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A Neutrosophic Student's t –Type of Statistic for AR(1) Random Processes

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Abstract

Neutrosophic statistics are used when one is dealing with imprecise and indeterminate data or parameters. In the present paper we propose a method for performing a neutrosophic Student's t –type of statistical test that concerns the population mean when data arise from an autoregressive process of order 1 (AR (1)). In classical statistics, data obtained through this process are not independent when the autocorrelation coefficient of the process is not equal to 0, and hence the usual Student's t distribution is inadequate for inferring about the population mean; however a result obtained in earlier literature states that a Student's t –type of statistic, which is asymptotically normally distributed, can be used instead. Our method is based on the neutrosophic version of this result and it is implemented using simulated data.

Keywords: Neutrosophic statistics, Neutrosophic hypotheses, Neutrosophic χ ^2, Indeterminate data, Autocorrelation.

1 | Introduction

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Since classical statistics are applicable only when all data are determinate, it has been argued in the literature that in statistical analyses where part of the data is indeterminate, classical statistics turn out to be inadequate and one should then refer to neutrosophic statistics instead, that is, a theory that deals with sets rather than spark values and can be considered in that respect as an extension of classical statistics (see [1]). A brief history of neutrosophic statistics provided together with a comprehensive reference list and a list of seminars can be found in [2], which is a seminal reference on the field, and, furthermore, a concise comparison between neutrosophic and classical statistics can be found in [3]. It is also worth mentioning that neutrosophic statistics are frequently applied to other fields like medicine [4], climatology [5], industry [6], and chemistry [7], to cite only a few among many others. In this context, techniques of classical statistics, used for inferential purposes, need to be adequately modified for coping with indeterminacy in the data. Smarandache [1], proposed the neutrosophic Student's t distribution as an extension for the well-known Student's t distribution used in classical statistics for testing hypotheses concerning the population mean. Note that the classical Student' s t distribution has many practical applications in several fields such as medicine (see [8]), finance (see [9]), and meteorology (see [10]) among many others, and therefore it is interesting to investigate its neutrosophic version. In that direction an interesting application of the neutrosophic version of the classical ANOVA, which extends Student's t test, was recently provided by Aslam [11].

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In classical statistics, asymptotic theory concerning independent random variables states that as the sample size n becomes sufficiently large, Student's t distribution can be approximated by the normal $N(0,1)$ distribution, and this holds in practice for $n \geq 30$ (see for example Salvatore and Reagle [12]). This result is also equivalent to approximating the square of Student's t by a χ^2 distribution. In the present paper we study the case where data arise from an autoregressive process of order one (denoted as AR (1)) and thus they are not independent any more. In this case, the usual Student's t distribution is no more adequate for statistical inference and thus we propose to use an asymptotic result from Van. Belle [13], or Polymenis [14] which also lies on the χ^2 distribution. Furthermore, we suppose that there is some indeterminacy in the data, thus requiring the use of neutrosophic statistics for inference. To the best of our knowledge, this case has not been investigated as yet in existing neutrosophic statistics literature.

This paper is organized as follows. In Section 2 the proposed model and simulated data are presented. In Section 3 a method for testing neutrosophic hypotheses is provided; this method is implemented for independent data in Subsection 3.1 and for the case of highly autocorrelated data in Subsection 3.2.

2| The Neutrosophic AR (1) Model

We first artificially generate 40 independent data from a standard normal distribution N (0,1) as independent random errors $\{\varepsilon_n\}$ for our AR (1) process using the random data generator provided from the link https://www.random.org/gaussian-distributions. The model of interest has the form

$$\mathrm{X_n} = \mathrm{aX_{n-1}} + \mathrm{\varepsilon_n}, \tag{1}$$

where a denotes the autocorrelation coefficient, $|a| < 1$, $X_0 = \varepsilon_0 = 0$, and ε_n are independently distributed $N(0,1)$ random errors. This model will be denoted in the sequel as *Model (1)* and it is equivalent to the AR (1) model appearing in *Eq. (1)* of Polymenis [15] with $s = 1$. We use data generated from process $\{\varepsilon_n\}$ in order to obtain data from model 1. Note that for *Model 1*, $E(X_n)=0$, but there is no lack of generality because if $E(X_n)$ were equal to $\mu \neq 0$, this model would then just amount to the corresponding "centered" expression $X_n - \mu = a(X_n - \mu) + \varepsilon_n$, with $E(X_n - \mu) = 0$, as reported in Polymenis [15]. We also mention that although we have considered for reasons of simplicity s to be 1, calculations that follow can easily be adapted to other values of s. Furthermore, we consider that some observations are indeterminate, that is, we do not know what their exact value is, but only an interval to which they belong. This in turn implies that classical statistics cannot be used for inferential purposes concerning population means. For that reason we use neutrosophic statistics which are based on intervals rather than crisp values. Our idea is to use a remark from Smarandache [1], for dealing with indeterminacy in the data, namely that distributions appearing in classical statistics like for example the normal, Student's t, χ^2 , etc. can be extended to their neutrosophic versions. Note that the neutrosophic normal, the binomial, the multinomial, and the t distributions have been described in detail in Smarandache [1], and the first three distributions in Patro and Smarandache [16]. Other well-known distributions like the uniform, the exponential, and Poisson were also extended to their neutrosophic versions by Alhabib et al. [17].

We now describe the data that will be used for our analysis. We will study the cases where $a = 0$ (corresponding to independent observations) and $a = 0.5$ (corresponding to high autocorrelation in *Model (1)*). The case where $a = 0$ is the particular situation where $\{X_n\}$ is just process $\{\varepsilon_n\}$. In order to construct a neutrosophic sample of size 40 (for the definition of neutrosophic sample see Smarandache, [1], we use the same approach as for the neutrosophic sweat data described in [7], that is, generated realizations from process $\{\varepsilon_n\}$ will appear as lower bounds ε_{il} $(i = 1, \dots, n)$ of intervals of the form $[\varepsilon_{il}, \varepsilon_{iu}]$, ε_{iu} being the upper bounds which express indeterminacy in the data. In case of indeterminacy concerning ε_i, we have that ε_{iu} is different from ε_{il} ; on the other hand, if ε_i is determinate then $\varepsilon_{iu} = \varepsilon_{il}$. Suppose that 11 values (out of 40) are indeterminate. Since process $\{\varepsilon_n\}$ is used in order to obtain corresponding values for *Model (1)*, we consider the first 29 observations $\varepsilon_1, \dots, \varepsilon_{29}$, obtained from the random number generator to be determinate and we suppose that the last 11 ones $\varepsilon_{30}, \dots, \varepsilon_{40}$ are indeterminate. The results appear in *Table*

Polymenis |J. Fuzzy. Ext. Appl. 2(4) (2021) 388-393

1; for example, $\varepsilon_{1l} = -0.32 = \varepsilon_{1u}$, $\varepsilon_{2l} = -0.51 = \varepsilon_{2u}$, $\varepsilon_{3l} = 1.2 = \varepsilon_{3u}$,---, $\varepsilon_{29l} = -1.2 = \varepsilon_{29u}$, and $\varepsilon_{30l} = -0.51, \ldots, \varepsilon_{40l} = 1$, $\varepsilon_{30u} = -0.3, \ldots, \varepsilon_{40u} = 1.2$. For the high autocorrelation case corresponding to $a = 0.5$ we also obtain 40 intervals from *Model (1)* by using the approach of Salvatore and Reagle [12], with random errors provided by ε_n, supposing that like for process $\{\varepsilon_n\}$ there is no indeterminacy concerning the first 29 observations and there is some form of indeterminacy for the remaining 11 ones. As a consequence corresponding lower and upper bounds X_{il} and X_{iu} $(i = 1, \ldots, 40)$ for X_i are obtained using ε_{il} and ε_{iu}. To be more specific we have $X_{il} = X_{iu}$ concerning the 29 first data since there is no indeterminacy in X_i, and for the remaining indeterminate 11 data we have $X_{il} \neq X_{iu}$. Then consecutive neutrosophic data for process $\{X_n\}$ of *Model (1)* are obtained in the form of intervals which are presented in *Table 2*; for example, $X_{1l} = (0.5)0 - 0.32 = -0.32 = X_{1u}$, $X_{2l} = (0.5)(-0.32) - 0.51 = -0.67 = X_{2u}$, $X_{3l} = (0.5)(-0.67) + 1.2 = 0.865 = X_{3u}$,---, $X_{29l} = -0.687 = X_{29u}$, $X_{30l} = -0.853$,..., $X_{40l} = 0.86$, and $X_{30u} = -0.643, \ldots, X_{40u} = 1.25$.

Table 1. Neutrosophic independent data from normal distribution.

Independent data (model 1 with $a = 0$
[−0.32, −0.32] [−0.51, −0.51] [1.2, 1.2] [−0.43, −0.43] [−0.69, −0.69] [0.033, 0.033] [−1, −1] [1.2, 1.2] [1.4, 1.4] [−0.3, −0.3] [−0.79, −0.79] [−0.11, −0.11] [0.11, 0.11] [−1.2, −1.2] [1.4, 1.4] [1, 1] [1.3, 1.3] [−0.3, −0.3] [−1, −1] [−0.12, −0.12] [1.3, 1.3] [−0.18, −0.18] [−0.77, −0.77] [0.72, 0.72] [−0.2, −0.2] [−2.1, −2.1] [−1.7, −1.7] [2.4, 2.4] [−1.2, −1.2] [−0.51, −0.3] [0.67, 0.8] [−0.3, −0.1] [−0.087, 0] [−0.81, −0.6] [−0.74, −0.5] [0.98, 1.2] [0.97, 1] [0.1, 0.3] [−0.62, −0.4] [1, 1.2]

Table 2. Neutrosophic AR (1) data from normal distribution.

AR(1) data (model 1 with $a = 0.5$)
[−0.32, −0.32] [−0.67, −0.67] [0.865, 0.865] [0.0025, 0.0025] [−0.689, −0.689] [−0.3114, −0.3114] [−1.3114, −1.3114] [0.5443, 0.5443] [1.6722, 1.6722] [0.536, 0.536] [−0.522, −0.522] [−0.371, −0.371] [−0.076, −0.076] [−1.238, −1.238] [0.781, 0.781] [1.39, 1.39] [2, 2] [0.7, 0.7] [−0.65, −0.65] [−0.45, −0.45] [1.08, 1.08] [0.358, 0.358] [−0.591, −0.591] [0.425, 0.425] [0.0122, 0.0122] [−2.094, −2.094] [−2.747, −2.747] [1.03, 1.03] [−0.687, −0.687] [−0.853, −0.643] [0.243, 0.478] [−0.178, 0.139] [−0.176, 0.076] [−0.898, −0.562] [−1.19, −0.781] [0.386, 0.81] [1.163, 1.4] [0.682, 1] [−0.279, 0.1] [0.86, 1.25]

3 | A Method for Testing Neutrosophic Hypotheses

In classical statistics, let us consider population data Υ which are normally and independently distributed $N(\mu, \sigma)$; also let a random sample $\Upsilon_1, \ldots, \Upsilon_n$ from population Υ with mean $\bar{\Upsilon}$ and variance $s_Y^2 = \frac{\sum_{i=1}^{n}(\Upsilon_i - \bar{\Upsilon})^2}{n-1}$. Then the ratio $t = \sqrt{n}\frac{(Y_i - \bar{Y})}{s_Y}$ has the usual Student's t distribution (see, for example, Grimmett and Stirzaker [18]). This result is no more valid for dependent data like for example those pertaining to model (1), with $a = 0.5$. In view of that we use an asymptotic result which is as follows. Let us consider a random sample of size n from process $\{X_n\}$ appearing in *Model (1)* with mean $\bar{X}$ and variance $s_X^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$, then the ratio $t = \sqrt{n}\frac{(X_i - \bar{X})}{s_X}$ is approximately normally distributed for large n, under the null hypothesis $X_n = aX_{n-1} + \varepsilon_n$, with mean 0 and variance equal to $\frac{1+a}{1-a}$ (see Van. Belle [13] or Polymenis [14]). This result implies in turn that $(\frac{1-a}{1+a})t^2$ is approximately χ_1^2 distributed for large n, under the aforementioned null hypothesis. An immediate consequence is that for $a = 0$, corresponding to the independence case, we obtain that t^2 is approximately χ_1^2 distributed under the null hypothesis, which coincides with standard theory of independent random variables (see Introduction - Section 1 or Theorem 5.2.3 of Anderson [19], with number of degrees of freedom $p = 1$). We propose to use the neutrosophic version of this result as a natural extension from classical statistics theory, namely that the neutrosophic sampling distribution *of* $(\frac{1-a}{1+a})t^2$ is approximated by a neutrosophic χ_1^2 curve for n large enough (in practice it is sufficient that $n \geq 30$ for this approximation to be valid like for classical statistics – see Smarandache [1]).

In view of this theory we implement the following method for a two-sided test between neutrosophic hypotheses $NH_0 : \mu \in [\mu_{0l}, \mu_{0u}]$ and $NH_1 : \mu < \mu_{0l}$ or $\mu > \mu_{0u}$, using the same rationale as that appearing in Smarandache [1].

I. Based on a large number of simulated neutrosophic data ($[\varepsilon_{il}, \varepsilon_{iu}]$ for $a = 0$, $[X_{il}, X_{iu}]$ for $a = 0.5$) compute the neutrosophic statistic $(\frac{1-a}{1+a})t^2 = [(\frac{1-a}{1+a})(t(l))^2, (\frac{1-a}{1+a})(t(u))^2]$ (with $(t(l))^2$ and $(t(u))^2$ corresponding to the lower and upper bounds for the neutrosophic t^2).

II. Decision rule: if $\chi^2_{1,\alpha/2} < [(\frac{1-a}{1+a})(t(l))^2, (\frac{1-a}{1+a})(t(u))^2]$ or $\chi^2_{1,\alpha/2} > [\left((\frac{1-a}{1+a})t(l)\right)^2, \left((\frac{1-a}{1+a})t(u)\right)^2]$ then reject NH_0 at significance level α.

Note that we can also easily apply the above method for one-sided tests. For example, in case the alternative hypothesis NH_1 is $\mu > \mu_{0u}$ then the decision rule becomes: if $\chi^2_{1,\alpha} < [(\frac{1-a}{1+a})(t(l))^2, (\frac{1-a}{1+a})(t(u))^2]$ then reject NH_0 at significance level α.

3.1| First Experiment: the Independence Case

We first consider the neutrosophic version of *Model (1)*, for which $a = 0$, that is, process $\{X_n\} = \{\varepsilon_n\}$. Using the simulated data of *Table 1*, we perform the neutrosophic statistical test with neutrosophic null hypothesis $NH_0 : \mu \in [1, 2]$ vs. the alternative $NH_1 : \mu < 1$ or $\mu > 2$ at $\alpha = [0.10, 0.10]$ significance level. Following a statistical test corresponding to the analysis provided by Smarandache [1], let $t = \frac{[\bar{X}(l),\bar{X}(u)]-[\mu_0(l),\mu_0(u)]}{[s(l),s(u)]/\sqrt{n}}$, where $\bar{X}(u) = 0.0516$ is the mean of the upper bounds, $\mu_0(u) = 2$ is the upper bound for the mean under the neutrosophic null hypothesis, $s(u) = 0.98116$ denotes the (unbiased) standard deviation of the data in the upper bound, $\bar{X}(l) = -0.00785$ is the mean of the lower bounds, $\mu_0(l) = 1$ is the lower bound for the mean under the neutrosophic null hypothesis, and $s(l) = 0.98$ denotes the estimated (unbiased) standard deviation of the data in the lower bound. More specifically $s^2(u) = \left(\sum X_i^2(u) - n(\bar{X}(u))^2\right)/(n-1)$, where $\sum X_i^2(u)$ is the sum of squares of the upper bounds, and $s^2(l) = (\sum X_i^2(l) - n(\bar{X}^2(l))/(n-1)$, where $\sum X_i^2(l)$ is the sum of squares of the lower bounds. Since the sample size $n = 40$ is large the well-known asymptotic χ^2 distribution will be used.

In order to perform our analysis we now compute the ratio $t = \frac{[\bar{X}(l),\bar{X}(u)]-[\mu_0(l),\mu_0(u)]}{\frac{[s(l),s(u)]}{\sqrt{40}}} =$

$$\frac{[-0.00785-2,\ 0.0516-1]}{[0.98/\sqrt{40},\ 0.98116/\sqrt{40}]} = \frac{[-2.00785,-0.9484]}{[0.155,\ 0.15536]} = \left[-\frac{2.00785}{0.155}, -\frac{0.9484}{0.15536}\right] = [-12.954, -6.11335].$$

Then we compute the square of this interval, that is, $[-12.954, -6.11335][-12.954, -6.11335] = [37.373, 167.8]$. Since the critical value for the two-sided test at $\alpha = [0.10, 0.10]$ significance level is $\chi^2_{1,0.05} = [3.84, 3.84] < [37.373, 167.8]$, the null hypothesis is then rejected at this significance level.

We digress to say that for independent random variables this method extends the usual asymptotic χ^2 distribution for t^2 to its neutrosophic version; furthermore, it is equivalent to using a neutrosophic $F(1, n-1)$ distribution. Neutrosophic F has already been used in the analysis of variance proposed by Aslam [11]. Concerning our data, the critical value is $F_{0.05}(1, 39) = [4.08, 4.08]$ at $[0.10, 0.10]$ significance level for the two-sided test, and since $[4.08, 4.08] < [37.373, 167.8]$ NH_0 is rejected at $[0.10, 0.10]$ significance level. As expected, conclusions concerning neutrosophic hypothesis testing using F coincide with conclusions obtained from our approach that uses the asymptotic χ_1^2, both leading to the rejection of the null hypothesis.

Let us now consider testing the null hypothesis $NH_0 : \mu \in [0,1]$ vs. the alternative $NH_1 : \mu < 0 \; or \; \mu > 1$ at $\alpha = [0.10, 0.10]$ significance level. Using the same method as previously we have $t = \frac{[-0.00785-1,\ 0.0516-0]}{[0.98/\sqrt{40},\ 0.98116/\sqrt{40}]} = \frac{[-1.00785, 0.0516]}{[0.155,\ 0.15536]} = \left[-\frac{1.00785}{0.155}, \frac{0.0516}{0.15536}\right] = [-6.5, 0.3326]$, and thus $t^2 = [0.1106, 42.25]$. Since now $[3.84, 3.84] \in [0.1106, 42.25]$, we conclude that NH_0 cannot be rejected at $[0.10, 0.10]$ significance level. On the other hand, like for the previous statistical test, neutrosophic F can also be used in this case, and since the critical value $[4.08, 4.08] \in [0.1106, 42.25]$ we conclude that, as expected, NH_0 cannot be rejected at $[0.10, 0.10]$ significance level.

3.2| Second Experiment: the High Autocorrelation Case

We now consider the AR (1) process $\{X_n\}$ with $a = 0.5$ corresponding to equation $X_n = 0.5X_{n-1} + \varepsilon_n$ and we perform the same neutrosophic hypothesis tests as in the first experiment using the simulated data of *Table 2*. We first consider testing between the null hypothesis $NH_0 : \mu \in [1, 2]$ and the alternative $NH_1 : \mu < 1$ or $\mu > 2$ at $= [0.10, 0.10]$. Using calculations like in the case of the previous application we find $\overline{X}(l) = -0.0393$, $\overline{X}(u) = 0.0484$, $s(l) = 0.98966$ and $s(u) = 0.99425$. In view of that the neutrosophic statistic is

$$t = \frac{[\bar{X}(l), \bar{X}(u)] - [\mu_0(l), \mu_0(u)]}{\frac{[s(l), s(u)]}{\sqrt{40}}} = \frac{[-0.0393-2,\ 0.0484-1]}{[0.98966/\sqrt{40},\ 0.99425/\sqrt{40}]} = \frac{[-2.0393, -0.9516]}{[0.15537,\ 0.1572]} = \left[-\frac{2.0393}{0.15537}, -\frac{0.9516}{0.1572}\right] =$$

$[-13.125, -6.0534]$. It results that $t^2 = [36.64, 172.26]$. Thus, for $a = 0.5$, $(\frac{1-a}{1+a})t^2 = \left(\frac{1-0.5}{1+0.5}\right)[36.64, 172.26] = \frac{[36.64,\ 172.26]}{3} = \left[\frac{36.64}{3}, \frac{172.26}{3}\right] = [12.21,\ 57.42]$. Since $[12.21,\ 57.42] > [3.84, 3.84]$ NH_0 is rejected at $[0.10, 0.10]$ significance level.

Like for the first experiment, we also consider testing between the null hypothesis $NH_0 : \mu \in [0,1]$ and the alternative $NH_1 : \mu < 0 \; or \; \mu > 1$ at $\alpha = [0.10, 0.10]$ significance level. Following the previously-used rationale we have that $t = \frac{[\bar{X}(l), \bar{X}(u)] - [\mu_0(l), \mu_0(u)]}{\frac{[s(l), s(u)]}{\sqrt{40}}} = \frac{[-0.0393-1,\ 0.0484-0]}{[0.15537,\ 0.1572]} = \left[-\frac{1.0393}{0.15537}, \frac{0.0484}{0.1572}\right] = [-6.6892, 0.3079]$, and so $t^2 = [0.0948,\ 44.74]$. For $a = 0.5$ we obtain $\left(\frac{1-a}{1+a}\right)t^2 = \left[\frac{0.0948}{3}, \frac{44.74}{3}\right] = [0.0316, 14.9]$. Since $[3.84, 3.84] \in [0.0316, 14.9]$ it results that NH_0 cannot be rejected at $[0.10, 0.10]$ significance level.

4| Conclusion

In the present paper a statistical test for the population mean of an AR (1) random process, that makes use of neutrosophic statistics, is proposed in order to deal with situations where there is indeterminacy in the data. The method uses a neutrosophic version of a result obtained from classical statistical theory which states that as the sample size increases a Student's $t-$ type of distribution approaches the normal distribution; consequently this method can only be applied for large sample sizes. Results are obtained from implementation of this statistical test on the basis of two experiments and they are encouraging. Thus we reckon that the proposed method provides an efficient way for testing for the population mean under uncertainty.

Conflicts of Interest

There are no conflicts of interest to declare.

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Polymenis |J. Fuzzy. Ext. Appl. 2(4) (2021) 388-393

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