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# Quadripartitioned Neutrosophic Pythagorean Lie Subalgebra

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## Abstract

A Quadripartitioned Neutrosophic Pythagorean (QNP) set is a powerful general format framework that generalizes the concept of Quadripartitioned Neutrosophic Sets and Neutrosophic Pythagorean Sets. In this paper, we apply the notion of quadripartitioned Neutrosophic Pythagorean sets to Lie algebras. We develop the concept of QNP Lie subalgebras and QNP Lie ideals. We describe some interesting results of QNP Lie ideals.

**Keywords:** QNP Lie ideal; QNP Lie subalgebra; Lie ideal; Lie subalgebra.

## 1 | Introduction



The concept of Lie groups was first introduced by Sophus Lie in nineteenth century through his studies in geometry and integration methods for differential equations. Lie algebras were also discovered by him when he attempted to classify certain smooth subgroups of a general linear group. The importance of Lie algebras in mathematics and physics has become increasingly evident in recent years. In mathematics, Lie theory remains a robust tool for studying differential equations, special functions and perturbation theory. It's noted that Lie theory has applications not only in mathematics and physics but also in diverse fields like continuum mechanics, cosmology and life sciences. Lie algebra has been utilized by electrical engineers, mainly within the mobile robot control [5].

Lie algebra has also been accustomed solve the problems of computer vision. Fuzzy structures are related to theoretical soft computing, especially Lie algebras and their different classifications, have numerous applications to the spectroscopy of molecules, atoms and nuclei. One amongst the key concepts within the applying of Lie algebraic method in physics is that of spectrum generating algebras and their associated dynamic symmetries. The most important advancements within the fascinating world of fuzzy sets started with the work of renowned scientist Zulqarnain et al. [14] with new directions and ideas.

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Wang et al. [6] defined SVN sets as a generalization of fuzzy sets and intuitionistic fuzzy sets [4]. Algebraic structures have a major place with vast applications in various disciplines.

Neutrosophic set has been applied to algebraic structures. Fuzzification of Lie algebras has been discussed in [1]–[3]. The idea of single valued neutrosophic Lie algebra was investigated by Akram et al. [7]. Quadripartitioned Neutrosophic Set and its properties were introduced by Smarandache [12]. During this case, indeterminacy is split into two components: contradiction and ignorance membership function. The Quadripartitioned Neutrosophic Set is a particular case of Refined Neutrosophic Set. Smarandache [12] extended the Neutrosophic Set to refined [n-valued] neutrosophic set, and to refined neutrosophic logic, and to refined neutrosophic probability, i.e. the truth value T is refined/split into types of sub-truths such as T1, T2, …, similarly indeterminacy I is refined/split into types of sub-indeterminacies I1, I2, …, and the falsehood F is refined/split into sub-falsehood F1, F2 ,...

We've now extended our research during this Pentapartitioned neutrosophic set as a space. Also we introduced the concept of Penta partitioned neutrosophic Pythagorean set [8]-[14] and establish variety of its properties in our previous work. During this paper, we apply the notion of Quadripartitioned Neutrosophic Pythagorean (QNP) sets to Lie algebras.

In this paper, we develop the concepts of QNP Lie subalgebras and investigated some of its properties. Furthermore, we have also studied the concept of QNP Lie ideals. We describe some interesting results of QNP Lie ideals.

## 2| Preliminaries

Lie algebra [1] is a vector space L over a field F (equal to R or C) on which $L \times L \rightarrow L$ denoted by (x, y) → [x, y] is defined satisfying the following axioms:

(L1) [x, y] is bilinear,

(L2) [x, x] = 0 for all x ∈L,

(L3) [[x, y], z] + [[y, z], x] + [[z, x], y] = 0 for all x, y, z ∈L (Jacobi identity).

Throughout this paper, L is a Lie algebra and F is a field. We note that the multiplication in a Lie algebra is not associative, i.e., it is not true in general that [[x, y], z] = [x, [y, z]]. But it is anti commutative, i.e., [x, y] = −[y, x]. A subspace H of L closed under [ · , · ] will be called a Lie subalgebra.

A fuzzy set μ: L → [0, 1] is called a fuzzy Lie ideal [1] of L if

I. $\mu(x + y) \geq min\{\mu(x), \mu(y)\}$,
II. $\mu(\alpha x) \geq \mu(x)$,
III. $\mu([x, y]) \geq \mu(x)$,

hold for all x, y ∈L and $\alpha$∈F.

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**Definition 1. [9]. Let** R be a space of points (objects). A QNP set on a non-empty R is characterized by truth membership function A1: R → [0, 1], contradiction membership function A2: R→ [0, 1], ignorance membership function A4: R → [0, 1] and false membership function A5: R → [0, 1].

Thus, R = { <r, A1( r) ,A2( r), A4 ( r),A5( r)>} satisfies with the following conditions A1+A2+A4+A5≤ 2, A1 + A5 ≤ 1,A2+A4 ≤ 1.Here A1,A2, A4, A5 are dependent neutrosophic components.

**Definition 2. [7].** An SVN set N = (TN, IN, FN) on Lie algebra L is called an SVN Lie subalgebra if the following conditions are satisfied:

I. $TN(x + y) \geq min(TN(x), TN(y)), IN(x + y) \geq min(IN(x), IN(y))$ and $FN(x + y) \leq max(FN(x), FN(y)),$
II. $TN(\alpha x) \geq TN(x), IN(\alpha x) \geq IN(x)$ and $FN(\alpha x) \leq FN(x),$
III. $TN ([x, y]) \geq min \{TN(x), TN(y)\}, IN ([x, y]) \geq min \{IN(x), IN(y)\}$ and $FN ([x, y]) \leq max \{FN(x), FN(y)\}$ for all x, $y \in L$ and $\alpha \in F$.

**Definition 3. [7].** A SVN set N = (TN, IN, FN) onL is called an SVN Lie ideal if it satisfies the *Conditions (I), (II)* and the following additional condition:

Single-valued Neutrosophic Lie algebras

IV. $TN([x, y]) \geq TN(x), IN([x, y]) \geq IN(x)$ and $FN([x, y]) \leq FN(x)$

for all x, y ∈L.

From *Condition (2)* it follows that:

V. $TN(0) \geq TN(x), IN(0) \geq IN(x), FN(0) \leq FN(x),$
VI. $TN (-x) \geq TN(x), IN (-x) \geq IN(x), FN (-x) \leq FN(x).$

## 3 | Quadripartitioned Neutrosophic Pythagorean Lie Subalgebra

We define here QNP Lie subalgebras and QNP Lie ideal.

**Definition 4.** A QNP set R = ($A1_R$, $A2_R$, $A4_R$, $A5_R$) on is called a QNP Lie subalgebra $\mathcal{L}$ if the following conditions are satisfied:

I. $A1_R (a + b) \geq min (A1_R (a), A1_R (b)), A2_R (a + b) \geq min (A2_R (a), A2_R (b)), A4_R (a + b) \leq max (A4_R (a), A4_R (b)), A5_R (a + b) \leq max (A5_R (a), A5_R (b)),$
II. $A1_R (\beta a) \geq A1_R (a), A2_R (\beta a) \geq A2_R (a), A4_R (\beta a) \leq A4_R (a)$ and $A5_R (\beta a) \leq A5_R (a).$
III. $A1_R ([a, b]) \geq min (A1_R (a), A1_R (b)), A2_R ([a, b]) \geq min (A2_R (a), A2_R (b)), A4_R ([a, b]) \leq max (A4_R (a), A4_R (b)), A5_R ([a, b]) \leq max (A5_R (a), A5_R (b)).$

For all $a, b \in \mathscr{L}$ and $\in \mathscr{F}$.

**Definition 5.** A QNP set R = ($A1_R$, $A2_R$, $A3_R$, $A4_R$, $A5_R$) on $\mathcal{L}$ is called an QNP Lie ideal if it satisfies the following *Conditions (I)* and *(II)* and the following additional conditions:

IV. $A1_R ([a, b]) \geq A1_R (a), A2_R ([a, b]) \geq A2_R (a), A4_R ([a, b]) \leq A4_R (a), A5_R ([a, b]) \leq A5_R (a),$

From (II), it follows that:

V. $A1_R (0) \geq A1_R (a), A2_R (0) \geq A2_R (a), A4_R (0) \leq A4_R (a)$ and $A5_R (0) \leq A5_R (a),$
VI. $A1_R (- a) \geq A1_R (a), A2_R (- a) \geq A2_R (a), A4_R (- a) \leq A4_R (a)$ and $A5_R (- a) \leq A5_R (a).$

**Proposition 1.** Every QNP Lie ideal is a QNP Lie subalgebra.

We note here that the converse of the above proposition does not hold in general as it can be seen in the following example.

**Example 1.** Consider $\mathscr{F} = \mathbb{R}$. Let $\mathscr{L} = \Re^3 = \{(a, b, c): a, b, c \in \mathbb{R}\}$ be the set of all three dimensional real vectors which forms a QNP Lie algebra and define

$$\Re^3 \text{x}\, \Re^3 \rightarrow \Re^3$$

$$[a, b] \rightarrow a \text{ x } b,$$

Where x is the usual cross product. We define an QNP set R = ($A1_R$, $A2_R$, $A4_R$, $A5_R$)$: \Re^3 \rightarrow$ [0,1] x [0,1] x [0,1] x [0,1] by

$$A1_R\,(a, b, c) = \begin{cases} 1, & \text{if } a = b = c = 0, \\ 0.3, & \text{if } a \neq 0, b = c = 0, \\ 0, & \text{otherwise} \end{cases}$$

$$A2_R\,(a, b, c) = \begin{cases} 1, & \text{if } a = b = c = 0, \\ 0.2, & \text{if } a \neq 0, b = c = 0, \\ 0, & \text{otherwise} \end{cases}$$

$$A4_R\,(a, b, c) = \begin{cases} 0, & \text{if } a = b = c = 0, \\ 0.3, & \text{if } a \neq 0, b = c = 0, \\ 1, & \text{otherwise} \end{cases}$$

$$A5_R\,(a, b, c) = \begin{cases} 0, & \text{if } a = b = c = 0, \\ 0.5, & \text{if } a \neq 0, b = c = 0. \\ 1, & \text{otherwise} \end{cases}$$

Then R = ($A1_R$, $A2_R$, $A4_R$, $A5_R$) is an QNP Lie subalgebra of $\mathscr{L}$ but R = ($A1_R$, $A2_R$, $A4_R$, $A5_R$) is not an QNP Lie ideal of $\mathscr{L}$ since

$$A1_R\,([1,0,0)\,(1,1,1)]) = A1_R\,(0, -1, 1) = 0,$$

$$A2_R\,([1,0,0)\,(1,1,1)]) = A2_R\,(0, -1, 1) = 0,$$

$$A4_R\,([1,0,0)\,(1,1,1)]) = A4_R\,(0, -1, 1) = 1,$$

$$A5_R\,([1,0,0)\,(1,1,1)]) = A5_R\,(0, -1, 1) = 1,$$

$$A1_R\,(1,0,0) = 0.2,\ A2_R\,(1,0,0) = 0.3,\ A4_R\,(1,0,0) = 0.3,\ A5_R\,(1,0,0) = 0.5.$$

That is,

$$A1_R\,([1,0,0)\,(1,1,1)]) \ngeq A1_R\,(1,0,0),$$

$$A2_R\,([1,0,0)\,(1,1,1)]) \ngeq A2_R\,(1,0,0),$$

$$A4_R\,([1,0,0)\,(1,1,1)]) \nleq A4_R\,(1,0,0),$$

$$A5_R\,([1,0,0)\,(1,1,1)]) \nleq A5_R\,(1,0,0).$$

**Proposition 2.** If R is an QNP Lie ideal of $\mathscr{L}$ , then

I. $A1_R\,(0) \geq A1_R\,(a)$, $A2_R\,(0) \geq A2_R\,(a)$, $A4_R\,(0) \leq A4_R\,(a)$ and $A5_R\,(0) \leq A5_R\,(a)$,
II. $A1_R\,([a, b]) \geq \max\,\{A1_R\,(a), A1_R\,(b)\}$,

III. $A2_R([a,b]) \geq \max\{A2_R(a), A2_R(b)\}$,
IV. $A4_R([a,b]) \leq \min\{A4_R(a), A4_R(b)\}$,
V. $A5_R([a,b]) \leq \min\{A5_R(a), A5_R(b)\}$,
VI. $A1_R([a,b]) = A1_R(-[b,a]) = A1_R([b,a])$,
VII. $A2_R([a,b]) = A2_R(-[b,a]) = A2_R([b,a])$,
VIII. $A4_R([a,b]) = A4_R(-[b,a]) = A4_R([b,a])$,
IX. $A5_R([a,b]) = A5_R(-[b,a]) = A5_R([b,a])$.

For all $a, b \in \mathscr{L}$.

**Proof.** The proof follows from *Definition 5*.

**Proposition 3.** If $\{R_i : i \in J\}$ is a family of QNP Lie algebra of $\mathscr{L}$, then $\bigcap R_i = (\wedge A1_{Ri} \wedge A2_{Ri} \vee A4_{Ri} \vee A5_{Ri})$ is an QNP Lie ideal of $\mathscr{L}$ where,

$$\wedge A1_{Ri}(a) = \inf\{\wedge A1_{Ri}(a) : i \in J,\ a \in \mathscr{L}\},$$

$$\wedge A2_{Ri}(a) = \inf\{\wedge A2_{Ri}(a) : i \in J,\ a \in \mathscr{L}\},$$

$$\vee A4_{Ri}(a) = \sup\{\vee A4_{Ri}(a) : i \in J,\ a \in \mathscr{L}\},$$

$$\vee A5_{Ri}(a) = \sup\{\vee A5_{Ri}(a) : i \in J,\ a \in \mathscr{L}\}.$$

**Proof.** The proof follows from *Definition 5*.

**Definition 6.** Let R = ($A1_R$, $A2_R$, $A4_R$, $A5_R$) be an QNP Lie subalgebra of $\mathscr{L}$ and let $(\alpha,\beta,\delta,\vartheta)$ [0,1] X [0,1] X [0,1] X [0,1] with $\alpha+\beta+\delta+\vartheta \leq 2$. Then level subset of R is defined as

$$R^{(\alpha,\beta,\delta,\vartheta)} = \{\ a \in \mathscr{L} : A1(a) \geq \alpha, A2(a) \geq \beta, A4(a) \leq \delta, A5(a) \leq \vartheta\},$$

are called $(\alpha,\beta,\gamma,\delta,\vartheta)$ level subsets of QNP set $R$. The set of all $(\alpha,\beta,\delta,\vartheta) \in$ Im($A1_R$) X Im($A2_R$) X Im($A4_R$) X Im($A5_R$) such that $\alpha+\beta+\delta+\vartheta \leq 2$ is known as image of R = ($A1_R$, $A2_R$, $A4_R$, $A5_R$).

Note:

$$R^{(\alpha,\beta,\delta,\vartheta)} = \{a \in \mathscr{L} : A1(a) \geq \alpha, A2(a) \geq \beta, A4(a) \leq \delta, A5(a) \leq \vartheta\},$$

$$R^{(\alpha,\beta,\delta,\vartheta)} = \{a \in \mathscr{L} : A1(a) \geq \alpha\} \cap \{a \in \mathscr{L} : A2(a) \geq \beta\} \cap \{a \in \mathscr{L} : A4(a) \leq \delta\} \cap \{a \in \mathscr{L} : A5(a) \leq \vartheta\},$$

$$R^{(\alpha,\beta,\delta,\vartheta)} = U(A1(a),\alpha) \cap U'(A2(a),\beta) \cap \gamma) \cap L'(A4(a),\delta) \cap L''(A5(a),\vartheta).$$

**Theorem 1.** An QNP set R = ($A1_R$, $A2_R$, $A4_R$, $A5_R$) of $\mathscr{L}$ is an QNP lie ideal of $\mathscr{L}$ *iff* $R^{(\alpha,\beta,\delta,\vartheta)}$ is a QNP Lie ideal of $\mathscr{L}$ for every $(\alpha,\beta,\gamma,\delta,\vartheta)$ [0,1] X [0,1] X [0,1] X [0,1] with $\alpha+\beta+\delta+\vartheta \leq 3$.

**Proposition 4.** Let R = ($A1_R$, $A2_R$, $A4_R$, $A5_R$) be an QNP Lie ideal of $\mathscr{L}$ and ($r_1$, $s_1$, $u_1$, $v_1$), ($r_2$, $s_2$, $u_2$, $v_2$) $\in$ Im(A1R) X Im($A2_R$) X Im($A4_R$) X Im($A5_R$) with ri+ si+ ui +vi$\leq 3$ for i = 1,2. Then $\mathscr{L}_R^{(r1,s1,u1,v1)} = \mathscr{L}_R^{(r2,s2,u2,v2)}$ if and only if (r1, s1, u1, v1) = (r2, s2, u2, v2)

**Theorem 2.** Let $K_0 \subset K_1 \subset K_2 \subset K_3 \ldots\ldots\ldots \subset K_n = L$ be a chain of QNP Lie ideals of a QNP Lie algebra $\mathscr{L}$. Then there exists an QNP ideal $A1_R$ of $\mathscr{L}$ for which level subsets U (A1 (a),$\alpha$), U′(A2(a), β), L′( A 4(a), δ) and L”(A5(a),$\vartheta$) coincide with this chain.

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**Proof.** Let { $r_k$: k = 0,1,2…,n}, {$s_k$:k = 0,1,….n} , {$u_k$: k = 0,1,2…n} and {$v_k$: k = 0,1,2…n} be finite decreasing and increasing sequences in [0,1]. Let Let R = ($A1_R$, $A2_R$, $A4_R$, $A5_R$) be a QNP set in $\mathscr{L}$ defined by $A1_R(K_0) = r_0$, $A2_R(K_0) = s_0$, $A4_R(K_0) = u_0$, $A5_R(K_0) = v_0$, $A1_R(K_l{:}K_{l-1}) = r_l$, $A2_R(K_l \backslash K_{l-1}) = s_l$, $A4_R(K_l \backslash K_{l-1}) = u_l$,$A5_R(K_l \backslash K_{l-1}) = v_l$, for $0 < l \leq n$. Let a, b $\in \mathscr{L}$. If a ,b$\in K_l \backslash K_{l-1}$, then a +b, $\beta$a, [a,b] $\in K_l$

$$A1_R(a + b) \geq r_k = \min\{A1_R(a), A1_R(b)\},$$

$$A2_R(a + b) \geq s_k = \min\{A2_R(a), A2_R(b)\},$$

$$A4_R(a + b) \leq u_k = \max\{A4_R(a), A4_R(b)\},$$

$$A5_R(a + b) \leq v_k = \max\{A5_R(a), A5_R(b)\},$$

$$A1_R(\alpha a) \geq r_k = A1_R(a),\ A2_R(\alpha a) \geq s_k = A2_R(a),$$

$$A4_R(\alpha a) \leq u_k = A4_R(a),\ A5_R(\alpha a) \leq v_k = A5_R(a),$$

$$A1_R([a, b]) \geq r_k = A1_R(a),\ A2_R([a, b]) \geq s_k = A2_R(a),$$

$$A4_R([a, b]) \leq u_k = A4_R(a),\ A5_R([a, b]) \leq v_k = A5_R(a).$$

For i> j, if a $\in K_i \setminus K_{i-1}$ and b $\in K_j \setminus K_{j-1}$, then $A1_R(a) = r_i = A1_R(b)$, $A2_R(a) = s_i = A2_R(b)$,

$A4_R(a) = u_j = A4_R(b)$, $A5_R(a) = v_j = A5_R(b)$ and a +b, $\alpha$a, [a, b] $\in K_I$. Thus

$$A1_R(a + b) \geq r_i = \min\{A1_R(a), A1_R(b)\},$$

$$A2_R(a + b) \geq s\,i = \min\{A2_R(a), A2_R(b)\},$$

$$A4_R(a + b) \leq u_j = \max\{A4_R(a), A4_R(b)\},$$

$$A5_R(a + b) \leq v\,j = \max\{A5_R(a), A5_R(b)\},$$

$$A1_R(\alpha a) \geq r_i = A1_R(a),\ A2_R(\alpha a) \geq s\,i = A2_R(a),$$

$$A4_R(\alpha a) \leq u_j = A4_R(a),\ A5_R(\alpha a) \leq v\,j = A5_R(a),$$

$$A1_R([a, b]) \geq r_i = A1_R(a),\ A2_R([a, b]) \geq s_i = A2_R(a),$$

$$A4_R([a, b]) \leq u_j = A4_R(a),\ A5_R([a, b]) \leq v_j = A5_R(a).$$

Thus, we conclude that R = ($A1_R$, $A2_R$, $A4_R$, $A5_R$) is a QNP Lie ideal of a QNP Lie algebra $\mathscr{L}$ and all its non-empty level subsets are QNP Lie ideals.

Since Im ($A1_R$) = {$r_0$, $r_1$, $r_2$….,$r_n$ }, Im ($A2_R$) = {$s_0$, $s_1$, $s_2$….,$s_n$ },

Im ($A4_R$) = {$u_0$, $u_1$, $u_2$….,$u_n$ }, Im ($A5_R$) = {$v_0$, $v_1$, $v_2$….,$v_n$ }, level subsets of R forms chains:

$$U(A1_R, r_0) \subset U(A1_R, r_1) \subset \ \ldots.. \subset U(A1_R, r_n) = L,$$

$$U'(A2_R, s_0) \subset U'(A2_R, s_1) \subset \ \ldots.. \subset U'(A2_R, s_n) = L,$$

$L'(A4_R, u_0) \subset L'(A4_R, u_1) \subset \ldots.. \subset L'(A4_R, u_n) = L$,

$L''(A5_R, v_0) \subset L''(A5_R, v_1) \subset \ldots.. \subset L''(A5_R, v_n) = L$.

Respectively. Indeed

$U(A1_R, r_0) = \{a \in \mathcal{L}: A1_R(a) \geq r_0\} = K_0$,

$U'(A2_R, s_0) = \{a \in \mathcal{L}: A2_R(a) \geq s_0\} = K_0$,

$L'(A4_R, u_0) = \{a \in \mathcal{L}: A4_R(a) \leq u_0\} = K_0$,

$L''(A5_R, v_0) = \{a \in \mathcal{L}: A5_R(a) \leq v_0\} = K_0$.

We prove that $U(A1_R, r_l) = U'(A2_R, s_l) = L'(A4_R, u_l) = L''(A5_R, v_l) = K_l$ for $0 \leq l \leq n$.

Clearly, $K_l \subseteq U(A1_R, r_l)$, $K_l \subseteq U'(A2_R, s_l)$, $K_l \subseteq L'(A4_R, u_l)$, $K_l \subseteq L''(A5_R, v_l)$.

If $a \in U(A1_R, r_l)$, then $A1_R(a) \geq r_l$ and for $a \notin K_j$, for $j > l$. Hence $A1_R(a) \in \{r_0, r_1, r_2 \ldots.., r_l\}$,

Which implies $a \in {}_j$ for some $j \leq l$. Since $K_j \subset K_l$, it follows that $a \in K_l$ .Consequently, $U(A1_R, r_l) = K_l$ for some $0 < l \leq n$.

If $a \in U'(A2_R, s_l)$, then $A2_R(a) \geq s_l$ and for $a \notin K_j$, for $j > l$. Hence $A2_R(a) \in \{s_0, s_1, s_2 \ldots.., s_l\}$,

Which implies $a \in K_j$ for some $j \leq l$. Since $K_j \subset K_l$ ,it follows that $a \in K_l$ .Consequently, $U'(A2_R, s_l) = K_l$ for some $0 < l \leq n$.

If $a \in L'(A4_R, u_l)$, then $A4_R(a) \leq u_l$ and for $a \notin K_m$ ,for $m > l$. Hence $A4_R(a) \in \{u_0, u_1, u_2 \ldots.., u_l\}$,

Which implies $a \in K_m$ for some $m \leq l$. Since $K_m \subset K_l$ ,it follows that $a \in K_l$ .

Consequently, $L'(A4_R, u_l) = K_l$ for some $0 < l \leq n$.

If $a \in L''(A5_R, v_l)$,then $A5_R(a) \leq v_l$ and for $a \notin K_m$ ,for $m > l$. Hence $A5_R(a) \in \{v_0, v_1, v_2 \ldots.., v_l\}$,

Which implies $a \in K_m$ for some $m \leq l$. Since $K_m \subset K_l$ ,it follows that $a \in K_l$ .

Consequently, $L''(A5_R, v_l) = K_l$ for some $0 < l \leq n$. This completes the proof.

**Theorem 3.** If $R = (A1_R, A2_R, A4_R, A5_R)$ is an QNP Lie ideal of a QNP Lie algebra $\mathscr{L}$, then

$A1_R(a) = \sup\{r \in [0,1] \setminus a \in U(A1_R, r)\}$,

$A2_R(a) = \sup\{s \in [0,1] \setminus a \in U'(A2_R, s)\}$,

$A4_R(a) = \inf\{u \in [0,1] \setminus a \in L'(A4_R, u)\}$,

$A5_R(a) = \inf\{v \in [0,1] \setminus a \in U(A5_R, v)\}$,

for every $a \in \mathscr{L}$.

**Proof.** The proof follows from *Definition 5*.

**Definition 7.** Let f be a map from a set $\mathscr{L}_1$ to a set $\mathscr{L}_2$. If R = ($A1_R$, $A2_R$, $A4_R$, $A5_R$) and

R = ($A1_R$, $A2_R$, $A4_R$, $A5_R$) are QNP sets in $\mathscr{L}_1$ and $\mathscr{L}_2$ respectively, then the preimage of R2 under f, denoted by $f^{-1}$(R2), is a QNP set defined by

$f^{-1}(R2) = (f^{-1}(A1_{R2}), f^{-1}(A2_{R2}), f^{-1}(A4_{R2}), f^{-1}(A5_{R2}))$.

**Theorem 4.** Let $f: \mathscr{L}_1 \rightarrow \mathscr{L}_2$ be an onto homomorphisms of Lie algebras. If R = ($A1_R$, $A2_R$, $A4_R$, $A5_R$) is a QNP Lie ideal of $\mathscr{L}_2$, then the preimage

$f^{-1}(R2) = (f^{-1}(A1_{R2}), f^{-1}(A2_{R2}), f^{-1}(A4_{R2}), f^{-1}(A5_{R2}))$ under f is a QNP Lie ideal of $\mathcal{L}_1$ .

**Proof.** The proof follows from *Definitions 5* and *7*.

**Theorem 5.** Let $f: \mathscr{L}_1 \rightarrow \mathscr{L}_2$ be an epimorphisms of QNP Lie algebras. If R = ($A1_R$, $A2_R$, $A4_R$, $A5_R$) is a QNP Lie ideal of $\mathscr{L}_2$, then the preimage $f^{-1}((R1)^C) = (f^{-1}(R1))^C$

**Proof.** The proof follows from *Definitions 5* and *7*.

**Theorem 6.** Let $f: \mathscr{L}_1 \rightarrow \mathscr{L}_2$ be an epimorphisms of QNP Lie algebras. If R = ($A1_R$, $A2_R$, $A4_R$, $A5_R$) is a QNP Lie ideal of $\mathscr{L}_2$ and R = ($A1_R$, $A2_R$, $A4_R$, $A5_R$) is the preimage of R = ($A1_R$, $A2_R$, $A4_R$, $A5_R$) under f. Then R2 is a QNP Lie ideal of $\mathscr{L}_1$.

**Proof.** The proof follows from *Definitions 5* and *7*.

**Definition 8.** Let $\mathscr{L}_1$ and $\mathscr{L}_2$ be two QNP Lie algebras and f be a mapping of $\mathscr{L}_1$ into $\mathscr{L}_2$. If R = ($A1_R$, $A2_R$, $A4_R$, $A5_R$) is a QNP set of $\mathscr{L}_1$, then the image of R1 under f is the QNP set in $\mathcal{L}_2$ defined by

$$f(A1_{R1})(b) = \begin{cases} \sup_{a \in f^{-1}(b)} A1_{R1}(a), & \text{if } f^{-1}(b) \neq 0, \\ 0, & \text{otherwise} \end{cases}$$

$$f(A2_{R1})(b) = \begin{cases} \sup_{a \in f^{-1}(b)} A2_{R1}(a), & \text{if } f^{-1}(b) \neq 0, \\ 0, & \text{otherwise} \end{cases}$$

$$f(A4_{R1})(b) = \begin{cases} \inf_{a \in f^{-1}(b)} A4_{R1}(a), & \text{if } f^{-1}(b) \neq 0, \\ 1, & \text{otherwise} \end{cases}$$

$$f(A5_{R1})(b) = \begin{cases} \inf_{a \in f^{-1}(b)} A5_{R1}(a), & \text{if } f^{-1}(b) \neq 0, \\ 1, & \text{otherwise} \end{cases}$$

for each $b \in \mathscr{L}_2$

**Theorem 7.** Let $f: \mathscr{L}_1 \rightarrow \mathscr{L}_2$ be an epimorphisms of QNP Lie algebras. If R = ($A1_R$, $A2_R$, $A4_R$, $A5_R$) is a QNP Lie ideal of $\mathscr{L}_1$, then f(R1) is a QNP Lie ideal of $\mathscr{L}_2$.

**Proof.** The proof follows from *Definitions 5* and *8*.

**Definition 9.** Let $f: \mathscr{L}_1 \rightarrow \mathscr{L}_2$ be an homomorphisms of QNP Lie algebras, For any QNP set, If R = ($A1_R$, $A2_R$, $A4_R$, $A5_R$) is a QNP Lie ideal of $\mathscr{L}_2$ we define a PNP set $R^f = (A1^f_R, A2^f_R, A4^f_R, A5^f_R)$ in $\mathscr{L}_1$ by

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$A1_R^f(a) = A1_R(f(a))$, $A2_R^f(a) = A2_R(f(a))$, $A4_R^f(a) = A4_R(f(a))$, $A5_R^f(a) = A5_R(f(a))$, for all $a \in \mathscr{L}_1$.

**Lemma 1.** Let $f: \mathscr{L}_1 \to \mathscr{L}_2$ be an homomorphisms of QNP Lie algebras, If $R = (A1_R, A2_R, A4_R, A5_R)$ is a QNP Lie ideal of $\mathscr{L}_2$, then $R^f = (A1_R^f, A2_R^f, A4_R^f, A5_R^f)$ is a QNP Lie ideal in $\mathscr{L}_1$.

**Proof.** Let $a, b \in \mathscr{L}_1$ *and* $\beta \in \mathscr{F}$. Then

$A1_R^f(a+b) = A1_R(f(a+b)) = A1_R(f(a)+f(b)) \geq \min\{A1_R(f(a)), A1_R(f(b))\} = \min\{A1_R^f(a), A1_R^f(b)\}$,

$A2_R^f(a+b) = A2_R(f(a+b)) = A2_R(f(a)+f(b)) \geq \min\{A2_R(f(a)), A2_R(f(b))\} = \min\{A2_R^f(a), A2_R^f(b)\}$,

$A4_R^f(a+b) = A4_R(f(a+b)) = A4_R(f(a)+f(b)) \leq \min\{A4_R(f(a)), A4_R(f(b))\} = \min\{A4_R^f(a), A4_R^f(b)\}$,

$A5_R^f(a+b) = A5_R(f(a+b)) = A5_R(f(a)+f(b)) \leq \min\{A5_R(f(a)), A5_R(f(b))\} = \min\{A5_R^f(a), A5_R^f(b)\}$,

$A1_R^f(\beta a) = A1_R(f(\beta a)) = A1_R(\beta f(a)) \geq A1_R(f(a)) = A1_R^f(a)$,

$A2_R^f(\beta a) = A2_R(f(\beta a)) = A2_R(\beta f(a)) \geq A2_R(f(a)) = A2_R^f(a)$,

$A4_R^f(\beta a) = A4_R(f(\beta a)) = A4_R(\beta f(a)) \leq A4_R(f(a)) = A4_R^f(a)$,

$A5_R^f(\beta a) = A5_R(f(\beta a)) = A5_R(\beta f(a)) \leq A5_R(f(a)) = A5_R^f(a)$.

Similarly,

$A1_R^f([a, b]) = A1_R(f[a, b]) = A1_R([f(a), f(b)]) \geq A1_R(f(a)) = A1_R^f(a)$,

$A2_R^f([a, b]) = A2_R(f([a,b]) = A2_R([f(a), f(b)]) \geq A2_R(f(a)) = A2_R^f(a)$,

$A4_R^f([a, b]) = A4_R(f([a, b]) = A4_R([f(a), f(b)]) \leq A4_R(f(a)) = A4_R^f(a)$,

$A5_R^f([a, b]) = A5_R(f([a, b]) = A5_R([f(a), f(b)]) \leq A5_R(f(a)) = A5_R^f(a)$.

This proves that $R^f = (A1_R^f, A2_R^f, A4_R^f, A5_R^f)$ is a QNP Lie ideal in $\mathscr{L}_1$.

We now characterize the QNP Lie ideals of Lie algebras.

**Theorem 8.** Let $f: \mathscr{L}_1 \to \mathscr{L}_2$ be an epimorphisms of QNP Lie algebras. Then $R^f = (A1_R^f, A2_R^f, A4_R^f, A5_R^f)$ is a QNP Lie ideal in $\mathscr{L}_1$ iff $R = (A1_R, A2_R, A4_R, A5_R)$is a QNP Lie ideal of $\mathscr{L}_2$.

**Definition 10.** Let $R = (A1_R, A2_R, A4_R, A5_R)$ be a QNP Lie ideal in $\mathscr{L}$. Define a inductively a sequences of QNP Lie ideals in $\mathscr{L}$ by $R^0 = R$, $R^1 = [R^0, R^0]$, $R^2 = [R^1, R^1]$,..... $R^n = [R^{n-1}, R^{n-1}]$.

$R^n$ is called the n th derived QNP Lie ideal of $\mathscr{L}$. A series $R^0 \supseteq R^1 \supseteq R^2 \supseteq ..... \supseteq R^n \supseteq \cdots$

is called derived series of a QNP Lie ideal R in $\mathscr{L}$.

**Definition 11.** A QNP Lie ideal R in is called a solvable QNP Lie ideal, if there exists a positive integer n such that $R^0 \supseteq R^1 \supseteq R^2 \supseteq .....\supseteq R^n = (0,0,0)$

**Theorem 9.** Homomorphic images of solvable QNP Lie ideals are solvable QNP Lie ideals.

**Proof.** Let $f : \mathcal{L}_1 \rightarrow \mathscr{L}_2$ be homomorphisms of QNP Lie algebras. Suppose that $R = (A1_R, A2_R, A4_R, A5_R)$ is a QNP Lie ideal of $\mathscr{L}_1$. We prove by induction on n that $f(R^n) \supseteq [f(R)]^n$, where n is any positive integer. First we claim that $f([R, A]) \supseteq [f(R), f(R)]$. Let $y \in \mathscr{L}_2$. Then

$$f(<<A1_R, A1_R>>)(y) = \sup\{<<A1_R, A1_R>>(y)\backslash f(x) = y\}$$

$$= \sup\{\sup\{\min(A1_R(a), A1_R(b))\backslash a,b \in \mathscr{L}_1,[a, b] = x, f(x) = y\}\}$$

$$= \sup\{\min(A1_R(a), A1_R(b))\backslash a, b \in \mathscr{L}_1,,[a, b] = x, f(x) = y\}\}$$

$$= \sup\{\min(A1_R(a), A1_R(b))\backslash a,b \in \mathscr{L}_1,,[f(a), f(b)] = x\}$$

$$= \sup\{\min(A1_R(a), A1_R(b))\backslash a,b \in \mathscr{L}_1,f(a) = u, f(b) = v,[u, v] = y\}\}$$

$$\geq \sup\{\min(\sup_{a\in f^{-1}(u)} A1_R(a), \min(\sup_{b\in f^{-1}(v)} A1_R(b)\backslash[u, v] = y\}$$

$$= \sup\{\min\{f(A1_R)(u), f(A1_R)(v))\backslash[u, v] = y\}$$

$$= <<f(A1_R), f(A1_R)>>(y),$$

$$f(<<A2_R, A2_R>>)(y) = \sup\{<<A2_R, A2_R>>(y)\backslash f(x) = y\}$$

$$= \sup\{\sup\{\min(A2_R(a), A2_R(b))\backslash a, b \in \mathscr{L}_1,[a, b] = x, f(x) = y\}\}$$

$$= \sup\{\min(A2_R(a), A2_R(b))\backslash a, b \in \mathscr{L}_1,,[a, b] = x, f(x) = y\}\}$$

$$= \sup\{\min(A2_R(a), A2_R(b))\backslash a, b \in \mathscr{L}_1,,[f(a), f(b)] = x\}$$

$$= \sup\{\min(A2_R(a), A2_R(b))\backslash a, b \in \mathscr{L}_1,,f(a) = u, f(b) = v,[u, v] = y\}\}$$

$$\geq \sup\{\min(\sup_{a\in f^{-1}(u)} A2_R(a), \min(\sup_{b\in f^{-1}(v)} A2_R(b)\backslash[u, v] = y\}$$

$$= \sup\{\min\{f(A2_R)(u), f(A2_R)(v))\backslash[u, v] = y\}$$

$$= <<f(A2_R), f(A2_R)>>(y),$$

$$f(<<A4_R, A4_R>>)(y) = \inf\{<<A4_R, A4_R>>(y)\backslash f(x) = y\}$$

$$= \inf\{\inf\{\max(A4_R(a), A4_R(b))\backslash a, b \in \mathcal{L}_1,[a, b] = x, f(x) = y\}\}$$

$$= \inf\{\max(A4_R(a), A4_R(b))\backslash a, b \in \mathcal{L}_1,[a, b] = x, f(x) = y\}\}$$

$$= \inf\{\max(A4_R(a), A4_R(b))\backslash a, b \in \mathcal{L}_1,[f(a), f(b)] = x\}$$

$$= \inf\{\max(A4_R(a), A4_R(b))\backslash a, b \in \mathcal{L}_1,f(a) = u, f(b) = v,[u, v] = y\}\}$$

$$\leq \inf\{\max(\inf_{a\in f^{-1}(u)} A4_R(a), \min(\inf_{b\in f^{-1}(v)} A4_R(b)\backslash[u, v] = y\}$$

$= \inf\{\max\{f(A4_R)(u), f(A4_R)(v))\backslash[u, v] = y\}$

$= <<f(A4_R), f(A4_R)>>(y),$

$f(<<A5_R, A5_R>>)(y) = \inf\{<<A5_R, A5_R>>(y)\backslash f(x) = y\}$

$= \inf\{\inf\{\max(A5_R(a), A5_R(b))\backslash a, b \in \mathcal{L}_1, [a, b] = x, f(x) = y\}\}$

$= \inf\{\max(A5_R(a), A5_R(b))\backslash a, b \in \mathcal{L}_1, [a, b] = x, f(x) = y\}\}$

$= \inf\{\max(A5_R(a), A5_R(b))\backslash a, b \in \mathcal{L}_1, [f(a), f(b)] = x\}$

$= \inf\{\max(A5_R(a), A5_R(b))\backslash a, b \in \mathcal{L}_1, f(a) = u, f(b) = v, [u, v] = y\}\}$

$\leq \inf\{\max(\inf_{a\in f^{-1}(u)} A5_R(a), \min(\inf_{b\in f^{-1}(v)} A5_R(b)\backslash[u, v] = y\}$

$= \inf\{\max\{f(A5_R)(u), f(A5_R)(v))\backslash[u, v] = y\}$

$= <<f(A5_R), f(A5_R)>>(y).$

Thus $f([R, R]) \supseteq f(<<A,A>>) \supseteq <<f(R), f(R)>> = [f(R), f(R)]$.

Now for n> 1, we get $f(R^n) = f([R^{n-1}, R^{n-1}]) \supseteq [f(R^{n-1}), f(R^{n-1})]$.

This completes the proof

**Definition 12.** Let $R = (A1_R, A2_R, A4_R, A5_R)$ be a QNP Lie ideal in $\mathscr{L}$. We define a inductively a sequences of QNP Lie ideals in $\mathscr{L}$ by $R_0 = R, R_1 = [R, R_0], R_2 = [R, R_1] \ldots . R_n = [R, R_{n-1}]$.

A series $R_0 \supseteq R_1 \supseteq R_2 \supseteq \ldots . \supseteq R_n \supseteq \cdots$

is called descending central series of a QNP Lie ideal R in $\mathscr{L}$.

**Definition 13.** An QNP Lie ideal R is called a nilpotent QNP Lie ideal in $\mathscr{L}$, if there exists a positive integer n such that $R_0 \supseteq R_1 \supseteq R_2 \supseteq \ldots . \supseteq R_n = (0,0,0)$.

**Theorem 10.** Homomorphic image of a nilpotent QNP Lie ideal is a nilpotent QNP Lie ideal.

**Proof.** It is obvious

**Theorem 14.** Let K be a QNP Lie ideal of a QNP Lie algebra $\mathscr{L}$. If $R = (A1_R, A2_R, A4_R, A5_R)$ is a QNP Lie ideal of $\mathscr{L}$, then the QNP set $*R = (*A1_R, *A2_R, *A4_R, *A5_R)$ of $\mathscr{L}/K$ defined by

$$*A1_R(a + K) = \sup_{x\in K} A1_R(a + x),$$

$$*A2_R(a + K) = \sup_{x\in K} A2_R(a + x),$$

$$*A4_R(a + K) = \inf_{x\in K} A4_R(a + x),$$

$$*A5_R(a + K) = \inf_{x\in K} A5_R(a + x),$$

is a QNP Lie ideal of the quotient QNP Lie algebra $\mathscr{L}/K$ of $\mathscr{L}$ with respect to K.

**Proof.** Clearly,*R is defined. Let x + K, y + K ∈ $\mathscr{L}$/ K. Then

$*A1_R((x + K) + (y + K)) = *A1_R((x + y) + K)$

$$= \sup_{z\in K} A1_R\big((x+y)+z\big),$$

$$= \sup_{z=s+t\in K} A1_R\big((x+y)+(s+t)\big),$$

$$\geq \sup_{s,t\in K} \min\{A1_R(x+s), A1_R(y+t)\},$$

$$= \min\{ \sup_{s\in K} A1_R(x+s), \sup_{t\in K} A1_R(y+t)\},$$

$$= \min\{* A1_R(x+s), * A1_R(y+t)\},$$

$*A1_R(\beta(x + K) = *A1_R(\beta x + K) = \sup_{z\in K} A1_R(\beta x + z) \geq \sup_{z\in K} A1_R(x+z) = *A1_R(x + K)$.

$*A1_R([x + K, *A1_R(a + K) = \sup_{x\in K} A1_R(a + x)$,

$y + K]) = *A1_R([x , y] + K) = \sup_{z\in K} A1_R([x,y]+z) \geq \sup_{z\in K} A1_R([x,y]+z) = *A1_R(x + K)$.

Thus $*A1_R$ is a PNP Lie ideal of $\mathscr{L}$/ K. In a similar way, we can verify that $*A2_R$, $*A4_R$ and $*A5_R$ PNP Lie ideals of $\mathscr{L}$ /K. Hence $*R = (*A1_R, *A2_R, *A4_R, *A5_R)$ is a QNP Lie ideal of $\mathscr{L}$ /K.

**Theorem 15.** Let K be a QNP Lie ideal of a QNP Lie algebra $\mathscr{L}$. Then there is a one-to=one correspondence between the set of QNP Lie ideals $R = (A1_R, A2_R, A4_R, A5_R)$ of $\mathscr{L}$ such that R(0) = A(s) for all s ∈ K and the set of all QNP Lie ideals $*R = (*A1_R, *A2_R, *A4_R, *A5_R)$ of $\mathscr{L}$ /K.

**Proof.** Let R = (A1 R, A2 R, A4R ,A5 R) be QNP Lie ideal of $\mathscr{L}$. Using Theorem 3.27, we prove that

*A1 R, *A2 R, *A4R ,*A5 R defined by

$*A1R(a + K) = \sup_{x\in K} A1\, R(a + x)$,

$*A2R(a + K) = \sup_{x\in K} A2\, R(a + x)$,

$*A4R(a + K) = \inf_{x\in K} A4\, R(a + x)$,

$*A5R(a + K) = \inf_{x\in K} A5\, R(a + x)$,

are QNP Lie ideals of $\mathscr{L}$ /K. Since A1R(0) = A1 R(s), A2 R(0) = A2 R(s),

A4R(0) = A4 R(s), A5 R(0) = A5 R(s) for all s ∈ K,

A1R(a + s) ≥ min(A1 R(a), A1 R(s)) = A1 R(a),

A2R(a + s) ≥ min(A2 R(a), A2 R(s)) = A2 R(a),

A4R(a + s) ≤ max(A4 R(a), A4 R(s)) = A4 R(a),

A5R(a + s) ≤ min(A5 R(a), A5 R(s)) = A5 R(a).

Again,

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$A1\ R(a) = A1R(a + s - s) \geq \min(A1\ R(a + s), A1\ R(s)) = A1\ R(a + s),$

$A2\ R(a) = A2R(a + s - s) \geq \min(A2\ R(a + s), A2\ R(s)) = A2\ R(a + s),$

$A4\ R(a) = A4R(a + s - s) \leq \max(A4\ R(a + s), A4\ R(s)) = A4\ R(a + s),$

$A5\ R(a) = A5R(a + s - s) \leq \max(A5\ R(a + s), A5\ R(s)) = A5\ R(a + s).$

Thus $R(a + s) = R(a)$ for all $s \in K$. Hence the correspondence $R \rightarrow {}^*R$ is one- to -one. Let $^*R$ be a QNP Lie ideal of $\mathscr{L}$ / K and define a PNP set R = (A1 R, A2 R, A4R ,A5 R) in $\mathscr{L}$ by

$A1\ R(a) = {}^*A1R(a + K), A2\ R(a) = {}^*A2\ R(a + K), A4\ R(a) = {}^*A4\ R(a + K), A5\ R(a) = {}^*A5\ R(a + K).$

For $a,b \in \mathscr{L}$, we have

$A1R(a + b) = {}^*A1\ R((a + b) + K) = {}^*A1\ R((a + K) + (b + K)),$

$\geq \min\{{}^*A1\ R(a + K), {}^*A1\ R(b + K)\},$

$= \min\{A1\ R(a), A1\ R(b)\},$

$A1\ R(\beta a) = {}^*A1R(\beta a + K) \geq {}^*A1\ R(a + K) = A1\ R(a),$

$A1R([a, b]) = {}^*A1\ R([a,b] + K) = {}^*A1\ R([a + K, b + K]),$

$\geq {}^*A1R(a + K) = A1\ R(a).$

Thus A1 R is a QNP lie ideal of $\mathscr{L}$. In a similar way, we can verify that A2 R, A4 R and A5R are QNP Lie ideals of $\mathscr{L}$. Hence R = (A1 R, A2 R, A4R ,A5 R) is a QNP Lie ideal of $\mathscr{L}$.

Note that $A1\ R(a) = {}^*A1R(a + K), A2\ R(a) = {}^*A2\ R(a + K), A4\ R(a) = {}^*A4\ R(a + K), A5\ R(a) = {}^*A5R(a + K).$

This completes the proof.

## 4| Conclusion

In this article, we have discussed above QNP Lie subalgebra and QNP Lie ideals of a QNP Lie Algebra. We have also investigated some of its properties of Quadripartitioned Neutrosophic Pythagorean Lie ideals. In future, we are planned to study on Lie rings. We may also develop for heptapartitioned neutrosophic sets and other hybrid sets.

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