E-ISSN: 2717-3453 | P-ISSN: 2783-1442

Journal of Fuzzy Extension and Applications

www.journal-fea.com

J. Fuzzy. Ext. Appl. Vol. 3, No. 2 (2022) 169–176.

Paper Type: Research Paper

A Novel Method for Solving Multi-Objective Linear Fractional Programming Problem under Uncertainty

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Citation:

Mekawy, I. M. (2022). A novel method for solving multi- objective linear fractional programming problem under uncertainty. *Journal of fuzzy extension and applications, 3*(2), 169-176.

Received: 23/02/2022 Reviewed: 13/03/2022 Revised: 29/03/2022 Accepted: 03/05/2022

Abstract

This paper deals with a multi-objective linear fractional programming problem in fuzzy environment. The problem is considered by introducing all the parameters as piecewise quadratic fuzzy numbers. Through the use of the associated real number of the close interval approximation and the order relation of the piecewise quadratic fuzzy numbers, the problem is transformed into the corresponding crisp problem. A proposed method introduces to generate ideals and the set of all fuzzy efficient solutions. The advantage of it helps the decision maker to handle the real life problem. A numerical example is given illustrate the method.

Keywords: Linear fractional programming, Multi-objective decision making, Piecewise quadratic fuzzy number, Close interval approximation, Proposed method, Hungarian method, Fuzzy optimal solution.

1 | Introduction

Fractional Problem (FP) is a decision problem arises to optimize the ratio subject to constraints. In real world decision situations Decision Maker (DM) sometimes may face to evaluate ratio between inventory and sales, actual cost and standard cost, output and employee etc., with both denominator and numerator are linear. If only one ratio is considered as an objective function then under linear constraints the problem is said to be Linear Fractional Programming (LFP) problem. The fractional programming problem, i.e., the maximization of a fraction of two functions subject to given conditions, arises in various decision making situations; for instance , fractional programming is used in the fields of traffic planning [6], network flows [1], and game theory [8]. A review of various applications is given by Schaible [13] and [14]. Hassian et al. [21] introduced a parametric approach for solving multi-criteria LFP problem. Tantawy [17] introduced two approaches to solve the LFP problem namely; a feasible direction approach and a duality approach.

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http://dx.doi.org/10.22105/jfea.2022.331180.1206

Odior [10] introduced an algebraic approach based on the duality concept and the partial fractions to solve the LFP problem. Pandey and Punnen [11] introduced a procedure based on the Simplex method developed by Dantzig [6] to solve the LFP problem. Gupta and Chakraborty [7] solved the LFP problem depending on the sign of the numerator under the assumption that the denominator is non -vanishing in the feasible region using the fuzzy programming approach. Chakraborty [5] studied nonlinear fractional programming problem with multiple constraints under fuzzy environment. Stanojevic and Stancu- Minasian [16] proposed a method for solving fully fuzzified LFP problem. Buckley and Feuring [3] studied fully fuzzified linear programming involving coefficients and decision variables as fuzzy quantities. Li and Chen [9] introduced a fuzzy LFP problem with fuzzy coefficients and present the concept of fuzzy optimal solution. Sakawa et al. [20] introduced an interactive satisficing method for solving multi- objective fuzzy LFP problems with fuzzy parameters both in the objective functions and constraints. Pop and Stancu [12] studied LFP problem with all parameters and decision variables are triangular fuzzy numbers.

In this paper, fuzzy multi- objective LFP problem is introduced. The problem is converted into the crisp problem using the associated real number of the close interval approximation and the order relation of Piecewise Quadratic Fuzzy Numbers (PQFN), and hence the optimal transportation is obtained by applying optimal flowing method.

The following is a summary of the rest of the paper.

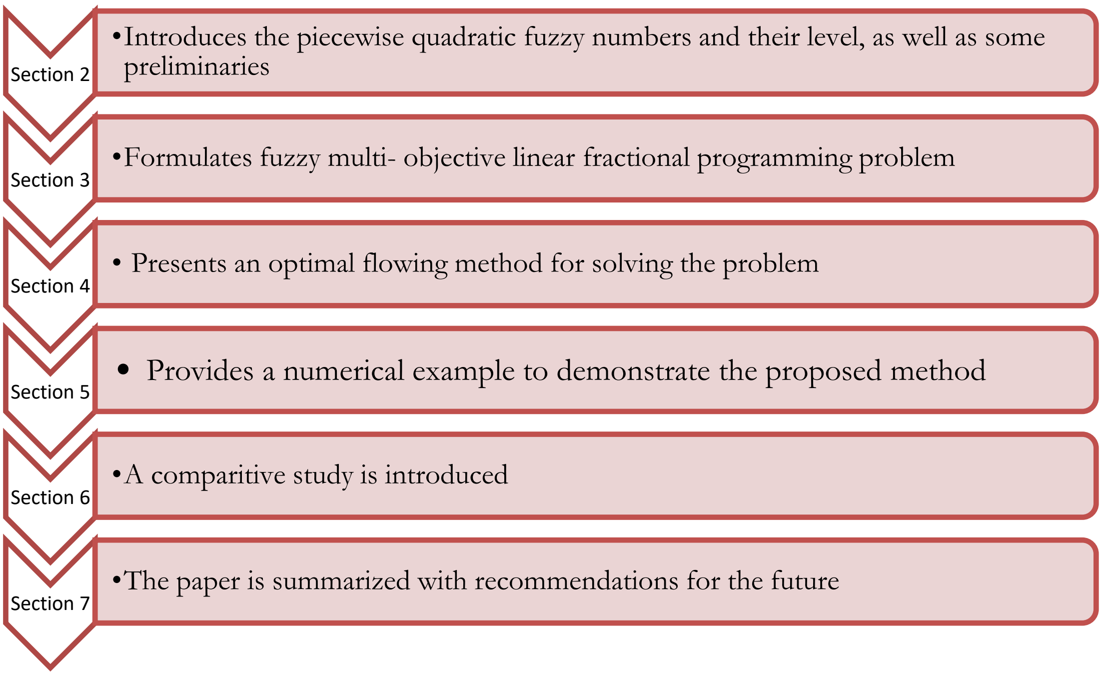

Fig. 1. Layout of remaining paper.

2 | Preliminaries

This section introduces some of basic concepts and results related to neutrosophic PQFN, close interval approximation, and their arithmetic operations are recalled.

Definition 1. [4]. A PQFN is denoted by $\tilde{A}_{PQ} = (a_1, a_2, a_3, a_4, a_5)$, where $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5$ are real numbers, and is defined by if its membership function $\mu_{\tilde{A}_{PQ}}$ is given by (see, *Fig. 1*).

$$\mu_{\tilde{A}_{PQ}} = \begin{cases} 0, & x < a_1; \\ \frac{1}{2}\frac{1}{(a_2-a_1)^2}(x-a_1)^2, & a_1 \le x \le a_2, \\ \frac{1}{2}\frac{1}{(a_3-a_2)^2}(x-a_3)^2+1, & a_2 \le x \le a_3, \\ \frac{1}{2}\frac{1}{(a_4-a_3)^2}(x-a_3)^2+1, & a_3 \le x \le a_4, \\ \frac{1}{2}\frac{1}{(a_5-a_4)^2}(x-a_5)^2, & a_4 \le x \le a_5, \\ 0, & x > a_5. \end{cases}$$

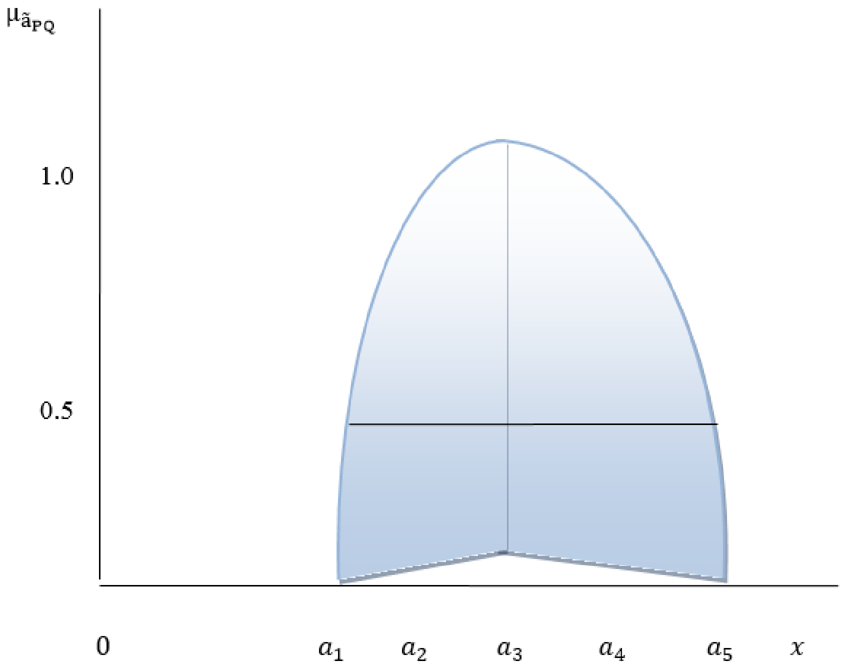

Fig. 1. Graphical representation of a Piecewise Quadratic Fuzzy Number (PQFN).

Definition 2. [4]. An interval approximation $[A] = \left[A_\alpha^L, A_\alpha^U\right]$ of a PQFN $\tilde{A}$ is called closed interval approximation if

$$A^L_\alpha = \inf\left\{x \in \mathbb{R}: \mu_{\tilde{A}} \ge 0.5\right\}, \text{ and } A^U_\alpha = \sup\left\{x \in \mathbb{R}: \mu_{\tilde{A}} \ge 0.5\right\}.$$

Definition 3. Associated ordinary number [4]. If $[A] = \left[A_\alpha^L, A_\alpha^U\right]$ is the close interval approximation of PQFN, the Associated ordinary number of $[A]$ is defined as $\hat{A} = \frac{A_\alpha^L + A_\alpha^U}{2}$.

Definition 4. [4]. Let $[A] = \left[A_\alpha^L, A_\alpha^U\right]$, and $[B] = \left[B_\alpha^L, B_\alpha^U\right]$ be two interval approximations of PQFN. Then the arithmetic operations are:

I. Addition: $[A] \oplus [B] = \left[A^L_\alpha + B^L_\alpha, A^U_\alpha + Bb^U_\alpha\right]$.

II. Subtraction: $[A] \ominus [B] = \left[A^L_\alpha - B^U_\alpha, A^U_\alpha - B^L_\alpha\right]$.

III. Scalar multiplication: $\alpha\,[A] = \begin{cases} \left[k\,A^L_\alpha,\ k\,A^U_\alpha\right], k > 0 \\ \left[\ k\,A^U_\alpha, k\,A^L_\alpha\right], k < 0. \end{cases}$

IV. Multiplication: $[A]\otimes[B]$

$$\left[\frac{A^U_\alpha B^L_\alpha + A^L_\alpha B^U_\alpha}{2}, \frac{A^L_\alpha B^L_\alpha + A^U_\alpha B^U_\alpha}{2}\right].$$

V. Division: $[A]\oslash[B]$

$$\begin{cases}\left[2\left(\frac{A^L_\alpha}{B^L_\alpha+B^U_\alpha}\right), 2\left(\frac{A^U_\alpha}{B^L_\alpha+B^U_\alpha}\right)\right], [B]>0, B^L_\alpha+B^U_\alpha\neq 0\\ \left[2\left(\frac{A^U_\alpha}{B^L_\alpha+B^U_\alpha}\right), 2\left(\frac{A^L_\alpha}{B^L_\alpha+B^U_\alpha}\right)\right], [B]<0, B^L_\alpha+B^U_\alpha\neq 0.\end{cases}$$

VI. The order relation: $[A](\lesssim)[B]$

$$[A](\lesssim)[B] \text{ if } A^L_\alpha \le B^L_\alpha \text{ and } A^U_\alpha \le B^U_\alpha \text{ or } A^L_\alpha + A^U_\alpha \le B^L_\alpha + B^U_\alpha.$$

It is noted that $P(\mathbb{R}) \subset F(\mathbb{R})$, where $F(\mathbb{R})$, and $P(\mathbb{R})$ are the sets of all PQFNs and close in interval approximation of PQFN, respectively.

3 | Problem Statement

Consider the following Fuzzy Multi-Objective Linear Fractional Programming (F-MOLFP) problem.

$$\max \tilde{Z}_{k_{PQ}} = \left(\tilde{Z}_{1_{PQ}}(x), \tilde{Z}_{2_{PQ}}(x), \dots, \tilde{Z}_{K_{PQ}}(x)\right), \quad K \ge 2$$

Subject to

$$\tilde{A}_{PQ}x \lesssim \tilde{b}_{PQ}.$$

$$x \ge 0.$$

Where, $\tilde{M}_{PQ} = \left\{x \in \mathbb{R}^2 : \tilde{A}_{PQ}x \lesssim \tilde{b}_{PQ}, x \ge 0\right\}$ is the fuzzy feasible domain,

$\tilde{A}_{PQ} = \left(\tilde{a}_{ij_{PQ}}\right)_{m\times n}, x \in \mathbb{R}^n$ and $\tilde{b}_{PQ} = \left(\tilde{b}_{i_{PQ}}\right)_{1\times m}$, $\tilde{Z}_{k_{PQ}}(x) = \frac{\tilde{f}_{k_{PQ}}(x)}{\tilde{g}_{k_{PQ}}(x)} = \frac{\tilde{c}^T_{k\,PQ}x + \tilde{\zeta}_{k_{PQ}}}{\tilde{d}^T_{k\,PQ}x + \tilde{\xi}_{k_{PQ}}}, \tilde{c}^T_k, \tilde{d}^T_k$ are $n-$dimensional PQF vectors; $\tilde{\zeta}_{k_{PQ}}, \tilde{\xi}_{k_{PQ}}$ are PQF fuzzy scalars and $\tilde{g}_{k_{PQ}}(x) > 0; \forall k = 1, K$ and for all $x \in \tilde{M}_{PQ}$.

Definition 5. [19]. A point $\bar{x} = \left\{\bar{x}_j : j = \overline{1, n}\right\}$ is said to be fuzzy feasible solution to F-MOLFP if $\bar{x}$ satisfies the constraints in it.

Definition 6. A fuzzy feasible point $\bar{x} = \left\{\bar{x}_j : j = \overline{1, n}\right\}$ is called a fuzzy efficient solution to F-MOLFP if and only if there does not exists an $x \in \tilde{M}_{PQ}$ and $\bigvee_{x\in\tilde{H}_{PQ}} \tilde{Z}_{k_{PQ}}(x) \le \tilde{Z}_{k_{PQ}}(\bar{x})$, where $\tilde{H}_{PQ}$ is the set of all fuzzy efficient solutions and $\bigvee$ is the maximum.

Based on the close interval approximation of PQFN, the F-MOLFP is converted into the following CIA-MOLFP and MOLFP (i.e., (P_k)), respectively as

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$$\max[Z_k(x)] = ([Z_1(x)], [Z_2(x)], \dots, [Z_K(x)]), K \geq 2$$

Subject to

$[A]x \precsim [b]$,

$x \geq 0$.

Where, $[Z_k(x)] = \frac{[c_k]^T x + [\zeta_k]}{[d_k]^T x + [\xi_k]}, k = 1, 2, \dots, K$, $[c_k] = \left(c_{k\alpha}^L, c_{k\alpha}^U\right), [d_k] = \left(d_{k\alpha}^L, d_{k\alpha}^U\right), [c\ \zeta_k], [\xi_k]$, $[b] = \left(b_\alpha^L, b_\alpha^U\right)$and $[A] = \left(A_\alpha^L, A_\alpha^U\right) \in P(\mathbb{R})$ ($P(\mathbb{R})$ is the set of PQFNs.

$$(P_k) \quad \max \widehat{Z}_k(x) = \left(\widehat{Z}_1(x), \widehat{Z}_2(x), \dots, \widehat{Z}_K(x)\right), \quad K \geq 2$$

Subject to

$$A_\alpha^L x \leq b_\alpha^L, A_\alpha^U x \leq b_\alpha^U,$$

$$x \geq 0.$$

4 | Solution Method

In this section, a method for finding the ideal and fuzzy efficient solution of the F-MOLFP is proposed as in the following steps:

Step1. Consider the F-MOLFP.

Step 2. Convert the F-MOLFP into the CIA-MOLFP and hence into (P_k).

Step 3. Construct " k" single objective from the (P_k).

Step 4. Solve each objective of the $(P_l), l = 1, 2, \dots, r, \dots, K$with respect to given constraints individually to obtain the optimal solution $X_l^*, l = 1, 2, \dots, K$ with the optimum value Z_l^* which is the ideal solution of the MOLFP.

Step 5. Use the optimal solution of problem (P_r), resulted from step4 in the problem $(P_l), l = 1, 2, \dots, r - 1, r + 1, \dots, K$.

Step 6. Repeat the setp5 for all $(P_l), l = 1, 2, \dots, r, \dots, K$ to obtain the efficient solution of the MOLFP.

5 | Numerical Example

Consider the following problem:

$$\max \tilde{Z}_{1PQ} = \frac{(1,4,7,10,12)x_1 \oplus (8,10,14,15,17)x_2 \oplus (1,2.5,4,7.5,11.5)x_3 \oplus (1,2,3,4,6)}{(10,14,20,22,24)x_1 \oplus (20,23.5,27.5,29,30)x_2 \oplus (16,18,20,25,28)x_3 \oplus (5,10,18,20,22)}.$$

$$\max \tilde{Z}_{2PQ} = \frac{(18,20,24,28,30)x_1 \oplus (14,16,18,25,30)x_2 \oplus (12,14,19,25,28)x_3 \oplus (0,1,6,10,12)}{(14,16,19,23,25)x_1 \oplus (16,18,21,25,27)x_2 \oplus (13,15,20,25,30)x_3 \oplus (8,10,15,20,25)}.$$

Subject to

$$(8,10,17,19,25)x_1 \oplus (12,14,16,22,24)x_2 \oplus (18,20,25,27,30)x_3$$
$$\leq (35,38,45,47,50),$$
$$(0.01,0.03,0,07,0.09,0.11)x_1 \oplus (0.03,0.05,0.08,0.1,0.15)\, x_2$$
$$\oplus (0.01,0.02,0.06,0.07,0.09)x_3 \leq (0.5,0.7,0.9163,.95,1.0),$$
$$(4,6,10,13,15)x_1 \oplus (0,5,10,15,18)x_2 \oplus (6,8,11,14,20)x_3 \leq (25,30,35,40,48),$$
$$x_1, x_2, x_3 \geq 0.$$

Using the close interval approximation of the PQFN, the problem (1) is converted into the following

$$\max Z_{CIA} = \frac{[4,10]x_1 \oplus [10,15]x_2 \oplus [2.5,7.5]x_3 \oplus [2,4]}{[14,22]x_1 \oplus [23.5,29.5]x_2 \oplus [18,25]x_3 \oplus [10,20]}.$$
$$\max Z_{2CIA} = \frac{[20,28]x_1 \oplus [16,25]x_2 \oplus [14,25]x_3 \oplus [1,10]}{[16,23]x_1 \oplus [18,25]x_2 \oplus [15,25]x_3 \oplus [10,20]}.$$

Subject to

$$[10,19]x_1 \oplus [14,22]x_2 \oplus [20,27]x_3 \leq [38,47],$$
$$[0.03,0.09]x_1 \oplus [0.05,0.1]\, x_2 \oplus [0.02,0.07]x_3 \leq [0.7,0.95],$$
$$[6,13]x_1 \oplus [5,15]x_2 \oplus [8,14]x_3 \leq [30,40],$$
$$x_1, x_2, x_3 \geq 0.$$

And hence

$$\max \widehat{Z}_1 = \frac{7x_1 + 12.5x_2 + 5x_3 + 3}{18x_1 + 32.5x_2 + 21.5x_3 + 15}.$$
$$\max \widehat{Z}_2 = \frac{24x_1 + 20.5x_2 + 19.5x_3 + 5.5}{19.5x_1 + 21.5x_2 + 20x_3 + 15}.$$

Subject to

$$10x_1 + 14x_2 + 20x_3 \leq 38,$$
$$19x_1 + 22x_2 + 27x_3 \leq 47,$$
$$0.03x_1 + 0.05\, x_2 + 0.07x_3 \leq 0.7,$$
$$0.09x_1 + 0.1\, x_2 + 0.07x_3 \leq 0.95,$$
$$6x_1 + 5x_2 + 8x_3 \leq 30,$$
$$13x_1 + 15x_2 + 14x_3 \leq 40,$$
$$x_1, x_2, x_3 \geq 0.$$

Step 4. $Z_1^* = 0.3518170$ at $x^* = (x_1^*, x_2^*, x_3^*) = (0, 2.136364, 0)$, and $Z_2^* = 1.02580$ at $x^* = (x_1^*, x_2^*, x_3^*) = (2.473684, 0,0)$.

Step 5. Using the solution $x^* = (x_1^*, x_2^*, x_3^*) = (0, 2.136364, 0)$ into the second problem Z_2 with respect to the given constraints, we have $Z_2^* = 0.809903$. The efficient solution is $(\widehat{Z}_1, \widehat{Z}_2) = (0.3518170, 1.02580)$. similarly, use the solution $x^* = (x_1^*, x_2^*, x_3^*) = (2.473684, 0,0)$ into Z_1 with respect to the given constraints, we obtain $Z_1^* = 0.341291$. The efficient solution is $(\widehat{Z}_1, \widehat{Z}_2) = (0.341291, 0.809903)$.

Therefore, the ideal solution is $(Z_1^*, Z_2^*) = (0.3518170, 1.02580)$, and the set of all efficient solution for $(\widehat{Z}_1, \widehat{Z}_2)$ are $(0.3518170, 1.02580)$ and $(0.341291, 0.809903)$.

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In close interval approximation of piecewise quadratic fuzzy: the ideal solution is $(Z_1^*, Z_2^*) =$ ([0.281410,0.59872], [0.65640,1.59873]), and the set of all efficient solution for $(\widehat{Z}_1, \widehat{Z}_2)$ are ([0.281410,0.65640]: [0.59872,1.59873]) and ([0.15983,0.64387]: [0.465635,1.10225]).

6 | Comparative Study

In this section, the proposed approach is compared with some existing literature to illustrate the advantages of the proposed approach. *Table 1* investigates this comparison in the case of some parameters.

Table 1. Comparison of different researcher's contributions.

Author's Name	Multiobjective Programming	Hungarian Method	Optimal Solution	Environment
Simi and Talukder [15]	NO	NO	YES	Crisp set
Borza and Rambely [2]	NO	NO	YES	Crisp set
Xiao and Tian [18]	NO	NO	YES	Crisp set
Proposed approach	YES	YES	YES	Fuzzy Set

7 | Conclusions and Future Works

In this paper, a new algorithm for solving fuzzy multi- objective LFP has proposed which based purely on the Hungarian method. The idea of the algorithm is to find the ideal solution and the set of all efficient solutions. The advantage of it helps the DM to handle the real life problem. Therefore, future work may include the study of the problem in different uncertainty environment as neutrosophic, stochastic, etc.

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