposed a data set that is suitable for selecting a plot of land by the decision-makers. Suppose ς be the set of decision-makers to decide the best plot. We consider $\varsigma = \left\{d^1, d^2, d^3, d^4, d^5\right\}$ and $A = \left\{d^1, d^3, d^5\right\} \subset \varsigma$.

Now we consider the sets of attributes as P^1 = plot size (in sq. ft), P^2 = plot location, P^3 = cost of the plot (in dollars), and P^4 = landmark surrounding of a plot. Their corresponding values are given as

$P^1 = \{2000, 1745, 1100, 900, 1245\}$, P^2 = {Agartala, Lucknow, Amritsar, Greater Noida, Hooghly},

$P^3 = \{4135, 3812, 3907, 2547\}$ and P^4 = {shopping mall, Railway Station, Airport, Multi specialist Hospital, Highway}.

Therefore, $F : P^1 \times P^2 \times P^3 \times P^4 \to P(\varsigma)$.

We consider the following tables to assign their membership values:

Table 1. Decision making fuzzy values for the size of the plot.

(Plot size in sq. ft)	d^1 d^5	d^2	d^3	d^4	
2000	0.6	0.5	0.7	0.4	0.8
1745	0.6	0.9	0.1	0.5	0.5
1100	0.6	0.7	0.4	0.8	0.5
900	0.3	0.2	0.5	0.6	0.2
1245	0.5	0.7	0.8	0.6	0.8

Table 2. Decision making fuzzy values for the location of the plot.

(Plot location)	d^1	d^2	d^3	d^4	d^5
Agartala	0.8	0.9	0.7	0.8	0.9
Lucknow	0.7	0.8	0.6	0.7	0.6
Amritsar	0.6	0.8	0.5	0.6	0.8
Greater Noida	0.4	0.3	0.2	0.3	0.2
Hooghly	0.8	0.6	0.5	0.7	0.9

Table 3. Decision making fuzzy values for the cost of the plot.

(Plot cost in sq. ft)	d^1	d^2	d^3	d^4	d^5
4135	0.5	0.4	0.8	0.5	0.7
3812	0.8	0.6	0.8	0.6	0.8
3907	0.8	0.4	0.8	0.6	0.9
2547	0.6	0.6	0.5	0.7	0.6

Table 4. Decision making fuzzy values for the landmark surrounding of a plot.

(Landmark surrounding of a plot)	d^1	d^2	d^3	d^4	d^5
Shopping Mall	0.7	0.6	0.8	0.5	0.8
Railway Station	0.6	0.4	0.5	0.8	0.7
Airport	0.5	0.7	0.6	0.8	0.7
Multispecialist Hospital	0.3	0.7	0.5	0.4	0.7
Highway	0.4	0.7	0.6	0.6	0.8

Then for the set $A=\{d^1,d^3,d^5\}$, we define the fuzzy hypersoft in the following way:

$$F = \{1100, \text{Agartala}, 3812, \text{shopping mall}\} = \left\{\begin{array}{l} \langle d^1,(1100,0.6),(Agartala,0.8),(3812,0.8),(Shopping\ Mall,0.7)\rangle, \\ \langle d^3,(1100,0.4),(Agartala,0.7),(3812,0.8),(Shopping\ Mall,0.8)\rangle, \\ \langle d^5,(1100,0.5),(Agartala,0.9),(3812,0.8),(Shopping\ Mall,0.8)\rangle \end{array}\right\}$$

Similarly, we can construct $5\times5\times4\times5=500$ such fuzzy hypersoft sets for the set A as per the attribute values are concerned. To find these 500 sets manually is a very tedious job but with the blessings of Data Science the computing and the storage process are very easy to practice and we use it for various practical purposes. So, with an aid of hypersoft set, there is a lot of choices made by the decision-makers among which we fix with one choice that is suitable for use in all perspective. Such type of facility is not available if we use the fuzzy soft set. So, the concept of a fuzzy hypersoft set gives us a scope to enhance our critical thinking systematically. Also, by using it we redefine or extend the earlier concepts by introducing various properties on fuzzy hypersoft sets and all these properties are more generalized as compared to the other existing set theories.

It is to be noted that the set of all fuzzy hypersoft sets over ς is denoted by $FHSS(\varsigma)$.

3 | Different Types of Fuzzy Hypersoft Sets and Their Properties

Definition 6. Let $\Gamma_S \in FHSS(\varsigma)$, where $L^1\times L^2\times.......\times L^n=S$. If $\forall x\in\varsigma$, $S=\varnothing$ then Γ_S is called an FHS-null set and it is denoted by $\Gamma_\varnothing$.

Definition 7. Let $\Gamma_S \in FHSS(\varsigma)$, where $L^1\times L^2\times.......\times L^n=S$. If S is a crisp set and $\forall x\in\varsigma$, $\Gamma_S=\varsigma$ then Γ_S is called the FHS-universal set and it is denoted by Γ_U.

Definition 8. Let $\Gamma_S,\Gamma_T \in FHSS(\varsigma)$. Then Γ_S is said to be an FHS-subset of Γ_T i.e $\Gamma_S\subseteq\Gamma_T$ iff $S\subseteq T$, and $\mu(F(S))\le\mu(F(T))$.

Proposition 1. For $\Gamma_S,\Gamma_T,\Gamma_R \in FHSS(\varsigma)$, we have the following results:

1) $\Gamma_S\subseteq\Gamma_T$ and $\Gamma_T\subseteq\Gamma_S \Rightarrow \Gamma_S=\Gamma_T$.
2) $\Gamma_S\subseteq\Gamma_T$ and $\Gamma_T\subseteq\Gamma_R \Rightarrow \Gamma_S\subseteq\Gamma_R$.
3) $\Gamma_\varnothing\subseteq\Gamma_S$ and $\Gamma_S\subseteq\Gamma_U$

Definition 9. Let $\Gamma_S \in FHSS(\varsigma)$. Then the complement of Γ_S is denoted by $(\Gamma_S)^c$ and it is defined as $(\Gamma_S)^c=\{\langle x,1-\mu(F(S))\rangle : x\in\varsigma\}$.

Definition 10. Let $\Gamma_S, \Gamma_T \in FHSS(\varsigma)$. Then their union is denoted by $\Gamma_S \cup \Gamma_T$ and is defined by

$$\Gamma_S \cup \Gamma_T = \begin{Bmatrix} \langle x, \max(\mu(F(S)), \mu(F(T)))\rangle : \forall x \in S \wedge T \\ \langle x, \mu(F(S))\rangle, \text{if } x \in S \\ \langle x, \mu(F(T))\rangle, \text{if } x \in T \end{Bmatrix}$$

Definition 11. Let $\Gamma_S, \Gamma_T \in FHSS(\varsigma)$. Then their union is denoted by $\Gamma_S \cap \Gamma_T$ and is defined by

$$\Gamma_S \cap \Gamma_T = \begin{Bmatrix} \langle x, \min(\mu(F(S)), \mu(F(T)))\rangle : \forall x \in S \wedge T \\ \langle x, \mu(F(S))\rangle \text{ if } x \in S \\ \langle x, \mu(F(T))\rangle \text{ if } x \in T \end{Bmatrix}.$$

Proposition 2. We have the following propositions that are based on FHS-complementary set:

1) $(\Gamma^c{}_S)^c = \Gamma_S$.
2) $(\Gamma_\varnothing)^c = \varsigma$ (in case of crisp set).
3) (iii) $(\Gamma_S \cup \Gamma_T)^c = \Gamma^c{}_S \cap \Gamma^c{}_T$ and $(\Gamma_S \cap \Gamma_T)^c = \Gamma^c{}_S \cup \Gamma^c{}_T$ (De Morgan Laws).
4) (iv) $\Gamma_S \cup (\Gamma_S \cap \Gamma_T) = \Gamma_S$ and $\Gamma_S \cap (\Gamma_S \cup \Gamma_T) = \Gamma_S$ (Absorption Laws).

Proposition 3. For $\Gamma_S, \Gamma_T, \Gamma_R \in FHSS(\varsigma)$, we have the following results:

1) $\Gamma_S \cup \Gamma_T = \Gamma_T \cup \Gamma_S$ and $\Gamma_S \cap \Gamma_T = \Gamma_T \cap \Gamma_S$ (Idempotent Laws).
2) $\Gamma_S \cup (\Gamma_T \cup \Gamma_R) = (\Gamma_S \cup \Gamma_T) \cup \Gamma_R$ and $\Gamma_S \cap (\Gamma_T \cap \Gamma_R) = (\Gamma_S \cap \Gamma_T) \cap \Gamma_R$ (Associative Laws).
3) $\Gamma_S \cup (\Gamma_T \cap \Gamma_R) = (\Gamma_S \cup \Gamma_T) \cap (\Gamma_S \cup \Gamma_R)$ and $\Gamma_S \cap (\Gamma_T \cup \Gamma_R) = (\Gamma_S \cap \Gamma_T) \cup (\Gamma_S \cap \Gamma_R)$ (Distributive Laws).

4 | Weightage of Fuzzy Hypersoft Set in Decision Making

In example 2, we have determined only one fuzzy hypersoft set though there are 500 different choices available. As we know that computer science is an integral part of Mathematics, then by using suitable software or programming we can easily enumerate all these 500 different sets within few minutes as it is quite difficult to do manually. That's why here we avoid such lengthy calculations. But we need an operator by which we can select the best alternative for the customer among these 500 cases. Without any proper decision-making of all the items we do not say, this or that is the best choice or there may be multiple choices for the customer. For that purpose, we need to find the weightage of each fuzzy hypersoft set by using the following operator.

$$\Omega = \frac{1}{|FHSS(\varsigma)|} \sum_{i=1}^{n} \max(\mu(d^i)) |\Xi_{FV}|.$$

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Where $|FHSS(\varsigma)|$=number of fuzzy hypersoft sets over ς, $max\left(\mu\left(d^i\right)\right)$=maximum fuzzy membership value concerning for to the decision-makers, and $|\Xi_{FV}|$=number of fuzzy attribute values corresponding to the set of attributes.

We have to determine Ω for every each possible fuzzy hypersoft set of the given problem. Afterward, we choose the best choice for the client which has the maximum weightage. In case of a tie, there may be multiple choices for a client. In such a case, he or she may choose any one of the suitable choices.

5 | Application

There is a huge scope of the use of hypersoft set in different areas such as forecasting, business management, traffic control, similarity measures, neural networking, data science, sociology, etc.

6 | Conclusions

Here we have used the novel concept known as a hypersoft set which was introduced by F. Smarandache in 2018. By combining the fuzzy set and hypersoft set a new theory called fuzzy hypersoft set is introduced in [23]. The Fuzzy hypersoft set is a more generalized form of fuzzy set, soft set, fuzzy soft set, etc. In this article, an attempt has been made to study fuzzy hypersoft set and their kinds and discuss some set-theoretic operations and propositions on them. With the help of a valid and concrete example, it also shown that how this concept is more effective in multi-criteria decision-making problems compared to other existing theories. In future, it has a wide range of application in the fields of topology, game theory, computer science, neural network, decision-making etc.

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Conflict of interest

Author declared no conflict of interest regarding the publication of the paper.

References

[1] Abbas, M., Murtaza, G., & Smarandache, F. (2020). Basic operations on hypersoft sets and hypersoft point. *Neutrosophic sets and systems, 35*(1), 407-421.

[2] Ajay, D., & Charisma, J. J. (2021). Neutrosophic hypersoft topological spaces. *Neutrosophic sets and systems, 40*(1), 11.27

[3] Aktaş, H., & Çağman, N. (2007). Soft sets and soft groups. *Information sciences, 177*(13), 2726-2735. https://doi.org/10.1016/j.ins.2006.12.008

[4] Ali, M. I., Feng, F., Liu, X., Min, W. K., & Shabir, M. (2009). On some new operations in soft set theory. *Computers and mathematics with applications, 57*(9), 1547-1553. https://doi.org/10.1016/j.camwa.2008.11.009

[5] Atanassov, K. (1986). Intuitionistic fuzzy sets. *Fuzzy sets and systems, 20*(1), 87-96.

[6] Atanassov, K., & Gargov, G. (1989). Interval valued intuitionistic fuzzy sets. *Fuzzy sets and systems, 3*, 343-349.

[7] Çağman, N., & Enginoğlu, S. (2010). Soft set theory and uni–int decision making. *European journal of operational research, 207*(2), 848-855. https://doi.org/10.1016/j.ejor.2010.05.004

[8] Çağman, N., & Enginoğlu, S. (2010). Soft matrix theory and its decision making. *Computers and mathematics with applications, 59*(10), 3308-3314. https://doi.org/10.1016/j.camwa.2010.03.015

[9] Cagman, N., Enginoglu, S., & Citak, F. (2011). Fuzzy soft set theory and its applications. *Iranian journal of fuzzy systems*, *8*(3), 137-147.

[10] Cuong, B. C., & Kreinovich, V. (2014). Picture fuzzy sets. *Journal of computer science and cybernetics*, *30*(4), 409-420.

[11] Goguen, J. A. (1967). L-fuzzy sets. *Journal of mathematical analysis and applications*, *18*(1), 145-174.

[12] Gorzałczany, M. B. (1987). A method of inference in approximate reasoning based on interval-valued fuzzy sets. *Fuzzy sets and systems*, *21*(1), 1-17. https://doi.org/10.1016/0165-0114(87)90148-5

[13] Maji, P. K., Biswas, R., & Roy, A. R. (2001). Fuzzy soft sets. *Journal of fuzzy mathematics*, *9*, 589-602.

[14] Maji, P. K., Roy, A. R., & Biswas, R. (2002). An application of soft sets in a decision-making problem. *Computers and mathematics with applications*, *44*(8-9), 1077-1083.

[15] Maji, P. K., Biswas, R., & Roy, A. R. (2003). Soft set theory. *Computers and mathematics with applications*, *45*(4-5), 555-562. https://doi.org/10.1016/S0898-1221(03)00016-6

[16] Molodtsov, D. (1999). Soft set theory—first results. *Computers and mathematics with applications*, *37*(4-5), 19-31. https://doi.org/10.1016/S0898-1221(99)00056-5

[17] Roy, A. R., & Maji, P. K. (2007). A fuzzy soft set theoretic approach to decision making problems. *Journal of computational and applied mathematics*, *203*(2), 412-418. https://doi.org/10.1016/j.cam.2006.04.008

[18] Saqlain, M., Moin, S., Jafar, M. N., Saeed, M., & Smarandache, F. (2020). *Aggregate operators of neutrosophic hypersoft set*. Infinite Study.

[19] Saeed, M., Ahsan, M., Siddique, M. K., & Ahmad, M. R. (2020). *A study of the fundamentals of hypersoft set theory*. Infinite Study.

[20] Smarandache, F. (2018). Extension of soft set to hypersoft set, and then to plithogenic hypersoft set. *Neutrosophic sets syst*, *22*, 168-170.

[21] Wang, F., Li, X., & Chen, X. (2014). Hesitant fuzzy soft set and its applications in multicriteria decision making. *Journal of applied mathematics*, 1-10. https://doi.org/10.1155/2014/643785

[22] Xu, W., Ma, J., Wang, S., & Hao, G. (2010). Vague soft sets and their properties. *Computers and mathematics with applications*, *59*(2), 787-794. https://doi.org/10.1016/j.camwa.2009.10.015

[23] Yolcu, A., & Ozturk, T. Y. (2021). Fuzzy hypersoft sets and its application to decision-making. In *Theory and application of hypersoft set*. https://www.researchgate.net/profile/Naveed-Jafar/publication/349455966_HyperSoftSet-book/links/6030c5424585158939b7c455/HyperSoftSet-book.pdf#page=58

[24] Zadeh, L. A. (1965). Fuzzy set. *Information and control*, *8*, 338-353.

[25] Zhang, X., & Xu, Z. (2014). Extension of TOPSIS to multiple criteria decision-making with Pythagorean fuzzy sets. *International journal of intelligent systems*, *29*(12), 1061-1078. https://doi.org/10.1002/int.21676

[26] Zimmermann, H. J. (1993). *Fuzzy set theory and its applications*. Kluwer academic publishers.

[27] Zulqarnain, R. M., Xin, X. L., & Saeed, M. (2020). Extension of TOPSIS method under intuitionistic fuzzy hypersoft environment based on correlation coefficient and aggregation operators to solve decision making problem. *AIMS mathematics*, *6*(3), 2732-2755.

[28] Zulqarnain, R. M., Xin, X. L., Saqlain, M., Saeed, M., Smarandache, F., & Ahamad, M. I. (2021). Some fundamental operations on interval valued neutrosophic hypersoft set with their properties. *Neutrosophic sets and systems*, *40*(1), 134-148. https://digitalrepository.unm.edu/nss_journal/vol40/iss1/8