E-ISSN: 2717-3453 | P-ISSN: 2783-1442

Journal of Fuzzy Extension and Applications

www.journal-fea.com

J. Fuzzy. Ext. Appl. Vol. 2, No. 4 (2021) 355–363.

Paper Type: Research Paper

Spherical Fuzzy Cross Entropy for Multiple Attribute Decision Making Problems

Princy Rayappan [1,*], Krishnaswamy Mohana [2]

[1] Department of Mathematics, Research Scholar, Nirmala College for Women, Coimbatore, India; Princy.pjs@gmai.com; riyaraju1116@gmail.com.

Citation:

Rayappan, P., & Mohana, K. (2021). Spherical fuzzy cross entropy for multiple attribute decision making problems. *Journal of fuzzy extension and application, 2* (4), 355-363.

Received: 15/04/2021 Reviewed: 07/05/2021 Revised: 21/06/2021 Accept: 22/07/2021

Abstract

In this paper, we investigate the multiple attribute decision making problems with spherical fuzzy information. The advantage of spherical fuzzy set is easily reflecting the ambiguous nature of subjective judgments because the spherical fuzzy sets are suitable for capturing imprecise, uncertain and inconsistent information in the multiple attribute decision making analysis. Thus, the cross- entropy of spherical fuzzy sets called, spherical fuzzy cross-entropy, is proposed as an extension of the cross-entropy of fuzzy sets. Then, a multiple attribute decision making method based on the proposed spherical fuzzy cross entropy is established in which attribute values for alternatives are spherical fuzzy numbers. In decision making process, we utilize the spherical fuzzy weighted cross entropy between the ideal alternative and an alternative to rank the alternatives corresponding to the cross entropy values and to select the most desirable one(s). Finally, a practical example for enterprise resource planning system selection is given to verify the developed approach and to demonstrate its practicality and effectiveness.

Keywords: Spherical fuzzy set, Spherical fuzzy cross entropy, Spherical fuzzy weighted cross entropy, Enterprise resource planning system selection.

1 | Introduction

Entropy is very important and effective tool for measuring uncertain information. Firstly, Zadeh [15], [16] introduced the fuzzy entropy. The starting point for the cross-entropy is information theory as developed by Shannon [8]. Kullback and Leibler [5] proposed the “cross-entropy distance” measure between two probability distributions. Later, Lin [6] proposed a modified cross-entropy measure. Shang and Jiang [7] proposed a fuzzy cross-entropy measure and a symmetric discrimination information measure between fuzzy sets. Vlachos and Sergiadis [9] developed the intuitionistic fuzzy-cross entropy based on the De Luca-Termini non- probabilistic entropy [2]. Zhang and Jiang [17] defined the cross-entropy between vague sets. According to the cross-entropy of vague sets, Ye [12] has investigated the fault diagnosis problem of turbine. Ye [13] has applied the intuitionistic fuzzy cross-entropy to multicriteria fuzzy decision-making problems.

Corresponding Author: Princy.pjs@gmai.com

http://dx.doi.org/10.22105/jfea.2021.281447.1120

Ye [14] proposed an interval-valued intuitionistic fuzzy cross-entropy for multiple attribute decision making problems on the basis of the vague-cross entrop Xia and Xu [10] proposed some cross-entropy and entropy formulas for intuitionistic fuzzy sets and applied them to group decision-making.

For interval-valued intuitionistic fuzzy sets, Zhang and Jiang [18] proposed the entropy and cross-entropy concepts and discussed the connections among some important measures. Xu and Xia [10] introduced the concepts of entropy and cross-entropy for hesitant fuzzy information, and discussed their desirable properties.

Antonov [1] introduced the new stereo metrical interpretation of the IFS elements and gave the geometrical interpretation of the distance between two intuitionistic fuzzy points. Yang and Chiclana [11] proposed a spherical representation, which allowed us to define a distance function between intuitionistic fuzzy sets. They have showed that the spherical distance is different from the existing distances in that it is non- linear with respect to the change of the corresponding fuzzy membership degrees, and thus it seems more appropriate than usual linear distances for non- linear contexts in 3D spaces. Their work is just on the usage of IFS on a sphere.

In the spherical representation, hesitancy can be calculated based on the given membership and non-membership values since they only consider the surface of the sphere. Besides, they measure the spherical arc distance between two IFSs. Furthermore, Gong et al. [4] introduced an approach generalizing Yang and Chiclana's work. They applied the spherical distance measure to obtain the difference between two IFSs. They first introduced an ideal opinion and each individual's opinion in group decision, they briefly constructed a non- linear optimization model.

The Spherical Fuzzy Sets (SFSs) are based on the fact that the hesitancy of a decision maker can be defined independently from membership and non-membership degrees, satisfying the following condition:

$$0 \leq \mu_{\tilde{A}}{}^2(u) + \vartheta_{\tilde{A}}{}^2(u) + \pi_{\tilde{A}}{}^2(u) \leq 1. \qquad \forall\ u \in U$$

On the surface of the sphere, above Equation becomes

$$\mu_{\tilde{A}}{}^2(u) + \vartheta_{\tilde{A}}{}^2(u) + \pi_{\tilde{A}}{}^2(u) = 1. \qquad \forall\ u \in U$$

Since Yang and Chiclana [11] and Gong et al. [4] measure the arc distance on the surface of the sphere, Euclidean distance is not measured in these works. In our spherical fuzzy sets approach, the sphere is not solid but a spherical volume. Based on this fact, Euclidean distance measurement is meaningful. This also means that any two points within the spherical volume are also on the surface of another sphere. Euclidean distance gives the shortest distance between two points in the sphere.

The rest of the paper is organized as follows: Section 2 introduces the concept of SFSs. Section 3 introduces the concept of cross entropy between spherical fuzzy sets and spherical fuzzy weighted cross entropy measure. Also a modal for spherical fuzzy cross entropy decision making environment is studied. Section 4 illustrates the spherical fuzzy cross entropy measures through a numerical example.

2| Preliminaries

Definition 1. [1]. An IFS *A* in *X* is given by,

$$A = \{< x, \mu_A(x), \vartheta_A(x) > | x \in X\},$$

Rayappan and Mohana J. Fuzzy. Ext. Appl. 2(4) (2021) 355-363

where $\mu_A: X \to [0,1]$ and $\vartheta_A: X \to [0,1]$, where $0 \le \mu_A(x) + \vartheta_A(x) \le 1, \forall\ x \in X$. The number $\mu_A(x)$ and $\vartheta_A(x)$ represents, respectively the membership degree and non-membership degree of the element x to the set A.

Definition 2. [3]. A Spherical Fuzzy Set (SFS) $\tilde{A}_s$ of the universe of discourse U is given by,

$$\tilde{A}_s = \left\{ \langle \mu_{\tilde{A}_s}(u), \vartheta_{\tilde{A}_s}(u), \pi_{\tilde{A}_s}(u) | u \in U \rangle \right\},$$

where

$$\mu_{\tilde{A}_s}: U \to [0,1],\ \vartheta_{\tilde{A}_s}: U \to [0,1],\ \pi_{\tilde{A}_s}: U \to [0,1] \text{ and}$$

$$0 \le \mu_{\tilde{A}_s}{}^2(u) + \vartheta_{\tilde{A}_s}{}^2(u) + \pi_{\tilde{A}_s}{}^2(u) \le 1. \qquad \forall\ u \in U$$

For each u, the numbers $\mu_{\tilde{A}_s}(u), \vartheta_{\tilde{A}_s}(u)$ *and* $\pi_{\tilde{A}_s}(u)$ are the degree of membership, non-membership and hesitancy of u to $\tilde{A}_s$, respectively.

Definition 3. [3]. Basic operators of Spherical fuzzy sets:

Union:

$$\tilde{A}_s \cup \tilde{B}_s = \left\{ \max\left\{\mu_{\tilde{A}_s}, \mu_{\tilde{B}_s}\right\}, \min\left\{\vartheta_{\tilde{A}_s}, \vartheta_{\tilde{B}_s}\right\}, \min\left\{\pi_{\tilde{A}_s}, \pi_{\tilde{B}_s}\right\} \right\}.$$

Intersection:

$$\tilde{A}_s \cap \tilde{B}_s = \left\{ \min\left\{\mu_{\tilde{A}_s}, \mu_{\tilde{B}_s}\right\}, \max\left\{\vartheta_{\tilde{A}_s}, \vartheta_{\tilde{B}_s}\right\}, \max\left\{\pi_{\tilde{A}_s}, \pi_{\tilde{B}_s}\right\} \right\}.$$

Addition:

$$\tilde{A}_s \oplus \tilde{B}_s = \left\{ \left(\mu_{\tilde{A}_s}{}^2 + \mu_{\tilde{B}_s}{}^2 - \mu_{\tilde{A}_s}{}^2 \mu_{\tilde{B}_s}{}^2\right)^{1/2}, \vartheta_{\tilde{A}_s}\vartheta_{\tilde{B}_s}, \pi_{\tilde{A}_s}\pi_{\tilde{B}_s} \right\}.$$

Multiplication:

$$\tilde{A}_s \otimes \tilde{B}_s = \left\{ \mu_{\tilde{A}_s}\mu_{\tilde{B}_s}, \left(\vartheta_{\tilde{A}_s}{}^2 + \vartheta_{\tilde{B}_s}{}^2 - \vartheta_{\tilde{A}_s}{}^2 \vartheta_{\tilde{B}_s}{}^2\right)^{1/2}, \pi_{\tilde{A}_s}\pi_{\tilde{B}_s} \right\}.$$

Multiplication by a scalar; $\lambda > 0$:

$$\lambda . \tilde{A}_s = \left\{ \left(1 - \left(1 - \mu_{\tilde{A}_s}{}^2\right)^{\lambda}\right)^{1/2}, \vartheta_{\tilde{A}_s}{}^{\lambda}, \pi_{\tilde{A}_s}{}^{\lambda} \right\}.$$

Power of $\tilde{A}_s$; $\lambda > 0$:

$$\tilde{A}_s{}^{\lambda} = \left\{ \mu_{\tilde{A}_s}{}^{\lambda}, \left(1 - \left(1 - \vartheta_{\tilde{A}_s}{}^2\right)^{\lambda}\right)^{1/2}, \pi_{\tilde{A}_s}{}^{\lambda} \right\}.$$

3| Cross-Entropy between Spherical Fuzzy Sets

In this section, we shall develop the cross-entropy and discrimination information measures between two SFSs based on the extension of the concept of cross-entropy between two fuzzy sets.

To do this, we firstly introduce the concepts of cross-entropy and symmetric discrimination information measures between two fuzzy sets which were proposed by Shang and Jiang [7].

Definition 4. [7]. Assume that $\alpha = \left(\alpha(x_1), \alpha(x_2), \dots, \alpha(x_n)\right)$ *and* $\beta = \left(\beta(x_1), \beta(x_2), \dots, \beta(x_n)\right)$ are two fuzzy sets in the universe of discourse $(x_1, x_2, \dots, x_n)$. The fuzzy cross-entropy of $\alpha$and β is defined as follows:

$$h(\alpha,\beta) = \sum_{j=1}^{n}\left(\alpha(x_j)\ln\frac{\alpha(x_j)}{\frac{1}{2}\left(\alpha(x_j)+\beta(x_j)\right)} + \left(1-\alpha(x_j)\right)\ln\frac{1-\alpha(x_j)}{1-\frac{1}{2}\left(\alpha(x_j)+\beta(x_j)\right)}\right), \tag{1}$$

which indicates the degree of discrimination of α and β.

However, $h(\alpha, \beta)$ is not symmetric with respect to its arguments. Shang and Jiang [7] proposed a symmetric discrimination information measure:

$$I(\alpha,\beta) = h(\alpha,\beta) + h(\beta,\alpha). \tag{2}$$

Moreover, there are $I(\alpha, \beta) \geq 0$ and $I(\alpha, \beta) = 0$ if and only if $\alpha = \beta$.

Then, the cross-entropy and symmetric discrimination information measures between two fuzzy sets are extended to these measures between SFSs. In order to do so, let us consider two groups of Spherical Fuzzy numbers $\alpha = \left(\mu_{\alpha_j}, \vartheta_{\alpha_j}, \pi_{\alpha_j}\right)$ and $\beta = \left(\mu_{\beta_j}, \vartheta_{\beta_j}, \pi_{\beta_j}\right), j = 1,2,..,n.$

Thus, based on *Eq. (3)*, the amount of information for discrimination of μ_{α_j} from μ_{β_j} $(j = 1,2, \dots, n)$ can be given by:

$$C^{\mu}(\alpha,\beta,\mu) = \mu_{\alpha_j}{}^2 \ln\frac{\mu_{\alpha_j}{}^2}{\frac{1}{2}\left(\mu_{\alpha_j}{}^2+\mu_{\beta_j}{}^2\right)} + \left(1-\mu_{\alpha_j}{}^2\right)\ln\frac{1-\mu_{\alpha_j}{}^2}{1-\frac{1}{2}\left(\mu_{\alpha_j}{}^2+\mu_{\beta_j}{}^2\right)}. \tag{3}$$

Therefore, the expected information based on the single degree of membership for discrimination of α against β is expressed by:

$$C^{\mu}(\alpha,\beta) = \sum_{j=1}^{n}\left(\mu_{\alpha_j}{}^2 \ln\frac{\mu_{\alpha_j}{}^2}{\frac{1}{2}\left(\mu_{\alpha_j}{}^2+\mu_{\beta_j}{}^2\right)} + \left(1-\mu_{\alpha_j}{}^2\right)\ln\frac{1-\mu_{\alpha_j}{}^2}{1-\frac{1}{2}\left(\mu_{\alpha_j}{}^2+\mu_{\beta_j}{}^2\right)}\right). \tag{4}$$

Similarly, considering the degree of non-membership and the degree of hesitancy, we have the following amounts of information:

$$C^{\vartheta}(\alpha,\beta) = \sum_{j=1}^{n}\left(\vartheta_{\alpha_j}{}^2 \ln\frac{\vartheta_{\alpha_j}{}^2}{\frac{1}{2}\left(\vartheta_{\alpha_j}{}^2+\vartheta_{\beta_j}{}^2\right)} + \left(1-\vartheta_{\alpha_j}{}^2\right)\ln\frac{1-\vartheta_{\alpha_j}{}^2}{1-\frac{1}{2}\left(\vartheta_{\alpha_j}{}^2+\vartheta_{\beta_j}{}^2\right)}\right). \tag{5}$$

$$C^{\pi}(\alpha,\beta) = \sum_{j=1}^{n}\left(\pi_{\alpha_j}{}^2 \ln\frac{\pi_{\alpha_j}{}^2}{\frac{1}{2}\left(\pi_{\alpha_j}{}^2+\pi_{\beta_j}{}^2\right)} + \left(1-\pi_{\alpha_j}{}^2\right)\ln\frac{1-\pi_{\alpha_j}{}^2}{1-\frac{1}{2}\left(\pi_{\alpha_j}{}^2+\pi_{\beta_j}{}^2\right)}\right). \tag{6}$$

Rayappan and Mohana J. Fuzzy. Ext. Appl. 2(4) (2021) 355-363

Hence, a novel spherical fuzzy cross- entropy measure between α and β is obtained as the sum of the three amounts:

$$C(\alpha,\beta)=\sum_{j=1}^{n}\left(\mu_{\alpha_j}{}^2\ln\frac{\mu_{\alpha_j}{}^2}{\frac{1}{2}\left(\mu_{\alpha_j}{}^2+\mu_{\beta_j}{}^2\right)}+\left(1-\mu_{\alpha_j}{}^2\right)\ln\frac{1-\mu_{\alpha_j}{}^2}{1-\frac{1}{2}\left(\mu_{\alpha_j}{}^2+\mu_{\beta_j}{}^2\right)}\right)+$$
$$\sum_{j=1}^{n}\left(\vartheta_{\alpha_j}{}^2\ln\frac{\vartheta_{\alpha_j}{}^2}{\frac{1}{2}\left(\vartheta_{\alpha_j}{}^2+\vartheta_{\beta_j}{}^2\right)}+\left(1-\vartheta_{\alpha_j}{}^2\right)\ln\frac{1-\vartheta_{\alpha_j}{}^2}{1-\frac{1}{2}\left(\vartheta_{\alpha_j}{}^2+\vartheta_{\beta_j}{}^2\right)}\right)+ \quad (7)$$
$$\sum_{j=1}^{n}\left(\pi_{\alpha_j}{}^2\ln\frac{\pi_{\alpha_j}{}^2}{\frac{1}{2}\left(\pi_{\alpha_j}{}^2+\pi_{\beta_j}{}^2\right)}+\left(1-\pi_{\alpha_j}{}^2\right)\ln\frac{1-\pi_{\alpha_j}{}^2}{1-\frac{1}{2}\left(\pi_{\alpha_j}{}^2+\pi_{\beta_j}{}^2\right)}\right),$$

which also indicates discrimination degree of α from β. According to Shannon's [8] inequality, one can easily prove that $C(\alpha,\beta)\geq 0$ and $C(\alpha,\beta)=0$ if and only if $\alpha=\beta$. Then, $C(\alpha,\beta)$ is not symmetric. So it should be modified to a symmetric discrimination information measure for SFSs as:

$$D(\alpha,\beta)=C(\alpha,\beta)+C(\beta,\alpha). \quad (8)$$

The larger the difference between α and β is, the larger $D(\alpha,\beta)$ is.

If we consider the weights of α and β, a spherical fuzzy weighted cross-entropy measure between α and β is proposed as follows:

$$C^{\omega}(\alpha,\beta)=\sum_{j=1}^{n}\omega_j\left(\mu_{\alpha_j}{}^2\ln\frac{\mu_{\alpha_j}{}^2}{\frac{1}{2}\left(\mu_{\alpha_j}{}^2+\mu_{\beta_j}{}^2\right)}+\left(1-\mu_{\alpha_j}{}^2\right)\ln\frac{1-\mu_{\alpha_j}{}^2}{1-\frac{1}{2}\left(\mu_{\alpha_j}{}^2+\mu_{\beta_j}{}^2\right)}\right)+$$
$$\sum_{j=1}^{n}\omega_j\left(\vartheta_{\alpha_j}{}^2\ln\frac{\vartheta_{\alpha_j}{}^2}{\frac{1}{2}\left(\vartheta_{\alpha_j}{}^2+\vartheta_{\beta_j}{}^2\right)}+\left(1-\vartheta_{\alpha_j}{}^2\right)\ln\frac{1-\vartheta_{\alpha_j}{}^2}{1-\frac{1}{2}\left(\vartheta_{\alpha_j}{}^2+\vartheta_{\beta_j}{}^2\right)}\right)+ \quad (9)$$
$$\sum_{j=1}^{n}\omega_j\left(\pi_{\alpha_j}{}^2\ln\frac{\pi_{\alpha_j}{}^2}{\frac{1}{2}\left(\pi_{\alpha_j}{}^2+\pi_{\beta_j}{}^2\right)}+\left(1-\pi_{\alpha_j}{}^2\right)\ln\frac{1-\pi_{\alpha_j}{}^2}{1-\frac{1}{2}\left(\pi_{\alpha_j}{}^2+\pi_{\beta_j}{}^2\right)}\right),$$

where $\omega=(\omega_1,\omega_2,\ldots,\omega_n)^T$ is the weight vector of $\alpha,\beta (j=1,2,\ldots,n)$, with $\omega_j\in[0,1], j=1,2,\ldots,n, \sum_{j=1}^{n}\omega_j=1$,which also indicated discrimination degree of α from β. According to *Shannon's inequality (3)*, one can easily prove that $C_\omega(\alpha,\beta)\geq 0$ and $C_\omega(\alpha,\beta)=0$ if and only if $\alpha=\beta$.

Then, $C_\omega(\alpha,\beta)$ is not symmetric. So it should be modified to a symmetric discrimination information measure for SFSs as

$$D_\omega(\alpha,\beta)=C_\omega(\alpha,\beta)+C_\omega(\beta,\alpha). \quad (10)$$

The larger the difference between α and β is, the larger $D_\omega(\alpha,\beta)$ is.

3.1| Models for Multiple Attribute Decision Making based on Cross-Entropy with Spherical Fuzzy Information

Based on cross- entropy with spherical fuzzy information, in this section, we shall propose the corresponding model for multiple attribute decision making with spherical fuzzy information. Let $A = \{A_1, A_2, \dots, A_m\}$ be a discrete set of alternatives, and $G = \{G_1, G_2, \dots, G_n\}$ be the set of attributes, $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ is the weighting vector of the attribute $G_j (j = 1,2, \dots, n)$, where $\omega_j \in [0,1]$, $\sum_{j=1}^{n} \omega_j = 1$. Suppose that $\tilde{R} = (\tilde{r}_{ij})_{m\times n} = (\mu_{ij}, \vartheta_{ij}, \pi_{ij})_{m\times n}$ is the spherical fuzzy decision matrix, where μ_{ij} indicates the degree of membership that the alternative A_i satisfies the attribute G_j given by the decision maker, ϑ_{ij} indicates the degree of non-membership that the alternative A_i doesn't satisfy the attribute G_j given by the decision maker and π_{ij} indicates the hesitancy degree that the alternative A_i doesn't satisfy the attribute G_j, $\mu_{ij} \in [0,1]$, $\vartheta_{ij} \in [0,1]$, $\pi_{ij} \in [0,1]$,

$$0 \le \mu_{ij}^2 + \vartheta_{ij}^2 + \pi_{ij}^2 \le 1, i = 1,2, \dots, m, j = 1,2, \dots, n.$$

In multiple attribute decision-making environments, the concept of ideal point has been used to help identify the best alternative in the decision set. Although the ideal alternative does not exist in real world, it does provide a useful theoretical construct against which to evaluate alternatives. Hence, we can define an ideal alternative value in the ideal alternative A^+.

Let $\tilde{r}_j^+ = (\mu_j^+, \vartheta_j^+, \pi_j^+) (j = 1,2, \dots, n)$ where

$$\mu_j^+ = \max\{\mu_{ij}\}, \vartheta_j^+ = \min\{\vartheta_{ij}\}, \pi_j^+ = \min\{\pi_{ij}\}.$$

Then, we call:

$$A^+ = (\tilde{r}_1^+, \tilde{r}_2^+, \dots, \tilde{r}_n^+). \tag{11}$$

The relative spherical fuzzy ideal alternative (RSFIA).

In the following, we apply the cross- entropy to the MADM problems.

Step 1. Define the alternatives A_i and A^+:

$$A_i = ((\mu_{i1}, \vartheta_{i1}, \pi_{i1}), (\mu_{i2}, \vartheta_{i2}, \pi_{i2}), \dots, (\mu_{in}, \vartheta_{in}, \pi_{in})), i = 1,2, \dots, m, \tag{12}$$

$$A^+ = ((\mu_1^+, \vartheta_1^+, \pi_1^+), (\mu_2^+, \vartheta_2^+, \pi_2^+), \dots, (\mu_n^+, \vartheta_n^+, \pi_n^+)),$$

$$\mu_j^+ = \max\{\mu_{ij}\}, \vartheta_j^+ = \min\{\vartheta_{ij}\}, \pi_j^+ = \min\{\pi_{ij}\}, j = 1,2, \dots, n.$$

Step 2. Calculate the spherical fuzzy weighted cross- entropy between alternatives A_i and A^+:

Step 3. Rank all the alternatives $A_i\,(i = 1,2, .. m)$ and select the best one(s) in accordance with $C(A_i, A^+)\ (i = 1,2, \dots, m)$. The smaller the value of $C(A_i, A^+)$ is, the better the alternative A_i is. In this case, the alternative A_i is close to the ideal alternative A^+. Through the cross-entropy $C(A_i, A^+)\ (i = 1,2, \dots, m)$ between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best one can be easily identified as well.

Rayappan and Mohana J. Fuzzy. Ext. Appl. 2(4) (2021) 355-363

4| Numerical Example

Thus, in this section we shall present a numerical example for potential evaluation of emerging technology commercialization with spherical fuzzy information in order to illustrate the method proposed in this paper. Let us suppose there is a problem to deal with the potential evaluation of emerging technology commercialization which is classical multiple attribute decision making problems. There is a panel with possible emerging technology enterprises A_i $(i = 1,2,3,4,5)$ to select. The experts selects six attribute to evaluate the five possible emerging technology enterprises:

I. G_1 is the technical advancement.
II. G_2 is the potential market and market risk.
III. G_3 is the industrialization infrastructure.
IV. G_4 is the development of science and technology.
V. G_5 is the financial conditions.
VI. G_6 is the employment creation.

In order to avoid the influence of each other, the decision makers are required to evaluate the five possible emerging technology enterprises A_i $(i = 1,2,3,4,5)$ under the above six attributes and the decision matrix $\tilde{R} = \left(\tilde{r}_{ij}\right)_{5\times 6}$ is presented in Table 1, where $\tilde{r}_{ij}(i = 1,2,3,4,5, j = 1,2,3,4,5,6)$ are in the form of SFSs. The weight vector of $x_i (i = 1,2, \dots ,6)$ is

$$\omega = (0.12, 0.25, 0.09, 0.16, 0.18, 0.20)^T.$$

Table 1. The spherical fuzzy decision matrix.

	A_1	A_2	A_3	A_4	A_5
x_1	[0.53,0.33,0.09]	[0.73,0.12,0.08]	[0.91,0.03,0.02]	[0.85,0.09,0.05]	[0.90,0.05,0.02]
x_2	[0.89,0.08,0.03]	[0.13,0.64,0.21]	[0.07,0.09,0.05]	[0.68,0.08,0.21]	[0.74,0.16,0.10]
x_3	[0.03,0.82,0.13]	[0.42,0.35,0.18]	[0.02,0.89,0.05]	[0.04,0.85,0.05]	[0.05,0.87,0.10]
x_4	[0.33,0.12,0.15]	[0.52,0.61,0.31]	[0.15,0.07,0.76]	[0.15,0.73,0.08]	[0.16,0.71,0.05]
x_5	[0.73,0.15,0.08]	[0.08,0.89,0.03]	[0.62,0.06,0.04]	[0.68,0.08,0.02]	[0.13,0.75,0.09]
x_6	[0.17,0.53,0.13]	[0.31,0.39,0.25]	[0.51,0.42,0.12]	[0.09,0.81,0.15]	[0.91,0.03,0.05]

$$C(A_i, A^+) = \sum_{j=1}^{n} \omega_j \left(\mu_{ij}^2 \ln \frac{\mu_{ij}^2}{\frac{1}{2}\left(\mu_{ij}^2 + \mu_j^{+2}\right)} + \left(1 - \mu_{ij}^2\right) \ln \frac{1 - \mu_{ij}^2}{1 - \frac{1}{2}\left(\mu_{ij}^2 + \mu_j^{+2}\right)} \right) +$$
$$\sum_{j=1}^{n} \omega_j \left(\vartheta_{ij}^2 \ln \frac{\vartheta_{ij}^2}{\frac{1}{2}\left(\vartheta_{ij}^2 + \vartheta_j^{+2}\right)} + \left(1 - \vartheta_{ij}^2\right) \ln \frac{1 - \vartheta_{ij}^2}{1 - \frac{1}{2}\left(\vartheta_{ij}^2 + \vartheta_j^{+2}\right)} \right) + \quad (15)$$
$$\sum_{j=1}^{n} \omega_j \left(\pi_{ij}^2 \ln \frac{\pi_{ij}^2}{\frac{1}{2}\left(\pi_{ij}^2 + \pi_j^{+2}\right)} + \left(1 - \pi_{ij}^2\right) \ln \frac{1 - \pi_{ij}^2}{1 - \frac{1}{2}\left(\pi_{ij}^2 + \pi_j^{+2}\right)} \right)$$

$$i = 1,2, \dots , m.$$

To get the most desirable emerging technology enterprises, the following steps are involved:

Step 1. Based on the *Table 1*, we denote the five possible emerging technology enterprises A_i $(i = 1,2,3,4,5)$ by:

$$A_1 = \{(0.53,0.33,0.09), (0.89,0.08,0.03), (0.03,0.82,0.13),$$

$$(0.33,0.12,0.15),(0.73,0.15,0.08),(0.17,0.53,0.13)\};$$

$$A_2 = \{(0.73,0.12,0.08),(0.13,0.64,0.21),(0.42,0.35,0.18),$$

$$(0.52,0.61,0.31),(0.08,0.89,0.03),(0.31,0.39,0.25)\};$$

$$A_3 = \{(0.91,0.03,0.02),(0.07,0.09,0.05),(0.02,0.89,0.05)$$

$$(0.15,0.07,0.76),(0.62,0.06,0.04),(0.51,0.42,0.12)\};$$

$$A_4 = \{(0.85,0.09,0.05),(0.68,0.08,0.21),(0.04,0.85,0.05),$$

$$(0.15,0.73,0.08),(0.68,0.08,0.02),(0.09,0.81,0.15)\};$$

$$A_5 = \{(0.90,0.05,0.02),(0.74,0.16,0.10),(0.05,0.87,0.10)$$

$$(0.16,0.71,0.05),(0.13,0.75,0.09),(0.91,0.03,0.05)\}.$$

Step 2. Based on the Table 1 and *Eqs. (12) – (14)*, we can get the RSFIA A^+:

$$A^+ = \{(0.91,0.03,0.02),(0.89,0.08,0.03),(0.42,0.35,0.05)$$

$$(0.52,0.07,0.05),(0.73,0.06,0.02),(0.91,0.03,0.05)\}.$$

Step 3. Calculate the cross-entropy $C_\omega(A_i, A^+)$ between $A_i (i = 1,2,3,4,5)$ and the RSFIA A^+ by using (15):

$$C_\omega(A_1, A^+) = 0.1499,$$

$$C_\omega(A_2, A^+) = 0.3655,$$

$$C_\omega(A_3, A^+) = 0.2354,$$

$$C_\omega(A_4, A^+) = 0.226,$$

$$C_\omega(A_5, A^+) = 0.1481.$$

Step 4. Rank the emerging technology enterprises $A_i (i = 1,2,3,4,5)$ in accordance with the cross-entropy $C_\omega(A_i, A^+)$ $(i = 1,2,3,4,5)$:

$$A_5 > A_1 > A_4 > A_3 > A_2.$$

Thus, the most desirable emerging technology enterprise is A_5.

5| Conclusion

In this paper, we investigate the multiple attribute decision making problems with spherical fuzzy information. The advantage of spherical fuzzy set is easily reflecting the ambiguous nature of subjective judgments because the spherical fuzzy sets are suitable for capturing imprecise, uncertain and inconsistent information in the multiple attribute decision making analysis. Thus, the cross- entropy of spherical fuzzy sets, called spherical fuzzy cross entropy, is proposed as an extension of the cross-entropy of fuzzy sets. Then, a multiple attribute decision making method based on the proposed spherical fuzzy cross entropy is established in which attribute values for alternatives are spherical fuzzy numbers. In decision making process, we utilize the spherical fuzzy weighted cross entropy between the ideal alternative and an alternative to rank the alternatives corresponding to the cross entropy values and to select the most desirable one(s). The plan of our future work is to apply the proposed method to some other practical decision-making problems, such as the performance evaluation, water resource schedule and risk investment.

References

[1] Antonov, I. (1995). On a new geometrical interpretation of the intuitionistic fuzzy sets. *Notes on Intuitionistic fuzzy sets*, *1*(1), 29-31.

[2] De Luca, A., & Termini, S. (1972). A definition of a nonprobabilistic entropy in the setting of fuzzy sets theory. *Information and control*, *20*(4), 301-312. http://dx.doi.org/10.1016/S0019-9958(72)90199-4

[3] Kutlu Gündoğdu, F., & Kahraman, C. (2019). Spherical fuzzy sets and spherical fuzzy TOPSIS method. *Journal of intelligent & fuzzy systems*, *36*(1), 337-352.

[4] Gong, Z., Xu, X., Yang, Y., Zhou, Y., & Zhang, H. (2016). The spherical distance for intuitionistic fuzzy sets and its application in decision analysis. *Technological and economic development of economy*, *22*(3), 393-415.

[5] Kullback, S., & Leibler, R. A. (1951). On information and sufficiency. *The annals of mathematical statistics*, *22*(1), 79-86. http://dx.doi.org/10.1214/aoms/1177729694

[6] Lin, J. (1991). Divergence measures based on the Shannon entropy. *IEEE transactions on information theory*, *37*(1), 145-151. http://dx.doi.org/10.1109/18.61115

[7] Shang, X. G., & Jiang, W. S. (1997). A note on fuzzy information measures. *Pattern recognition letters*, *18*(5), 425-432. http://dx.doi.org/10.1016/S0167-8655(97)00028-7.

[8] Shannon, C. E. (1948). A mathematical theory of communication. *The bell system technical journal*, *27*(3), 379-423. http://dx.doi.org/10.1002/j.1538-7305.1948.tb01338.x

[9] Vlachos, I. K., & Sergiadis, G. D. (2007). Intuitionistic fuzzy information–applications to pattern recognition. *Pattern recognition letters*, *28*(2), 197-206. http://dx.doi.org/10.1016/j.patrec.2006.07.004

[10] Xia, M., & Xu, Z. (2012). Entropy/cross entropy-based group decision making under intuitionistic fuzzy environment. *Information fusion*, *13*(1), 31-47. http://dx.doi.org/10.1016/j.inffus.2010.12.001

[11] Yang, Y., & Chiclana, F. (2009). Intuitionistic fuzzy sets: spherical representation and distances. *International journal of intelligent systems*, *24*(4), 399-420.

[12] Ye, J. (2009). Fault diagnosis of turbine based on fuzzy cross entropy of vague sets. *Expert systems with applications*, *36*(4), 8103-8106. http://dx.doi.org/10.1016/j.eswa.2008.10.017

[13] Ye, J. (2009, August). Multicriteria fuzzy decision-making method based on the intuitionistic fuzzy cross-entropy. *2009 international conference on intelligent human-machine systems and cybernetics* (Vol. 1, pp. 59-61). IEEE. http://dx.doi.org/10.1109/ihmsc.2009.23

[14] Ye, J. (2011). Fuzzy cross entropy of interval-valued intuitionistic fuzzy sets and its optimal decision-making method based on the weights of alternatives. *Expert systems with applications*, *38*(5), 6179-6183. http://dx.doi.org/10.1016/j.eswa.2010.11.052

[15] Zadeh, L. A. (1965). Fuzzy sets. *Information control*, 8, 338–353. http://dx.doi.org/10.1016/S0019-9958(65)90241-X

[16] Zadeh, L. A. (1968). Probability measures of fuzzy events. *Journal of mathematical analysis and applications*, 23, 421–427. http://dx.doi.org/10.1016/0022-247X(68)90078-4

[17] Zhang, Q. S., & Jiang, S. Y. (2008). A note on information entropy measures for vague sets and its applications. *Information sciences*, *178*(21), 4184-4191. http://dx.doi.org/10.1016/j.ins.2008.07.003

[18] Zhang, Q., & Jiang, S. (2010). Relationships between entropy and similarity measure of interval-valued intuitionistic fuzzy sets. *International journal of intelligent systems*, *25*(11), 1121-1140.