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Theoretical Approaches of Interval-Valued Fuzzy Code and Fuzzy Soft Code

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Abstract

In this study, we attempted to demonstrate the interval-valued fuzzy code by extending the concept of an interval-valued fuzzy set. Further, we discussed the operations of the interval-valued fuzzy code. The interval-valued fuzzy soft code is introduced, and various related properties are investigated in this paper. Finally, we show that the operations of interval-valued fuzzy soft code are discussed. Through this paper, we use the set of integers modulo 2, that is $Z_2 = \{0, 1\}$.

Keywords: Interval-valued fuzzy set, Interval-valued fuzzy soft set, Interval-valued fuzzy code, Interval-valued fuzzy soft code.

1 | Introduction

According to Zadeh [1], a "fuzzy set" concept is proposed to represent a class of objects with a range of membership ratings. Zadeh was the first to propose the concept of interval-valued fuzzy sets [2]. Molodtsov [3] established soft-set theories and models as a new mathematical tool for dealing with uncertainties that existing mathematical tools cannot control. Using the concept of Molodtsov, Maji et al. [4] presented the theoretical investigation of the soft set theory in more detail. Many scholars have presented an application of fuzzy soft sets in a decision-making problem for real-life activities [5]-[8]. Ali et al. [9] designed and developed a new class of linear algebraic codes defined as soft linear algebraic codes for the first time, and they developed methods for generating and decoding the new classes of soft linear algebraic codes. The detection and correction of errors in soft linear algebraic codes were discussed. Soft codes (soft linear codes) were introduced by Smarandache [10] through the use of soft sets, which are an approximated collection of codes. He also discussed various kinds of soft codes. A soft code is a code set that has been parameterized.

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Humberto [11] discussed interval-valued fuzzy sets in soft computing, which is dependent on the uncertainty (complexity) associated with different levels of models to solve a problem. Gehrke et al. [12] developed the fundamental theory of interval-valued fuzzy sets. Hong and Qin [13] mentioned the algorithmic structure as well as the characteristics of fuzzy soft sets.

Yang et al. [14] developed the interval-valued fuzzy soft set concept by combining the interval-valued fuzzy set and soft set models. A generalized interval-valued fuzzy soft set and its operations were proposed by Alkhazaleh and Salleh [15]. They also discussed using an interval-valued fuzzy soft set to solve a decision-making problem and how to use this theory to solve medical diagnosis problems.

By combining the interval-valued intuitionistic fuzzy theory of sets and a soft set theory, Jiang et al. [16] established the concept of the interval-valued intuitionistic fuzzy soft theory. In other words, the interval-valued intuitionistic fuzzy soft set theory combines the intuitionistic fuzzy soft set theory and the interval-valued fuzzy soft set theory. Mohamed et al. [17] proposed a novel hybrid method based on two modern metaheuristic algorithms. Ali et al. [18] developed a novel approach for the three-way decision model by utilizing decision-theoretic rough sets and power aggregation operators. Ali et al. [19] created the Generalized Intuitionistic Decision-Theoretic Rough Set (GI-DTRS), a fusion that melds the principles of decision-theoretic rough sets and intuitionistic fuzzy sets. Kane et al. [20] proposed solving semi-fully linear programming problems. Arif et al. [2] investigated the concept of the (α, β) -interval-valued set as well as the order relation on the (α, β)-interval-valued set. Shikhi and Ebadi [21] presented an efficient method for solving linear interval fractional transportation problems. Chetia and Das [22] investigated the use of interval-valued fuzzy soft sets in medical diagnosis. Soltanifar [23] presented a new interval for ranking alternatives in multi attribute decision making problems.

From the above discussion, a number of works are based on the classical fuzzy sets and fuzzy soft sets. However, in this paper, we developed a new concept of theoretical approaches to interval-valued fuzzy codes and fuzzy soft codes. This paper aims to integrate the interval-valued fuzzy soft set and fuzzy codes, from which a novel fuzzy soft set model can be derived: interval-valued fuzzy code and Interval-Valued Fuzzy Soft Code (IVFSC). The application of IVFSC is used to design hypothetical case studies, medical diagnoses, and software engineering.

In Section 2, we first introduce the interval-valued fuzzy set and interval-valued fuzzy soft set to help with our explanation. The ideas of interval-valued fuzzy code and IVFSC are covered in Section 3. We then provide a summary at the end of the publication and suggest additional research.

2 | Preliminary

This section will review some fundamental concepts applied to our results.

Molodtsov [3] defined the soft set as follows. Let $\overline{\mathrm{U}}$ represent the initial universe set, and E represents the parameters set.

Definition 1 ([4]). A soft set is a pair $(\overline{\mathrm{F}}, \overline{\mathrm{E}})$ if it is the case that $\overline{\mathrm{F}}$ is a mapping of $\overline{\mathrm{E}}$ into the collection of all sub-sets of the set $\overline{\mathrm{U}}$.

In other words, the soft set is a parameterized family of subsets of the universe $\overline{\mathrm{U}}$, not a type of set. If we consider $\mathrm{P}(\overline{\mathrm{U}})$ to be a set of all subsets of $\overline{\mathrm{U}}$, then $\overline{\mathrm{F}}$ is a mapping such that

$$\overline{\mathrm{F}}: \overline{\mathrm{E}} \mapsto \mathrm{P}(\overline{\mathrm{U}}). \tag{1}$$

Such a mapping represents the inherent nature of the concept of a soft set, namely that a soft set is a mapping from parameters to $\mathrm{P}(\overline{\mathrm{U}})$. $\overline{\mathrm{F}}(\mathrm{e})$ can be thought of as the set of e -approximate elements of the soft set $(\overline{\mathrm{F}}, \overline{\mathrm{E}})$ for all $\mathrm{e} \in \overline{\mathrm{E}}$.

Definition 2 ([3]). If $\mathrm{P}(\overline{\mathrm{U}})$ represents the set of all fuzzy subsets of $\overline{\mathrm{U}}$, then a pair $(\overline{\mathrm{F}}, \overline{\mathrm{E}})$ is called a fuzzy soft set over $\mathrm{P}(\overline{\mathrm{U}})$, where $\overline{\mathrm{F}}$ is a mapping given by

$$\bar{F}: \bar{E} \mapsto P(\bar{U}). \quad (2)$$

Definition 3 ([14], [15]). An interval-valued fuzzy set $\mathcal{X}_{int}$ on a universe $\bar{U}$ is a mapping such that

$$\mathcal{X}_{int}: \bar{U} \mapsto int([0,1]), \quad (3)$$

where $int([0,1])$ stands for the set of all closed sub-intervals of $[0,1]$, the set of all interval-valued fuzzy sets on $\bar{U}$ is denoted by $P(\bar{U})$.

Suppose that $\mathcal{X}_{int} \in P(\bar{U})$, for all $x \in \bar{U}$, $\sigma_{\mathcal{X}_{int}}(x) = [\sigma^{-}_{\mathcal{X}_{int}}(x), \sigma^{+}_{\mathcal{X}_{int}}(x)]$ is called the degree of membership element x to $\mathcal{X}_{int}$. Where $\sigma^{-}_{\mathcal{X}_{int}}(x)$ and $\sigma^{+}_{\mathcal{X}_{int}}(x)$ referred to as the lower and upper degree of membership x to $\mathcal{X}_{int}$, where $0 \leq \sigma^{-}_{\mathcal{X}_{int}}(x) \leq \sigma^{+}_{\mathcal{X}_{int}}(x) \leq 1$.

Definition 4 ([14]). Let $X_{int}, Y_{int} \in P(\bar{U})$ be two interval-valued fuzzy sets:

I. The complement of an interval-valued fuzzy set $X_{int} \in P(\bar{U})$ is denoted by X^{C}_{int} is an interval-valued fuzzy set defined as follows:

$$\sigma_{X^{C}_{int}}(x) = 1 - \sigma_{X_{int}}(x) = \left[1 - \sigma^{+}_{X_{int}}(x), 1 - \sigma^{-}_{X_{int}}(x)\right]. \quad (4)$$

II. The intersection of X_{int} and Y_{int} is an interval-valued fuzzy set denoted by $X_{int} \sqcap Y_{int}$ and defined as

$$\sigma_{X_{int} \sqcap Y_{int}}(x) = \inf[\sigma_{X_{int}}(x), \quad \sigma_{Y_{int}}(x)]$$
$$= [\inf\left(\sigma^{-}_{X_{int}}(x), \sigma^{-}_{Y_{int}}(x)\right), \inf(\sigma^{+}_{X_{int}}(x), \sigma^{+}_{Y_{int}}(x)]. \quad (5)$$

III. The union of two interval-valued fuzzy sets X_{int} and Y_{int} is an interval-valued fuzzy set and denoted by $X_{int} \sqcup Y_{int}$ defined as

$$\sigma_{X_{int} \sqcup Y_{int}}(x) = \sup\left[\sigma_{X_{int}}(x), \quad \sigma_{Y_{int}}(x)\right]$$
$$= [\sup\left(\sigma^{-}_{X_{int}}(x), \sigma^{-}_{Y_{int}}(x)\right), \sup(\sigma^{+}_{X_{int}}(x), \sigma^{+}_{X_{int}}(x)]. \quad (6)$$

Definition 5 ([14]). Let $\bar{U}$ be an initial universe and $\bar{E}$ be the set of parameters, a pair $(\bar{F}, \bar{E})$ is called an interval-valued fuzzy soft set over $P(\bar{U})$, where $\bar{E}$ is a mapping given by

$$\bar{F}: \bar{E} \mapsto P(\bar{U}).$$

An interval-valued fuzzy soft set is a parametrized family of interval-valued fuzzy subsets of $\bar{U}$, thus, its universe is the set of all interval-valued fuzzy sets of $\bar{U}$, i.e., $P(\bar{U})$. An interval-valued fuzzy soft set is also a special case of a soft set because it is still a mapping from parameters to $P(\bar{U})$.

$\bar{F}(e)$ is referred to as the interval fuzzy value set of parameters e, it is actually an interval-valued fuzzy set of $\bar{U}$ where $x \in \bar{U}$ and $e \in \bar{E}$, and it can be written as

$$\bar{F}(e) = \left\{\left(x, \sigma_{\bar{F}(e)_{int}}(x)\right) : x \in \bar{U}\right\}, \quad (7)$$

where, $\bar{F}(e)$ is the interval-valued fuzzy membership degree that object x holds on parameter e. If $\sigma^{-}_{\bar{F}(e)_{int}}(x) = \sigma^{+}_{\bar{F}(e)_{int}}(x)$, for all $e \in \bar{E}$, for all $x \in \bar{U}$, then $\bar{F}(e)$ called a standard interval-valued fuzzy set, and then the pair $(\bar{F}, \bar{E})$ is called a standardized interval-valued fuzzy soft set.

Definition 6 ([14]). Suppose that $(\bar{F}, \bar{E})$ is an interval-valued fuzzy soft set over $P(\bar{U})$, $\bar{F}(e)$ is the interval value set of parameter e, then all interval fuzzy value sets in interval-valued fuzzy soft sets $(\bar{F}, \bar{E})$ are referred to as the interval fuzzy valued class of $(\bar{F}, \bar{E})$ and is denoted by $\hat{C}_{(\bar{F},\bar{E})}$, then we have

$$\hat{C}_{(\bar{F},\bar{E})} = \{\bar{F}(e_i): e_i \in \bar{E}\}. \quad (8)$$

Definition 7 ([14]). Let $\overline{U}$ be an initial universe and $\overline{E}$ be a set of parameters, assume that $\overline{A}, \overline{B} \sqsubseteq \overline{E}$, $(\overline{F}, \overline{A})$ and $(\overline{G}, \overline{B})$ are two interval-valued fuzzy soft sets, then we say that $(\overline{F}, \overline{A})$ is an interval-valued fuzzy soft subset of $(\overline{G}, \overline{B})$ if and only if the following condition are satisfied:

I. $\overline{A} \subseteq \overline{B}$.

II. For all $e \in \overline{E}$, $\overline{F}(e)$ is an interval-valued fuzzy subset of $\overline{G}(e)$, which can be written as

$$(\overline{F}, \overline{A}) \sqsubseteq (\overline{G}, \overline{B}).$$

$(\overline{F}, \overline{A})$ is said to be an interval-valued fuzzy soft super set of $(\overline{G}, \overline{B})$, if $(\overline{G}, \overline{B})$ is an interval-valued fuzzy soft subset of $(\overline{F}, \overline{A})$ and is denoted by $(\overline{F}, \overline{A}) \sqsupseteq (\overline{G}, \overline{B})$.

Definition 8 ([14]). Let $(\overline{F}, \overline{A})$ and $\overline{G}, \overline{B})$ be two interval-valued fuzzy soft sets, $(\overline{F}, \overline{A})$ and $(\overline{G}, \overline{B})$ are said to be interval-valued fuzzy soft equal if and only if $(\overline{F}, \overline{A})$ is an interval-valued fuzzy soft subset of $\overline{G}, \overline{B})$ and vice versa and defined by

$$(\overline{F}, \overline{A}) \cong \overline{G}, \overline{B}).$$

Definition 9 ([14]). The complement of an interval-valued fuzzy soft set $(\overline{F}, \overline{A})$ is denoted by $(\overline{F}, \overline{A})^C$ and is defined as

$$(\overline{F}, \widetilde{A})^c = \left(\overline{F}^C, \quad \neg\overline{A}\right), \tag{9}$$

where for all $e \in \overline{A}, \neg e = \text{not}\, e$ is the not set of the parameter e, which holds the opposite meanings of parameter e, and $\overline{F}^C$ is a mapping given by

$$\overline{F}^C: \neg\overline{A} \mapsto P(\overline{U}). \tag{10}$$

Such that $\overline{F}^C(e) = (\overline{F}(\neg e))^C$, for all $e \in \neg\overline{A}$.

Definition 10 ([14]). The "AND" or " ∧" or " MEET" operation on the two interval-valued fuzzy soft sets $(\overline{F}, \overline{C}_1)$ and $(\overline{G}, \overline{C}_2)$ is denoted by $(\overline{F}, \overline{C}_1)\text{AND}(\overline{G}, \overline{C}_2)$ or we can rewritten as $(\overline{F}, \overline{C}_1) \wedge (\overline{G}, \overline{C}_2)$ and defined as follows:

$$(\overline{F}, \overline{C}_1) \wedge (\overline{G}, \overline{C}_2) = \overline{F}(e_i) \cap \overline{G}(\nabla_j) = \min\{\overline{F}(e_i), \overline{G}(\nabla_j), \text{for all } i, j\}. \tag{11}$$

Definition 11 ([14]). The "OR" or " ∨" or " JOIN" operation on the two interval-valued fuzzy soft sets $(\overline{F}, \overline{C}_1)$ and $(\overline{G}, \overline{C}_2)$ is denoted by $(\overline{F}, \overline{C}_1)\text{OR}(\overline{G}, \overline{C}_2)$ or we can rewritten as $(\overline{F}, \overline{C}_1) \vee ((\overline{G}, \overline{C}_2)$ and defined as follows:

$$(\overline{F}, \overline{C}_1) \vee (\overline{G}, \overline{C}_2) = \overline{F}(e_i) \cup \overline{G}(\nabla_j) = \max\{\overline{F}(e_i), \overline{G}(\nabla_j), \text{for all } i, j\}. \tag{12}$$

3 | Interval-Valued Fuzzy Code

This section introduced an interval-valued fuzzy code by combining interval-valued fuzzy soft sets with fuzzy codes.

Definition 12. Let $C = \mathbb{Z}_2$ be a finite field with characteristic two and assume $K = F_q^n$ be a vector space over a field C and μ_C is a fuzzy subset of the universal set K, then an interval-valued fuzzy code is a mapping given by

$$\mu_C: K \to \text{int}([0,1]), \tag{13}$$

defined by

$$\mu_C(x) = \frac{\sum_{i=1}^{n} \tilde{p}_i}{\frac{k}{2}(k+1)}, \tag{14}$$

where $\mathrm{int}([0,1])$ is the set of all closed sub-intervals of the closed interval [0,1], and $\tilde{p}_i{}'s$ are the positions of 1's in the code word, and $\frac{k}{2}(k+1)$ is the maximum relative weight of a code word C in F_2^n, which is the same as the sum of the first n positive integers.

For instance, $111\ldots1$ is a code word in a vector space F_2^n, then its maximum relative weight is given by

$$1+2+3+\ldots+n=\frac{k}{2}(k+1).$$

Assume that $C \in P(K)$ for all $x \in C$, then the degree of memberships of x to C is given by

$$\mu_C(x)=[\mu_C{}^-(x),\ \mu_C{}^+(x)], \tag{15}$$

where $0 \le \mu_C{}^-(x) \le \mu_C{}^+(x) \le 1$.

Here, Eq. (14) can be modified in the following ways:

$$\mu_C(x)=[\mathrm{lb}\mu_C(x),\mathrm{upb}\mu_C(x)], \tag{16}$$

where $\mu_C{}^-(x)=\mathrm{lb}\mu_C(x)$ and $\mu_C{}^+(x)=\mathrm{upb}\mu_C(x)$ are lower and upper bound degrees of membership x to C, respectively.

Example 1. Let $0001111 \in F_2^n$. Then, to calculate the interval-valued fuzzy code, we first solve the fuzzy code by applying Eq. (14). The string 1 is located in the position of $4\text{th}, 5\text{th}$, and 6th, and the length of the string is 7, so we have

$$\mu_C(x)=\frac{4+5+6}{\frac{7}{2}(7+1)}=0.78571,$$

$$0.7 \le 0.78571 \le 0.8,$$

$$\mu_C(x)=[\mathrm{lb}\mu_C(x),\mathrm{upb}\mu_C(x)],$$

$$\mu_C(0001111)=[\mathrm{lb}\mu_C(0001111),\mathrm{upb}\mu_C(0001111)]$$

$$=[0.7,0.8],$$

that is an interval-valued fuzzy code.

Definition 13. The complement, intersection, and union of interval-valued fuzzy code are defined as follows:

Let $C_1,\ C_2 \in P(K)$, then

I. The complement of μ_{C_1} is denoted by $\mu^C{}_{C_1}$ and defined as

$$\mu_{C_1}{}^C(x)=1-\mu_{C_1}(x)=\left[1-\mathrm{upb}\mu_{C_1}(x),1-\mathrm{lb}\mu_{C_1}(x)\right]. \tag{17}$$

II. The intersection of two interval-valued fuzzy codes μ_{C_1} and μ_{C_2} is denoted by $\mu_{C_1} \sqcap \mu_{C_2}$ and defined as

$$\mu_{C_1}(x) \sqcap \mu_{C_2}(y)=\inf\left[\mu_{C_1}(x),\quad \mu_{C_2}(y)\right]$$
$$=\left[\inf\left(\mu_{C_1}{}^-(x),\quad \mu_{C_2}{}^-(y)\right),\inf\left(\mu_{C_1}{}^+(x),\quad \mu_{C_2}{}^+(y)\right)\right]. \tag{18}$$

III. The union of two interval-valued fuzzy codes μ_{C_1} and μ_{C_2} is denoted by $\mu_{C_1} \sqcup \mu_{C_2}$ and defined as

$$\mu_{C_1}(x) \sqcup \mu_{C_2}(y)=\sup\left[\mu_{C_1}(x),\quad \mu_{C_2}(y)\right]$$
$$=\left[\sup\left(\mu_{C_1}{}^-(x),\quad \mu_{C_2}{}^-(y)\right),\sup\left(\mu_{C_1}{}^+(x),\quad \mu_{C_2}{}^+(y)\right)\right]. \tag{19}$$

Proposition 1. Consider an interval fuzzy code to be μ_C, then

$$(\mu_C{}^C)^C=\mu_C.$$

Proof: Since ${\mu_C}^C(x) = 1 - \mu_C(x)$ from the definition, we have

$$\left({\mu_C}^C\right)^C(x) = (1 - {\mu_C}^C)^C$$

$$= 1 - (1 - \mu_C\,(x))$$

$$= 1 - 1 + \mu_C\,(x)$$

$$= \mu_C\,(x).$$

Here is the complete proof.

Proposition 2. Let μ_{C_1} and μ_{C_2} be two interval-valued fuzzy coeds. Then, the following outcomes are valid:

I. $(\mu_{C_1}(x) \sqcap \mu_{C_2}(y))^{\,C} = {\mu_{C_1}}^C(x) \sqcup \mu^C{}_{C_2}(y)$.

II. $(\mu_{C_1}(x) \sqcup \mu_{C_2}(y))^{\,C} = {\mu_{C_1}}^C(x) \sqcap \mu^C{}_{C_2}(y)$.

The proof is directly from the definition.

Example 2. From the above example, assume $0001111 \in F_2^7$. Then

$${\mu_C}^C(x) = 1 - \mu_C(x)$$

$$= [1 - \text{upb}\mu_C(x), 1 - \text{lb}\mu_C(x)],$$

$${\mu_C}^C\,(0001111) = 1 - \mu_C(0001111)$$

$$= [1 - \text{upb}\mu_C(0001111), 1 - \text{lb}\mu_C(0001111)]$$

$$= [1 - 0.8, 1 - 0.7] = [0.2, 0.3].$$

Hence, $[0.2, 0.3]$ is the complement of the interval-valued fuzzy code of $\mu_C(0001111)$.

Example 3. Let $0001111, 1001111 \in F_2^7$,

$$\mu_{C_1}(x) \sqcap \mu_{C_2}(y) = \inf[\mu_{C_1}(x),\ \mu_{C_2}(y)$$

$$= \left[\inf\left({\mu_{C_1}}^-(x),\ {\mu_{C_2}}^-(y)\right), \inf\left({\mu_{C_1}}^+(x),\ {\mu_{C_2}}^+(y)\right)\right],$$

$$\mu_{C_1}(0001111) \sqcap\ \mu_{C_2}(1001111) = \inf\left[\mu_{C_1}(0001111), \mu_{C_2}(1001111)\right].$$

But,

$$\mu_{C_1}(0001111) = \frac{4+5+6}{\frac{7}{2}(7+1)} = 0.78571,$$

and

$$\mu_{C_2}(1001111) = \frac{1+4+5+6+7}{\frac{7}{2}(7+1)} = 0.82143,$$

are fuzzy codes.

Now,

$$\mu_{C_1}(0001111) \sqcap \mu_{C_2}(1001111) = [\inf[0.7, 0.8], \qquad \inf[0.8, 0.9]] = [0.7, 0.8].$$

4 | Interval-Valued Fuzzy Soft Code

Definition 14. Let $K \sqsubseteq F_q^n$ be an initial universe, and $P(K)$ be the power set of K. Assume $\overline{E}$ be the parameter set, then a pair $(\overline{F}_\mu, \overline{E})$ is called an IVFSC over $P(K)$ if and only if $\overline{F}_\mu$ is a mapping given by

$$\overline{F}_\mu: \overline{E} \mapsto P(K) \times \text{int}([0,1]). \tag{20}$$

An IVFSC is a parameterized family of interval-valued fuzzy code subsets of all interval-valued fuzzy code subsets of K. i.e., $P(K)$. An IVFSC is a special case of the soft set. Since a mapping is from the parameter to $P(K)$.

$\overline{F}_\mu(e)$ is referred to as the IVFSC set of parameters e, for all $e \in \overline{E}$ and which can be written as

$$\overline{F}_\mu(e) = \left\{\left(x, \mu_{\overline{F}_\mu(e)}(x)\right): x \in K\right\}, \tag{21}$$

where $\mu_{\overline{F}_\mu(e)}(x) = [lb\mu_{\overline{F}_\mu(e)}(x), upb\mu_{\overline{F}_\mu(e)}(x)]$.

Here, *Eq. (21)* can be modified in the following ways:

$$\overline{F}_\mu(e) = \left\{\left(x, \left[lb\mu_{\overline{F}_\mu(e)}(x), \quad upb\mu_{\overline{F}_\mu(e)}(x)\right]\right): x \in K\right\}. \tag{22}$$

In other words, $\overline{F}_\mu(e)$ is the interval-valued fuzzy degree of membership every element x holds on parameter e. Thus, the interval-valued soft code $(\overline{F}_\mu, \overline{E})$ is the family

$$(\overline{F}_\mu, \overline{E}) = \left\{\left(e, \left\{\left(x, \left[lb\mu_{\overline{F}_\mu(e)}(x), \quad upb\mu_{\overline{F}_\mu(e)}(x)\right]\right)\right\}\right): e \in \overline{E}, x \in K\right\}.$$

If $e \in \overline{E}$, for all $x \in \mathbf{K}$, $lb\mu_{\overline{F}_\mu(e)}(x) = upb\mu_{\overline{F}_\mu(e)}(x)$, then $\overline{F}_\mu(e)$ will generate a standard fuzzy code subset and then $(\overline{F}_\mu, \overline{E})$ is generated to a standard fuzzy soft code.

Example 4. Let $K = \{001011, 10111, 111101, 111001, 100101, 101111, 101011, 010110\}$ be the set of universals. Let $\overline{E} = \{e_1, e_2, e_3\}$ be the set of parameters such that

$$\overline{F}_\mu(e_1) = \{(001011, \mu_{\overline{F}_\mu(e_1)}(001011)), (111101, \mu_{\overline{F}_\mu(e_1)}(111101)), (101011, \mu_{\overline{F}_\mu(e_1)}(101011)),$$
$$(101111, \mu_{\overline{F}_\mu(e_1)}(101111))\},$$

$$\overline{F}_\mu(e_2) = \{(010110, \mu_{\overline{F}_\mu(e_2)}(010110)), (101011, \mu_{\overline{F}_\mu(e_2)}(101011)), (111101, \mu_{\overline{F}_\mu(e_2)}(111101)),$$
$$(100101, \mu_{\overline{F}_\mu(e_2)}(100101))\},$$

$$\overline{F}_\mu(e_3) = \{(001011, \mu_{\overline{F}_\mu(e_3)}(001011)), (111001, \mu_{\overline{F}_\mu(e_3)}(111001)), (101111, \mu_{\overline{F}_\mu(e_3)}(101111)),$$
$$(111001, \mu_{\overline{F}_\mu(e_3)}(111001))\}.$$

Then

$\bar{F}_{\mu}(e_1)$
$= \{ (001011, [0.6, 0.7]), (111101, [0.7, 0.8]), (101011, [0.7, 0.8]), (101111, [0.9, 1])\},$

$\bar{F}_{\mu}(e_2)$
$= \{ (010110, [0.5, 0.6]), (101011, [0.7, 0.8]), (111101, [0.7, 0.8]), (100101, [0.5, 0.6])\},$

$\bar{F}_{\mu}(e_3)$
$= \{ (001011, [0.6, 0.7]), (111001, [0.5, 0.6]), (101111, [0.9, 1]), (111001, [0.5, 0.6])\}.$

Hence, the IVFSC $(\bar{F}_{\mu}, \bar{E})$ is given by

$$(\bar{F}_{\mu}, \bar{E}) = \{\bar{F}_{\mu}(e_1), \bar{F}_{\mu}(e_2), \bar{F}_{\mu}(e_3)\}.$$

Definition 15. Suppose that $(\bar{F}_{\mu}, \bar{E})$ be the interval-valued fuzzy code over $P(K)$. Let $\bar{F}_{\mu}(e)$ be the interval value set of parameter e; then all interval fuzzy value sets in IVFSCs $(\bar{F}_{\mu}, \bar{E})$ are referred to as the interval fuzzy value class of $(\bar{F}_{\mu}, \bar{E})$and is denoted by $\bar{C}_{(\bar{F}_{\mu}, \bar{E})}$, then we have

$$\bar{C}_{(\bar{F}_{\mu}, \bar{E})} = \{\bar{F}_{\mu}(e): e \in \bar{E}\}. \quad \textbf{(23)}$$

5 | Operations on Interval-Valued Fuzzy Soft Codes

Definition 16. The complement of the IVFSC $(\bar{F}_{\mu}, \bar{A})$ is denoted by $(\bar{F}_{\mu}, \bar{A})^C$ and is defined

$$(\bar{F}_{\mu}, \bar{E})^C = \left(\bar{F}_{\mu}{}^C, \quad : \neg\bar{A}\right), \quad \textbf{(24)}$$

where for all $e \in \bar{A}, \neg e = \text{not } e$ is the not set of the parameter e, which holds the opposite meanings of parameter e, and $\bar{F}_{\mu}{}^C$ is a mapping given by

$$\bar{F}_{\mu}{}^C: \neg\bar{A} \mapsto P(K). \quad \textbf{(25)}$$

Such that $\bar{F}_{\mu}{}^C(e) = (\bar{F}_{\mu}(\neg e))^C$, for all $e \in \neg\bar{A}$.

Proposition 3. Let $(\bar{F}_{\mu}, \bar{A})$ be the IVFSC over (K, E). Then

$$((\bar{F}_{\mu}, \bar{A})^C)^C = (\bar{F}_{\mu}, \bar{A}).$$

Proof: Since $(\bar{F}_{\mu}, \bar{E})^C = \left(\bar{F}_{\mu}{}^C, \ \neg\bar{A}\right)$. Then we have

$$((\bar{F}_{\mu}, \bar{A})^C)^C = \left(\bar{F}_{\mu}{}^C, \quad \neg\bar{A}\right)^C$$

$$= \left((\bar{F}_{\mu}{}^C)^C, \ \neg(\neg\bar{A})\right)$$

$$= (\bar{F}_{\mu}, \bar{A}).$$

Definition 17. The $''\wedge''$ or the" MEEt" operator on the two IVFSC $(\bar{F}_{\mu}, \bar{A})$and $(\bar{G}, \bar{B})$ is denoted by $(\bar{F}_{\mu}, \bar{A})\text{MEET}(\bar{G}_{\delta}, \bar{B})$ or we can rewrite it as $(\bar{F}_{\mu}, \bar{A}) \wedge (\bar{G}_{\delta}, \bar{B})$and defined as follows:

$$(\bar{F}_{\mu}, \bar{A}) \wedge (\bar{G}_{\delta}, \bar{B}) = \bar{F}_{\mu}(e_i) \sqcap \bar{G}_{\delta}(\nabla_j) = \min\{\bar{F}_{\mu}(e_i), \bar{G}_{\delta}(\nabla_j), \text{for all } i, j\}, \quad \textbf{(26)}$$

where,

$$\bar{F}_{\mu}(e_i) \sqcap \bar{G}_{\delta}(\nabla_j) = \left(\bar{H}_{\pi}, (e_i, \nabla_j)\right).$$

Definition 18. The $''\vee''$ or the "JOIN" operator on the two IVFSC $(\bar{F}_\mu, \bar{A})$ and $(\bar{G}_\delta, \bar{B})$ is denoted by $(\bar{F}_\mu, \bar{A})\text{JOIN}(\bar{G}_\delta, \bar{B})$ or we can rewrite it as $(\bar{F}_\mu, \bar{A}) \vee (\bar{G}_\delta, \bar{B})$ and defined as follows:

$$(\bar{F}_\mu, \bar{A}) \vee (\bar{G}_\delta, \widetilde{B}) = \bar{F}_\mu(e_i) \sqcup \bar{G}_\delta(\nabla_j) = \max\{\bar{F}_\mu(e_i), \bar{G}_\delta(\nabla_j), \text{for all i, j}\}, \quad \textbf{(27)}$$

where,

$$\bar{F}_\mu(e_i) \sqcup \bar{G}_\delta(\nabla_j) = \left(\bar{H}_\pi, (e_i, \nabla_j)\right).$$

Example 5. Consider there are three Houses $K = \{h_1, h_2, h_3\}$ in F_2^5, $C = \{0100, 0110, 0101\}$ and think about the collection of parameters such that the length of their strings according to a certain task is described by $E_1 = \{e_1, e_2, e_3\} = \{011, 10110, 0111010\}$ and $E_2 = \{\nabla_1, \nabla_2, \nabla_3\} = \{0101, 11110, 0111011\}$. Assume a business wants to buy a single House of this type based on just two parameters.

Let the IVFSCs $(\bar{F}_\mu, \bar{A})$ and $(\bar{G}_\delta, \widetilde{B})$, respectively are as follows:

$$\bar{F}_\mu(e_1) = \left(0.83, \quad \left\{\frac{h_1}{[0.1,0.2]}, \frac{h_2}{[0.5,0.6]}, \frac{h_3}{[0.3,0.4]}\right\}\right),$$

$$\bar{F}_\mu(e_2) = \left(0.53, \quad \left\{\frac{h_1}{[0,0.1]}, \frac{h_2}{[0.2,0.3]}, \frac{h_3}{[0.4,0.5]}\right\}\right),$$

$$\bar{F}_\mu(e_3) = \left(0.54, \quad \left\{\frac{h_1}{[0.5,0.6]}, \frac{h_2}{[0.8,0.9]}, \frac{h_3}{[0.7,0.8]}\right\}\right).$$

And

$$\bar{G}_\delta(\nabla_1) = \left(0.6, \quad \left\{\frac{h_1}{[0.3,0.4]}, \frac{h_2}{[0.2,0.3]}, \frac{h_3}{[0.4,0.5]}\right\}\right),$$

$$\bar{G}_\delta(\nabla_2) = \left(0.67, \quad \left\{\frac{h_1}{[0.4,0.5]}, \frac{h_2}{[0.6,0.7]}, \frac{h_3}{[0.8,0.9]}\right\}\right),$$

$$\bar{G}_\delta(\nabla_3) = \left(0.82, \quad \left\{\frac{h_1}{[0.1,0.2]}, \frac{h_2}{[0.5,0.6]}, \frac{h_3}{[0.3,0.4]}\right\}\right).$$

To find the $''\wedge''$ between the two IVFSCs over (K, E) as follows:

$(\bar{F}_\mu, \bar{A}) \wedge (\bar{G}_\delta, \bar{B}) = (\bar{H}_\pi, \bar{A} \times \bar{B})$, where $(\bar{H}_\pi, \bar{A} \times \bar{B})$, such that

$$\bar{H}_\pi, (e_1, \nabla_1) = \left(0.6, \quad \left\{\frac{h_1}{[0.1,0.2]}, \frac{h_2}{[0.2,0.3]}, \frac{h_3}{[0.3,0.4]}\right\}\right),$$

$$\bar{H}_\pi, (e_1, \nabla_2) = \left(0.67, \quad \left\{\frac{h_1}{[0.1,0.2]}, \frac{h_2}{[0.5,0.6]}, \frac{h_3}{[0.3,0.4]}\right\}\right),$$

$$\bar{H}_\pi, (e_1, \nabla_3) = \left(0.82, \quad \left\{\frac{h_1}{[0.1,0.2]}, \frac{h_2}{[0.5,0.6]}, \frac{h_3}{[0.3,0.4]}\right\}\right),$$

$$\bar{H}_\pi, (e_2, \nabla_1) = \left(0.53, \quad \left\{\frac{h_1}{[0,0.1]}, \frac{h_2}{[0.2,0.3]}, \frac{h_3}{[0.4,0.5]}\right\}\right),$$

$$\bar{H}_\pi, (e_2, \nabla_2) = \left(0.53, \quad \left\{\frac{h_1}{[0,0.1]}, \frac{h_2}{[0.2,0.3]}, \frac{h_3}{[0.4,0.5]}\right\}\right),$$

$$\overline{H}_{\pi}, (e_2, \nabla_3) = \left(0.53, \quad \left\{\frac{h_1}{[0,0.1]}, \frac{h_2}{[0.2,0.3]}, \frac{h_3}{[0.3,0.4]}\right\}\right),$$

$$\overline{H}_{\pi}, (e_3, \nabla_1) = \left(0.54, \quad \left\{\frac{h_1}{[0.3,0.4]}, \frac{h_2}{[0.2,0.3]}, \frac{h_3}{[0.4,0.5]}\right\}\right),$$

$$\overline{H}_{\pi}, (e_3, \nabla_2) = \left(0.54, \quad \left\{\frac{h_1}{[0.4,0.5]}, \frac{h_2}{[0.6,0.7]}, \frac{h_3}{[0.7,0.8]}\right\}\right),$$

$$\overline{H}_{\pi}, (e_3, \nabla_2) = \left(0.54, \quad \left\{\frac{h_1}{[0.1,0.2]}, \frac{h_2}{[0.5,0.6]}, \frac{h_3}{[0.3,0.4]}\right\}\right).$$

Table 1. Matrix.

$\overline{H}_{\pi}$	π	h_1	h_2	h_3
(e_1, ∇_1)	0.6	[0.1, 0.2]	[0.2,0.3]	[0.3,0.4]
(e_1, ∇_2)	0.67	[0.1, 0.2]	[0.5,0.6]	[0.3,0.4
(e_1, ∇_3)	0.82	[0.1, 0.2]	[0.5,0.6]	[0.3,0.4]
(e_2, ∇_1)	0.53	[0, 0.1]	[0.2,0.3]	[0.4,0.5]
(e_2, ∇_2)	0.53	[0, 0.1]	[0.2,0.3]	[0.4,0.5]
(e_2, ∇_3)	0.53	[0, 0.1]	[0.2,0.3]	[0.3,0.4]
(e_3, ∇_1)	0.54	[0.3, 0.4]	[0.2,0.3]	[0.4,0.5]
(e_3, ∇_2)	0.54	[0.4, 0.5]	[0.5,0.6]	[0.7,0.8]
(e_3, ∇_3)	0.54	[0.1, 0.2]	[0.5,0.6]	[0.3,0.4]

Now, for each $N \in P$, first, we determine the statistical grade. $R_N \in P(h_i)$ to identify the optimal house.

$$R_N \in P(h_i) = \sum\left[\left(h^{-}{}_{i} - h^{-}{}_{j}\right) + \left(h^{+}{}_{i} - h^{+}{}_{j}\right)\right]. \quad \textbf{(28)}$$

Table 2. Numerical grade $N_g \in P(c_i)$.

	N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8	N_9
x_1	−0.6	−1.2	−1.2	−1.2	−1.2	−1	(0)	−0.8	−1.2
x_2	0	(1.2)	(1.2)	0	0	(0.2)	−0.6	−0.2	(1.2)
x_3	(0.6)	0	0	(1.2)	(1.2)	(0.8)	(0.6)	(1)	0
π	0.6	0.67	0.82	0.53	0.53	0.53	0.54	0.54	0.54

The outcome is displayed in *Tables 1* and *2*.

Let $P = \{N_1 = (e_1, \nabla_1), N_2 = (e_1, \nabla_2), \ldots, N_9 = (e_3, \nabla_3)\}$.

We now record the best possible score in numbers, denoted by parentheses, in each row, with the exception of the final row, which represents the House's grade of such belongingness in relation to each set of parameters. Currently, all of these numerical grades' items combined with the House value are used to calculate each machine's result. The House that is desired is the one with the greatest result. Since both of the parameters are identical, we do not take into account the machine's numerical grades against the pair $(e_i, \nabla_j), i = 1,2,3, j = 1,2,3$.

$$\text{Result}(h_1) = 0,$$

$$\text{Result}(h_2) = (1.2 * 0.67) + (1.2 * 0.82) + (0.2 * 0.53) + (1.2 * 0.54) = 2.542,$$

$$\text{Result}(h_3) = (0.6 * 0.6) + (1.2 * 0.53) + (1.2 * 0.53) + (0.8 * 0.53) + (0.6 * 0.54) + (1 * 0.54) = 2.92.$$

The company will choose the House with the best outcome. They will, therefore, purchase House h_3.

Theorem 4 (De Morgan's law). Let $(\bar{F}_\mu, \bar{A})$ and $(\bar{G}_\delta, \bar{B})$ be two IVFSCs, and then the following properties are held:

$$((\bar{F}_\mu, \bar{A}) \wedge (\bar{G}_\delta, \bar{B}))^C = (\bar{F}_\mu, \bar{A})^C \vee (\bar{G}_\delta, \bar{B})^C.$$

$$((\bar{F}_\mu, \bar{A}) \vee (\bar{G}_\delta, \bar{B}))^C = (\bar{F}_\mu, \bar{A})^C \wedge (\bar{G}_\delta, \bar{B})^C.$$

Proof:

I. Let $(\bar{F}_\mu, \bar{A})$ and $(\bar{G}_\delta, \bar{B})$ be two IVFSCs, then we have

$$(\bar{F}_\mu, \bar{A})^C \vee (\bar{G}_\delta, \bar{B})^C = \left(\bar{F}_\mu{}^C, \neg\bar{A}\right) \vee \left(\bar{G}_\delta{}^C, \neg\bar{B}\right),$$

$$(\bar{H}_\pi, \neg\bar{A} \times \neg\bar{B}),$$

where

$$\bar{H}_\pi(\neg\epsilon, \neg\varepsilon) = \bar{F}_\mu{}^C(\epsilon) \sqcup \bar{G}_\delta{}^C(\varepsilon)$$

$$= \left(\bar{H}_\pi, \neg(\bar{A} \times \bar{B})\right).$$

Assume that

$$(\bar{F}_\mu, \bar{A}) \wedge (\bar{G}_\delta, \bar{B}) = (\bar{H}_\pi, \bar{A} \times \bar{B}).$$

Then we have

$$((\bar{F}_\mu, \bar{A}) \wedge (\bar{G}_\delta, \bar{B}))^C = (\bar{H}_\pi, \bar{A} \times \bar{B})^C$$

$$= \left(\bar{H}_\pi{}^C, \neg(\bar{A} \times \bar{B})\right), \text{for all } (\epsilon, \varepsilon) \in \bar{A} \times \bar{B}.$$

$$\bar{H}_\pi{}^C(\neg\epsilon, \neg\varepsilon) = (\bar{H}_\pi(\epsilon, \varepsilon))^C$$

$$= (\bar{F}_\mu(\epsilon) \sqcap \bar{G}_\delta(\varepsilon))^C$$

$$= (\bar{F}_\mu(\epsilon))^C \sqcup (\bar{G}_\delta(\varepsilon))^C$$

$$= \bar{F}_\mu{}^C(\neg\epsilon) \sqcup \bar{G}_\delta{}^C(\neg\varepsilon)$$

$$= \left(\bar{H}_\pi, \neg(\bar{A} \times \bar{B})\right).$$

Thus, from our discussion, we get the result. To prove the second one, we use the same fashion.

6 | Conclusion

In this study, we used an interval-valued fuzzy soft set to introduce the idea of IVFSC and explore some of its properties. The operations complement, intersection, and union have been defined on the IVFSC. This study is used when making decisions. This study will serve as an introduction for future researchers working in this field. The study of interval-valued fuzzy soft linear codes, interval-valued fuzzy soft cyclic codes, interval-valued generalized fuzzy soft codes, and other related topics is possible in future research.

Author Contribution

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Data Availability

All data analyzed during this study are included in the text.

Conflict of Interest

The authors declared no conflict of interest or common interest.

References

[1] Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, *8*(3), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X

[2] Arif, B. A. L., Çuvalcioğlu, G., & Altinci, C. (2023). (α, β)-interval valued intuitionistic fuzzy sets defined on (A, B)-interval valued set. *Journal of universal mathematics*, *6*(1), 114–130.

[3] Molodtsov, D. (1999). Soft set theory—first results. *Computers & mathematics with applications*, *37*(4-5), 19–31. https://doi.org/10.1016/S0898-1221(99)00056-5

[4] Maji, P. K., Biswas, R., & Roy, A. R. (2003). Soft set theory. *Computers and mathematics with applications*, *45*(4–5), 555–562. DOI:10.1016/S0898-1221(03)00016-6

[5] Kong, Z., Wang, L., & Wu, Z. (2011). Application of fuzzy soft set in decision making problems based on grey theory. *Journal of computational and applied mathematics*, *236*(6), 1521–1530.

[6] Cağman, N., Enginoğlu, S., & Citak, F. (2011). Fuzzy soft set theory and its applications. *Iranian journal of fuzzy systems*, *8*(3), 137–147.

[7] Roy, A. R., & Maji, P. K. (2007). A fuzzy soft set theoretic approach to decision making problems. *Journal of computational and applied mathematics*, *203*(2), 412–418. DOI:10.1016/j.cam.2006.04.008

[8] Gogoi, K., Kr. Dutta, A., & Chutia, C. (2014). Application of fuzzy soft set theory in day to day problems. *International journal of computer applications*, *85*(7), 27–31. DOI:10.5120/14854-3221

[9] Ali, M., Khan, H., Son, L. H., Smarandache, F., & Kandasamy, W. B. V. (2018). New soft set based class of linear algebraic codes. *Symmetry*, *10*(10), 510. DOI:10.3390/sym10100510

[10] Smarandache, F. (2016). New class of soft linear algebraic codes and their properties using soft sets. *University of new mexico digital repository*. https://digitalrepository.unm.edu/math_fsp/396/

[11] Bustince, H. (2010). Interval-valued fuzzy sets in soft computing. *International journal of computational intelligence systems*, *3*(2), 215–222. DOI:10.2991/ijcis.2010.3.2.9

[12] Gehrke, M., Walker, C., & Walker, E. (2001). Some basic theory of interval-valued fuzzy sets. *Proceedings joint 9th IFSA world congress and 20th NAFIPS international conference (Cat. No. 01TH8569)* (Vol. 3, pp. 1332–1336). IEEE. DOI: 10.1109/nafips.2001.943741

[13] Hong, Z., & Qin, K. (2014). Algebraic structure of fuzzy soft sets. *Rough sets and knowledge technology: 9th international conference, RSKT 2014, Shanghai, China, October 24-26, 2014, proceedings* (pp. 569–576). Springer International Publishing. https://doi.org/10.1007/978-3-319-11740-9_52

[14] Yang, X., Lin, T. Y., Yang, J., Li, Y., & Yu, D. (2009). Combination of interval-valued fuzzy set and soft set. *Computers and mathematics with applications*, *58*(3), 521–527. DOI:10.1016/j.camwa.2009.04.019

[15] Alkhazaleh, S., & Salleh, A. R. (2012). Generalised interval-valued fuzzy soft set. *Journal of applied mathematics*, *2012*. https://doi.org/10.1155/2012/870504

[16] Jiang, Y., Tang, Y., Chen, Q., Liu, H., & Tang, J. (2010). Interval-valued intuitionistic fuzzy soft sets and their properties. *Computers and mathematics with applications*, *60*(3), 906–918. DOI:10.1016/j.camwa.2010.05.036

[17] Mohamed, A. awad, Abualigah, L., Alburaikan, A., & Khalifa, H. A. E. W. (2023). AOEHO: a new hybrid data replication method in fog computing for iot application. *Sensors*, *23*(4), 2189. DOI:10.3390/s23042189

[18] Ali, W., Shaheen, T., Haq, I. U., Toor, H. G., Alballa, T., & Khalifa, H. A. E. W. (2023). A novel interval-valued decision theoretic rough set model with intuitionistic fuzzy numbers based on power aggregation operators and their application in medical diagnosis. *Mathematics*, *11*(19), 4153. DOI:10.3390/math11194153

[19] Ali, W., Shaheen, T., Toor, H. G., Alballa, T., Alburaikan, A., & Khalifa, H. A. E. W. (2023). An improved intuitionistic fuzzy decision-theoretic rough set model and its application. *Axioms*, *12*(11), 1003. DOI:10.3390/axioms12111003

[20] Kane, L., Bado, H., Diakite, M., Konate, M., Kane, S., & Traore, K. (2021). Solving semi-fully fuzzy linear programming problems. *International journal of research in industrial engineering*, *10*(3), 251–275.

[21] Sheikhi, A., & Ebadi, M. J. (In Press). An efficient method for solving linear interval fractional transportation problems. *Journal of applied research on industrial engineering*. https://www.journal-aprie.com/article_181720.html

[22] Chetia, B., & Das, P. K. (2010). An application of interval-valued fuzzy soft sets in medical diagnosis. *International journal of contemporary mathematical sciences*, *5*(38), 1887–1894.

[23] Soltanifar, M. (2024). A new interval for ranking alternatives in multi attribute decision making problems. *Journal of applied research on industrial engineering*, *11*(1), 37–56. https://www.journal-aprie.com/article_158340.html