E-ISSN: 2717-3453 | P-ISSN: 2783-1442

Journal of Fuzzy Extension and Applications

www.journal-fea.com

J. Fuzzy. Ext. Appl. Vol. 3, No. 2 (2022) 152–157.

Paper Type: Research Paper

A Hybrid Method for the Assessment of Analogical Reasoning Skills

Michael Voskoglou[1,*], **Said Broumi**[2]

[1] Mathematical Sciences, Graduate TEI of Western Greece; mvoskoglou@gmail.com.

[2] Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, Casablanca, Morocco; broumisaid@gmail.com.

Citation:

Voskoglou, M., & Broumi, S. (2022). A hybrid method for the assessment of analogical reasoning skills. *Journal of fuzzy extension and applications*, *3*(2), 152-157.

Received: 13/03/2022 Reviewed: 01/04/2022 Revised: 16/05/2022 Accepted: 20/05/2022

Abstract

Much of a person's cognitive activity depends on the ability to reason analogically. Analogical reasoning (AR) compares the similarities between new and past knowledge and uses them to obtain an understanding of the new knowledge. The mechanisms, however, under which the human mind works are not fully investigated and as a result AR is characterized by a degree of fuzziness and uncertainty. Probability theory has been proved sufficient for dealing with the cases of uncertainty due to randomness. During the last 50-60 years, however, various mathematical theories have been introduced for tackling effectively the other forms of uncertainty, including fuzzy sets, intuitionistic fuzzy sets, neutrosophic sets, rough sets, etc. The combination of two or more of those theories gives frequently better results for the solution of the corresponding problems. A hybrid assessment method of AR skills under fuzzy conditions is developed in this work using Grey Numbers (GN) and soft sets as tools, which is illustrated by an application on evaluating student analogical problem solving skills.

Keywords: Soft set, Grey Number, Analogical Reasoning, Assessment under fuzzy conditions.

1 | Introduction

Analogical Reasoning (AR) is a way of reasoning that compares the similarities between new and past knowledge and uses these similarities to obtain an understanding of the new knowledge. Polya, who introduced the use of heuristic strategies for problem solving, suggests: "When you are not sure how to solve a given problem, a good hint would be to look to a similar problem solved in the past and try to exploit its solution for use with the new problem" [1]. In Artificial Intelligence the case-based Reasoning approach includes a range of methods based on AR for retrieving and utilizing the knowledge of past cases, usually stored in a computer's memory, for solving new problems or for producing new knowledge [2] and [3]. As many specialists report, much of the human cognitive activity depends in general on the ability to reason analogically [4]-[6]. Within cognitive science mental processes are simulated in computer programs (e.g. neural networks) and serve as models to support reasoning in new domains.

Corresponding Author: mvoskoglou@gmail.com

http://dx.doi.org/10.22105/jfea.2022.342139.1220

It becomes evident therefore, that the assessment of the AR skills is a very important process. The mechanisms, however, under which the human mind works have not fully investigated yet. As a result, human reasoning in general and AR in particular are characterized by a degree of fuzziness and uncertainty. The first of the present authors proposed several methods in the past for assessment under fuzzy conditions including the measurement of a system's uncertainty, the use of the center of gravity defuzzification technique, the use of triangular fuzzy numbers or of GNs, etc. All these methods are reviewed in [7] accompanied by suitable examples. Recently he has also used soft sets for assessment with respect to a finite set of parameters [8].

In this work we introduce a hybrid method for assessing AR skills that uses soft sets and GNs as tools. The rest of the paper is organized as follows: Section 2 contains the elements from the theory of GNs and soft sets that are necessary for understanding the paper. The hybrid assessment method is presented in Section 3, illustrated by an example of assessing student analogical problem solving skills. The article closes with the final conclusion and some hints for future research included in Section 4.

2 | Background

The frequently appearing in real world, in science and in everyday life uncertainty is due to several reasons, like randomness, imprecise data, vague information, etc. Probability theory has been proved sufficient for dealing with the cases of uncertainty due to randomness. During the last 50-60 years, however, various mathematical theories have been introduced for tackling effectively the other forms of uncertainty, including fuzzy sets [9], intuitionistic fuzzy sets [10], neutrosophic sets [11], rough sets [12] and several others [13]. The combination of two or more of those theories gives frequently better results for the solution of the corresponding problems and that is what we are going to attempt here for the assessment of AR skills.

2.1 | Grey Numbers (GNs)

Ju-Long [14] introduced in 1982 the theory of grey systems as a new tool for dealing with the uncertainty created by the use of approximate data. A system is characterized as grey if it lacks information about its structure, operation and/or its behavior. The use of GNs [15] is the tool for performing the necessary calculations in grey systems.

A GN, say A, is an interval estimate [x, y] of a real number, whose exact value within [x, y] is not known. We symbolize then A by writing A ∈ [x, y]. A GN A is frequently accompanied by a whitenization function g: [x, y] → [0, 1], such that, if a ∈ [x, y], then the closer is g(a) to 1, the better a approximates the unknown real value of A. If no whitenization function has been defined, we usually consider as the crisp approximation of A the real number

$$W(A)=\frac{x+y}{2}. \tag{1}$$

The arithmetic operations between GNs are defined with the help of the known arithmetic of the real intervals [16]. In this work we make use only of the addition A+B of GNs and the scalar product of a GN with a positive number, say r, which are defined as follows: if A ∈ [x1, y1] and B ∈ [x2, y2] are GNs, then:

$$A+B\in\left[x_1+y_1,\ x_2+y_2\right]. \tag{2}$$

$$rG_1\in\left[rx_1,\ ry_1\right]. \tag{3}$$

2.2 | Soft Sets

All the theories, mentioned in the beginning of this section, which have been developed for dealing with the several types of the existing uncertainty, have their own difficulties. The reason of these difficulties is usually the inadequacy of the parameterization tools used. In fuzzy set theory, for example, there is a difficulty with the membership function, the definition of which is not unique depending on the observers' personal criteria [17]. To overpass these difficulties Molodtsov [18] introduced the concept of soft set for tackling the uncertainty in a parametric number.

Let U be the set of the discourse, let E be a set of parameters and let f: E → P(U) be a map from E to the power set P(U) of U. Then the soft set on U depending on f and E, symbolized by (f, E), is defined as the set of the ordered pairs

$$(f, E) = \{(e, f(e)): e \in E\}. \quad (4)$$

The set (f, E), being actually a family of subsets of U, is called "soft" because its form depends on the parameters. For general facts on soft sets we refer to [19].

3 | The Hybrid Assessment Method

Several researches ([20]-[24], etc.) have provided detailed models for the AR process which are broadly consistent with problem solving strategies. According to these models the main steps of AR include Representation (R) of the problem under solution (target problem), Search and Retrieval (S-R) of the related past problem (source problem), Mapping (M) of the representations of the target to the source problem for finding the corresponding similarities between them and adaptation (A) of the solution of the source for use with the target problem.

The purpose of the following classroom application was the assessment of AR skills of a class of 20 students. For this, four sets of problems were given to students for solution. Each set included the target problem, the source problem which was a remote analogue to the target problem sharing structure but not surface characteristics with it, a distractor problem sharing surface but not structure characteristics with the target problem and an unrelated to the target problem. The solution procedures of the last three problems were given to students, who were asked to solve the target problem with the help of the given solutions. As an example, we present below one of the sets of those problems [24].

Target problem. A box contains 8 balls numbered from 1 to 8. One makes three successive drawings, putting back the corresponding ball to the box before the next drawing. Find the probability of getting all the balls drawn out of the box different to each other.

Solution. The probability is equal to the quotient of the total number of the ordered samples of 3 objects from 8 (favourable outcomes) to the total number of the corresponding samples with replacement (possible outcomes).

Remote analogue. How many numbers of 2 digits can be formed by using the digits from 1 to 6 and how many of them have their digits different?

Solution's sketch given to students. Find the total number of the ordered samples of 2 objects from 6 with and without replacement respectively.

Distractor problem. A box contains 3 white, 4 blue and 6 black balls. If we draw out 2 balls, what is the probability to be of the same colour?

Solution's sketch given to students. The number of all favourable outcomes is equal to the sum of the total number of combinations of 3, 4 and 6 objects taken 2 at each time respectively, while the number of all possible outcomes is equal to the total number of combinations of 13 objects taken 2 at each time.

Unrelated problem. Find the number of all possible anagrams of the word "SIMPLE". How many of them start with S and how many of them start with S and end with E?

Voskoglou and Broumi|J. Fuzzy. Ext. Appl. 3(2) (2022) 152-157

Solution's sketch given to students. The number of all possible anagrams is equal to the total number 6! of the permutations of 6 objects. The anagrams starting with S are 5! and the anagrams starting with S and ending with E are 4!

The student AR skills were assessed by using the linguistic grades A = excellent, B= very good, C = good, D = fair and F = unsatisfactory (failed). It becomes evident that such kind of assessment involves a degree of fuzziness caused by the existence of the linguistic grades, which are less accurate than the numerical scores.

Let U = $\{S_1, S_2, \ldots, S_{20}\}$ be the set of the students of the class under assessment and let E= {A, B, C, D, F} be the set of the linguistic grades (parameters) mentioned before. Then, according to the results of the student assessment, we defined a map f: E → P(U) as follows: f(A) = $\{S_5, S_{14}, S_{18}\}$, f(B) = $\{S_1, S_3, S_9, S_{12}, S_{15}, S_{17}\}$, f(C) = $\{S_2, S_6, S_8, S_{11}, S_{13}, S_{20}\}$, f(D) = $\{S_4, S_7, S_{16}\}$, f(F) = $\{S_{10}, S_{19}\}$. Consequently, the soft set

$$(f, E) = \{(A, f(A)), (B, f(B)), (C, f(C)), (D, f(D)), (F, f(F))\}, \quad (5)$$

represents mathematically the student class overall performance in a parametric way.

Another advantage of using softs sets as tools for the assessment of student AR skills is the mathematical representation of each student's profile with respect to the steps of the AR process. In fact, consider a particular student of the class and let V = {R, S-R, M, A} be the set of the steps of the AR process mentioned in the beginning of this section. Define a map g: E → P(V) assigning to each linguistic grade (parameter) of E the steps of the AR process in which the performance of the particular student was assessed by this grade. Then the soft set (g, E) represents mathematically the student's profile. For example, the soft set

$$(g, E) = \{(A, \{R, S-R\}), (B,\varnothing), (C, \{M, A\}), (D,\varnothing), (F,\varnothing)\}. \quad (6)$$

corresponds to a student who demonstrated excellent performance at the steps of representation and search-retrieval and good performance at the steps of mapping and adaptation.

Sometimes, however, it is necessary to assess the mean performance of a student class. This is needed, for example, when one wants to compare the performance of two different student classes. It becomes evident that, when using linguistic grades, the mean performance cannot be assessed by calculating the mean value of the student scores in the classical way. In order to overpass this difficulty we assigned a scale of numerical scores from 1 to 100 to the linguistic grades A, B, C, D and F as follows: A (85–100), B (75–84), C (60–74), D (50–59) and F (0–49)[1]. For simplifying our notation we used the same letters to correspond to each of those grades the GNs A ∈ [85, 100], B ∈ [75, 84], C ∈[60, 74], D ∈ [50, 59] and F ∈ [0, 49], respectively. This enabled us to assign one of the GNs A, B, C, D, F to each student of the class. Then, using equation (5), it is easy to check that the "mean value" M of all those GNs is equal to the GN $M = \frac{1}{20}$ (3A+6B+6C+3D+2F), or by (3) $M = \frac{1}{20}$ ([255, 300]+[450, 504]+[360, 444]+[150, 177]+[0, 98]), or by (2) $M = \frac{1}{20}$ [1215, 1523]. Thus, by (3) M ∈ [60.75, 76.15]. Therefore, by (1) W(M)=68.45, which means that the student class demonstrated a good mean performance.

[1] The scores assigned to the linguistic grades are not standard and may differ from case to case. For instance, in a more rigorous assessment one could take A(90-100), B (80-89), C(70-79), D (60-69), F(<60), etc.

4 | Discussion and Conclusions

A hybrid method using soft sets and GNs as tools was developed in this work for the assessment of student AR skills. The use of soft sets enabled a mathematical representation of the overall student performance in terms of the linguistic grades (parameters) used for their assessment, whereas the use of GNs enabled to obtain a crisp representative evaluating their mean performance. The second is useful when one wants to compare the performance of two or more student groups.

The new hybrid assessment method developed here has a general character. This means that, apart from student assessment, it could be utilized for assessing a great variety of other human or machine (e.g. case–based reasoning or decision making systems) activities. This is an important direction for future research. The theory of fuzzy systems, however, and the alternative theories related to it which were mentioned in the beginning of Section 2 of the present work, give also many and good other opportunities of applied and theoretical research in almost all sectors of the human activity, e.g. see [25]-[28], etc.

References

[1] Polya, G. (1954). *Induction and analogy in mathematics: volume I of mathematics and pausible reasoning*. Princeton University Press.

[2] Voskoglou, M. G. (2008, September). Case-based reasoning: a recent theory for problem-solving and learning in computers and people. In *world summit on knowledge society* (pp. 314-319). Springer, Berlin, Heidelberg. https://link.springer.com/chapter/10.1007/978-3-540-87783-7_40

[3] Voskoglou, M. Gr. & Salem, A. B. (2014). Analogy-based and case–based reasoning: two sides of the same coin. *International journal of applications of fuzzy sets and artificial intelligence*, *4*, 5-51.

[4] Holyoak, K. J. (1985). The pragmatics of analogical transfer. In *psychology of learning and motivation* (Vol. 19, pp. 59-87). Academic Press.

[5] Bugaiska, A., & Thibaut, J. P. (2015). Analogical reasoning and aging: the processing speed and inhibition hypothesis. *Aging, neuropsychology, and cognition*, *22*(3), 340-356.

[6] Thibaut, J. P., & French, R. M. (2016). Analogical reasoning, control and executive functions: a developmental investigation with eye-tracking. *Cognitive development*, *38*, 10-26.

[7] Voskoglou, M. G. (2019). Methods for assessing human–machine performance under fuzzy conditions. *Mathematics*, *7*(3), 230. https://doi.org/10.3390/math7030230

[8] Voskoglou, M. G. (2022). Soft sets as tools for assessing human-machine performance. *Egyptian computer science journal*, *46*(1), 1-6.

[9] Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, *8*(3), 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X

[10] Atanassov, K. T. (1986). Intuitionistic Fuzzy Sets. *Fuzzy sets and systems*, 20, 87-96

[11] Smarandache, F. (1998). *Neutrosophy: neutrosophic probability, set, and logic: analytic synthesis & synthetic analysis*. American Research Press.

[12] Pawlak, Z. (1991). *Rough sets: Theoretical aspects of reasoning about data* (Vol. 9). Springer Science & Business Media.

[13] Voskoglou, M. G. (2019). Fuzzy systems, extensions and relative theories. *WSEAS transactions on advances in engineering education*, *16*, 63-69.

[14] Ju-Long, D. (1982). Control problems of grey systems. *Systems & control letters*, *1*(5), 288-294.

[15] Liu, S., & Forrest, J. Y. L. (Eds.). (2010). *Advances in grey systems research*. Springer.

[16] Moore, R. E., Kearfott, R. B., & Cloud, M. G. (2009). *Introduction to interval analysis*. Society for Industrial and Applied Mathematics.

[17] Klir, G. J. & Folger, T. A. (1988). *Fuzzy sets, Uncertainty and information*. Prentice-Hall.

[18] Molodtsov, D. (1999). Soft set theory—first results. *Computers & mathematics with applications*, *37*(4-5), 19-31.

[19] Maji, P. K., Biswas, R., & Roy, A. R. (2003). Soft set theory. *Computers & mathematics with applications*, *45*(4-5), 555-562.

Voskoglou and Broumi|J. Fuzzy. Ext. Appl. 3(2) (2022) 152-157

[20] Hayes, J. R. (1977). Psychological differences among problem isomorphs. *Cognitive theory*, *2*, 21-41.

[21] Gentner, D., & Toupin, C. (1986). Systematicity and surface similarity in the development of analogy. *Cognitive science*, *10*(3), 277-300.

[22] Holyoak, K. J., & Koh, K. (1987). Surface and structural similarity in analogical transfer. *Memory & cognition*, *15*(4), 332-340.

[23] Novick, L. R. (1988). Analogical transfer, problem similarity, and expertise. *Journal of experimental psychology: learning, memory, and cognition*, *14*(3), 510. https://psycnet.apa.org/doiLanding?doi=10.1037/0278-7393.14.3.510

[24] Niss, M. (2003, January). Mathematical competencies and the learning of mathematics: The Danish KOM project. *3rd Mediterranean conference on mathematical education* (pp. 115-124). Hellenic Mathematical Society, Athen.

[25] Shafi Salimi, P., & Edalatpanah, S. A. (2020). Supplier selection using fuzzy AHP method and D-Numbers. *Journal of fuzzy extension and applications*, *1*(1), 1-14.

[26] Smarandache, F. (2020). Generalisations and alternatives of classical algebraic structures to neutroAlgebraic structures and antiAlgebraic structures. *Journal of fuzzy extension and applications*, 1(2), 81-83.

[27] Polymenis, A. (2021). A neutrosophic student'st–type of statistic for AR (1) random processes. *Journal of fuzzy extension and applications*, *2*(4), 388-393.

[28] Kouatli, I. (2022). Modelling fuzzymetric cognition of technical analysis decisions: reducing emotional trading. *Journal of fuzzy extension and applications,* 3(1), 45-63.