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A note on somewhat fuzzy continuity

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Abstract

Thangaraj and Balasubramanian introduced the so-called somewhat fuzzy semicontinuous and somewhat fuzzy semiopen f0unctions. Two years later, the same authors defined two other types of functions called somewhat fuzzy continuous and somewhat fuzzy open without indicating connections between them. At first glance, we may easily conclude (from their definitions) that every somewhat fuzzy continuous (resp. open) function is slightly fuzzy semicontinuous (resp. semiopen) but not conversely. In this note, we show that they are equivalent. We further prove that somewhat fuzzy continuous functions are weaker than fuzzy semicontinuous functions.

Keywords: Fuzzy continuous, somewhat continuous, somewhat fuzzy continuous, somewhat fuzzy semicontinuous, somewhat fuzzy open.

1 | Introduction and Preliminaries

One of the fundamental concepts of all mathematics, particularly topology and analysis, is continuity of functions, which means that "small" changes in the input result in "small" changes in the output. After introducing the fuzzy set, Chang defined the notion of fuzzy continuity [4] of functions between fuzzy topological spaces in 1968. Following that, several generalizations of fuzzy continuity were introduced. In 1981, Azad [3] presented some classes of generalized fuzzy continuous functions. Fuzzy semi-continuity is one of them that is weaker than fuzzy continuity. In 2001-2003, Thangaraj and Balasubramanian introduced somewhat fuzzy semicontinuous [6] and somewhat fuzzy continuous functions [7]. They only discussed their relationship with fuzzy continuous functions. In the present note, we show that somewhat fuzzy semi-continuity and somewhat fuzzy continuity are the same, and somewhat fuzzy continuity is weaker than fuzzy semi-continuity. It is worth stating somewhat continuity appeared in different topological structures (see [1, 5, 7]). These classes of functions have a significant role in characterizing (soft) Baire spaces [2, 8] and weakly equivalence spaces [5].

Let X be a universe (domain). A mapping μ from X to the unit interval $\mathbb{I}$ is named a fuzzy set in X. The value $\mu(x)$ is called the degree of the membership of x in μ for each $x \in X$. The support of μ

is the set $\{x \in X: \mu(x) > 0\}$. The complement of μ is, denoted by μ^c (or $1 - \mu$ if there is no confusion), given by $\mu^c(x) = 1 - \mu(x)$ for all $x \in X$. Let $\{\mu_j: j \in J\}$ be a collection of fuzzy sets in X,

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where J is any index set. Then $\vee\mu_j(x) = \sup\{\mu_j(x): j \in J\}$ and $\wedge\mu_j(x) = \inf\{\mu_j(x): j \in J\}$ for each $x \in X$.

Definition 1.1 *[4] A collection $\mathcal{T}$ of fuzzy sets in X is said to have a fuzzy topology on X if*

1. $0,1 \in \mathcal{T}$;
2. $\mu \wedge \lambda \in T$ whenever $\mu, \lambda \in \mathcal{T}$; and
3. $\vee\mu_j \in \mathcal{T}$ for any subcollection $\{\mu_j: j \in J\}$ of $\mathcal{T}$.

The pair $(X, \mathcal{T})$ is called a fuzzy topological space. Members of $\mathcal{T}$ are called fuzzy open subsets of X, and their complements are called fuzzy closed sets.

Let λ be a fuzzy set in X and let $(X, \mathcal{T})$ be a fuzzy topological space. The fuzzy interior of λ is defined by $Int(\lambda) := \sup\{\mu: \mu \leq \lambda, \mu \in T\}$ and the fuzzy closure of λ is given by $Cl(\lambda) := \inf\{\mu: \lambda \leq \mu, 1 - \mu \in T\}$. The set λ is called fuzzy semiopen [3] if $\lambda \leq Cl(Int(\lambda))$. The complement of every fuzzy semiopen set is fuzzy semiclosed. Evidently, every fuzzy open set is fuzzy semiopen but not the opposite (see, Example 2.4). The fuzzy semi-interior of λ is defined by $Int_s(\lambda) := \sup\{\mu: \mu \leq \lambda, \mu$ is fuzzy semiopen$\}$ and the fuzzy semi-closure of λ is given by $Cl_s(\lambda) := \inf\{\mu: \lambda \leq \mu, \mu$ is fuzzy semiclosed$\}$.

Lemma 1.2 *[10] Let λ be a fuzzy set in X and let $(X, \mathcal{T})$ be a fuzzy topological space. The following hold:*

1. $Int(\lambda) \leq Int_s(\lambda)$.
2. $Cl_s(\lambda) \leq Cl(\lambda)$.

To make our task easier, we introduce the following notions:

Definition 1.3 *Let λ be a fuzzy set in X and let $(X, \mathcal{T})$ be a fuzzy topological space. Then*

1. λ is called a somewhat fuzzy open set if $\lambda = 0$ or $Int(\lambda) \neq 0$.
2. λ is called a somewhat fuzzy semiopen set if $\lambda = 0$ or $Int_s(\lambda) \neq 0$.

Definition 1.4 *A function f from a fuzzy topological space $(X, \mathcal{T})$ into another fuzzy topological space $(Y, \mathcal{U})$ is said to be*

1. fuzzy continuous [4] if $f^{-1}(\beta) \in \mathcal{T}$ for each $\beta \in \mathcal{U}$.
2. fuzzy semicontinuous [3] if $f^{-1}(\beta)$ is fuzzy semiopen for each $\beta \in \mathcal{U}$.
3. somewhat fuzzy continuous [7] if $f^{-1}(\beta)$ is somewhat fuzzy open for each $\beta \in \mathcal{U}$.
4. somewhat fuzzy semicontinuous [6] if $f^{-1}(\beta)$ is somewhat fuzzy semiopen for each $\beta \in \mathcal{U}$.

Definition 1.5 *A function f from a fuzzy topological space $(X, \mathcal{T})$ into another fuzzy topological space $(Y, \mathcal{U})$ is said to be*

1. fuzzy open [9] if $f(\alpha) \in \mathcal{U}$ for each $\alpha \in \mathcal{T}$.

2. fuzzy semiopen [3] if $f(\alpha)$ is fuzzy semiopen for each $\alpha \in \mathcal{T}$.

3. somewhat fuzzy open [7] if $f(\alpha)$ is somewhat fuzzy open for each $\alpha \in \mathcal{T}$.

4. somewhat fuzzy semiopen [6] if $f(\alpha)$ is somewhat fuzzy semiopen for each $\alpha \in \mathcal{T}$.

2 | Main results

We start by proving several lemmas that help us achieve our goal.

Lemma 2.1 *Let λ be a non-zero fuzzy set in a fuzzy topological space $(X, \mathcal{T})$. Then λ is fuzzy semiopen iff $Cl(\lambda) = Cl(Int(\lambda))$.*

Proof. Given a fuzzy semiopen set λ such that $\lambda \neq 0$. Then $\lambda \leq Cl(Int(\lambda))$ and so $Cl(\lambda) \leq Cl(Int(\lambda))$. On the other hand, we always have $Int(\lambda) \leq \lambda$. Therefore $Cl(Int(\lambda)) \leq Cl(\lambda)$. Hence $Cl(\lambda) = Cl(Int(\lambda))$.

Conversely, assume that $Cl(\lambda) = Cl(Int(\lambda))$, but $\lambda \leq Cl(\lambda)$ always, so $\lambda \leq Cl(Int(\lambda))$. Thus λ is fuzzy semiopen.

Lemma 2.2 *Let λ be a non-zero fuzzy set in the fuzzy topological space $(X, \mathcal{T})$. If λ is fuzzy semiopen, then $Int(\lambda) \neq 0$.*

Proof. Assume, if possible, that λ is a fuzzy semiopen set for which $Int(\lambda) = 0$, by Lemma 2.1, $Cl(\lambda) = 0$ implies that $\lambda = 0$, a contradiction.

Lemma 2.3 *Let λ be a fuzzy set in a fuzzy topological space $(X, \mathcal{T})$. Then λ is somewhat fuzzy open if it is somewhat fuzzy semiopen.*

Proof. Without loss of generality, we assume $\lambda \neq 0$; otherwise, the equivalence of these concepts is apparent. Since, by Lemma 1.2, $Int(\lambda) \leq Int_s(\lambda)$, so this concludes that the somewhat fuzzy openness of λ implies somewhat fuzzy semiopenness.

Conversely, suppose that λ is somewhat fuzzy semiopen. By definition, for some $x \in \lambda$, there is a fuzzy semiopen set μ in X such that $x \in \mu \leq \lambda$. By Lemma 2.2, $Int(\mu) \neq 0$. Therefore, for some $x \in \lambda$, one can find a fuzzy open set β such that $x \in \beta \leq \mu \leq \lambda$. Hence, $Int(\lambda) \neq 0$. This shows that λ is somewhat fuzzy open.

From the above lemmas, perhaps a relationship among the stated classes of fuzzy open sets can be concluded.

fuzzy open $\Longrightarrow$ fuzzy semiopen $\Longrightarrow$ somewhat fuzzy open $\Longleftrightarrow$ somewhat fuzzy semiopen

Diagram 1. Relationships between generalized fuzzy open sets.

None of the arrows in the diagram above can be reversed, as shown in the following example:

Example 2.4 *Let $\mu, \lambda, \sigma, \alpha, \beta$ be fuzzy sets on $\mathbb{I}$ defined as:*

$$\mu(x) = \begin{cases} 0, & if \ \ 0 \leq x \leq 1/2 \\ 2x-1, & if \ \ 1/2 \leq x \leq 1, \end{cases}$$

$$\lambda(x) = \begin{cases} 1, & if \ \ 0 \leq x \leq 1/4 \\ -4x+2, & if \ \ 1/4 \leq x \leq 1/2 \\ 0, & if \ \ 1/2 \leq x \leq 1, \end{cases}$$

$$\sigma(x) = \begin{cases} 1, & if \ \ 0 \leq x \leq 1/4 \\ -4x+2, & if \ \ 1/4 \leq x \leq 1/2 \\ 2x-1, & if \ \ 1/2 \leq x \leq 1, \end{cases}$$

$$\alpha(x) = \begin{cases} 0, & if \ \ 0 \leq x \leq 1/4 \\ 4x/3-1/3, & if \ \ 1/4 \leq x \leq 1, \end{cases}$$

and

$$\beta(x) = \begin{cases} 2x, & if \ \ 0 \leq x \leq 1/2 \\ 1, & if \ \ 1/2 \leq x \leq 1. \end{cases}$$

The family $\mathcal{T} = \{0, \mu, \lambda, \sigma, 1\}$ forms a fuzzy topology on $\mathbb{I}$. The set α is fuzzy semiopen but not fuzzy open (see, Example 4.5 in [3]). One can quickly check that the set β is somewhat fuzzy open but not fuzzy semiopen.

Thangaraj and Balasubramanian showed that fuzzy continuity (openness) of a function implies somewhat fuzzy continuity (openness), but in reality, more than that is true

fuzzy continuous (fuzzy open) $\Longrightarrow$ fuzzy semicontinuous (fuzzy semiopen) $\Longrightarrow$ somewhat fuzzy continuous (somewhat fuzzy open)

Diagram 2. The connections between the generalized fuzzy functions.

Theorem 2.5 *For a function* $f: (X, \mathcal{T}) \to (Y, \mathcal{U})$, *the following statements hold:*

1. f is somewhat fuzzy continuous if it is somewhat fuzzy semicontinuous.

2. f is somewhat fuzzy open if it is somewhat fuzzy semiopen.

Proof. The Lemma 2.3 guarantees that $Int(\mu) \neq 0$ iff $Int_s(\mu) \neq 0$ for every fuzzy set μ, and the proof can be concluded.

We finish this note by pointing out that the same result holds for the functions mentioned above in general and soft topology.

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Conflicts of Interest

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