Journal of Fuzzy Extension and Applications

www.journal-fea.com

J. Fuzzy. Ext. Appl. Vol. 5, No. 1 (2024) 60–68.

Paper Type: Research Paper

Profitable Portfolio Using Fermatean Fuzzy Numbers

Amal Kumar Adak[1,*], Manish Kumar Gunjan[2]

[1] Department of Mathematics, Ganesh Dutt College, Begusarai, India; amaladak17@gmail.com.
[2] Department of Mathematics, Lalit Narayan Mithila University, Darbhanga, India; mgunjanmaths@gmail.com.

Citation:

Received: 04 August 2023 *Revised: 07 October 2023* *Accepted: 08 December 2023*	Adak, A. K., & Gunjan, M. K. (2023). Profitable portfolio using fermatean fuzzy numbers. *Journal of fuzzy extension and applications, 5*(1), 60-68.

Abstract

Stock portfolio problems are one of the most relevant real-world problems. In this study, we discuss the portfolio's risk amount, rate of risk-return, and expected return rate under a Fermatean fuzzy environment. A linear programming problem is used to formulate a Fermatean fuzzy portfolio. The Fermatean fuzzy portfolio is converted to a deterministic form using the score function. Lingo software is used to solve these deterministic portfolio problems. The main feature of this model is that investors can select a risk coefficient to enhance predicted returns and customize their strategies according to their circumstances. An example is offered that illustrates the effectiveness and dependability of the proposed approach.

Keywords: Portfolio problems, Fermatean fuzzy sets, Score function, Optimization problem.

1 | Introduction

Information uncertainty is one of the invertible characteristics of dealing with decision-making problems. This uncertainty usually originates from decision-makers' opinions and expressions.

Various methods exist for defining and quantifying the uncertainty associated with information. To address practical issues like economic risk management, researchers have shown a lot of enthusiasm for the Fuzzy Sets (FSs) theory that Zadeh [1] proposed. This theory allows us to interpret and handle unpredictability in decision support systems effectively. Fuzzy values or constraints can be employed to account for the imprecise characteristics of financial market activity; merely assigning a membership value is not always sufficient for the purpose of decision-making.

To address indecision in the human mind, FSs lack the capability to resolve this predicament. Atanassov [2] introduced the concept of Intuitionistic Fuzzy Sets (IFSs) to provide a clearer definition of hesitations, which are a significant extension of FSs. In this technique, membership and non-membership levels are utilized to represent ambiguity and perception while ensuring that their sum lies in the unit interval [0,1]. The primary benefit of IFSs is their ability to handle uncertainty resulting from insufficient knowledge. Its ability to manage uncertainty has led to its success in several domains.

Corresponding Author: amaladak17@gmail.com
https://doi.org/10.22105/jfea.2024.428028.1338

IFSs are unable to illustrate circumstances in which they fail to meet limitations. Pythagorean Fuzzy Sets (PFSs) were introduced as a solution to overcome this limitation of intuitionistic FSs. Yager [3] were the first to introduce PFSs, a relaxation of IFSs. He imposed the condition that the sum of the squares of the membership and non-membership grades should be between 0 and 1. Looking at IFSs and PFSs side by side, you can see that PFSs are better at showing uncertainty because they have a wider range of membership and non-membership degrees.

While PFSs provide a generalization of IFSs, they are unable to convey the decision information mentioned below. Given a degree of membership of 0.7 and a degree of non-membership of 0.8, it is evident that it does not satisfy the conditions of IFSs. Again, it does not satisfy the conditions of PFSs. IFSs and PFSs are unable to represent this situation. To illustrate this situation, Senapati and Yager [4] derived the notion of Fermatean Fuzzy Sets (FFSs). FFSs have a membership degree and a non-membership degree that must satisfy the condition that the sum of the cubes of the grades of membership and non-membership lie between 0 and 1. In the above scenarios, the sum of the cubes of 0.7 and 0.8 is less than 1. The membership space of FFSs is larger than that of PFSs and IFSs (refer to *Fig. 1*). Several investigations have been conducted on IFSs, PFSs, and FFSs in recent years. Abbasi Shureshjani and Shakouri [5] invented a novel parametric ranking method for intuitionistic fuzzy numbers. Zeb et al. [6] discussed aggregation operators of Pythagorean fuzzy bi-polar soft sets with application in multiple attribute decision-making. Sahoo [7] presented a new score function-based Fermatean fuzzy transportation problem.

Since the precise return of any security can not be predicted, much of the data is uncertain. Jing et al. [8] presented the optimal selection of stock portfolios using multi-criteria decision-making methods. Goldfarb et al. [9] introduced robust portfolio selection problems. Khalifa and Kumar [10] solved a fully neutrosophic linear programming problem with application to stock portfolio selection. Liu and Qin [11] investigated the mean semi-absolute deviation model for an uncertain portfolio optimization problem. Markowitz [12] developed the concepts of optimal portfolios and proposed the mean-variance models. Simamora et al. [13] present a fuzzy portfolio considering stock returns and downside risk. Since all securities returns are linear constants, the portfolio selection problem is often an LPP. Rasoulzadeh et al. [14] invented Markowitz and DEA cross-efficiency models for intuitionistic fuzzy portfolio selection problems. Saberhoseini et al. [15] discussed some results on choosing the best private sector partner according to the risk factors in a neutrosophic environment. Sardou et al. [16] presented results on optimal portfolio selection using the fuzzy genetic method. A multi-period mean-variance portfolio was demonstrated by [17]. Yin [18] applies linear programming problems in stock portfolio optimization. Several investigations into portfolio selection have been conducted in recent years.

The subsequent section outlines the organization of the remaining content in the article. Section 2 comprehensively examines the fundamental concepts and terminology related to FSs, including intuitionistic FSs, PFSs, and FFSs. The formulation and solution techniques for portfolio issues are presented in Section 3. Section 4 should showcase a collection of challenges in a portfolio that effectively employs the suggested technique. The conclusion is presented in Section 5.

2 | Basic Concepts

This section explores the interconnection and dedication of FSs, IFSs, and PFSs. Some operations on FFSs have been discussed.

Definition 1 ([1]). The FS F on X, the universal set is laid out as

$$F = \{\langle \xi, \mu_F(\xi) \rangle : \xi \in X\},$$

where $\mu_F(\xi)$ indicates fuzzy membership levels and that assigns the values in between 0 and 1.

Supplement of μ is stipulated as $\bar{\mu}(\xi) = 1 - \mu(\xi)$ for each $\xi \in X$ and is denoted by $\bar{\mu}$.

Definition 2 ([2]). In X, an IFS I is outlined as

$$I = \{\langle \xi, \alpha_I(\xi), \beta_I(\xi) \rangle : \xi \in X\},$$

where the $\alpha_A(\xi)$ indicates membership value and $\beta_A(\xi)$ indicates non-membership value of $\xi \in X$ respectively.

Also, α_I and β_I assigns values between 0 and 1. These fulfils $0 \le \alpha_I(\xi) + \beta_I(\xi) \le 1$, for every $\xi \in X$.

$h_I(\xi) = 1 - \alpha_I(\xi) - \beta_I(\xi)$ is known as the indeterminacy degree. In some circumferences, for whatever reason, $0 \le \alpha(\xi) + \beta(\xi) \le 1$ this may not be the hold. We take some situations, where $\alpha = 0.7$ and $\beta = 0.5$ such that $0.7 + 0.5 = 1.2 > 1$, but $0.7^2 + 0.5^2 < 1$. Again, if $\alpha = 0.6$ and $\beta = 0.6$ where $0.6 + 0.6 = 1.2 > 1$, but $0.6^2 + 0.6^2 < 1$. To deal with this situation, Yager [19] proposed the perspective assumption of the PFS.

Definition 3 ([19]). In the universe of discourse X, a PFS P is specified as

$$P = \{\langle \xi, \mu_P(\xi), v_P(\xi) \rangle : \xi \in X\}.$$

$\mu_P(\xi) : X \to [0,1]$ refers to membership value and $v_P(\xi) : X \to [0,1]$ refers to the value to which the element $\xi \in X$ is not a member of the set P, that satisfies

$$0 \le (\mu_P(\xi))^2 + (v_P(\xi))^2 \le 1, \text{ for all } \xi \in X.$$

$h_P(\xi) = \sqrt{1 - (\mu_P(\xi))^2 - (v_P(\xi))^2}$ indicates indeterminacy.

In practice, the condition $0 \le \mu^2(\xi) + v^2(\xi) \le 1$ may not be true for any reason. For example, if we consider $u = 0.9$, $v = 0.6$ where $0.9^2 + 0.6^2 = 1.17 > 1$, but $0.9^3 + 0.6^3 = 0.945 < 1$. Again $0.8^2 + 0.7^2 = 1.13 > 1$, but $0.8^3 + 0.7^3 = 0.855 < 1$. The concept of FFSs was introduced to address this issue.

Definition 4 ([6]). A FFS A in X is defined as

$$A = \{\langle \xi, u_A(\xi), v_A(\xi) \rangle \mid \xi \in X\}.$$

$u_A(\xi) : X \to [0,1]$ signifies the membership value and $v_A(\xi) : X \to [0,1]$ stands for the non-membership value to which the element $\xi \in X$ is not a member of the set A, with the condition that

$$0 \le (u_A(\xi))^3 + (v_A(\xi))^3 \le 1.$$

For every $\xi \in X$, $h_A(\xi) = \sqrt[3]{1 - (u_A(\xi))^3 - (v_A(\xi))^3}$ corresponds to the degree of indeterminacy.

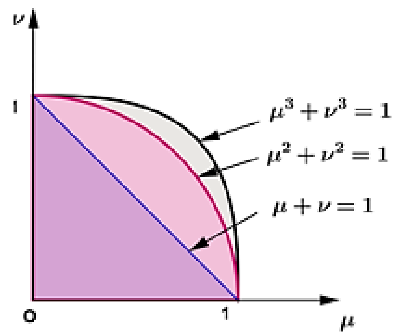

Fig. 1. Spaces for IFSs, PFSs, FFSs.

2.1 | Some Operations of FFSs

Let A_1 and A_2 be two FFSs, then the following operations and relations can be defined as:

I. $A_1 \subseteq A_2$ iff $(u_{A_1}(\xi) \le u_{A_2}(\xi))$ and $(v_{A_1}(\xi) \ge v_{A_2}(\xi))$ (for all $\xi \in X$).

II. $A_1 = A_2$ iff $(u_{A_1}(\xi) = u_{A_2}(\xi))$ and $(v_{A_1}(\xi) = v_{A_2}(\xi))$ (for all $\xi \in X$).

III. $A_1 \cap A_2 = \{\langle \xi, \min(u_{A_1}(\xi), u_{A_2}(\xi)), \max(v_{A_1}(\xi), v_{A_2}(\xi))\rangle : \xi \in X\}$.

IV. $A_1 \cup A_2 = \{\langle \xi, \max(u_{A_1}(\xi), u_{A_2}(\xi)), \min(v_{A_1}(\xi), v_{A_2}(\xi))\rangle : \xi \in X\}$.

The score function is used to rank the FFSs.

Definition 5 ([4]). The Score function of an FFS $A = (u_A, v_A)$ is denoted by $\text{score}(A)$ and is defined as $\text{score}(A) = u_A^3 - v_A^3$.

It is obvious that the value of the score function lies in the interval $[-1,1]$.

In particular, if $A = (1,0)$ then $\text{score}(A) = 1$ and for $A = (0,1)$, $\text{score}(A) = -1$. Otherwise, its value is in the open interval $(-1,1)$.

Definition 6. Let $A_1 = (u_{A_1}, v_{A_1})$ and $A_2 = (u_{A_2}, v_{A_2})$ be two FFSs, then

I. If $\text{score}(A_1) < \text{score}(A_2)$ then $A_1 < A_2$.

II. If $\text{score}(A_1) > \text{score}(A_2)$ then $A_1 > A_2$.

III. If $\text{score}(A_1) = \text{score}(A_2)$ then $A_1 = A_2$.

It is seen that the score function is not enough to rank the FFSs. Sometimes, it is impossible to rank the FFSs using the score function, e.g., let $A_1 = (0.56, 0.56)$ and $A_2 = (0.44, 0.44)$ be two FFSs. Then, $\text{score}(A_1) = 0 = \text{score}(A_2)$, although $A_1 \ne A_2$.

In this situation, the accuracy function is used for rank FFSs, which is defined as follows.

Definition 7. The accuracy function of an FFS $A = (u_A, v_A)$ is denoted by $\text{accr}(A)$ and is defined as $\text{accr}(A) = u_A^3 + v_A^3$. It is obvious that the value of $\text{accr}(A)$ will be in $[0,1]$.

Using the score function and accuracy function on FFSs, the ranking of two FFSs is as follows.

Definition 8. Let $A_1 = (u_{A_1}, v_{A_1})$ and $A_2 = (u_{A_2}, v_{A_2})$ be two FFSs, then

I. If $\text{score}(A_1) < \text{score}(A_2)$, then $A_1 < A_2$.

II. If $\text{score}(A_1) > \text{score}(A_2)$, then $A_1 > A_2$.

III. If $\text{score}(A_1) = \text{score}(A_2)$, then

- *If* $\text{accr}(A_1) < \text{accr}(A_2)$, *then* $A_1 < A_2$.
- *If* $\text{accr}(A_1) > \text{accr}(A_2)$, *then* $A_1 > A_2$.
- *If* $\text{accr}(A_1) = \text{accr}(A_2)$, *then* $A_1 = A_2$.

3 | Portfolio Problems

Portfolios are very sensitive to investing scenarios. Slight changes in the environment may affect the expected return. This section discussed assumptions and notions used to formulate a portfolio model. We formulate a portfolio in a Fermatean fuzzy environment by using the concept of linear programming problems and present the solution procedure of the proposed model.

3.1 | Assumptions

Given the high sensitivity of the investment climate, even small changes can significantly affect portfolio choices. To enhance the clarity of the problem formulation, we made the following assumptions:

I. The securities analysis involves utilizing both the predicted return rate and the loss risk rate.

II. Securities can be distinguished from one another and possess uncertain values.

III. Transactions do not necessitate any payment.

IV. Investors prioritize the avoidance of risk and dissatisfaction above all else.

V. The bank's interest rate remains constant throughout the investment period.

VI. Risk security comprises nine distinct components.

VII. Participation in short sales is prohibited.

Notations

α_0 : Interest rates at banks.

α_n : Rates of return to be anticipated, $n = 1:k$.

B_{nm} : Returns at risk, $n = 1:k$, $m = 1:j$.

ξ_0 : The overall proportion of investments made over the investing terms.

ξ_n : The proportion of investments financed by secondary securities, expressed as $n = 1:k$.

T : Total return expected.

β : Expenditures portfolio risk factor.

V : The highest possible score for all safety risks.

3.2 | Proposed Model

The proposed portfolio problem is formulated utilizing the concept of linear programming problems in a Fermatean fuzzy environment. The expected return of this model has been displayed as

$$T = \sum_{n=0}^{k} \alpha_n \xi_n .$$

Those who want to invest always try to minimize risk in securities while maximizing investment interest. The market risk is represented by the portfolio risk coefficient, which is the ratio of the average market risk to the risk of the stock portfolio. The total amount associated with all securities risks is indicated as

$$V = \max\{B_1\xi_1, B_2\xi_2, \ldots, B_k\xi_k\}.$$

Utilizing the classical linear programming, the proposed model is formulated as follows:

$$\max T = \sum_{n=0}^{k} \alpha_n \xi_n,$$
$$\text{subject to} \begin{cases} B\xi \le \beta, \\ \sum_{n=0}^{k} \xi_n = 1, \\ \xi_n \ge 0, n = 1:k. \end{cases} \quad (1)$$

To be more generic and adaptable, we consider α_n, β_n and B_n as Fermatean Fuzzy Numbers (FFNs). So, we construct the following model as

$$\max \tilde{T} = \alpha_0 \xi_0 + \sum_{n=1}^{k} \tilde{\alpha}_n \xi_n,$$
$$\text{subject to} \begin{cases} \tilde{B}\xi \le \tilde{\beta}, \\ \sum_{n=0}^{k} \xi_n = 1, \\ \xi_m \ge 0, m = 1:j, n = 1:k. \end{cases} \quad (2)$$

3.3 | Solution Procedure for Proposed Portfolio Problems

In this section, the solution procedure for the portfolio problem involves FFNs in expected return rates, risk loss rates, and risk coefficients.

$\tilde{B} = \left(\tilde{B}_{nm}\right)_{k\times j}$, $\tilde{\beta} = \left(\tilde{\beta}_1, \tilde{\beta}_2, \ldots, \tilde{\beta}_j\right)$, $\tilde{\alpha} = \left(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_j\right)$ and $\xi = \left(\xi_1, \xi_2, \ldots, \xi_j\right)^T$.

Then

$$\max \tilde{T} = \alpha_0 \xi_0 + \sum_{n=1}^{k} \tilde{\alpha}_n \xi_n,$$
$$\text{subject to} \begin{cases} \tilde{B}_{nm}\xi_m \le \tilde{\beta}_n, \\ \sum_{m=0}^{k} \xi_m = 1, \\ \xi_m \ge 0, m = 1:j, n = 1:k. \end{cases} \quad (3)$$

By applying the score function, *Model (3)* can be transformed into a deterministic linear programming program, which can be solved by Lingo software.

$$\max \tilde{T} = \alpha_0 \xi_0 + \sum_{n=1}^{k} S(\tilde{\alpha}_n) \xi_n,$$
$$\text{subject to} \begin{cases} S(\tilde{B}_{nm})\xi_m \le S(\tilde{\beta}_n), \\ \sum_{m=0}^{k} \xi_m = 1, \\ \xi_m \ge 0, m = 1:j, n = 1:k. \end{cases} \quad (4)$$

4 | Illustrative Example

This section highlights an example to demonstrate the proposed portfolio model in a Fermatean fuzzy environment. Suppose a person wants to give his preference for security as [0.8, 0.3], a FFN. It means that the investor may accept the degree to which the particular security satisfaction grade is 0.8 and the dissatisfaction grade is 0.3. In this problem, we consider five available equities, with the first stock being a bank savings that earns an annual rate of return of $\alpha_0 = 7\%$.

Tables 1-3 present the data for the remaining four equalities, which are FFNs, with each having a membership grade and a non-membership grade, also satisfying the condition of FFSs.

Table 1. Risked loss rate %.

$\tilde{B}_{nm}$	Risk loss rate
$\tilde{B}_{11}$	$\langle 0.6, 0.3\rangle$
$\tilde{B}_{12}$	$\langle 0.5, 0.4\rangle$
$\tilde{B}_{13}$	$\langle 0.7, 0.2\rangle$
$\tilde{B}_{14}$	$\langle 0.5, 0.3\rangle$
$\tilde{B}_{21}$	$\langle 0.6, 0.4\rangle$
$\tilde{B}_{22}$	$\langle 0.8, 0.5\rangle$
$\tilde{B}_{23}$	$\langle 0.6, 0.4\rangle$
$\tilde{B}_{24}$	$\langle 0.7, 0.4\rangle$

Table 2. Risked loss rate.

$\tilde{\beta}_n$	Risk coefficient
$\tilde{\beta}_1$	$\langle 0.5, 0.3\rangle$
$\tilde{\beta}_2$	$\langle 0.6, 0.4\rangle$

Table 3. Expected rate return.

Stocks	$\tilde{\alpha}$
$\tilde{\alpha}_1$	$\langle 0.7, 0.4\rangle$
$\tilde{\alpha}_2$	$\langle 0.6, 0.4\rangle$
$\tilde{\alpha}_3$	$\langle 0.8, 0.3\rangle$
$\tilde{\alpha}_4$	$\langle 0.7, 0.3\rangle$

The problem can be formulated using the following model:

$$\max \tilde{T} = \alpha_0\xi_0 + \sum_{n=1}^{k} S(\tilde{\alpha}_n)\xi_n,$$
$$\text{subject to} \begin{cases} S(\tilde{B}_{11})\xi_1 + S(\tilde{B}_{12})\xi_1 + S(\tilde{B}_{13})\xi_1 + S(\tilde{B}_{14})\xi_1 \le S(\tilde{\beta}_1), \\ S(\tilde{B}_{21})\xi_1 + S(\tilde{B}_{22})\xi_1 + S(\tilde{B}_{23})\xi_1 + S(\tilde{B}_{24})\xi_1 \le S(\tilde{\beta}_2), \\ \xi_0 + \xi_1 + \xi_2 + \xi_3 + \xi_4 = 1, \\ \xi_m \ge 0, m = 0:4. \end{cases} \tag{5}$$

Applying the score function, the deterministic portfolio problem is as follows:

$$\max \tilde{T} = 0.07\xi_0 + 0.33\xi_1 + 0.20\xi_2 + 0.55\xi_3 + 0.40\xi_4,$$
$$\text{subject to} \begin{cases} 0.27\xi_1 + 0.09\xi_2 + 0.45\xi_3 + 0.16\xi_4 \le 0.16, \\ 0.20\xi_1 + 0.16\xi_2 + 0.20\xi_3 + 0.33\xi_4 \le 0.20, \\ \xi_0 + \xi_1 + \xi_2 + \xi_3 + \xi_4 = 1, \\ \xi_m \ge 0, m = 0:4. \end{cases} \tag{6}$$

Lingo software is used to solve converted deterministic portfolio problems.

The optimal solution is

$$\xi_0 = 0.25, \xi_1 = 0.0, \xi_2 = 0.28, \xi_3 = 0.0, \xi_4 = 0.47.$$

The optimal value $\tilde{T} = 31\%$.

The results of the study indicate that 25% of the entire funds in the bank are invested at an interest rate of 7%, 28% of the whole capital is invested in the S_2 security, and 47% of the total capital may be invested in the S_4 security, which is considered the most favorable investment based on the given facts. Under the premise that risk coefficients $\tilde{\beta}_1$ and $\tilde{\beta}_2$ are present, this technique results in a maximum projected return of 31%.

5 | Conclusion

A portfolio focuses on implementing an optimal investment strategy for investors that gives a maximum return with minimum risks. This paper introduces a formulation of the stock portfolio that takes into account FFNs in the risky return rate, portfolio risk amount, and expected turn rate. To form a portfolio, the concept of a linear programming problem is used, as it involves the task of optimizing a linear objective function in addition to several constraints. First, to construct the solution procedure, convert a Fermatean fuzzy portfolio to a deterministic form using the score function. Then, Lingo software is employed to obtain the optimal solution. The proposed method is demonstrated using a numerical example of Fermatean fuzzy portfolios. The main feature of this study is that investors can select risk-efficient strategies to enhance predicted returns and customize their strategies according to their personal views. Investigation of different fuzzy structures, such as interval-valued FSs, neutrosophic sets, and spherical FSs, can be conducted in the future in conjunction with in-depth discussions and suggestions.

Acknowledgments

The author would like to express their thanks to the referees for their valuable suggestions and helpful comments in improving this paper.

Author Contribution

All authors contributed equally to writing this article. All authors read and approved the final manuscript.

Funding

This section indicates any support not included in the Author Contribution or Funding sections.

Conflicts of Interest

The authors declare no conflict of interest.

References

[1] Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, *8*(3), 338–353.

[2] Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy sets and systems*, *20*(1), 87–96. DOI:10.1016/S0165-0114(86)80034-3

[3] Yager, R. R. (2014). Pythagorean membership grades in multicriteria decision making. *IEEE transactions on fuzzy systems*, *22*(4), 958–965. DOI:10.1109/TFUZZ.2013.2278989

[4] Senapati, T., & Yager, R. R. (2020). Fermatean fuzzy sets. *Journal of ambient intelligence and humanized computing*, *11*(2), 663–674. DOI:10.1007/s12652-019-01377-0

[5] Abbasi Shureshjani, R., & Shakouri, B. (2021). A comment on "A novel parametric ranking method for intuitionistic fuzzy numbers." *Big data and computing visions*, *1*(3), 156–160.

[6] Zeb, A., Khan, A., Fayaz, M., & Izhar, M. (2022). Aggregation operators of Pythagorean fuzzy bi-polar soft sets with application in multiple attribute decision making. *Granular computing*, *7*(4), 931–950. DOI:10.1007/s41066-021-00307-w

[7] Sahoo, L. (2021). A new score function based Fermatean fuzzy transportation problem. *Results in control and optimization*, *4*, 100040. https://doi.org/10.1016/j.rico.2021.100040

[8] Jing, D., Imeni, M., Edalatpanah, S. A., Alburaikan, A., & Khalifa, H. A. E. W. (2023). Optimal selection of stock portfolios using multi-criteria decision-making methods. *Mathematics*, *11*(2), 415. DOI:10.3390/math11020415

[9] Goldfarb, D., & Iyengar, G. (2003). Robust portfolio selection problems. *Mathematics of operations research*, *28*(1), 1–38. DOI:10.1287/moor.28.1.1.14260

[10] Khalifa, H. A. E. W., & Kumar, P. (2020). Solving fully neutrosophic linear programming problem with application to stock portfolio selection. *Croatian operational research review*, *11*(2), 165–176. DOI:10.17535/CRORR.2020.0014

[11] Liu, Y., & Qin, Z. (2012). Mean semi-absolute deviation model for uncertain portfolio optimization problem. *Journal of uncertain systems*, *6*(4), 299–307.

[12] Markowitz, H. M. (1952). Portfolio selection. *the journal of finance, 7*(1), 71–91.

[13] Simamora, I., & Sashanti, R. (2016). Optimization of fuzzy portfolio considering stock returns and downside risk. *International journal of science and research (IJSR)*, *5*(4), 141–145. DOI:10.21275/v5i4.nov162491

[14] Rasoulzadeh, M., Edalatpanah, S. A., Fallah, M., & Najafi, S. E. (2022). A multi-objective approach based on markowitz and dea cross-efficiency models for the intuitionistic fuzzy portfolio selection problem. *Decision making: applications in management and engineering*, *5*(2), 241–259. DOI:10.31181/dmame0324062022e

[15] Saberhoseini, S. F., Edalatpanah, S. A., & Sorourkhah, A. (2022). Choosing the best private-sector partner according to the risk factors in neutrosophic environment. *Big data and computing visions*, *2*(2), 61–68. DOI:10.22105/bdcv.2022.334005.1075

[16] Sardou, I. G., Nazari, A., Ghodsi, E., & Bagherzadeh, E. (2015). Optimal portfolio selection using multi-objective fuzzy-genetic method. *International journal*, *3*(2), 99–103.

[17] Wu, H., & Li, Z. (2011). Multi-period mean-variance portfolio selection with Markov regime switching and uncertain time-horizon. *Journal of systems science and complexity*, *24*(1), 140–155. DOI:10.1007/s11424-011-9184-z

[18] Yin, D. (2018). Application of interval valued fuzzy linear programming for stock portfolio optimization. *Applied mathematics*, *09*(02), 101–113. DOI:10.4236/am.2018.92007

[19] Yager, R. R. (2013). Pythagorean fuzzy subsets. *Proceedings of the 2013 joint IFSA world congress and NAFIPS annual meeting, IFSA/NAFIPS 2013* (pp. 57–61). IEEE. DOI: 10.1109/IFSA-NAFIPS.2013.6608375