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A Fuzzy Multi Objective Inventory Model of Demand Dependent Deterioration Including Lead Time

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Abstract

In this article, we have developed a deteriorated multi-item inventory model in a fuzzy environment. Here the demand rate is constant. Production cost and set-up cost are the most vital issue in the inventory system of the market world. Here production cost and set-up- cost are continuous functions of demand. Set-up-cost is also dependent on average inventory level. Deterioration cost is the most challenging issue in the business world. So here deterioration cost is dependent on inventory level and demand. Lead time crashing cost is considered the continuous function of leading time. In the real world all cost are not fixed. Due to uncertainty all cost parameters of the proposed model are taken as Generalized Triangular Fuzzy Number (GTFN). The formulated multi objective inventory problem has been solved by various techniques like as Geometric Programming (GP) technique, Fuzzy Programming Technique with Hyperbolic Membership Function (FPTHMF), Fuzzy Non-Linear Programming (FNLP) technique. Numerical example is taken to illustrate the model. Sensitivity analysis and graphical representation have been shown to test the parameters of the model.

Keywords: Inventory, Deterioration, Multi-item, Leading time, Generalized triangular fuzzy number, Fuzzy and GP techniques.

1 | Introduction

Inventory models deal with decisions that minimize the total average cost or maximize the total average profit. In that way to construct a real life mathematical inventory model on base on various assumptions and notations and approximation. The basic well known Economic Order Quantity (EOQ) model was first introduced by Harri [19]. In the business management systems deterioration is a most important key factor. In general, deterioration is defined as the damage, spoilage, dryness, vaporization etc., that results in decrease of usefulness of the original. A deteriorating model that is a model for exponentially decaying inventories was first introduced by Chare and Schrader [16]. Afterword many researchers have developed inventory models with deteriorating items.

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Aggarwal [2] studied a note on an order-level inventory model for a system with constant rate of deterioration. Dave and Patel [10] developed (T. Si) policy inventory for deteriorating items with time proportional demand. Dave [11] developed a order level inventory model for deteriorating items with variable instantaneous demand and discrete opportunities for replacement. Chen [8] studied an optimal determination of quality level, selling quantity and purchasing price for intermediate firms. Goyal and Giri [17] presented recent trends in modeling of deteriorating inventory. Wee et al. [33] considered a multi-objective joint replenishment inventory model of deteriorated items in a fuzzy environment. Tripathi et al. [32] discussed an inventory model with exponential time-dependent demand rate, variable deterioration, shortages and production cost. Shaikh et al. [31] established a fuzzy inventory model for a deteriorating item with variable demand, permissible delay in payments and partial backlogging with Shortage Follows Inventory (SFI) policy. Das and Islamn [13] proposed two warehouse inventory model for deteriorating items and Stock dependent demand under conditionally permissible delay in payment. Chakraborty et al. [7] derived two-warehouse partial backlogging inventory model with ramp type demand rate, three-parameter Weibull distribution deterioration under inflation and permissible delay in payments. Panda et al. [26] explored a credit policy approach in a two-warehouse inventory model for deteriorating items with price-and stock-dependent demand under partial backlogging.

In the real situation of the business world many inventory control model involves the deterministic lead times. Ben-Daya and Raouf [4] studied an inventory model involve the lead-time as a decision variable. Hariga and Ben-Daya [18] developed some stochastic inventory model with deterministic variable lead time. Chuang et al. [9] presented a note on periodic review inventory model with controllable setup cost and lead time. Ouyang [24] considered a mixture inventory model with backorders and lost sales for variable lead time also in [25] developed a min-max distribution free procedure for mixed inventory model with variable lead time. Sarkar et al. [29] established an integrated inventory model with variable lead time, defective units and delay in payments. Sarkar et al. [30] discussed a quality improvement and backorder price discount under controllable lead time in an inventory model. Multi items are used to fulfill customers demands as well as to increase the maximum profit of business men. Kotb and Fergany [22] developed a multi-item EOQ model with both demand dependet unit cost and varying lead time. Abou-El-Ata and Kotb [1] used a multi-item EOQ inventory model with varying holding cost under two restrictions. Saha and Sen [36] published a paper on inventory model with negative exponential demand and probabilistic deterioration under backlogging. Das and Islam [15] considered multi-objective two echelon supply chain inventory model with customer demand dependent purchase cost and production rate dependent production cost. Das [37] developed multi item inventory model include lead time with demand dependent production cost and set-up-cost in fuzzy environment.

The concept of fuzzy set theory was first introduced by Zadeh [34]. Afterward Zimmermann [35] applied the fuzzy set theory concept with some useful membership functions to solve the linear programming problem with some objective functions. Bit [3] studied fuzzy programming with hyperbolic membership functions for multi objective capacitated transportation problem. Maiti [23] considered fuzzy inventory model with two warehouse under possibility measure in fuzzy goal. Also Geometric Programming (GP) is a powerful optimization technique developed to solve a class of non-linear optimization programming problems especially found in engineering design and manufacturing. Multi objective GP techniques are also interesting in the EOQ model. GP was introduced by Duffin et al. [12]. Beightler and Phillips [5] applied GP. Biswal [6] considered fuzzy programming technique to solve multi- objective GP problems. Das et al. [14] established a multi-item inventory model with quantity dependent inventory costs and demand-dependent unit cost under imprecise objective and restrictions in GP approach. Mandal et al. [27] presented a multi-objective fuzzy inventory model with three constraints in a GP approach also used this method in [28] developed an inventory model of deteriorating items with a constraint. Islam [21] solved multi-objective marketing planning inventory model. Islam [20] discussed multi-objective geometric-programming problem and its application. Barman et al. [39] developed an analysis of retailer's inventory in a two-echelon centralized supply chain co-ordination under price-sensitive demand.

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In this paper, we have developed a deteriorated multi-item inventory model in a fuzzy environment. Here Production cost, set-up- cost and deterioration cost are continuous functions of constant demand. Set-up-cost and deterioration costs are also dependent on average inventory level. Lead time crashing cost is considered the continuous function of leading time. Due to uncertainty all cost parameters of the proposed model are taken as Generalized Triangular Fuzzy Number (GTFN). The formulated multi objective inventory problem has been solved by various techniques like as GP, FPTHMF, and FNLP. Numerical example is taken to illustrate the model. Sensitivity analysis and graphical representation have been shown to test the feasibility of the model for various parameters of the model. Finally conclusions have been drowning.

2 | Mathematical Model

2.1 | Notations

h_i: Holding cost per unit per unit time for ith item.

T_i: The length of cycle time for i^{th}item, $T_i > 0$.

D_i: Demand rate per unit time for the ith item.

L_i: Length of leading time for the ith item.

SS_i: Safety stock for the ith item.

$I_i(t)$: Inventory level of the ith item at time t.

Q_i: The order quantity for the duration of a cycle of length T_i for ith item.

$TAC_i(D_i, Q_i, L_i)$: Total average profit per unit for the ith item.

w_i: Storage space per unit time for the ith item.

W: Total area of space.

k: Safety factor.

θ_i: Constant deterioration rate for the ith item.

$\widetilde{w_\iota}$: Fuzzy storage space per unit time for the ith item.

$\widetilde{h_\iota}$: Fuzzy holding cost per unit per unit time for the ith item.

$\widetilde{TAC_\iota}(D_i, Q_i, L_i)$: Fuzzy total average cost per unit for the ith item.

$\widehat{w_\iota}$: Defuzzyfication of the fuzzy number $\widetilde{w_\iota}$.

$\widehat{h_\iota}$: Defuzzyfication of the fuzzy number $\widetilde{h_\iota}$.

$\widehat{TAC_\iota}(D_i, Q_i, L_i)$: Defuzzyfication of the fuzzy number $\widetilde{TAC_\iota}(D_i, Q_i, L_i)$.

2.2 | Assumptions

I. Multi-item is considered.
II. The replenishment rate is instantaneously and infinite.
III. The lead time is allowed.
IV. Shortage is not considered.
V. The unit production cost C_p^i of i^{th} item is inversely related to the demand rate D_i. So we take the following form $C_p^i(D_i) = \alpha_i D_i^{-\beta_i}$, where $\alpha_i > 0, \beta_i > 1$ are real constant.
VI. The set up cost is related to the average inventory level as well as demand. So we take the form $S_c^{\,i}(Q_i, D_i) = \gamma_i \left(\frac{Q_i}{2}\right)^{\delta_i} D_i^{\sigma_i}$ where $0 < \gamma_i, 0 < \delta_i \ll 1$ & $0 < \sigma_i \ll 1$ are real constant.
VII. Lead time crashing cost is related to the lead time by a function of the form $R^i(L_i) = \rho_i L_i^{-\tau_i}$, where $\rho_i > 0$ and $0 < \tau_i \leq 0.5$ are real constant.
VIII. $SS_i = k\omega\sqrt{L_i}$.

The deterioration cost is proporsonaly related to the average inventory level. So we take the

$$\theta_c^{\,i}(Q) = \upsilon_i \left(\frac{Q_i}{2}\right)^{\phi_i} D_i^{-\varphi_i}.$$

form where$0 < \upsilon_i \ll 1, \varphi_i > 1$ and $0 < \phi_i \ll 1$ are constant real numbers.

2.3 | Formulation of the Model

The inventory situation for i^{th} item is illustrated in *Fig. 1*. During the period the inventory level decreases due to demand rate and the deterioration rate. In the time interval $[0, T_i]$ the satisfying differential equation is

$$\frac{dI_i(t)}{dt} + \theta_i I_i(t) = -D_i, 0 \leq t \leq T_i \, . \quad (1)$$

With boundary conditions, $I_i(0) = Q_i$, $I_i(T_i) = 0$.

Solves the *Eq (1)* using the boundary conditions we have,

$$I_i(t) = e^{-\theta_i t}\left(\frac{D_i}{\theta_i} + Q_i\right) - \frac{D_i}{\theta_i}, \ 0 \leq t \leq T_i,$$

$$= Q_i - (D_i + Q_i\theta_i)t \ \text{(Neglected square and higher power of } \theta_i \text{ since } \theta_i \ll 1). \quad (2)$$

$$T_i = \frac{Q_i}{(D_i + Q_i\theta_i)}. \quad (3)$$

Now various average costs for i^{th} item are

I. Average production cost $(PC_i) = \frac{Q_i C_p^i(D_i)}{T_i} = \alpha_i\left(D_i^{(1-\beta_i)} + Q_i\theta_i D_i^{-\beta_i}\right)$.
II. Average inventory holding cost $(HC_i) = \frac{1}{T_i}\int_0^{T_i} h_i I_i(t)dt + h_i k\omega\sqrt{L_i} = h_i\left(\frac{Q_i}{2} + k\omega\sqrt{L_i}\right)$.
III. Average set-up-cost $(SC_i) = \frac{1}{T_i}\left[\gamma_i\left(\frac{Q_i}{2}\right)^{\delta_i} D_i^{\sigma_i}\right] = \frac{\gamma_i Q_i^{\delta_i - 1}\left(D_i^{\sigma_i+1} + Q_i\theta_i D_i^{\sigma_i}\right)}{2^{\delta_i}}$.
IV. Average lead time crashing cost $(CC_i) = \frac{\rho_i L_i^{-\tau_i}}{T_i} = \frac{\rho_i L_i^{-\tau_i}(D_i + Q_i\theta_i)}{Q_i}$.

V. Average deterioration cost $(DC_i) = \theta_i\, \upsilon_i \left(\frac{Q_i}{2}\right)^{\phi_i} D_i^{-\varphi_i}$.

Total average cost per unit time for ith item is

$$\mathrm{TAC_i(D_i, Q_i, L_i) = (PC_i + HC_i + SC_i + CC_i + DC_i)} = \alpha_i\left(D_i^{(1-\beta_i)} + Q_i\theta_i D_i^{-\beta_i}\right) + h_i\left(\frac{Q_i}{2} + k\omega\sqrt{L_i}\right) + \frac{\gamma_i Q_i^{\delta_i-1}\left(D_i^{\sigma_i+1} + Q_i\theta_i D_i^{\sigma_i}\right)}{2^{\delta_i}} + \frac{\rho_i L_i^{-\tau_i}\,(D_i + Q_i\theta_i)}{Q_i} + \theta_i\, \upsilon_i \left(\frac{Q_i}{2}\right)^{\phi_i} D_i^{-\varphi_i}. \quad (4)$$

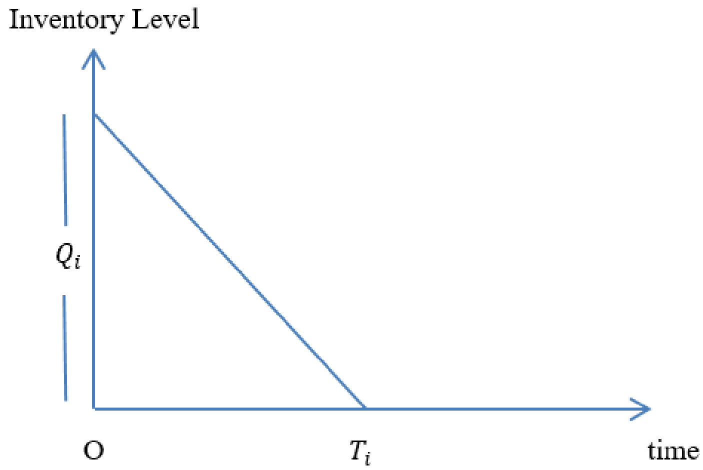

Fig. 1. For the i^th item.

So our Multi-Objective Inventory Model (MOIM) is defined as:

$$\text{Min } \{TAC_1, TAC_2, TAC_3, \ldots \ldots \ldots \ldots \ldots, TAC_n\},$$

$$\mathrm{TAC_i(D_i, Q_i, L_i)} = \alpha_i\left(D_i^{(1-\beta_i)} + Q_i\theta_i D_i^{-\beta_i}\right) + h_i\left(\frac{Q_i}{2} + k\omega\sqrt{L_i}\right) + \frac{\gamma_i Q_i^{\delta_i-1}\left(D_i^{\sigma_i+1} + Q_i\theta_i D_i^{\sigma_i}\right)}{2^{\delta_i}} + \frac{\rho_i L_i^{-\tau_i}\,(D_i + Q_i\theta_i)}{Q_i} + \theta_i\, \upsilon_i \left(\frac{Q_i}{2}\right)^{\phi_i} D_i^{-\varphi_i}, \quad (5)$$

Subject to, $D_i > 0, Q_i > 0, L_i > 0$, for $i = 1,2, \ldots \ldots \ldots .. n$.

2.4 | Fuzzy Model

For uncertainty, all the cost parameters of the model are taken as GTFN. The GTFN are assume as

$\widetilde{\alpha_i} = \left(\alpha_i^1, \alpha_i^2, \alpha_i^3; \psi_{\alpha_i}\right), 0 < \psi_{\alpha_i} \le 1;\ \widetilde{h_i} = \left(h_i^1, h_i^2, h_i^3; \psi_{h_i}\right), 0 < \psi_{h_i} \le 1,$

$\widetilde{\beta_i} = \left(\beta_i^1, \beta_i^2, \beta_i^3; \varphi_{\beta_i}\right), 0 < \varphi_{\beta_i} \le 1;\ \widetilde{\rho_i} = \left(\rho_i^1, \rho_i^1, \rho_i^1; \psi_{\rho_i}\right), 0 < \psi_{\rho_i} \le 1,$

$\widetilde{\theta_i} = \left(\theta_i^1, \theta_i^2, \theta_i^3; \psi_{\theta_i}\right), 0 < \psi_{\theta_i} \le 1;\ \widetilde{\tau_i} = \left(\tau_i^1, \tau_i^2, \tau_i^3; \psi_{\tau_i}\right), 0 < \psi_{\tau_i} \le 1,$

$\widetilde{\gamma_i} = \left(\gamma_i^1, \gamma_i^2, \gamma_i^3; \psi_{\gamma_i}\right), 0 < \psi_{\gamma_i} \le 1;\ \widetilde{\upsilon_i} = \left(\upsilon_i^1, \upsilon_i^2, \upsilon_i^3; \psi_{\upsilon_i}\right), 0 < \psi_{\upsilon_i} \le 1,$

$\widetilde{\delta_i} = \left(\delta_i^1, \delta_i^2, \delta_i^3; \psi_{\delta_i}\right), 0 < \psi_{\delta_i} \le 1;\ \widetilde{\sigma_i} = \left(\sigma_i^1, \sigma_i^2, \sigma_i^3; \psi_{\sigma_i}\right), 0 < \psi_{\sigma_i} \le 1,$

$\widetilde{\phi_\iota} = \left(\phi_i^{\ 1}, \phi_i^{\ 2}, \phi_i^{\ 3}; \psi_{\phi_i}\right), 0 < \psi_{\phi_i} \le 1; \widetilde{\varphi_\iota} = \left(\varphi_i^{\ 1}, \varphi_i^{\ 2}, \varphi_i^{\ 3}; \psi_{\varphi_i}\right), 0 < \psi_{\varphi_i} \le 1,$

For $i = 1,2, \dots \dots \dots, n$.

So our inventory *Model (5)* becomes the fuzzy model as

$$\text{Min} \quad \left\{\widetilde{TAC_1}, \widetilde{TAC_2}, \widetilde{TAC_3}, \dots \dots \dots \dots \dots \dots \dots, \widetilde{TAC_n}\right\},$$
$$\text{Subject to, } D_i > 0, Q_i > 0, L_i > 0, \text{ for } i = 1,2, \dots \dots \dots . . n.$$

$$\text{Where} \quad \widetilde{TAC_1(D_1, Q_1, L_1)} = \widetilde{\alpha_1}\left(D_i^{(1-\widetilde{\beta_1})} + Q_i\widetilde{\theta_1}D_i^{-\beta_i}\right) + \widetilde{h_1}\left(\frac{Q_i}{2} + k\omega\sqrt{L_i}\right) + \tag{6}$$
$$\frac{\widetilde{\gamma_1}Q_i^{\widetilde{\delta_1}-1}\left(D_i^{\widetilde{\sigma_1}+1} + Q_i\widetilde{\theta_1}D_i^{\widetilde{\sigma_1}}\right)}{2^{\widetilde{\delta_1}}} + \frac{\widetilde{\rho_1}L_i^{-\widetilde{\tau_1}}\left(D_i + Q_i\widetilde{\theta_1}\right)}{Q_i} + \widetilde{\theta_1}\,\widetilde{\upsilon_1}\left(\frac{Q_i}{2}\right)^{\widetilde{\phi_1}} D_i^{-\widetilde{\varphi_1}}.$$

It is positioned that ranking fuzzy number is very important in the fuzzy programming system. Bortolan and Degani [38] established a number of techniques of ranking fuzzy numbers. λ- integer method is used for defuzzification of fuzzy numbers. We know that approximated value of a GTFN $\tilde{A} = (a, b, c; \psi)$ is given by $\psi\left(\frac{a+2b+c}{4}\right)$ when $\lambda = 0.5$.

Therefore the fuzzy inventory *Model (6)* converted to

$$\text{Min} \left\{\widehat{TAC_1}, \widehat{TAC_2}, \widehat{TAC_3}, \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots, \widehat{TAC_n}\right\},$$
$$\text{Subject to, } D_i > 0, Q_i > 0, L_i > 0,$$

$$\text{Where} \quad \widehat{TAC_1(D_1, Q_1, L_1)} = \widehat{\alpha_1}\left(D_i^{(1-\widehat{\beta_1})} + Q_i\widetilde{\theta_1}D_i^{-\widehat{\beta_1}}\right) + \widehat{h_1}\left(\frac{Q_i}{2} + k\omega\sqrt{L_i}\right) + \tag{7}$$
$$\frac{\widehat{\gamma_1}Q_i^{\widehat{\delta_1}-1}\left(D_i^{\widehat{\sigma_1}+1} + Q_i\widetilde{\theta_1}D_i^{\widehat{\sigma_1}}\right)}{2^{\widehat{\delta_1}}} + \frac{\widehat{\rho_1}L_i^{-\widehat{\tau_1}}\left(D_i + Q_i\widetilde{\theta_1}\right)}{Q_i} + \widehat{\theta_1}\widehat{\upsilon_1}\left(\frac{Q_i}{2}\right)^{\widehat{\phi_1}} D_i^{-\widehat{\varphi_1}} \quad \text{for } i = 1,2, \dots \dots \dots . . n.$$

3 | Fuzzy Programming Techniques to Solve MOIM

Solve the *MOIM (7)* as a single objective NLP using only one objective at a time and we ignoring the others. So we get the ideal solutions. Using the ideal solutions we have got prepared the pay-off matrix as follows:

$$\begin{pmatrix} & TAC_1(D_1,Q_1,L_1) & TAC_2(D_2,Q_2,L_2) & \dots & TAC_n(D_n,Q_n,L_n) \\ (D_1^1,Q_1^1,L_1^1) & TAC_1^*(D_1^1,Q_1^1,L_1^1) & TAC_2(D_1^1,Q_1^1,L_1^1) & \dots & TAC_n(D_1^1,Q_1^1,L_1^1) \\ (D_2^2,Q_2^2,L_2^2) & TAC_1(D_2^2,Q_2^2,L_2^2) & TAC_2^*(D_2^2,Q_2^2,L_2^2) & \dots & TAC_n(D_2^2,Q_2^2,L_2^2) \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ (D_n^n,Q_n^n,L_n^n) & TAC_1(D_n^n,Q_n^n,L_n^n) & TAC_2(D_n^n,Q_n^n,L_n^n) & \dots & TAC_n^*(D_n^n,Q_n^n,L_n^n) \end{pmatrix} \tag{8}$$

Let $U^k = max\left\{TAC_k\left(D_i^i, Q_i^i, L_i^i\right), i = 1,2, \dots, n\right\}$ $for\ k = 1,2, \dots, n$ and $L^k = TAC_k^*\left(D_k^k, Q_k^k, L_k^k\right) for\ k = 1,2, \dots, n.$

Hence U^k, L^k are identified, $L^k \le TAP_k(D_i^i, Q_i^i, L_i^i) \le U^k$, $for\ i = 1,2,\dots,n\ ; k = 1,2,\dots,n.$

3.1 | Fuzzy Programming Technique Using Hyperbolic Membership Function (FPTHMF)

Now fuzzy non-linear hyperbolic membership functions $\mu^H_{TAC_k}(TAC_k(D_k, Q_k, L_k))$ for the k[th] objective function $TAC_k(D_k, Q_k, L_k)$ respectively for $k = 1.2. \dots, . n$ are defined as follows:

$$\mu^H_{TAC_k}(TAC_k(D_k, Q_k, L_k)) = \frac{1}{2}\tanh\left(\left(\frac{U^k + L^k}{2} - TAC_k(D_k, Q_k, L_k)\right)\sigma_k\right) + \frac{1}{2}.$$

Where α_k is a parameter, $\sigma_k = \frac{3}{(U^k - L^k)/2} = \frac{6}{U^k - L^k}$.

Using the above membership function, fuzzy non-linear programming problem is formulated as follows:

Max λ

Subject to $\frac{1}{2}\tanh\left(\left(\frac{U^k+L^k}{2} - TAC_k(D_k, Q_k, L_k)\right)\sigma_k\right) + \frac{1}{2} \ge \lambda \quad , \lambda \ge 0.$

After simplification the above non-linear programming problem can be written as

Max y

Subject to $y + \sigma_k TAC_k(D_k, Q_k, L_k) \le \frac{U^k+L^k}{2}\sigma_k,\ y \ge 0, D_k >, Q_k > 0, L_k > 0.$

Now the above problem can be easily solved by suitable mathematical programming algorithm and then we shall get the solution of the *MOIM (7)*.

3.2 | Fuzzy Non-Linear Programming Technique (FNLP) Based on Max-Min Operator

In this technique fuzzy membership function $\mu_{TAC_k}(TAC_k(Q_k. D_k))$ for the k[th] objective function $TAC_k(D_k, Q_k, L_k)$ respectively for $k = 1.2. \dots, . n$ are defined as follows:

$$\mu_{TAC_k}(TAC_k(D_k, Q_k, L_k)) = \begin{cases} 1 & \text{for } TAC_k(D_k, Q_k, L_k) < L^k \\ \dfrac{U^k - TAC_k(D_k, Q_k, L_k)}{U^k - L^k} & \text{for } L^k \le TAC_k(D_k, Q_k, L_k) \le U^k, \\ 0 & \text{for } TAC_k(D_k, Q_k, L_k) > U^k \end{cases}$$

for $k = 1.2. \dots, . n,$

Using the above membership function, fuzzy non-linear programming problem is formulated as

Max α'

subject to $TAC_k(D_k, Q_k, L_k) + \alpha'(U^k - L^k) \le U^k \quad . \quad$ for $k = 1.2. \dots, . n.$

$0 \le \alpha' \le 1,\ D_k >, Q_k > 0, L_k > 0.$

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Now the above problem can be easily solved by suitable mathematical programming algorithm and then we shall get the solution of the *MOIM (7)*.

4 | Geometric Programming Technique

Let us consider a unconstrainted Multi Objective Geometric Programming (MOGP) problem is as follows

$$\text{Minimize } g_s(t) = \sum_{k=1}^{T_0} c_{sk} \prod_{j=1}^{m} t_j^{\alpha_{skj}},\ s = 1.2.3.\dots\dots\dots,,.n.$$

$$\text{Subject to } t_j \geq 0.\, j = 1.2.\dots.m,$$

Where $c_{sk}(>0)$ and α_{skj} $(j = 1,2,\dots.,m;\ k = 1,2,\dots\dots.T_0;\ s = 1,2,3,\dots\dots\dots..,n)$ are all real numbers. Now introducing the weights w_i $(i = 1,2,3,\dots\dots\dots..,n)$, the above MOGP converted into the single objective GP problem as following

Primal Problem:

$$\text{Minimize } g(t) = \sum_{s=1}^{n} w_s g_s(t),\ s = 1.2.3.\dots\dots\dots,,.n,$$

$$\text{i.e} = \sum_{s=1}^{n} \sum_{k=1}^{T_0} w_s c_{sk} \prod_{j=1}^{m} t_j^{\alpha_{skj}},$$

$$\text{Subject to } t_j \geq 0.\, j = 1.2.\dots.m. \tag{9}$$

$$\sum_{i=1}^{n} w_i = 1.\, w_i > 0.\, i = 1.2.3.\dots\dots\dots,,.n,$$

Let T be the total numbers of terms and m is the number of variables. Then the degree of the difficulty (DD) is $T-(m+1)$.

Dual Program: The dual problem of *Eq. (9)* is given as follows:

$$\text{Maximize } v(\theta) = \prod_{s=1}^{n} \prod_{k=1}^{T_o} \left(\frac{w_s c_{sk}}{\theta_{sk}}\right)^{\theta_{sk}}.$$

Subject to

$$\sum_{s=1}^{n} \sum_{k=1}^{T_0} \theta_{sk} = 1. \qquad \text{(Normality condition)}$$

$$\sum_{s=1}^{n} \sum_{k=1}^{T_{s0}} \alpha_{skj}\theta_{sk} = 0. \qquad \text{(Orthogonality conditions)}$$

$$\theta_{sk} > 0. \qquad \text{(Positivity conditions)}$$

$$(j = 1.2.\dots,.m;\ k = 1.2.,,\dots,T_o;\ s = 1.2.3.\dots\dots\dots,,.n),$$

Now here three cases may arises

Case I: $T_0 = m+1$, (i.e. DD=0). So DP presents a system of linear equations for the dual variables. So we have a unique solution vector of dual variable.

Case II: $T_0 > m+1$, So a system of linear equations is presented for the dual variables, where the number of linear equations is less than the number of dual variables. So it is concluded that dual variable vector has many solutions.

Case III: $T_0 < m+1$, so a system of linear equations is presented for the dual variables, where the number of linear equations is greater than the number of dual variables. It is seen that generally no solution vector exists for the dual variables here.

4.1 | Solution Procedure of My Proposed Problem

Primal Problem:

$$\text{Minimize TAC(D.Q.L)} = \sum_{i=1}^{n} w_i \left(\widehat{\alpha}_1 \left(D_i^{(1-\widehat{\beta}_1)} + Q_i\widetilde{\theta}_1 D_i^{-\widehat{\beta}_1}\right) + \widehat{h}_1\left(\frac{Q_i}{2} + k\omega\sqrt{L_i}\right) + \frac{\widehat{\gamma}_1 Q_i^{\widehat{\delta}_1 - 1}\left(D_i^{\widehat{\sigma}_1+1} + Q_i\widetilde{\theta}_1 D_i^{\widehat{\sigma}_1}\right)}{2^{\widehat{\delta}_1}} + \frac{\widehat{\rho}_1 L_i^{-\widehat{\tau}_1}\left(D_i + Q_i\widetilde{\theta}_1\right)}{Q_i} + \widehat{\theta}_1\widehat{\upsilon}_1\left(\frac{Q_i}{2}\right)^{\widehat{\phi}_1} D_i^{-\widehat{\varphi}_1}\right), \tag{10}$$

Subject to, $D_i > 0, Q_i > 0, L_i > 0,$

$$\sum_{i=1}^{n} w_i = 1.\, w_i > 0.\, i = 1.2.3. \dots \dots \dots ,,.n,$$

Dual Program: The dual problem of the primal problem *Eq. (10)* is as follows:

$$\text{Maximize } v(\theta) = \prod_{i=1}^{n}\left(\frac{w_i\widehat{\alpha}_1}{\theta_{i1}}\right)^{\theta_{i1}}\left(\frac{w_i\widetilde{\theta}_1\widehat{\alpha}_1}{\theta_{i2}}\right)^{\theta_{i2}}\left(\frac{w_i\widehat{h}_1}{2\theta_{i3}}\right)^{\theta_{i3}}\left(\frac{w_i k\omega\widehat{h}_1}{\theta_{i4}}\right)^{\theta_{i4}}\left(\frac{w_i\widehat{\gamma}_1}{2^{\widehat{\delta}_1}\theta_{i5}}\right)^{\theta_{i5}}\left(\frac{w_i\widehat{\gamma}_1\widetilde{\theta}_1}{2^{\widehat{\delta}_1}\theta_{i6}}\right)^{\theta_{i6}}\left(\frac{w_i\widehat{\rho}_1}{\theta_{i7}}\right)^{\theta_{i7}}\left(\frac{w_i\widehat{\rho}_1\widetilde{\theta}_1}{\theta_{i8}}\right)^{\theta_{i8}}\left(\frac{w_i\widehat{\theta}_1\widehat{\upsilon}_1}{2^{\widehat{\phi}_1}\theta_{i9}}\right)^{\theta_{i9}}.$$

$$\begin{aligned}\text{Subject to}\quad & \theta_{i1} + \theta_{i2} + \theta_{i3} + \theta_{i4} + \theta_{i5} + \theta_{i6} + \theta_{i7} + \theta_{i8} + \theta_{i9} = 1.\\ & \left(1 - \widehat{\beta}_1\right)\theta_{i1} - \widehat{\beta}_1\theta_{i2} + (\widehat{\sigma}_1 + 1)\theta_{i5} + \widehat{\sigma}_1\theta_{i6} + \theta_{i7} - \widehat{\varphi}_1\theta_{i9} = 0.\\ & \theta_{i2} + \theta_{i3} + \left(\widehat{\delta}_1 - 1\right)\theta_{i5} + \widehat{\delta}_1\theta_{i6} - \theta_{i7} + \widehat{\phi}_1\theta_{i9} = 0.\\ & \frac{\theta_{i4}}{2} - \widehat{\tau}_1(\theta_{i7} + \theta_{i8}) = 0.\\ & \sum_{i=1}^{n} w_i = 1.\, w_i > 0.\end{aligned} \tag{11}$$

$$\theta_{i1}.\theta_{i2}.\theta_{i3}.\theta_{i4}.\theta_{i5}.\theta_{i6}.\theta_{i7}.\theta_{i8}.\theta_{i9} \geq 0 \text{ for } i = 1.2.3. \dots \dots \dots ,,.n.$$

Solving the above linear equations we have

$$\theta_{i2} = \left[-\widehat{\varphi}_1\theta_{i3} + \left\{\widehat{\phi}_1(\widehat{\sigma}_1 + 1) - \widehat{\varphi}_1\left(\widehat{\delta}_1 - 1\right)\right\}\theta_{i5} + \left(\widehat{\phi}_1\widehat{\sigma}_1 - \widehat{\delta}_1\widehat{\varphi}_1\right)\theta_{i6} + \left(\widehat{\varphi}_1 + \widehat{\phi}_1\right)\theta_{i7} + \left(1 - \widehat{\beta}_1\right)\widehat{\phi}_1\theta_{i1}\right], \tag{12}$$

$$\begin{aligned}\theta_{i4} = 2\widehat{\tau}_1\theta_{i7} + \frac{2\widehat{\tau}_1}{(1 + 2\widehat{\tau}_1)}\Bigg[1 - \theta_{i1}\left\{1 + \frac{2\left(1 - \widehat{\beta}_1\right)}{\left(\widehat{\beta}_1\widehat{\phi}_1 + \widehat{\varphi}_1\right)}\right\} - \frac{\left(\widehat{\beta}_1\widehat{\phi}_1 + \widehat{\beta}_1\right)}{\left(\widehat{\beta}_1\widehat{\phi}_1 + \widehat{\varphi}_1\right)}\theta_{i3}\\ - \frac{\widehat{\phi}_1(\widehat{\sigma}_1 + 1) - \widehat{\varphi}_1\left(\widehat{\delta}_1 - 1\right) + \widehat{\beta}_1\widehat{\phi}_1 + \widehat{\varphi}_1 + \widehat{\beta}_1\left(\widehat{\delta}_1 - 1\right) + (\widehat{\sigma}_1 + 1)}{\left(\widehat{\beta}_1\widehat{\phi}_1 + \widehat{\varphi}_1\right)}\theta_{i5}\\ - \frac{\widehat{\phi}_1\widehat{\delta}_1 - \widehat{\varphi}_1\widehat{\delta}_1 + \widehat{\beta}_1\widehat{\phi}_1 + \widehat{\varphi}_1 + \widehat{\beta}_1\widehat{\delta}_1 + \widehat{\sigma}_1}{\left(\widehat{\beta}_1\widehat{\phi}_1 + \widehat{\varphi}_1\right)}\theta_{i6}\\ - \frac{2\widehat{\varphi}_1 + \widehat{\phi}_1 + \widehat{\beta}_1\widehat{\phi}_1 + 1 - \widehat{\beta}_1 + 2\widehat{\tau}_1\left(\widehat{\beta}_1\widehat{\phi}_1 + \widehat{\varphi}_1\right)}{\left(\widehat{\beta}_1\widehat{\phi}_1 + \widehat{\varphi}_1\right)}\theta_{i7}\Bigg],\end{aligned} \tag{13}$$

$$\theta_{i8}=\frac{1}{(1+2\widehat{\tau}_1)}\Bigg[1-\theta_{i1}\left\{1+\frac{2\left(1-\widehat{\beta}_1\right)}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}\right\}-\frac{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\beta}_1\right)}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}\theta_{i3}$$
$$-\frac{\widehat{\phi}_1(\widehat{\sigma}_1+1)-\widehat{\varphi}_1\left(\widehat{\delta}_1-1\right)+\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1+\widehat{\beta}_1\left(\widehat{\delta}_1-1\right)+(\widehat{\sigma}_1+1)}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}\theta_{i5}$$
$$-\frac{\widehat{\phi}_1\widehat{\delta}_1-\widehat{\varphi}_1\widehat{\delta}_1+\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1+\widehat{\beta}_1\widehat{\delta}_1+\widehat{\sigma}_1}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}\theta_{i6}$$
$$-\frac{2\widehat{\varphi}_1+\widehat{\phi}_1+\widehat{\beta}_1\widehat{\phi}_1+1-\widehat{\beta}_1+2\widehat{\tau}_1\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}\theta_{i7}\Bigg], \quad (14)$$

$$\theta_{i9}=\frac{1}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}\left[\left(1-\widehat{\beta}_1\right)\theta_{i1}+\widehat{\beta}_1\theta_{i3}+\left\{\widehat{\beta}_1\left(\widehat{\delta}_1-1\right)+(\widehat{\sigma}_1+1)\right\}\theta_{i5}+\left(\widehat{\beta}_1\widehat{\delta}_1+\widehat{\sigma}_1\right)\theta_{i6}\right.$$
$$\left.+\left(1-\widehat{\beta}_1\right)\theta_{i7}\right], \quad (15)$$

Using *Eqs. (12)-(15)* the dual problem is converted into

$$\text{Maximize } v(x.y.z.s.t)=\prod_{i=1}^{n}\left(\frac{w_i\widehat{\alpha}_1}{x_i}\right)^{x_i}\left(\frac{w_i\widetilde{\theta}_1\widehat{\alpha}_1}{X_i}\right)^{X_i}\left(\frac{w_i\widehat{h}_1}{2y_i}\right)^{y_i}\left(\frac{w_i k\omega\widehat{h}_1}{Y_i}\right)^{Y_i}\left(\frac{w_i\widehat{\gamma}_1}{2^{\widehat{\delta}_1}z_i}\right)^{z_i}$$
$$\left(\frac{w_i\widehat{\gamma}_1\widetilde{\theta}_1}{2^{\widehat{\delta}_1}s_i}\right)^{s_i}\left(\frac{w_i\widehat{\rho}_1}{t_i}\right)^{t_i}\left(\frac{w_i\widehat{\rho}_1\widetilde{\theta}_1}{Z_i}\right)^{Z_i}\left(\frac{w_i\widehat{\theta}_1\widetilde{\upsilon}_1}{2^{\widehat{\phi}_1}S_i}\right)^{S_i}. \quad (16)$$

$\sum_{i=1}^{n} w_i=1.\ w_i.\ x_i,\ y_i.\ z_i,\ s_i.\ t_i\geq 0.$

Where

$$X_i=\left[-\widehat{\varphi}_1 y_i+\left\{\widehat{\phi}_1(\widehat{\sigma}_1+1)-\widehat{\varphi}_1\left(\widehat{\delta}_1-1\right)\right\}z_i+\left(\widehat{\phi}_1\widehat{\sigma}_1-\widehat{\delta}_1\widehat{\varphi}_1\right)s_i+\left(\widehat{\varphi}_1+\widehat{\phi}_1\right)t_i+\left(1-\widehat{\beta}_1\right)x_i\right].\quad Y_i=2\widehat{\tau}_1 t+$$
$$\frac{2\widehat{\tau}_1}{(1+2\widehat{\tau}_1)}\left[\begin{array}{l}1-x_i\left\{1+\frac{2(1-\widehat{\beta}_1)}{(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1)}\right\}-\frac{(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\beta}_1)}{(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1)}y_i-\frac{\widehat{\phi}_1(\widehat{\sigma}_1+1)-\widehat{\varphi}_1(\widehat{\delta}_1-1)+\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1+\widehat{\beta}_1(\widehat{\delta}_1-1)+(\widehat{\sigma}_1+1)}{(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1)}z_i\\ -\frac{\widehat{\phi}_1\widehat{\delta}_1-\widehat{\varphi}_1\widehat{\delta}_1+\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1+\widehat{\beta}_1\widehat{\delta}_1+\widehat{\sigma}_1}{(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1)}s_i-\frac{2\widehat{\varphi}_1+\widehat{\phi}_1+\widehat{\beta}_1\widehat{\phi}_1+1-\widehat{\beta}_1+2\widehat{\tau}_1(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1)}{(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1)}t_i\end{array}\right].$$

$$Z_i=\frac{1}{(1+2\widehat{\tau}_1)}\Bigg[1-x_i\left\{1+\frac{2\left(1-\widehat{\beta}_1\right)}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}\right\}-\frac{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\beta}_1\right)}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}y_i$$
$$-\frac{\widehat{\phi}_1(\widehat{\sigma}_1+1)-\widehat{\varphi}_1\left(\widehat{\delta}_1-1\right)+\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1+\widehat{\beta}_1\left(\widehat{\delta}_1-1\right)+(\widehat{\sigma}_1+1)}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}z_i$$
$$-\frac{\widehat{\phi}_1\widehat{\delta}_1-\widehat{\varphi}_1\widehat{\delta}_1+\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1+\widehat{\beta}_1\widehat{\delta}_1+\widehat{\sigma}_1}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}s_i$$
$$-\frac{2\widehat{\varphi}_1+\widehat{\phi}_1+\widehat{\beta}_1\widehat{\phi}_1+1-\widehat{\beta}_1+2\widehat{\tau}_1\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}t_i\Bigg].$$

$$S_i=\frac{1}{(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1)}\left[\left(1-\widehat{\beta}_1\right)x_i+\widehat{\beta}_1 y_i+\left\{\widehat{\beta}_1\left(\widehat{\delta}_1-1\right)+(\widehat{\sigma}_1+1)\right\}z_i+\left(\widehat{\beta}_1\widehat{\delta}_1+\widehat{\sigma}_1\right)s_i+\left(1-\widehat{\beta}_1\right)t_i\right],$$

for $i=1.2.3.\dots,.n.$

To solve *Eq. (16)* and we get the following

$$x^* = (x_1^*. x_2^*. x_3^*. \dots,, x_n^*),\ y^* = (y_1^*. y_2^*. y_3^*. \dots,, y_n^*),\ z^* = (z_1^*. z_2^*. z_3^*. \dots,, z_n^*),$$
$$s^* = (s_1^*. s_2^*. s_3^*. \dots,, s_n^*),\ t^* = (t_1^*. t_2^*. t_3^*. \dots,, t_n^*).$$

Now using the following primal-dual relation we shall get the requird solution.

$$TAC^*(D.Q.L) = n\left(v^*(x.y.z.s.t)\right)^{1/n}.$$

$$w_i'\widehat{\alpha}_1 D_i^{*(1-\widehat{\beta}_1)} = x_i^*\left(v^*(x.y.z.s.t)\right)^{1/n}.$$

$$\frac{w_i'\widehat{h}_1 Q_i^*}{2} = y_i^*\left(v^*(x.y.z.s.t)\right)^{1/n}.$$

$$w_i'\widehat{h}_1 k\omega\sqrt{L_i^*} = Y_i^*\left(v^*(x.y.z.s.t)\right)^{1/n} \text{ for } i = 1.2.3.\dots\dots\dots,,.n,$$

5 | Numerical Example

Here we consider two items an inventory system with all parametric values in proper units. Taking $k = 4.\omega = 6$,

Table 1. Input imprecise data for shape parameters.

Parameters	Items I	II
$\widetilde{\alpha}_1$	(250,260,270; 0.8)	(280,290,300; 0.7)
$\widetilde{\beta}_1$	(8,9,10; 0.8)	(6,7,8; 0.8)
$\widetilde{h}_1$	(4,5,6; 0.9)	(7,8,9; 0.8)
$\widetilde{\rho}_1$	(11,12,13; 0.7)	(10,11,12; 0.8)
$\widetilde{\gamma}_1$	(95,100,105; 0.7)	(95,98,101; 0.8)
$\widetilde{\delta}_1$	(0.03,0.04,0.05; 0.8)	(0.05,0.06,0.07; 0.8)
$\widetilde{\sigma}_1$	(0.7,0.8,0.9; 0.7)	(0.6,0.7,0.8; 0.9)
$\widetilde{\theta}_1$	(0.05,0.06,0.07; 0.9)	(0.07,0.08,0.09; 0.8)
$\widetilde{\tau}_1$	$\left(\frac{1}{5},\frac{1}{4},\frac{1}{3};0.8\right)$	$\left(\frac{1}{6},\frac{1}{5},\frac{1}{4};0.7\right)$
$\widetilde{\vartheta}_1$	(0.25,0.26,0.27; 0.8)	(0.26,0.27,0.28; 0.9)
$\widetilde{\varphi}_1$	(11,12,13; 0.8)	(10,11,12; 0.9)
$\widetilde{\phi}_1$	(0.30,0.32,0.34; 0.9)	(0.44,0.45,0.46; 0.7)

Using defuzzification the approximate values of the above parameter are:

Table 2. approximate values of table 1.

Defuzzification of the Fuzzy Numbers	Items I	Items II
$\widehat{\alpha}_1$	208	203
$\widehat{\beta}_1$	7.2	5.6
$\widehat{h}_1$	4.5	6.4
$\widehat{\rho}_1$	8.4	8.8
$\widehat{\gamma}_1$	70	78.4
$\widehat{\delta}_1$	0.032	0.048
$\widehat{\sigma}_1$	0.56	0.63
$\widehat{\tau}_1$	0.20666667	0.1429166667
$\widehat{\theta}_1$	0.054	0.064
$\widehat{\upsilon}_1$	0.208	0.243
$\widehat{\varphi}_1$	9.6	9.9
$\widehat{\phi}_1$	0.288	0.315

Table 3. Optimal solution of MOIM using different methods.

Methods	D_1^*	Q_1^*	L_1^*	TAC_1^*	D_2^*	Q_2^*	L_2^*	TAC_2^*
FPTHMF	1.88	9.10	0.12×10^{-2}	57.15	2.03	8.09	0.28×10^{-3}	74.74
FNLP	1.89	9.16	0.12×10^{-2}	57.15	2.03	8.09	0.28×10^{-3}	74.74
GP	1.80	11.84	0.13×10^{-2}	59.43	2.03	7.94	0.17×10^{-3}	74.86

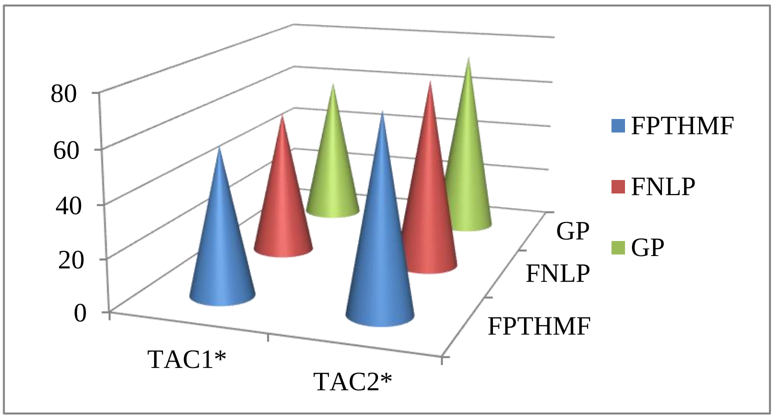

Fig. 2. Minimizing cost of both items using different methods.

From the above figure suggests that GP, FPTHMF and FNLP methods nearly provide the same result.

6 | Sensitivity Analysis

In the sensitivity analysis all optimal solutions have been found by using the FNLP method.

Table 4. Optimal solution of MOIM for different values of α_1, α_2.

Method	α_1, α_2	D_1^*	Q_1^*	L_1^*	TAC_1^*	D_2^*	Q_2^*	L_2^*	TAC_2^*
FNLP	-20%	1.83	8.90	$0{,}11\times10^{-2}$	56.02	1.95	7.84	$0{,}27\times10^{-3}$	72.61
	-10%	1.86	9.00	$0{,}12\times10^{-2}$	56.61	1.99	7.97	$0{,}28\times10^{-3}$	73.73
	+10%	1.91	9.18	$0{,}12\times10^{-2}$	57.64	2.07	8.19	$0{,}28\times10^{-3}$	75.67
	+20%	1.94	9.26	$0{,}12\times10^{-2}$	58.09	2.10	8.29	$0{,}28\times10^{-3}$	76.54

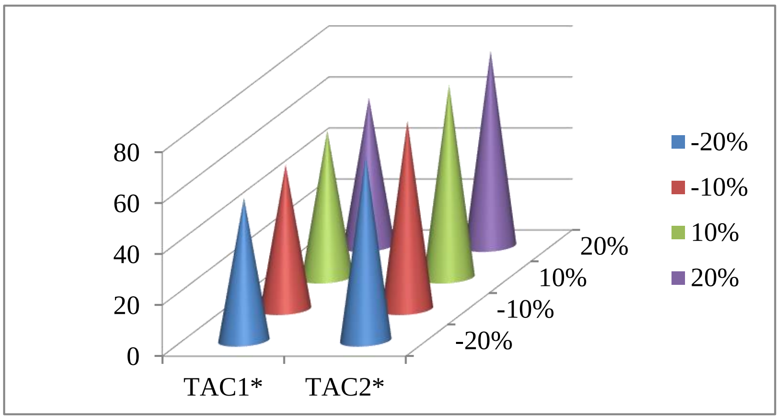

Fig. 3. Minimizing cost of both items for different values of α_1, α_2.

The above *Fig. 3* suggests that the minimum cost of the both items is proportionally related to the parameter α_1, α_2.

Table 5. Optimal solution of MOIM for different values of β_1, β_2.

Method	β_1, β_2	D_1^*	Q_1^*	L_1^*	TAC_1^*	D_2^*	Q_2^*	L_2^*	TAC_2^*
FNLP	-20%	2.12	9.79	$0{,}12 \times 10^{-2}$	63.21	2.29	8.75	$0{,}29 \times 10^{-3}$	84.79
	-10%	1.99	9.40	$0{,}12 \times 10^{-2}$	59.79	2.15	8.39	$0{,}28 \times 10^{-3}$	79.11
	+10%	1.80	8.84	$0{,}11 \times 10^{-2}$	55.05	1.94	7.84	$0{,}27 \times 10^{-3}$	71.29
	+20%	1.74	8.63	$0{,}11 \times 10^{-2}$	53.34	1.86	7.63	$0{,}27 \times 10^{-3}$	68.48

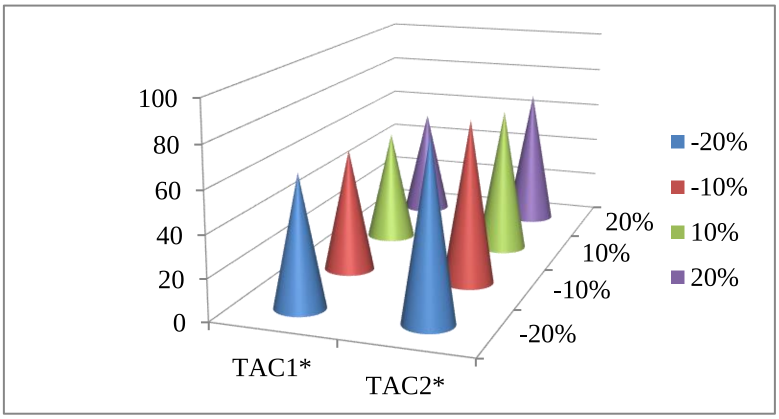

Fig. 4. Minimizing cost of both items for different values of β_1, β_2.

The above *Fig. 4* suggests that minimum cost of the both items is inversely proportional to the parameter β_1, β_2.

Table 6. Optimal solution of MOIM for different values of γ_1, γ_2.

Method	γ_1, γ_2	D_1^*	Q_1^*	L_1^*	TAC_1^*	D_2^*	Q_2^*	L_2^*	TAC_2^*
FNLP	-20%	1.91	8.52	$0{,}13 \times 10^{-2}$	53.22	2.07	7.56	$0{,}31 \times 10^{-3}$	69.00
	-10%	1.90	8.82	$0{,}12 \times 10^{-2}$	55.22	2.05	7.83	$0{,}29 \times 10^{-3}$	71.93
	+10%	1.88	9.36	$0{,}11 \times 10^{-2}$	59.02	2.02	8.33	$0{,}26 \times 10^{-3}$	77.46
	+20%	1.87	9.62	$0{,}11 \times 10^{-2}$	60.84	2.00	8.57	$0{,}25 \times 10^{-3}$	80.10

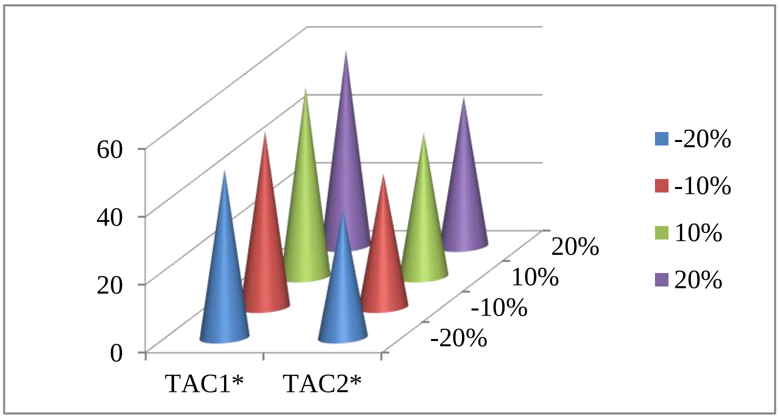

Fig. 5. Minimizing cost of both items for different values of γ_1, γ_2.

The above *Fig. 5* suggests that the minimum cost of the both items is proportionally related to the parameter γ_1, γ_2.

Table 7. Optimal solution of MOIM for different values of δ_1, δ_2.

Method	δ_1, δ_2	D_1^*	Q_1^*	L_1^*	TAC_1^*	D_2^*	Q_2^*	L_2^*	TAC_2^*
FNLP	-20%	1.89	9.09	$0{,}12 \times 10^{-2}$	56.88	2.04	8.09	$0{,}28 \times 10^{-3}$	74.19
	-10%	1.89	9.09	$0{,}12 \times 10^{-2}$	57.02	2.03	8.08	$0{,}27 \times 10^{-3}$	74.47
	+10%	1.88	9.10	$0{,}12 \times 10^{-2}$	57.28	2.03	8.07	$0{,}27 \times 10^{-3}$	75.02
	+20%	1.88	9.10	$0{,}12 \times 10^{-2}$	57.42	2.02	8.09	$0{,}27 \times 10^{-3}$	75.30

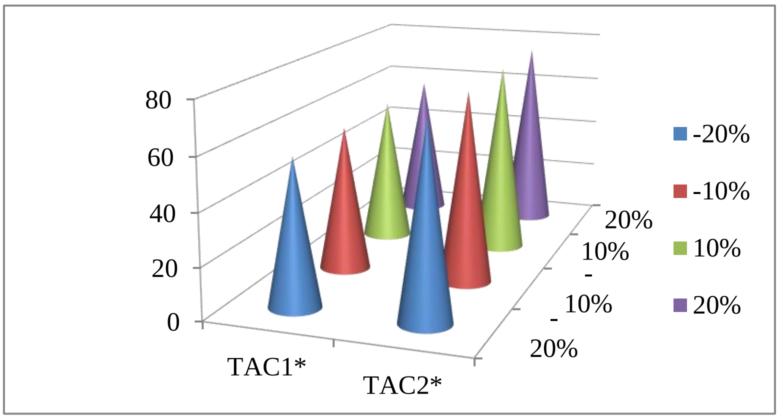

Fig. 6. Minimizing cost of both items for different values of δ_1, δ_2.

The above *Fig. 6* suggests that the minimum cost of the both items is proportionally related to the parameter δ_1, δ_2.

Table 8. Optimal solution of MOIM for different values of σ_1, σ_2.

Method	σ_1, σ_2	D_1^*	Q_1^*	L_1^*	TAC_1^*	D_2^*	Q_2^*	L_2^*	TAC_2^*
FNLP	-20%	1.90	9.19	$0{,}12 \times 10^{-2}$	57.26	2.07	8.19	$0{,}28 \times 10^{-3}$	74.66
	-10%	1.90	9.14	$0{,}12 \times 10^{-2}$	57.21	2.05	8.14	$0{,}28 \times 10^{-3}$	74.70
	+10%	1.88	9.05	$0{,}12 \times 10^{-2}$	57.08	2.02	8.04	$0{,}28 \times 10^{-3}$	74.77
	+20%	1.87	9.00	$0{,}12 \times 10^{-2}$	57.02	2.01	7.98	$0{,}28 \times 10^{-3}$	74.78

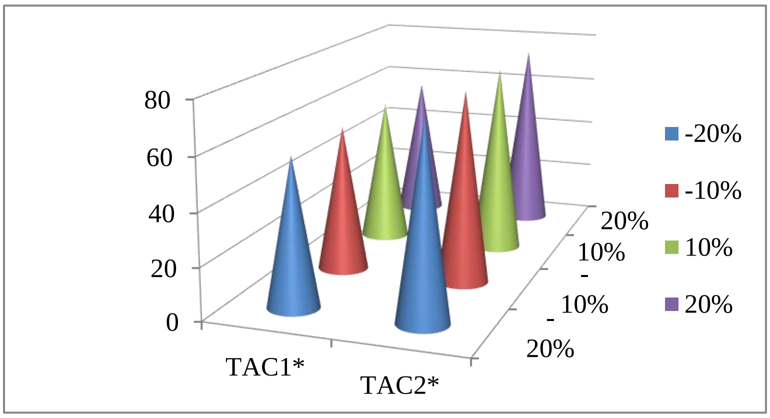

Fig. 7. Minimizing cost of both items for different values of σ_1, σ_2.

The above *Fig. 7* suggests that the minimum cost of the both items is proportionally related to the parameter σ_1, σ_2.

Table 9. Optimal solution of MOIM for different values of ρ_1, ρ_2.

Method	ρ_1, ρ_2	D_1^*	Q_1^*	L_1^*	TAC_1^*	D_2^*	Q_2^*	L_2^*	TAC_2^*
FNLP	-20%	1.89	8.87	$0{,}87 \times 10^{-3}$	55.30	2.04	7.92	$0{,}20 \times 10^{-3}$	72.90
	-10%	1.89	8.99	$0{,}10 \times 10^{-2}$	56.24	2.03	8.00	$0{,}24 \times 10^{-3}$	73.83
	+10%	1.89	9.20	$0{,}13 \times 10^{-2}$	58.02	2.03	8.17	$0{,}32 \times 10^{-3}$	75.63
	+20%	1.89	9.31	$0{,}15 \times 10^{-2}$	58.87	2.03	8.25	$0{,}36 \times 10^{-3}$	76.49

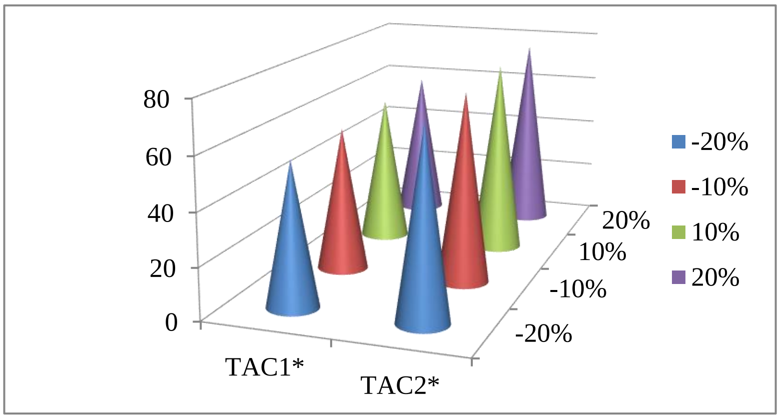

Fig. 8. Minimizing cost of both items for different values of ρ_1, ρ_2.

The above *Fig. 8* suggests that the minimum cost of the both items is proportionally related with the parameter ρ_1, ρ_2.

Table 10. Optimal solution of MOIM for different values of τ_1, τ_2.

Method	τ_1, τ_2	D_1^*	Q_1^*	L_1^*	TAC_1^*	D_2^*	Q_2^*	L_2^*	TAC_2^*
FNLP	-20%	1.89	8.88	$0{,}56 \times 10^{-3}$	54.71	2.04	7.94	$0{,}14 \times 10^{-3}$	72.70
	-10%	1.89	8.99	$0{,}82 \times 10^{-3}$	55.92	2.03	8.01	$0{,}20 \times 10^{-3}$	73.71
	+10%	1.89	9.20	$0{,}16 \times 10^{-2}$	58.40	2.03	8.16	$0{,}38 \times 10^{-3}$	75.80
	+20%	1.89	9.30	$0{,}21 \times 10^{-2}$	59.66	2.03	8.23	$0{,}51 \times 10^{-3}$	76.88

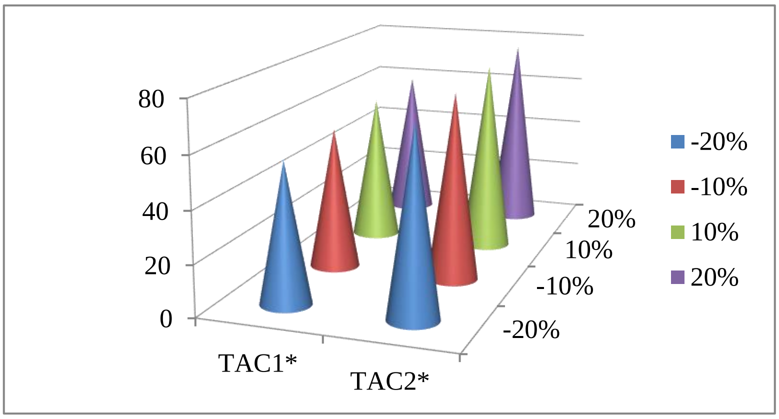

Fig. 9. Minimizing cost of both items for different values of τ_1, τ_2.

The above *Fig. 9* suggests that the minimum cost of the both items is proportionally related with the parameter τ_1, τ_2.

7 | Conclusion

In this paper, we have developed a deteriorated multi-item inventory model in a fuzzy environment. Production cost, set-up- cost and deterioration cost are continuous functions of demand. Set-up-cost and deterioration cost are also dependent on average inventory level. Lead time crashing cost is considered the continuous function of leading time. Due to uncertainty all cost parameters of the proposed model are taken as GTFNs. The formulated multi objective inventory problem has been solved by various techniques like as GP, FPTHMF, and FNLP. Numerical example is solved by using LINGO13 software. This paper will be extended by using linear, quadratic demand, ramp type demand, power demand etc. Inflation performs a crucial position in the inventory systems, but here it is not considered. So inflation can be used in this model for practical. Also another types of fuzzy numbers like as Generalized Trapezoidal Fuzzy Number (GTrFN), PfFN, pFN etc. may be used for all cost parameters of the model.

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