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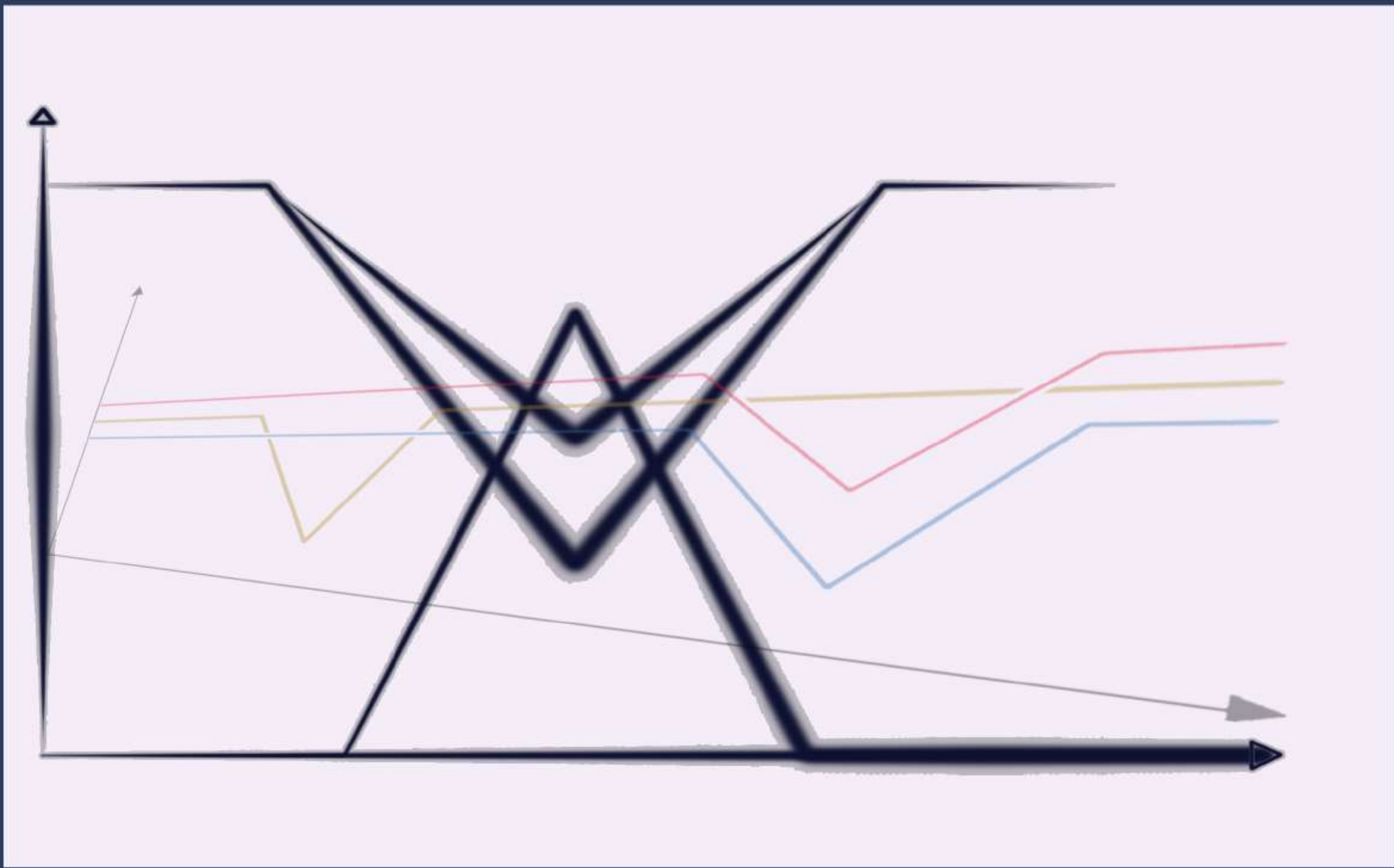

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A Fuzzy Multi Objective Inventory Model of Demand Dependent Deterioration Including Lead Time

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Abstract

In this article, we have developed a deteriorated multi-item inventory model in a fuzzy environment. Here the demand rate is constant. Production cost and set-up cost are the most vital issue in the inventory system of the market world. Here production cost and set-up- cost are continuous functions of demand. Set-up-cost is also dependent on average inventory level. Deterioration cost is the most challenging issue in the business world. So here deterioration cost is dependent on inventory level and demand. Lead time crashing cost is considered the continuous function of leading time. In the real world all cost are not fixed. Due to uncertainty all cost parameters of the proposed model are taken as Generalized Triangular Fuzzy Number (GTFN). The formulated multi objective inventory problem has been solved by various techniques like as Geometric Programming (GP) technique, Fuzzy Programming Technique with Hyperbolic Membership Function (FPTHMF), Fuzzy Non-Linear Programming (FNLP) technique. Numerical example is taken to illustrate the model. Sensitivity analysis and graphical representation have been shown to test the parameters of the model.

Keywords: Inventory, Deterioration, Multi-item, Leading time, Generalized triangular fuzzy number, Fuzzy and GP techniques.

1 | Introduction

Inventory models deal with decisions that minimize the total average cost or maximize the total average profit. In that way to construct a real life mathematical inventory model on base on various assumptions and notations and approximation. The basic well known Economic Order Quantity (EOQ) model was first introduced by Harri [19]. In the business management systems deterioration is a most important key factor. In general, deterioration is defined as the damage, spoilage, dryness, vaporization etc., that results in decrease of usefulness of the original. A deteriorating model that is a model for exponentially decaying inventories was first introduced by Chare and Schrader [16]. Afterword many researchers have developed inventory models with deteriorating items.

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Aggarwal [2] studied a note on an order-level inventory model for a system with constant rate of deterioration. Dave and Patel [10] developed (T. Si) policy inventory for deteriorating items with time proportional demand. Dave [11] developed a order level inventory model for deteriorating items with variable instantaneous demand and discrete opportunities for replacement. Chen [8] studied an optimal determination of quality level, selling quantity and purchasing price for intermediate firms. Goyal and Giri [17] presented recent trends in modeling of deteriorating inventory. Wee et al. [33] considered a multi-objective joint replenishment inventory model of deteriorated items in a fuzzy environment. Tripathi et al. [32] discussed an inventory model with exponential time-dependent demand rate, variable deterioration, shortages and production cost. Shaikh et al. [31] established a fuzzy inventory model for a deteriorating item with variable demand, permissible delay in payments and partial backlogging with Shortage Follows Inventory (SFI) policy. Das and Islamn [13] proposed two warehouse inventory model for deteriorating items and Stock dependent demand under conditionally permissible delay in payment. Chakraborty et al. [7] derived two-warehouse partial backlogging inventory model with ramp type demand rate, three-parameter Weibull distribution deterioration under inflation and permissible delay in payments. Panda et al. [26] explored a credit policy approach in a two-warehouse inventory model for deteriorating items with price-and stock-dependent demand under partial backlogging.

In the real situation of the business world many inventory control model involves the deterministic lead times. Ben-Daya and Raouf [4] studied an inventory model involve the lead-time as a decision variable. Hariga and Ben-Daya [18] developed some stochastic inventory model with deterministic variable lead time. Chuang et al. [9] presented a note on periodic review inventory model with controllable setup cost and lead time. Ouyang [24] considered a mixture inventory model with backorders and lost sales for variable lead time also in [25] developed a min-max distribution free procedure for mixed inventory model with variable lead time. Sarkar et al. [29] established an integrated inventory model with variable lead time, defective units and delay in payments. Sarkar et al. [30] discussed a quality improvement and backorder price discount under controllable lead time in an inventory model. Multi items are used to fulfill customers demands as well as to increase the maximum profit of business men. Kotb and Fergany [22] developed a multi-item EOQ model with both demand dependet unit cost and varying lead time. Abou-El-Ata and Kotb [1] used a multi-item EOQ inventory model with varying holding cost under two restrictions. Saha and Sen [36] published a paper on inventory model with negative exponential demand and probabilistic deterioration under backlogging. Das and Islam [15] considered multi-objective two echelon supply chain inventory model with customer demand dependent purchase cost and production rate dependent production cost. Das [37] developed multi item inventory model include lead time with demand dependent production cost and set-up-cost in fuzzy environment.

The concept of fuzzy set theory was first introduced by Zadeh [34]. Afterward Zimmermann [35] applied the fuzzy set theory concept with some useful membership functions to solve the linear programming problem with some objective functions. Bit [3] studied fuzzy programming with hyperbolic membership functions for multi objective capacitated transportation problem. Maiti [23] considered fuzzy inventory model with two warehouse under possibility measure in fuzzy goal. Also Geometric Programming (GP) is a powerful optimization technique developed to solve a class of non-linear optimization programming problems especially found in engineering design and manufacturing. Multi objective GP techniques are also interesting in the EOQ model. GP was introduced by Duffin et al. [12]. Beightler and Phillips [5] applied GP. Biswal [6] considered fuzzy programming technique to solve multi- objective GP problems. Das et al. [14] established a multi-item inventory model with quantity dependent inventory costs and demand-dependent unit cost under imprecise objective and restrictions in GP approach. Mandal et al. [27] presented a multi-objective fuzzy inventory model with three constraints in a GP approach also used this method in [28] developed an inventory model of deteriorating items with a constraint. Islam [21] solved multi-objective marketing planning inventory model. Islam [20] discussed multi-objective geometric-programming problem and its application. Barman et al. [39] developed an analysis of retailer's inventory in a two-echelon centralized supply chain co-ordination under price-sensitive demand.

In this paper, we have developed a deteriorated multi-item inventory model in a fuzzy environment. Here Production cost, set-up- cost and deterioration cost are continuous functions of constant demand. Set-up-cost and deterioration costs are also dependent on average inventory level. Lead time crashing cost is considered the continuous function of leading time. Due to uncertainty all cost parameters of the proposed model are taken as Generalized Triangular Fuzzy Number (GTFN). The formulated multi objective inventory problem has been solved by various techniques like as GP, FPTHMF, and FNLP. Numerical example is taken to illustrate the model. Sensitivity analysis and graphical representation have been shown to test the feasibility of the model for various parameters of the model. Finally conclusions have been drowning.

2 | Mathematical Model

2.1 | Notations

h_i: Holding cost per unit per unit time for ith item.

T_i: The length of cycle time for i^{th}item, $T_i > 0$.

D_i: Demand rate per unit time for the ith item.

L_i: Length of leading time for the ith item.

SS_i: Safety stock for the ith item.

$I_i(t)$: Inventory level of the ith item at time t.

Q_i: The order quantity for the duration of a cycle of length T_i for ith item.

$TAC_i(D_i, Q_i, L_i)$: Total average profit per unit for the ith item.

w_i: Storage space per unit time for the ith item.

W: Total area of space.

k: Safety factor.

θ_i: Constant deterioration rate for the ith item.

$\widetilde{w}_i$: Fuzzy storage space per unit time for the ith item.

$\widetilde{h}_i$: Fuzzy holding cost per unit per unit time for the ith item.

$\widetilde{TAC}_i(D_i, Q_i, L_i)$: Fuzzy total average cost per unit for the ith item.

$\widehat{w}_i$: Defuzzyfication of the fuzzy number $\widetilde{w}_i$.

$\widehat{h}_i$: Defuzzyfication of the fuzzy number $\widetilde{h}_i$.

$\widehat{TAC}_i(D_i, Q_i, L_i)$: Defuzzyfication of the fuzzy number $\widetilde{TAC}_i(D_i, Q_i, L_i)$.

2.2 | Assumptions

I. Multi-item is considered.
II. The replenishment rate is instantaneously and infinite.
III. The lead time is allowed.
IV. Shortage is not considered.
V. The unit production cost C_p^i of i^{th} item is inversely related to the demand rate D_i. So we take the following form $C_p^i(D_i) = \alpha_i D_i^{-\beta_i}$, where $\alpha_i > 0, \beta_i > 1$ are real constant.
VI. The set up cost is related to the average inventory level as well as demand. So we take the form $S_c{}^i(Q_i, D_i) = \gamma_i \left(\frac{Q_i}{2}\right)^{\delta_i} D_i^{\sigma_i}$ where $0 < \gamma_i, 0 < \delta_i \ll 1$ & $0 < \sigma_i \ll 1$ are real constant.
VII. Lead time crashing cost is related to the lead time by a function of the form $R^i(L_i) = \rho_i L_i{}^{-\tau_i}$, where $\rho_i > 0$ and $0 < \tau_i \leq 0.5$ are real constant.
VIII. $SS_i = k\omega\sqrt{L_i}$.

The deterioration cost is proporsonaly related to the average inventory level. So we take the

$$\theta_c{}^i(Q) = \upsilon_i \left(\frac{Q_i}{2}\right)^{\phi_i} D_i{}^{-\varphi_i}.$$

form where$0 < \upsilon_i \ll 1, \varphi_i > 1$ and $0 < \phi_i \ll 1$ are constant real numbers.

2.3 | Formulation of the Model

The inventory situation for i^{th} item is illustrated in *Fig. 1*. During the period the inventory level decreases due to demand rate and the deterioration rate. In the time interval $[0, T_i]$ the satisfying differential equation is

$$\frac{dI_i(t)}{dt} + \theta_i I_i(t) = -D_i, 0 \leq t \leq T_i\ . \tag{1}$$

With boundary conditions, $I_i(0) = Q_i$, $I_i(T_i) = 0$.

Solves the *Eq (1)* using the boundary conditions we have,

$$I_i(t) = e^{-\theta_i t}\left(\frac{D_i}{\theta_i} + Q_i\right) - \frac{D_i}{\theta_i},\ 0 \leq t \leq T_i,$$

$$= Q_i - (D_i + Q_i\theta_i)t \text{ (Neglected square and higher power of } \theta_i \text{ since } \theta_i \ll 1). \tag{2}$$

$$T_i = \frac{Q_i}{(D_i + Q_i\theta_i)}. \tag{3}$$

Now various average costs for i^{th} item are

I. Average production cost $(PC_i) = \frac{Q_i C_p^i(D_i)}{T_i} = \alpha_i\left(D_i^{(1-\beta_i)} + Q_i\theta_i D_i{}^{-\beta_i}\right)$.
II. Average inventory holding cost $(HC_i) = \frac{1}{T_i}\int_0^{T_i} h_i I_i(t)dt + h_i k\omega\sqrt{L_i} = h_i\left(\frac{Q_i}{2} + k\omega\sqrt{L_i}\right)$.
III. Average set-up-cost $(SC_i) = \frac{1}{T_i}\left[\gamma_i\left(\frac{Q_i}{2}\right)^{\delta_i} D_i^{\sigma_i}\right] = \frac{\gamma_i Q_i{}^{\delta_i - 1}\left(D_i^{\sigma_i+1} + Q_i\theta_i D_i{}^{\sigma_i}\right)}{2^{\delta_i}}$.
IV. Average lead time crashing cost $(CC_i) = \frac{\rho_i L_i{}^{-\tau_i}}{T_i} = \frac{\rho_i L_i{}^{-\tau_i}(D_i + Q_i\theta_i)}{Q_i}$.

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V. Average deterioration cost $(DC_i) = \theta_i\, \upsilon_i \left(\frac{Q_i}{2}\right)^{\phi_i} D_i^{-\varphi_i}$.

Total average cost per unit time for ith item is

$$\mathrm{TAC_i}(\mathrm{D_i}, \mathrm{Q_i}, \mathrm{L_i}) = (\mathrm{PC_i} + \mathrm{HC_i} + \mathrm{SC_i} + \mathrm{CC_i} + \mathrm{DC_i}) = \alpha_i\left(\mathrm{D_i}^{(1-\beta_i)} + \mathrm{Q_i}\theta_i\mathrm{D_i}^{-\beta_i}\right) + \mathrm{h_i}\left(\frac{\mathrm{Q_i}}{2} + \mathrm{k}\omega\sqrt{\mathrm{L_i}}\right) + \frac{\gamma_i \mathrm{Q_i}^{\delta_i - 1}\left(\mathrm{D_i^{\sigma_i+1}} + \mathrm{Q_i}\theta_i\mathrm{D_i}^{\sigma_i}\right)}{2^{\delta_i}} + \frac{\rho_i \mathrm{L_i}^{-\tau_i}\,(\mathrm{D_i} + \mathrm{Q_i}\theta_i)}{\mathrm{Q_i}} + \theta_i\, \upsilon_i \left(\frac{\mathrm{Q_i}}{2}\right)^{\phi_i} \mathrm{D_i}^{-\varphi_i}. \tag{4}$$

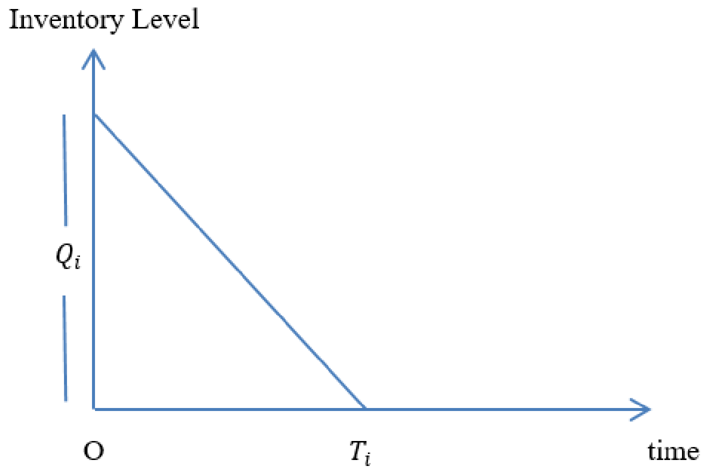

Fig. 1. For the ith item.

So our Multi-Objective Inventory Model (MOIM) is defined as:

$$\mathrm{Min}\ \{\mathrm{TAC_1}, \mathrm{TAC_2}, \mathrm{TAC_3}, \dots \dots \dots \dots \dots, \mathrm{TAC_n}\},$$

$$\mathrm{TAC_i}(\mathrm{D_i}, \mathrm{Q_i}, \mathrm{L_i}) = \alpha_i\left(\mathrm{D_i}^{(1-\beta_i)} + \mathrm{Q_i}\theta_i\mathrm{D_i}^{-\beta_i}\right) + \mathrm{h_i}\left(\frac{\mathrm{Q_i}}{2} + \mathrm{k}\omega\sqrt{\mathrm{L_i}}\right) + \frac{\gamma_i \mathrm{Q_i}^{\delta_i - 1}\left(\mathrm{D_i^{\sigma_i+1}} + \mathrm{Q_i}\theta_i\mathrm{D_i}^{\sigma_i}\right)}{2^{\delta_i}} + \frac{\rho_i \mathrm{L_i}^{-\tau_i}\,(\mathrm{D_i} + \mathrm{Q_i}\theta_i)}{\mathrm{Q_i}} + \theta_i\, \upsilon_i \left(\frac{\mathrm{Q_i}}{2}\right)^{\phi_i} \mathrm{D_i}^{-\varphi_i}, \tag{5}$$

$$\text{Subject to, } \mathrm{D_i} > 0, \mathrm{Q_i} > 0, \mathrm{L_i} > 0, \text{ for } \mathrm{i} = 1, 2, \dots \dots \dots \dots \mathrm{n}.$$

2.4 | Fuzzy Model

For uncertainty, all the cost parameters of the model are taken as GTFN. The GTFN are assume as

$$\widetilde{\alpha_i} = \left(\alpha_i^{1}, \alpha_i^{2}, \alpha_i^{3}; \psi_{\alpha_i}\right), 0 < \psi_{\alpha_i} \le 1; \widetilde{h_i} = \left(h_i^{1}, h_i^{2}, h_i^{3}; \psi_{h_i}\right), 0 < \psi_{h_i} \le 1,$$

$$\widetilde{\beta_i} = \left(\beta_i^{1}, \beta_i^{2}, \beta_i^{3}; \varphi_{\beta_i}\right), 0 < \varphi_{\beta_i} \le 1; \widetilde{\rho_i} = \left(\rho_i^{1}, \rho_i^{1}, \rho_i^{1}; \psi_{\rho_i}\right), 0 < \psi_{\rho_i} \le 1,$$

$$\widetilde{\theta_i} = \left(\theta_i^{1}, \theta_i^{2}, \theta_i^{3}; \psi_{\theta_i}\right), 0 < \psi_{\theta_i} \le 1; \widetilde{\tau_i} = \left(\tau_i^{1}, \tau_i^{2}, \tau_i^{3}; \psi_{\tau_i}\right), 0 < \psi_{\tau_i} \le 1,$$

$$\widetilde{\gamma_i} = \left(\gamma_i^{1}, \gamma_i^{2}, \gamma_i^{3}; \psi_{\gamma_i}\right), 0 < \psi_{\gamma_i} \le 1; \widetilde{\upsilon_i} = \left(\upsilon_i^{1}, \upsilon_i^{2}, \upsilon_i^{3}; \psi_{\upsilon_i}\right), 0 < \psi_{\upsilon_i} \le 1,$$

$$\widetilde{\delta_i} = \left(\delta_i^{1}, \delta_i^{2}, \delta_i^{3}; \psi_{\delta_i}\right), 0 < \psi_{\delta_i} \le 1; \widetilde{\sigma_i} = \left(\sigma_i^{1}, \sigma_i^{2}, \sigma_i^{3}; \psi_{\sigma_i}\right), 0 < \psi_{\sigma_i} \le 1,$$

$\widetilde{\phi_i} = \left(\phi_i^{\,1}, \phi_i^{\,2}, \phi_i^{\,3}; \psi_{\phi_i}\right), 0 < \psi_{\phi_i} \le 1; \widetilde{\varphi_i} = \left(\varphi_i^{\,1}, \varphi_i^{\,2}, \varphi_i^{\,3}; \psi_{\varphi_i}\right), 0 < \psi_{\varphi_i} \le 1,$

For $i = 1,2, \dots \dots \dots, n.$

So our inventory *Model (5)* becomes the fuzzy model as

$$\text{Min} \quad \left\{\widetilde{TAC_1}, \widetilde{TAC_2}, \widetilde{TAC_3}, \dots \dots \dots \dots \dots \dots \dots, \widetilde{TAC_n}\right\},$$

$$\text{Subject to, } D_i > 0, Q_i > 0, L_i > 0, \text{ for } i = 1,2, \dots \dots \dots . . n.$$

$$\text{Where} \quad \widetilde{TAC_1(D_1, Q_1}, L_1) = \widetilde{\alpha_1}\left(D_i^{(1-\widetilde{\beta_1})} + Q_i\widetilde{\theta_1}D_i^{-\beta_i}\right) + \widetilde{h_1}\left(\frac{Q_i}{2} + k\omega\sqrt{L_i}\right) + \tag{6}$$

$$\frac{\widetilde{\gamma_1}Q_i^{\widetilde{\delta_1}-1}\left(D_i^{\widetilde{\sigma_1}+1} + Q_i\widetilde{\theta_1}D_i^{\widetilde{\sigma_1}}\right)}{2^{\widetilde{\delta_1}}} + \frac{\widetilde{\rho_1}L_i^{-\widetilde{\tau_1}}\left(D_i + Q_i\widetilde{\theta_1}\right)}{Q_i} + \widetilde{\theta_1}\,\widetilde{\upsilon_1}\left(\frac{Q_i}{2}\right)^{\widetilde{\phi_1}} D_i^{-\widetilde{\varphi_1}}.$$

It is positioned that ranking fuzzy number is very important in the fuzzy programming system. Bortolan and Degani [38] established a number of techniques of ranking fuzzy numbers. λ- integer method is used for defuzzification of fuzzy numbers. We know that approximated value of a GTFN $\tilde{A} = (a, b, c; \psi)$ is given by $\psi\left(\frac{a+2b+c}{4}\right)$ when $\lambda = 0.5$.

Therefore the fuzzy inventory *Model (6)* converted to

$$\text{Min} \left\{\widehat{TAC_1}, \widehat{TAC_2}, \widehat{TAC_3}, \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots, \widehat{TAC_n}\right\},$$

$$\text{Subject to, } D_i > 0, Q_i > 0, L_i > 0,$$

$$\text{Where } \widehat{TAC_1(D_1, Q_1}, L_1) = \widehat{\alpha_1}\left(D_i^{(1-\widehat{\beta_1})} + Q_i\widetilde{\theta_1}D_i^{-\widehat{\beta_1}}\right) + \widehat{h_1}\left(\frac{Q_i}{2} + k\omega\sqrt{L_i}\right) + \tag{7}$$

$$\frac{\widehat{\gamma_1}Q_i^{\widehat{\delta_1}-1}\left(D_i^{\widehat{\sigma_1}+1} + Q_i\widehat{\theta_1}D_i^{\widehat{\sigma_1}}\right)}{2^{\widehat{\delta_1}}} + \frac{\widehat{\rho_1}L_i^{-\widehat{\tau_1}}\left(D_i + Q_i\widehat{\theta_1}\right)}{Q_i} + \widehat{\theta_1}\widehat{\upsilon_1}\left(\frac{Q_i}{2}\right)^{\widehat{\phi_1}} D_i^{-\widehat{\varphi_1}} \quad \text{for } i = 1,2, \dots \dots \dots . . n.$$

3 | Fuzzy Programming Techniques to Solve MOIM

Solve the *MOIM (7)* as a single objective NLP using only one objective at a time and we ignoring the others. So we get the ideal solutions. Using the ideal solutions we have got prepared the pay-off matrix as follows:

$$\begin{pmatrix} & TAC_1(D_1, Q_1, L_1) & TAC_2(D_2, Q_2, L_2) & \dots & TAC_n(D_n, Q_n, L_n) \\ (D_1^1, Q_1^1, L_1^1) & TAC_1^*(D_1^1, Q_1^1, L_1^1) & TAC_2(D_1^1, Q_1^1, L_1^1) & \dots & TAC_n(D_1^1, Q_1^1, L_1^1) \\ (D_2^2, Q_2^2, L_2^2) & TAC_1(D_2^2, Q_2^2, L_2^2) & TAC_2^*(D_2^2, Q_2^2, L_2^2) & \dots & TAC_n(D_2^2, Q_2^2, L_2^2) \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ (D_n^n, Q_n^n, L_n^n) & TAC_1(D_n^n, Q_n^n, L_n^n) & TAC_2(D_n^n, Q_n^n, L_n^n) & \dots & TAC_n^*(D_n^n, Q_n^n, L_n^n) \end{pmatrix} \tag{8}$$

Let $U^k = max\{TAC_k\left(D_i^i, Q_i^i, L_i^i\right), i = 1,2, \dots, n\}$ $for\ k = 1,2, \dots, n$ and $L^k = TAC_k^*\left(D_k^k, Q_k^k, L_k^k\right) for\ k = 1,2, \dots, n.$

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Hence U^k, L^k are identified, $L^k \leq TAP_k(D_i^i, Q_i^i, L_i^i) \leq U^k$, $for\ i = 1,2, \dots, n\ ; k = 1,2, \dots, n$.

3.1 | Fuzzy Programming Technique Using Hyperbolic Membership Function (FPTHMF)

Now fuzzy non-linear hyperbolic membership functions $\mu_{TAC_k}^{H}\big(TAC_k(D_k, Q_k, L_k)\big)$ for the k[th] objective function $TAC_k(D_k, Q_k, L_k)$ respectively for $k = 1,2, \dots, n$ are defined as follows:

$$\mu_{\mathrm{TAC_k}}^{\mathrm{H}}\big(\mathrm{TAC_k(D_k, Q_k, L_k)}\big) = \frac{1}{2}\tanh\left(\left(\frac{\mathrm{U^k + L^k}}{2} - \mathrm{TAC_k(D_k, Q_k, L_k)}\right)\sigma_k\right) + \frac{1}{2}.$$

Where α_k is a parameter, $\sigma_k = \frac{3}{(U^k - L^k)/2} = \frac{6}{U^k - L^k}$.

Using the above membership function, fuzzy non-linear programming problem is formulated as follows:

$$\text{Max } \lambda$$

$$\text{Subject to } \frac{1}{2}\tanh\left(\left(\frac{\mathrm{U^k+L^k}}{2} - \mathrm{TAC_k(D_k, Q_k, L_k)}\right)\sigma_k\right) + \frac{1}{2} \geq \lambda \quad , \lambda \geq 0.$$

After simplification the above non-linear programming problem can be written as

$$\text{Max y}$$

$$\text{Subject to } \mathrm{y} + \sigma_k \mathrm{TAC_k(D_k, Q_k, L_k)} \leq \frac{\mathrm{U^k+L^k}}{2}\sigma_k, \ \mathrm{y} \geq 0, \mathrm{D_k} >, \mathrm{Q_k} > 0, \mathrm{L_k} > 0.$$

Now the above problem can be easily solved by suitable mathematical programming algorithm and then we shall get the solution of the *MOIM (7)*.

3.2 | Fuzzy Non-Linear Programming Technique (FNLP) Based on Max-Min Operator

In this technique fuzzy membership function $\mu_{TAC_k}\big(TAC_k(Q_k, D_k)\big)$ for the k[th] objective function $TAC_k(D_k, Q_k, L_k)$ respectively for $k = 1,2, \dots, n$ are defined as follows:

$$\mu_{\mathrm{TAC_k}}\big(\mathrm{TAC_k(D_k, Q_k, L_k)}\big) = \begin{cases} 1 & \text{for } \mathrm{TAC_k(D_k, Q_k, L_k)} < \mathrm{L^k} \\ \dfrac{\mathrm{U^k - TAC_k(D_k, Q_k, L_k)}}{\mathrm{U^k - L^k}} & \text{for } \mathrm{L^k} \leq \mathrm{TAC_k(D_k, Q_k, L_k)} \leq \mathrm{U^k}, \\ 0 & \text{for } \mathrm{TAC_k(D_k, Q_k, L_k)} > \mathrm{U^k} \end{cases}$$

$$\text{for } \mathrm{k} = 1,2, \dots, \mathrm{n}.$$

Using the above membership function, fuzzy non-linear programming problem is formulated as

$$\text{Max } \alpha'$$

$$\text{subject to } \mathrm{TAC_k(D_k, Q_k, L_k)} + \alpha'\big(\mathrm{U^k - L^k}\big) \leq \mathrm{U^k} \quad , \quad \text{for } \mathrm{k} = 1,2, \dots, \mathrm{n},$$

$$0 \leq \alpha' \leq 1, \ \mathrm{D_k} >, \mathrm{Q_k} > 0, \mathrm{L_k} > 0.$$

Now the above problem can be easily solved by suitable mathematical programming algorithm and then we shall get the solution of the *MOIM (7)*.

4 | Geometric Programming Technique

Let us consider a unconstrainted Multi Objective Geometric Programming (MOGP) problem is as follows

$$\text{Minimize } g_s(t) = \sum_{k=1}^{T_0} c_{sk} \prod_{j=1}^{m} t_j^{\alpha_{skj}},\ s = 1,2,3, \dots \dots \dots .., n,$$

$$\text{Subject to } t_j \geq 0, j = 1,2, \dots , m.$$

Where $c_{sk}(> 0)$ and α_{skj} $(j = 1,2, \dots .., m;\ k = 1,2, \dots . T_0\ ;\ s = 1,2,3, \dots \dots \dots .., n)$ are all real numbers. Now introducing the weights w_i $(i = 1,2,3, \dots \dots \dots .., n)$, the above MOGP converted into the single objective GP problem as following

Primal Problem:

$$\text{Minimize } g(t) = \sum_{s=1}^{n} w_s g_s(t),\ s = 1,2,3, \dots \dots \dots .., n,$$

$$\text{i.e} = \sum_{s=1}^{n} \sum_{k=1}^{T_0} w_s c_{sk} \prod_{j=1}^{m} t_j^{\alpha_{skj}},$$

$$\text{Subject to } t_j \geq 0, j = 1,2, \dots , m, \tag{9}$$

$$\sum_{i=1}^{n} w_i = 1, w_i > 0, i = 1,2,3, \dots \dots \dots .., n.$$

Let T be the total numbers of terms and m is the number of variables. Then the degree of the difficulty (DD) is $T - (m + 1)$.

Dual Program: The dual problem of *Eq. (9)* is given as follows:

$$\text{Maximize } v(\theta) = \prod_{s=1}^{n} \prod_{k=1}^{T_o} \left(\frac{w_s c_{sk}}{\theta_{sk}}\right)^{\theta_{sk}},$$

Subject to

$$\sum_{s=1}^{n} \sum_{k=1}^{T_0} \theta_{sk} = 1, \qquad \text{(Normality condition)}$$

$$\sum_{s=1}^{n} \sum_{k=1}^{T_{s0}} \alpha_{skj} \theta_{sk} = 0, \qquad \text{(Orthogonality conditions)}$$

$$\theta_{sk} > 0, \qquad \text{(Positivity conditions)}$$

$$(j = 1,2, \dots .., m;\ k = 1,2, \dots . T_o\ ;\ s = 1,2,3, \dots \dots \dots .., n).$$

Now here three cases may arises

Case I: $T_0 = m + 1$, (i.e. DD=0). So DP presents a system of linear equations for the dual variables. So we have a unique solution vector of dual variable.

Case II: $T_0 > m + 1$, So a system of linear equations is presented for the dual variables, where the number of linear equations is less than the number of dual variables. So it is concluded that dual variable vector has many solutions.

Case III: $T_0 < m + 1$, so a system of linear equations is presented for the dual variables, where the number of linear equations is greater than the number of dual variables. It is seen that generally no solution vector exists for the dual variables here.

4.1 | Solution Procedure of My Proposed Problem

Primal Problem:

$$\text{Minimize } TAC(D,Q,L) = \sum_{i=1}^{n} w_i \Big(\widehat{\alpha}_1 \left(D_i^{(1-\widehat{\beta}_1)} + Q_i\widetilde{\theta}_1 D_i^{-\widehat{\beta}_1}\right) + \widehat{h}_1\left(\frac{Q_i}{2} + k\omega\sqrt{L_i}\right) + \frac{\widehat{\gamma}_1 Q_i^{\widehat{\delta}_1-1}\left(D_i^{\widehat{\sigma}_1+1} + Q_i\widetilde{\theta}_1 D_i^{\widehat{\sigma}_1}\right)}{2^{\widetilde{\widehat{\delta}}_1}} + \frac{\widehat{\rho}_1 L_i^{-\widehat{\tau}_1}\left(D_i + Q_i\widetilde{\theta}_1\right)}{Q_i} + \widehat{\theta}_1\widehat{\upsilon}_1\left(\frac{Q_i}{2}\right)^{\widehat{\phi}_1} D_i^{-\widehat{\varphi}_1}\Big), \tag{10}$$

Subject to, $D_i > 0, Q_i > 0, L_i > 0,$

$\sum_{i=1}^{n} w_i = 1, w_i > 0, i = 1,2,3, \ldots \ldots \ldots .., n.$

Dual Program: The dual problem of the primal problem *Eq. (10)* is as follows:

$$\text{Maximize } v(\theta) = \prod_{i=1}^{n} \left(\frac{w_i\widehat{\alpha}_1}{\theta_{i1}}\right)^{\theta_{i1}} \left(\frac{w_i\widetilde{\theta}_1\widetilde{\alpha}_1}{\theta_{i2}}\right)^{\theta_{i2}} \left(\frac{w_i\widehat{h}_1}{2\theta_{i3}}\right)^{\theta_{i3}} \left(\frac{w_i k\omega\widehat{h}_1}{\theta_{i4}}\right)^{\theta_{i4}} \left(\frac{w_i\widehat{\gamma}_1}{2^{\widehat{\delta}_1}\theta_{i5}}\right)^{\theta_{i5}} \left(\frac{w_i\widehat{\gamma}_1\widetilde{\theta}_1}{2^{\widehat{\delta}_1}\theta_{i6}}\right)^{\theta_{i6}} \left(\frac{w_i\widehat{\rho}_1}{\theta_{i7}}\right)^{\theta_{i7}} \left(\frac{w_i\widehat{\rho}_1\widetilde{\theta}_1}{\theta_{i8}}\right)^{\theta_{i8}} \left(\frac{w_i\widehat{\theta}_1\widehat{\upsilon}_1}{2^{\widehat{\phi}_1}\theta_{i9}}\right)^{\theta_{i9}},$$

$$\begin{aligned} \text{Subject to } \quad & \theta_{i1} + \theta_{i2} + \theta_{i3} + \theta_{i4} + \theta_{i5} + \theta_{i6} + \theta_{i7} + \theta_{i8} + \theta_{i9} = 1, \\ & \left(1-\widehat{\beta}_1\right)\theta_{i1} - \widehat{\beta}_1\theta_{i2} + (\widehat{\sigma}_1+1)\theta_{i5} + \widehat{\sigma}_1\theta_{i6} + \theta_{i7} - \widehat{\varphi}_1\theta_{i9} = 0, \\ & \theta_{i2} + \theta_{i3} + \left(\widehat{\delta}_1 - 1\right)\theta_{i5} + \widehat{\delta}_1\theta_{i6} - \theta_{i7} + \widehat{\phi}_1\theta_{i9} = 0, \\ & \frac{\theta_{i4}}{2} - \widehat{\tau}_1(\theta_{i7} + \theta_{i8}) = 0, \\ & \sum_{i=1}^{n} w_i = 1, w_i > 0, \end{aligned} \tag{11}$$

$\theta_{i1}, \theta_{i2}, \theta_{i3}, \theta_{i4}, \theta_{i5}, \theta_{i6}, \theta_{i7}, \theta_{i8}, \theta_{i9} \geq 0$ for $i = 1,2,3, \ldots \ldots \ldots .., n.$

Solving the above linear equations we have

$$\theta_{i2} = \Big[-\widehat{\varphi}_1\theta_{i3} + \left\{\widehat{\phi}_1(\widehat{\sigma}_1+1) - \widehat{\varphi}_1\left(\widehat{\delta}_1 - 1\right)\right\}\theta_{i5} + \left(\widehat{\phi}_1\widehat{\sigma}_1 - \widehat{\delta}_1\widehat{\varphi}_1\right)\theta_{i6} + \left(\widehat{\varphi}_1 + \widehat{\phi}_1\right)\theta_{i7} + \left(1-\widehat{\beta}_1\right)\widehat{\phi}_1\theta_{i1}\Big]. \tag{12}$$

$$\begin{aligned} \theta_{i4} = 2\widehat{\tau}_1\theta_{i7} + \frac{2\widehat{\tau}_1}{(1+2\widehat{\tau}_1)}\Bigg[& 1 - \theta_{i1}\left\{1 + \frac{2\left(1-\widehat{\beta}_1\right)}{\left(\widehat{\beta}_1\widehat{\phi}_1 + \widehat{\varphi}_1\right)}\right\} - \frac{\left(\widehat{\beta}_1\widehat{\phi}_1 + \widehat{\beta}_1\right)}{\left(\widehat{\beta}_1\widehat{\phi}_1 + \widehat{\varphi}_1\right)}\theta_{i3} \\ & - \frac{\widehat{\phi}_1(\widehat{\sigma}_1+1) - \widehat{\varphi}_1\left(\widehat{\delta}_1 - 1\right) + \widehat{\beta}_1\widehat{\phi}_1 + \widehat{\varphi}_1 + \widehat{\beta}_1\left(\widehat{\delta}_1 - 1\right) + (\widehat{\sigma}_1 + 1)}{\left(\widehat{\beta}_1\widehat{\phi}_1 + \widehat{\varphi}_1\right)}\theta_{i5} \\ & - \frac{\widehat{\phi}_1\widehat{\delta}_1 - \widehat{\varphi}_1\widehat{\delta}_1 + \widehat{\beta}_1\widehat{\phi}_1 + \widehat{\varphi}_1 + \widehat{\beta}_1\widehat{\delta}_1 + \widehat{\sigma}_1}{\left(\widehat{\beta}_1\widehat{\phi}_1 + \widehat{\varphi}_1\right)}\theta_{i6} \\ & - \frac{2\widehat{\varphi}_1 + \widehat{\phi}_1 + \widehat{\beta}_1\widehat{\phi}_1 + 1 - \widehat{\beta}_1 + 2\widehat{\tau}_1\left(\widehat{\beta}_1\widehat{\phi}_1 + \widehat{\varphi}_1\right)}{\left(\widehat{\beta}_1\widehat{\phi}_1 + \widehat{\varphi}_1\right)}\theta_{i7}\Bigg]. \end{aligned} \tag{13}$$

$$\theta_{i8}=\frac{1}{(1+2\widehat{\tau}_1)}\left[1-\theta_{i1}\left\{1+\frac{2\left(1-\widehat{\beta}_1\right)}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}\right\}-\frac{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\beta}_1\right)}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}\theta_{i3}\right.$$
$$-\frac{\widehat{\phi}_1(\widehat{\sigma}_1+1)-\widehat{\varphi}_1\left(\widehat{\delta}_1-1\right)+\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1+\widehat{\beta}_1\left(\widehat{\delta}_1-1\right)+(\widehat{\sigma}_1+1)}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}\theta_{i5}$$
$$-\frac{\widehat{\phi}_1\widehat{\delta}_1-\widehat{\varphi}_1\widehat{\delta}_1+\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1+\widehat{\beta}_1\widehat{\delta}_1+\widehat{\sigma}_1}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}\theta_{i6}$$
$$\left.-\frac{2\widehat{\varphi}_1+\widehat{\phi}_1+\widehat{\beta}_1\widehat{\phi}_1+1-\widehat{\beta}_1+2\widehat{\tau}_1\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}\theta_{i7}\right]. \tag{14}$$

$$\theta_{i9}=\frac{1}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}\left[\left(1-\widehat{\beta}_1\right)\theta_{i1}+\widehat{\beta}_1\theta_{i3}+\left\{\widehat{\beta}_1\left(\widehat{\delta}_1-1\right)+(\widehat{\sigma}_1+1)\right\}\theta_{i5}+\left(\widehat{\beta}_1\widehat{\delta}_1+\widehat{\sigma}_1\right)\theta_{i6}\right.$$
$$\left.+\left(1-\widehat{\beta}_1\right)\theta_{i7}\right]. \tag{15}$$

Using *Eqs. (12)-(15)* the dual problem is converted into

$$\text{Maximize } v(x,y,z,s,t)=\prod_{i=1}^{n}\left(\frac{w_i\widehat{\alpha}_1}{x_i}\right)^{x_i}\left(\frac{w_i\widetilde{\theta}_1\widehat{\alpha}_1}{X_i}\right)^{X_i}\left(\frac{w_i\widehat{h}_1}{2y_i}\right)^{y_i}\left(\frac{w_i k\omega\widehat{h}_1}{Y_i}\right)^{Y_i}\left(\frac{w_i\widehat{\gamma}_1}{2^{\widehat{\delta}_1}z_i}\right)^{z_i}$$
$$\left(\frac{w_i\widehat{\gamma}_1\widetilde{\theta}_1}{2^{\widehat{\delta}_1}s_i}\right)^{s_i}\left(\frac{w_i\widehat{\rho}_1}{t_i}\right)^{t_i}\left(\frac{w_i\widehat{\rho}_1\widetilde{\theta}_1}{Z_i}\right)^{Z_i}\left(\frac{w_i\widehat{\theta}_1\widetilde{\upsilon}_1}{2^{\widehat{\phi}_1}S_i}\right)^{S_i}, \tag{16}$$

$\sum_{i=1}^{n} w_i=1, w_i, x_i, y_i, z_i, s_i, t_i \geq 0.$

Where

$$X_i=\left[-\widehat{\varphi}_1 y_i+\left\{\widehat{\phi}_1(\widehat{\sigma}_1+1)-\widehat{\varphi}_1\left(\widehat{\delta}_1-1\right)\right\}z_i+\left(\widehat{\phi}_1\widehat{\sigma}_1-\widehat{\delta}_1\widehat{\varphi}_1\right)s_i+\left(\widehat{\varphi}_1+\widehat{\phi}_1\right)t_i+\right.$$
$$\left.\left(1-\widehat{\beta}_1\right)x_i\right],\quad Y_i=2\widehat{\tau}_1 t+$$
$$\frac{2\widehat{\tau}_1}{(1+2\widehat{\tau}_1)}\left[1-x_i\left\{1+\frac{2\left(1-\widehat{\beta}_1\right)}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}\right\}-\frac{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\beta}_1\right)}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}y_i-\frac{\widehat{\phi}_1(\widehat{\sigma}_1+1)-\widehat{\varphi}_1\left(\widehat{\delta}_1-1\right)+\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1+\widehat{\beta}_1\left(\widehat{\delta}_1-1\right)+(\widehat{\sigma}_1+1)}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}z_i\right.$$
$$\left.-\frac{\widehat{\phi}_1\widehat{\delta}_1-\widehat{\varphi}_1\widehat{\delta}_1+\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1+\widehat{\beta}_1\widehat{\delta}_1+\widehat{\sigma}_1}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}s_i-\frac{2\widehat{\varphi}_1+\widehat{\phi}_1+\widehat{\beta}_1\widehat{\phi}_1+1-\widehat{\beta}_1+2\widehat{\tau}_1\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}t_i\right],$$

$$Z_i=\frac{1}{(1+2\widehat{\tau}_1)}\left[1-x_i\left\{1+\frac{2\left(1-\widehat{\beta}_1\right)}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}\right\}-\frac{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\beta}_1\right)}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}y_i\right.$$
$$-\frac{\widehat{\phi}_1(\widehat{\sigma}_1+1)-\widehat{\varphi}_1\left(\widehat{\delta}_1-1\right)+\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1+\widehat{\beta}_1\left(\widehat{\delta}_1-1\right)+(\widehat{\sigma}_1+1)}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}z_i$$
$$-\frac{\widehat{\phi}_1\widehat{\delta}_1-\widehat{\varphi}_1\widehat{\delta}_1+\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1+\widehat{\beta}_1\widehat{\delta}_1+\widehat{\sigma}_1}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}s_i$$
$$\left.-\frac{2\widehat{\varphi}_1+\widehat{\phi}_1+\widehat{\beta}_1\widehat{\phi}_1+1-\widehat{\beta}_1+2\widehat{\tau}_1\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}t_i\right],$$

$$S_i=\frac{1}{\left(\widehat{\beta}_1\widehat{\phi}_1+\widehat{\varphi}_1\right)}\left[\left(1-\widehat{\beta}_1\right)x_i+\widehat{\beta}_1 y_i+\left\{\widehat{\beta}_1\left(\widehat{\delta}_1-1\right)+(\widehat{\sigma}_1+1)\right\}z_i+\left(\widehat{\beta}_1\widehat{\delta}_1+\widehat{\sigma}_1\right)s_i+\left(1-\widehat{\beta}_1\right)t_i\right],$$

for $i=1,2,3,\ldots,n$.

To solve *Eq. (16)* and we get the following

$$x^* = (x_1{}^*, x_2{}^*, x_3{}^*, \ldots \ldots x_n{}^*),\ y^* = (y_1{}^*, y_2{}^*, y_3{}^*, \ldots \ldots y_n{}^*),\ z^* = (z_1{}^*, z_2{}^*, z_3{}^*, \ldots \ldots z_n{}^*),$$
$$s^* = (s_1{}^*, s_2{}^*, s_3{}^*, \ldots \ldots s_n{}^*),\ t^* = (t_1{}^*, t_2{}^*, t_3{}^*, \ldots \ldots t_n{}^*).$$

Now using the following primal-dual relation we shall get the requird solution.

$$TAC^*(D, Q, L) = n\left(v^*(x, y, z, s, t)\right)^{1/n},$$
$$w_i' \widehat{\alpha}_1 D_i{}^{*(1-\widehat{\beta}_1)} = x_i{}^*\left(v^*(x, y, z, s, t)\right)^{1/n},$$
$$\frac{w_i' \widehat{h}_1 Q_i{}^*}{2} = y_i{}^*\left(v^*(x, y, z, s, t)\right)^{1/n},$$
$$w_i' \widehat{h}_1 k\omega\sqrt{L_i{}^*} = Y_i{}^*\left(v^*(x, y, z, s, t)\right)^{1/n} \text{ for } i = 1,2,3, \ldots \ldots \ldots \ldots, n.$$

5 | Numerical Example

Here we consider two items an inventory system with all parametric values in proper units. Taking $k = 4, \omega = 6$.

Table 1. Input imprecise data for shape parameters.

Parameters	Items I	II
$\widetilde{\alpha}_1$	(250,260,270; 0.8)	(280,290,300; 0.7)
$\widetilde{\beta}_1$	(8,9,10; 0.8)	(6,7,8; 0.8)
$\widetilde{h}_1$	(4,5,6; 0.9)	(7,8,9; 0.8)
$\widetilde{\rho}_1$	(11,12,13; 0.7)	(10,11,12; 0.8)
$\widetilde{\gamma}_1$	(95,100,105; 0.7)	(95,98,101; 0.8)
$\widetilde{\delta}_1$	(0.03,0.04,0.05; 0.8)	(0.05,0.06,0.07; 0.8)
$\widetilde{\sigma}_1$	(0.7,0.8,0.9; 0.7)	(0.6,0.7,0.8; 0.9)
$\widetilde{\theta}_1$	(0.05,0.06,0.07; 0.9)	(0.07,0.08,0.09; 0.8)
$\widetilde{\tau}_1$	$\left(\frac{1}{5}, \frac{1}{4}, \frac{1}{3}; 0.8\right)$	$\left(\frac{1}{6}, \frac{1}{5}, \frac{1}{4}; 0.7\right)$
$\widetilde{\vartheta}_1$	(0.25,0.26,0.27; 0.8)	(0.26,0.27,0.28; 0.9)
$\widetilde{\varphi}_1$	(11,12,13; 0.8)	(10,11,12; 0.9)
$\widetilde{\phi}_1$	(0.30,0.32,0.34; 0.9)	(0.44,0.45,0.46; 0.7)

Using defuzzification the approximate values of the above parameter are:

Table 2. approximate values of table 1.

Defuzzification of the Fuzzy Numbers	Items I	Items II
$\widehat{\alpha}_1$	208	203
$\widehat{\beta}_1$	7.2	5.6
$\widehat{h}_1$	4.5	6.4
$\widehat{\rho}_1$	8.4	8.8
$\widehat{\gamma}_1$	70	78.4
$\widehat{\delta}_1$	0.032	0.048
$\widehat{\sigma}_1$	0.56	0.63
$\widehat{\tau}_1$	0.20666667	0.1429166667
$\widehat{\theta}_1$	0.054	0.064
$\widehat{\upsilon}_1$	0.208	0.243
$\widehat{\varphi}_1$	9.6	9.9
$\widehat{\phi}_1$	0.288	0.315

Table 3. Optimal solution of MOIM using different methods.

Methods	D_1^*	Q_1^*	L_1^*	TAC_1^*	D_2^*	Q_2^*	L_2^*	TAC_2^*
FPTHMF	1.88	9.10	0.12×10^{-2}	57.15	2.03	8.09	0.28×10^{-3}	74.74
FNLP	1.89	9.16	0.12×10^{-2}	57.15	2.03	8.09	0.28×10^{-3}	74.74
GP	1.80	11.84	0.13×10^{-2}	59.43	2.03	7.94	0.17×10^{-3}	74.86

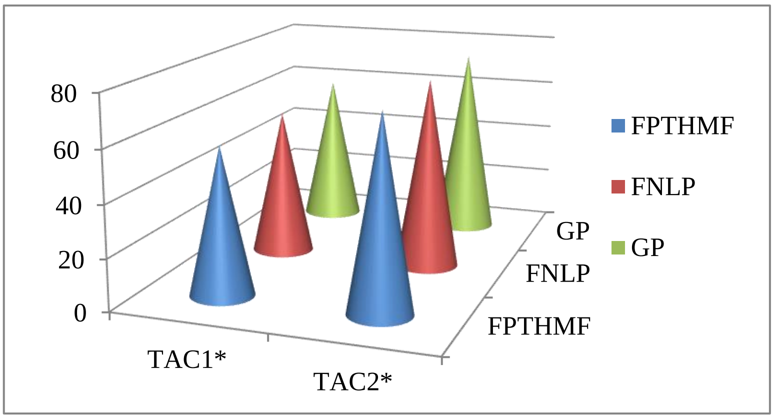

Fig. 2. Minimizing cost of both items using different methods.

From the above figure suggests that GP, FPTHMF and FNLP methods nearly provide the same result.

6 | Sensitivity Analysis

In the sensitivity analysis all optimal solutions have been found by using the FNLP method.

Table 4. Optimal solution of MOIM for different values of α_1, α_2.

Method	α_1, α_2	D_1^*	Q_1^*	L_1^*	TAC_1^*	D_2^*	Q_2^*	L_2^*	TAC_2^*
FNLP	-20%	1.83	8.90	0.11×10^{-2}	56.02	1.95	7.84	0.27×10^{-3}	72.61
	-10%	1.86	9.00	0.12×10^{-2}	56.61	1.99	7.97	0.28×10^{-3}	73.73
	+10%	1.91	9.18	0.12×10^{-2}	57.64	2.07	8.19	0.28×10^{-3}	75.67
	+20%	1.94	9.26	0.12×10^{-2}	58.09	2.10	8.29	0.28×10^{-3}	76.54

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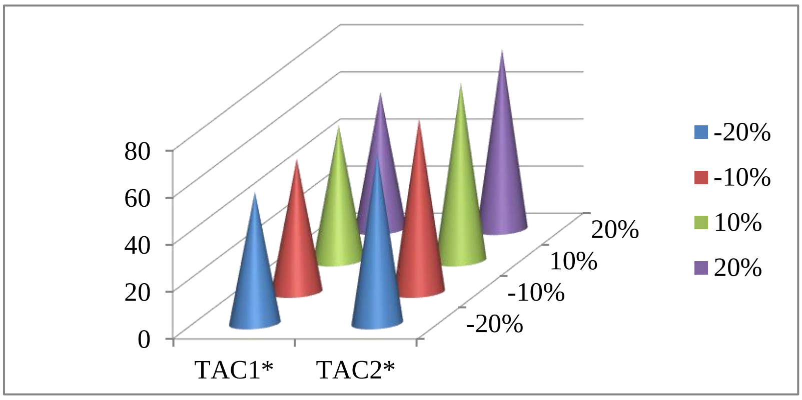

Fig. 3. Minimizing cost of both items for different values of α_1, α_2.

The above *Fig. 3* suggests that the minimum cost of the both items is proportionally related to the parameter α_1, α_2.

Table 5. Optimal solution of MOIM for different values of β_1, β_2.

Method	β_1, β_2	D_1^*	Q_1^*	L_1^*	TAC_1^*	D_2^*	Q_2^*	L_2^*	TAC_2^*
FNLP	-20%	2.12	9.79	0.12×10^{-2}	63.21	2.29	8.75	0.29×10^{-3}	84.79
	-10%	1.99	9.40	0.12×10^{-2}	59.79	2.15	8.39	0.28×10^{-3}	79.11
	+10%	1.80	8.84	0.11×10^{-2}	55.05	1.94	7.84	0.27×10^{-3}	71.29
	+20%	1.74	8.63	0.11×10^{-2}	53.34	1.86	7.63	0.27×10^{-3}	68.48

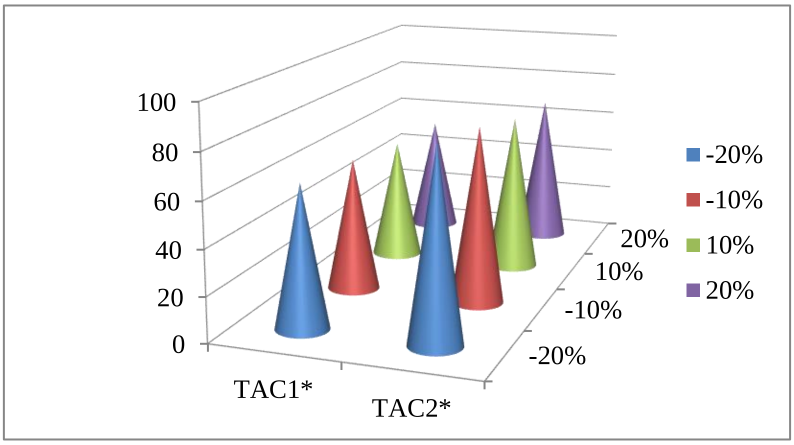

Fig. 4. Minimizing cost of both items for different values of β_1, β_2.

The above *Fig. 4* suggests that minimum cost of the both items is inversely proportional to the parameter β_1, β_2.

Table 6. Optimal solution of MOIM for different values of γ_1, γ_2.

Method	γ_1, γ_2	D_1^*	Q_1^*	L_1^*	TAC_1^*	D_2^*	Q_2^*	L_2^*	TAC_2^*
FNLP	-20%	1.91	8.52	0.13×10^{-2}	53.22	2.07	7.56	0.31×10^{-3}	69.00
	-10%	1.90	8.82	0.12×10^{-2}	55.22	2.05	7.83	0.29×10^{-3}	71.93
	+10%	1.88	9.36	0.11×10^{-2}	59.02	2.02	8.33	0.26×10^{-3}	77.46
	+20%	1.87	9.62	0.11×10^{-2}	60.84	2.00	8.57	0.25×10^{-3}	80.10

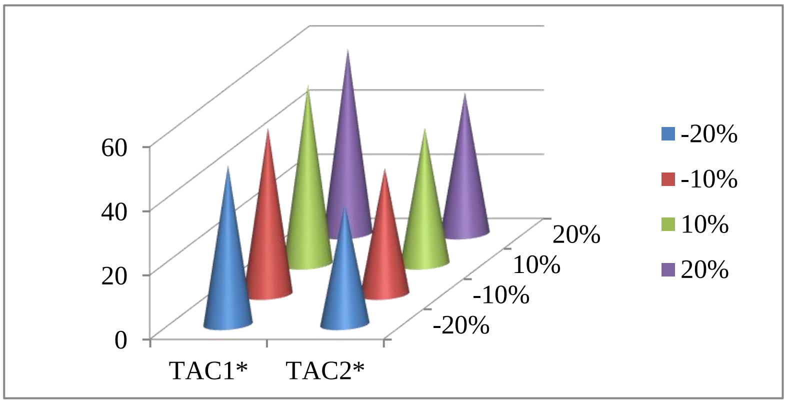

Fig. 5. Minimizing cost of both items for different values of γ_1, γ_2.

The above *Fig. 5* suggests that the minimum cost of the both items is proportionally related to the parameter γ_1, γ_2.

Table 7. Optimal solution of MOIM for different values of δ_1, δ_2.

Method	δ_1, δ_2	D_1^*	Q_1^*	L_1^*	TAC_1^*	D_2^*	Q_2^*	L_2^*	TAC_2^*
FNLP	-20%	1.89	9.09	0.12×10^{-2}	56.88	2.04	8.09	0.28×10^{-3}	74.19
	-10%	1.89	9.09	0.12×10^{-2}	57.02	2.03	8.08	0.27×10^{-3}	74.47
	+10%	1.88	9.10	0.12×10^{-2}	57.28	2.03	8.07	0.27×10^{-3}	75.02
	+20%	1.88	9.10	0.12×10^{-2}	57.42	2.02	8.09	0.27×10^{-3}	75.30

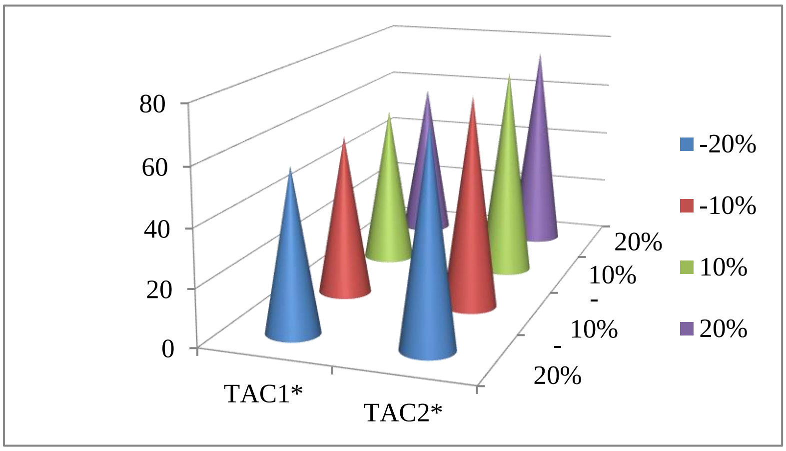

Fig. 6. Minimizing cost of both items for different values of δ_1, δ_2.

The above *Fig. 6* suggests that the minimum cost of the both items is proportionally related to the parameter δ_1, δ_2.

Table 8. Optimal solution of MOIM for different values of σ_1, σ_2.

Method	σ_1, σ_2	D_1^*	Q_1^*	L_1^*	TAC_1^*	D_2^*	Q_2^*	L_2^*	TAC_2^*
FNLP	-20%	1.90	9.19	0.12×10^{-2}	57.26	2.07	8.19	0.28×10^{-3}	74.66
	-10%	1.90	9.14	0.12×10^{-2}	57.21	2.05	8.14	0.28×10^{-3}	74.70
	+10%	1.88	9.05	0.12×10^{-2}	57.08	2.02	8.04	0.28×10^{-3}	74.77
	+20%	1.87	9.00	0.12×10^{-2}	57.02	2.01	7.98	0.28×10^{-3}	74.78

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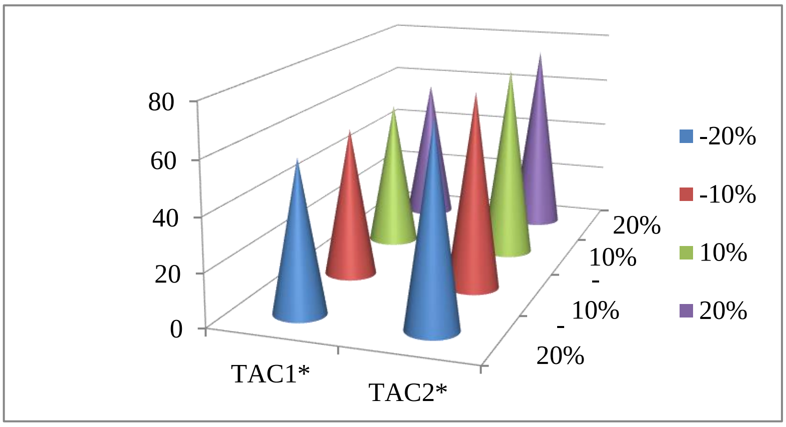

Fig. 7. Minimizing cost of both items for different values of σ_1, σ_2.

The above *Fig. 7* suggests that the minimum cost of the both items is proportionally related to the parameter σ_1, σ_2.

Table 9. Optimal solution of MOIM for different values of ρ_1, ρ_2.

Method	ρ_1, ρ_2	D_1^*	Q_1^*	L_1^*	TAC_1^*	D_2^*	Q_2^*	L_2^*	TAC_2^*
FNLP	-20%	1.89	8.87	0.87×10^{-3}	55.30	2.04	7.92	0.20×10^{-3}	72.90
	-10%	1.89	8.99	0.10×10^{-2}	56.24	2.03	8.00	0.24×10^{-3}	73.83
	+10%	1.89	9.20	0.13×10^{-2}	58.02	2.03	8.17	0.32×10^{-3}	75.63
	+20%	1.89	9.31	0.15×10^{-2}	58.87	2.03	8.25	0.36×10^{-3}	76.49

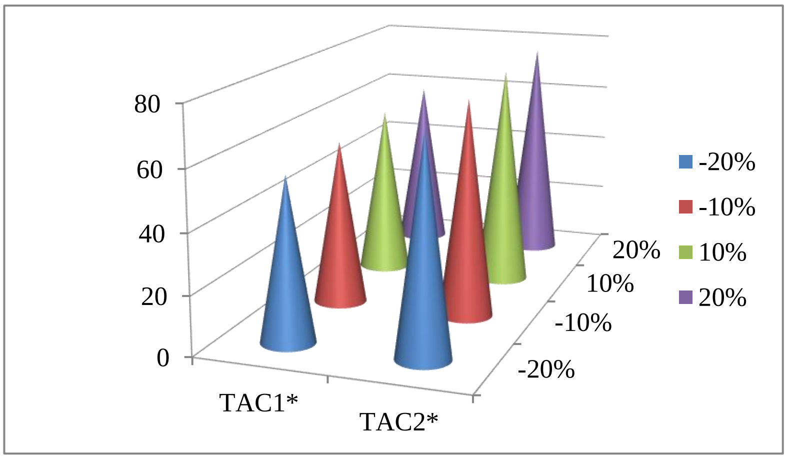

Fig. 8. Minimizing cost of both items for different values of ρ_1, ρ_2.

The above *Fig. 8* suggests that the minimum cost of the both items is proportionally related with the parameter ρ_1, ρ_2.

Table 10. Optimal solution of MOIM for different values of τ_1, τ_2.

Method	τ_1, τ_2	D_1^*	Q_1^*	L_1^*	TAC_1^*	D_2^*	Q_2^*	L_2^*	TAC_2^*
FNLP	-20%	1.89	8.88	0.56×10^{-3}	54.71	2.04	7.94	0.14×10^{-3}	72.70
	-10%	1.89	8.99	0.82×10^{-3}	55.92	2.03	8.01	0.20×10^{-3}	73.71
	+10%	1.89	9.20	0.16×10^{-2}	58.40	2.03	8.16	0.38×10^{-3}	75.80
	+20%	1.89	9.30	0.21×10^{-2}	59.66	2.03	8.23	0.51×10^{-3}	76.88

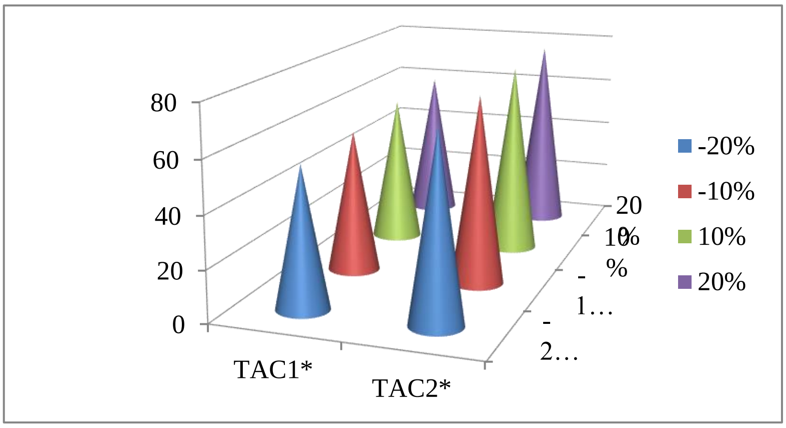

Fig. 9. Minimizing cost of both items for different values of τ_1, τ_2.

The above *Fig. 9* suggests that the minimum cost of the both items is proportionally related with the parameter τ_1, τ_2.

7 | Conclusion

In this paper, we have developed a deteriorated multi-item inventory model in a fuzzy environment. Production cost, set-up- cost and deterioration cost are continuous functions of demand. Set-up-cost and deterioration cost are also dependent on average inventory level. Lead time crashing cost is considered the continuous function of leading time. Due to uncertainty all cost parameters of the proposed model are taken as GTFNs. The formulated multi objective inventory problem has been solved by various techniques like as GP, FPTHMF, and FNLP. Numerical example is solved by using LINGO13 software. This paper will be extended by using linear, quadratic demand, ramp type demand, power demand etc. Inflation performs a crucial position in the inventory systems, but here it is not considered. So inflation can be used in this model for practical. Also another types of fuzzy numbers like as Generalized Trapezoidal Fuzzy Number (GTrFN), PfFN, pFN etc. may be used for all cost parameters of the model.

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Fuzzy FMEA Model: A Case Study to Identify Rejection and Losses in Fibre Industry

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Abstract

The growing competition between fibre producing industry and the standards to which, it requires high quality standards. ABC company's procurement department data shows N of number of defects in cellulose pulp sheet uncurl every month. Cellulose sheet is an important raw material in the fibre (Staple) producing industry. Quality tools such as Failure Mode and Effects Analysis (FMEA) applied to admeasure the risk of potential miscarriages. This study aims to determine the most dominant activity as the cause of rejection and losses of cellulose sheets and evince improvements that can be made by using the fuzzy FMEA model. Data collection techniques in the study are using the method of observations, interviews as well as assessment of experts to identity it. This study is based on the four criterion which dominates the defect of cellulose pulp sheet vis. Processing activities, acceptance, examination and delivery. Solicitation for overcoming these problems is presented.

Keywords: Fibre industry, Quality, FMEA, Fuzzy FMEA, Cellulose pulp.

1 | Introduction

In recent years, fibre manufacturing companies have faced an increasing number of competitive environments. With the enlargement of competitors in the market forces industries are constantly improve their processes and forces them to adopt innovative strategies for enlarging their product range and offer more and more personalised product. One of the main raw materials that focused in the fulfilment of quality is the quality of the cellulose pulp sheet raw material. Generally, the cellulose pulp sheet is made of hardwood, the wood chips go through a process of purification and separation in series of steps with require steam and chemicals (sodium hydroxide, sulphur dioxide) [1]. The role and function of cellulose sheet as one of the key raw materials in the staple fibre manufacturing industry makes the fulfilment of the quality and quantity of cellulose sheet as per the need. It is always said that the quality always proportional to productivity [2].

Risk management is the primordial part of the any organizations' strategy in which they propound the risks associated with the processes in order to achieve benefits.

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The main objective of the risk management is to maximize the sustainable value to all the activities, by enhancing the likelihood of their success and alleviating the likelihood of failures and uncertainties in conjunction with fulfilling or not fulfilling the objectives. One of the important tools for the risk management, is the FMEA [3].

FMEA is the method used to identify and analyse the possible failure modes of the process [4] and [5]. It is risk management methodology used for identification of the root causes. It is basically preventive method, by which risk will eliminate at the minimum level. FMEA can be used stand alone as well as part of the any quality management technique [6].

Company gets cellulose pulp sheet with its subsidiary industry. The use of cellulose pulp sheet a day is around 50-150tons & if consider a per month usage can be reaches up to 1500-4500tons. Hereby cellulose pulp sheet becomes an essential component in fibre manufacturing industry. On the basis of studies, we conducted research to identify cardinal cause of failure in achieving the quality and quantity of cellulose pulp sheet by using FMEA, Fuzzy FMEA methods. All these tools are very powerful methods for measuring the reliability of product and processes. These methods are helpful in identifying us to which risk has more concern and so that the action to prevent the loss before its arrival, hence reduces the loss of money and time of the industry [7] and [8]. In this paper, the critical failure mode factors are examined by using the FMEA in the fuzzy environment with the trapezoidal and triangular membership functions. The Fuzzy FMEA approach is applied to identifying, prioritizing and tracking the key potential failure effect, causes and controlling factors. This research is done for the fibre manufacturing industry for its raw material i.e., cellulose sheet which is come from the outside industry. The main motivation behind this research paper is to reduce the failures during handling with cellulose sheet and improve the process of handling by the reduction in the losses of the industry. The final results of this case study were to determine the most dominant activity for the cause of rejection and losses in the cellulose pulp sheet.

2 | Literature Review

Many studies indicate to use FMEA for the risk management. In 2004, Carbone and Tippett [29] put an application of project risk management by evaluating the risk score and RPN value to identify the most critical risk events which needs immediate risk responses. As per the management view, the sequential RPN calculations are very easy to realize the outcomes o the results. But we talk about the technical perspective, there are number of writers who hold concerns related to apply the traditional FMEA approach for the calculation of the Risk Priority Number (RPN). Exemplification of the Bowles and Peláez [23] and Puente et al. [30] focused number of loop holes in both the ways in which the calculations are made and the processes in which the results should have interpreted. By illustrating, with the different failure mode with assessment of severity say (8), occurrence say (6) and detection say (4), may have lower RPN (192) than that of with the high severity, high occurrence and moderate level of detection (say 7,7 and 5 achieves a RPN of 245). So far, the management point of view, the for most failure instigates higher priority for corrective action. The fuzzy model is first introduced by Zadeh [28], gives flexibility and expressive way to reach the risk associated with the substantive failure modes. The recent work within the cellulose pulp sheet is seen in apparent inspections in the fibre manufacturing industry, the usage of the fuzzy FMEA is shown in many varied sectors of the activity. The fuzzy FMEA is the improvement over the classical FMEA, in an ordinary method that is to be used as to fuzzing the risk parameters with appropriate holding functions. Many studies proposed the implementation of the Fuzzy FMEA to improve the efficiency of FMEA and overcome its limitations [9] and [10].

Fuzzy FMEA has been implemented to many distinct industries for different types of applications. A risk based fuzzy evidential outlook is tendering in by employing interval based Dempster- Shafer theory and fuzzy axiomatic design in order the analyse the risk of failure modes with fuzzy logic structures [11]. The competency of the proposed model investigated by the researchers by putting example and the results when they compared with riskless evaluations. An FMEA risk management outlook is proposed in [12] by fuzzy approach-based interface system with the intention of curtailing the failures of Load, Haul, Dump (LHD) Machine. An extended FMEA approach by catch hold of fuzzy best-worst method and multi-objective optimization by ratio analysis based on Z-number theory (Z-MOORA) method [13]. These methods are used to overcoming of the various traditional RPN pitfalls. Riaz and Hashmi [14] established new extension of fuzzy sets to the Linear Diophantine Fuzzy Set (LDFS) for efficient and flexible structure to deal with uncertainties. They presented geometrical properties of LDFS to compare the fuzzy sets. In [15], they created Spherical Linear Diophantine Fuzzy Set (SLDFS) which is more efficient to address various uncertainties in a parametric view. Spherical linear Diophantine fuzzy information includes additional features of reference or control parameters. They defined operations on picture fuzzy numbers and smooth aggregation operators. Riaz et al. [16] extended the conventional orthopair fuzzy sets to the q-Rung Orthopair Fuzzy Sets (q-ROFSs) so that their can analyse wider membership function which will help decision makers to put rational perception. Khan et al. [17] defined the linear Diophantine fuzzy numbers, they find ranking function for triangular linear Diophantine fuzzy number with no such limitations take grades generally in q-ROFS, Pythagorean Fuzzy Sets (PFS). The problems allied with healthcare are prioritize with the implementation of the fuzzy FMEA system [10]. They used FMEA along with linguistic variables and fuzzy system. Inputs like S & O were explained according the five linguistic conditions and trapezoidal function. Enabler D and output RPN ere explained by trapezoidal, linguistic terms and triangular functions. Considering the vast modes of failure comes in healthcare institutions, their prioritization is need of an hour. FMEA is best suited for identifying the potential failures. Nevertheless, the implementation of the fuzzy FMEA technique vindicated to be the more flexible alternative of evaluation by providing the image of the uncertainty associated with the variables [18].

The risk assessment model in the green supply chain applying the fuzzy approach to FMEA is focused in [19] and being implemented in Indian plastic industries. In different areas of application in management failure factors were examined through an intuitionistic fuzzy environment as a case study of Iran oil and gas service [20]. The outcomes of this study have shown are lacking behind the leadership and management commitments of the company. In a fuzzy number method for FMEA proposes to cater the drawbacks of concise FMEA and fuzzy based FMEA methods. A specific methodology is developed that combines with the similarities of fuzzy numbers and possibility doctrine. All these above studies have visualized those copulations of previous studies were not exceptionally important but applying the fuzzy FMEA is seen to be lackadaisical. Thereby, due to the contribution of fuzzy logics, it is probabilistic to improve the understanding of complex dynamic problems by considering the subjective and inappropriate information. This approach helps to all possible accurate risk and overcomes the limitations of FMEA. The fuzzy rule-based system was applied widely for as much that put distinct advantages. As compared to the traditional methods of FMEA, the fuzzy FMEA system provided following advantages [21], [22] and [23].

I. Helps the researcher to use linguistic terms in criticality assessment for assessing directly the failure modes associated with it.
II. Haziness of data or information not explicitly present, could be used in the assessment and management in a well organised way.
III. The more flexibility of the structural combination of Severity (S), Occurrence (O) and Detection (D).

3 | Methodology

The fuzzy FMEA system follows a basic structure of the fuzzy FMEA epitome system consists of three chief modules: input module, knowledge base module and output interface model, as shown in *Fig. 1* [22]. As it can be observing that in *Fig. 1*, the inputs variables concur to the parameters of S, O and D [24]. The output variables equal to the RPN. S, O and D have to fuzzified by using the membership functions to identify the degree of membership among each input classes. The resulting fuzzy inputs will be evaluated in the fuzzy environment, which uses a well-defined rule base. These rules are fall under the “IF- THEN” type and together with fuzzy logic operations are used to identify the level of risk of failure. The fuzzy conclusion is then defuzzified to get RPN. The higher the value of the RPN, the greater the risk and vice versa.

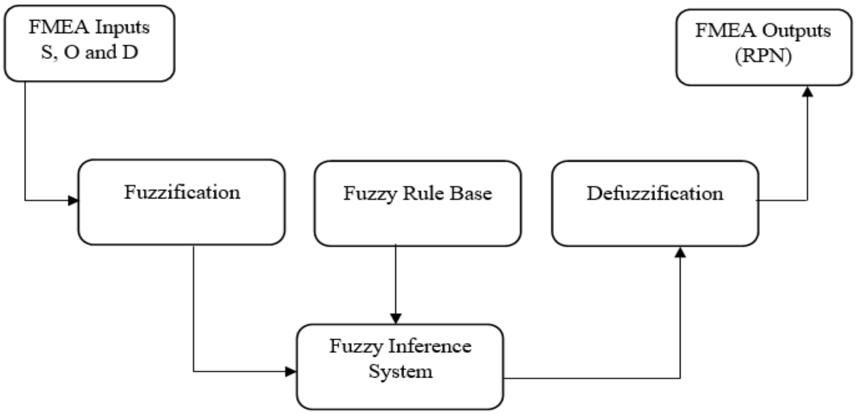

Fig. 1. Fuzzy inference system.

Fuzzy FMEA is legitimate technique which is employed to evaluate the output response from the input data. There are multiply reasons for using the Fuzzy FMEA are beneficial that's why the business commentators suggest the Fuzzy FMEA, these being, among others [25].

The fuzzy FMEA logic concept is very easy to understand. The fundamentals of mathematics are also less complicated in the fuzzy interface environment.

I. This is flexible and can endure the data if any undue error exists in the databases.
II. This technique has potential to model complex non-linear functions in the very short span of time.
III. This approach can also form the experience of specialists in absentia for the need of surplus training.
IV. This technique doesn't require an advance language, it works on the basis of simple language.

FMEA was used as a conjugation method with the other quality tools for alienating the potential risk and fabricate confidence in the system. Besides, the FMEA implementation used the RPN for visualizing the result of the assessment. The implementation process of the FMEA has to cast the correct evaluation of RPN was important because it was an intimation of the stiff severity to take appropriate actions to reduce or eliminate the risk that might occur. When FMEA used in the operable work was found., the RPN methods exhibited some drawbacks. Hereof, there are many researchers proposed FMEA implementation to step up its efficiency as a way to fix above mentioned drawbacks pertinent in real work [26].

3.1 | Fuzzification of the Inputs and Outputs

In this process, the S, O and D variables are modified into the linguistic terms and membership functions [27]. Several experts with varying degrees of competence are used to create the membership functions [22]. In this case, S, O and D are assigned to linguistic terms, rooted on FMEA's scales [24]. In FMEA, S, O and D are ceded in the values from 1 to 10. RPN will computed by the equation:

$$RPN = S \times O \times D.$$

As per the above mathematical expression, the minimum and maximum values which will be computed for RPN is 1 to 1000, respectively. Since the for the fuzzy FMEA will based on the traditional FMEA data, we were adopting the same values to define the universal for each variable. Hence, it is considered a universal value from 1 to 10 for S, O and D; and from 1 to 1000 for RPN. Then membership functions were designed in pursuance of the weight of the every FMEA classification. The data collection techniques in this study were using the method of observations, interviews and group discussion as well as the evaluation of the experts to identify it. Tables represent below the linguistic variables and membership functions of S, O, D and RPN.

Table 1. Linguistic variables and membership function for severity (S).

Input	Severity (S)
None	[0, 1, 2, 3]
Low	[2, 3, 4, 5]
Average	[4, 5, 6, 7]
High	[6, 7, 8, 9]
dangerous	[7, 8, 9, 10]

Table 2. Linguistic variables and membership function for Occurrence (O) event.

Input	Occurrence (O)
Nearly Impossible	[0, 1.5, 2.5]
Low	[1.5, 3, 4.5]
Average	[3.5, 4.5, 5.5]
High	[6.5, 7, 8.5]
Almost Few	[7.5, 8.5, 10]

Table 3. Linguistic variables and membership function for detection (D).

Input	Detection (D)
Almost Few	[0, 1.5, 2.5]
High	[1.5, 3, 4.5]
Average	[3, 4.5, 6]
Low	[4.5, 5.5, 7]
Nearly Impossible	[7, 8,10]

Table 4. Linguistic variables and membership function for RPN.

Input	RPN
No Important	[0, 100, 200]
Very Few Important	[150, 250, 400]
Few Important	[300, 450, 600]
Average important	[400, 550, 700]
Important	[600, 750, 900]
Very Important	[800, 950, 1000]

For defining the functions of variable S came to the less concern that the lowest values of S have for the process, and the way this is the reason that's why the trapezoidal functions with large belonging intervals were used. For above the values of average, we tried to redefine the criterion by using triangular function for the term "high". Thus, greater importance is obtained with the help of greater variability.

The O variable is represented with a set of symmetric functions, highlighten the terms "low" and "High", to define them precisely. This reflects the context in which the model is applied, since it is understand that the greatest variability should exist in the intervals that both terms represented.

The variable D emphasis for the "High" and "Average" terms, with the assignment of triangular functions. Because it is in the value range of these two functions are most concentrated, hence it deserves to be more accurately defined.

The output variable four triangular and two trapezoidal membership functions were selected. The range for the output variable defined by the set of [0, 1000], thus permitting in a more advance phase, comparing the output obtained by the implementation of Fuzzy RPN model with one obtained by basic FMEA RPN.

3.2 | Fuzzy Inference Process

In this paper, minimum inference engine used with the help of MATLAB to combine the fuzzy IF-THEN rules in the fuzzy rule base and being implication the fuzzy conclusions. The minimum inference engine uses:

I. Min operator for "AND" in the IF part of rules and rules and max operator for the "OR" in the IF part rules.
II. The prime combination to aggregate the consequences of the individual rules.

An example is offered to explain the process of minimum inference engine.

There are multiple defuzzification algorithms have been developed. In this paper the centre of gravity method defuzzification will be adopted. For determining the defuzzification value an expression is:

$$\mathrm{COG} = \frac{\sum \mu_i(x) \times x_i}{\sum \mu_i(x)}.$$

Where; x_i= The membership function reaches maximum value and $\mu_i(x)$= degree of membership function.

Trapezoidal Membership Function. The trapezoidal membership function is used in the Severity (S) for expressing the vagueness of the information which in generally caused due to linguistic assessments through the transformation into the numerical variables.

$$\mu_i(x) = \begin{cases} 0 & \text{if} \quad x < a \\ \dfrac{(x-a)}{(b-a)} & \text{if} \quad a \le x \le b \\ 1 & \text{if} \quad b \le x \le c \\ \dfrac{(d-x)}{(d-c)} & \text{if} \quad c \le x \le d \\ 0 & \text{if} \quad x > d \end{cases}.$$

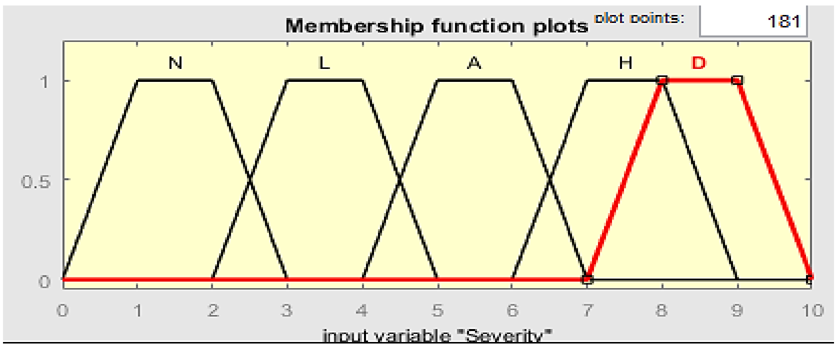

Fig. 2. Trapezoidal membership function for severity (S).

Triangular Membership Function. The Triangular Membership Function is used in the sets except in the Severity (S). It is elaborated by the three parameters (a, b, c) where for every value of x the membership function $\mu_i(x)$ is described in the *Fig. 3* and *Fig. 4*.

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$$\mu_i(x) = \begin{cases} 0 & \text{if} \quad x < a \\ \dfrac{(x-a)}{(b-a)} & \text{if} \quad a \le x \le b \\ \dfrac{(c-x)}{(c-b)} & \text{if} \quad b \le x \le c \\ 0 & \text{if} \quad x > c \end{cases}.$$

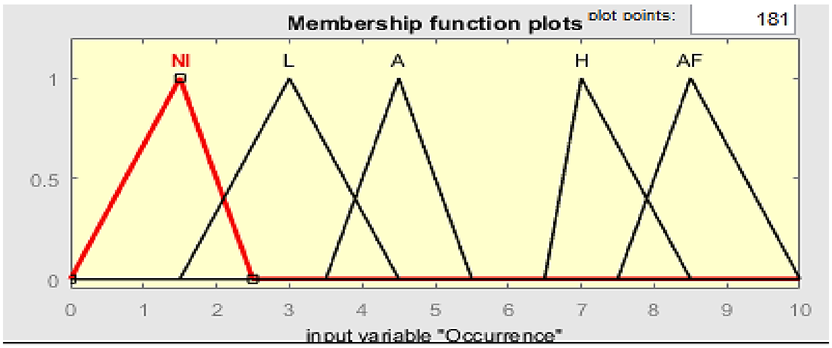

Fig. 3. Triangular membership function for occurrence (O).

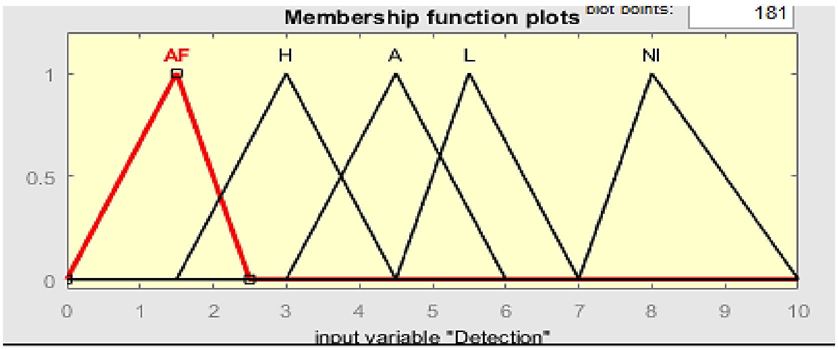

Fig. 4. Triangular membership function for detection (D).

If we compare the trapezoidal membership function is slightly complex to the triangular membership function. It needs more memory size for variable. Furthermore, it is complex process, the performance of trapezoidal function is better than that of triangular membership function. Severity plays an important role in this research paper so, for widening the spectrum of severity we choose trapezoidal function and others are operated at triangular membership function.

4 | Results and Discussion

4.1 | Identification of the Ordering Business Process for Cellulose Pulp Sheet

Fig. 5 gives a descriptive understanding of the activities for ordering supply chain to use of the cellulose pulp sheet. Few activities are carried out by the company and suppliers. But number of the activities can be identified by the type of failure and potential failure modes that can be occur.

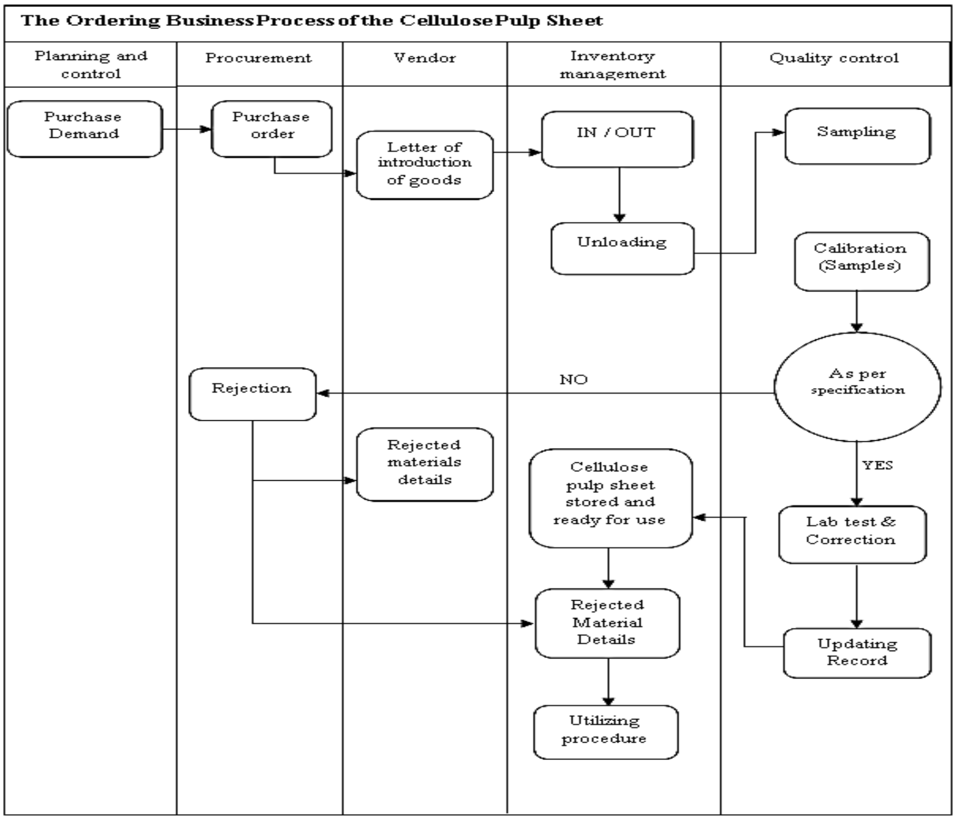

Fig. 5. Identification of the ordering business process in respect of cellulose pulp sheet.

4.2 | Identification of Failure Modes

The identification of failure modes was completed by using the analysis of previous years data and based on the interviews and group discussions with the procurement department engineers responsible for the management of the raw material handling processes. The potential failure for the cellulose pulp sheet rejections and losses are shown below in *Table 5*.

Table 5. Failure modes for the improvement.

Activity	Failure Mode	Notation	Causes	Impact	Control
Processing	Cellulose pulp sheet Spoilage	E1	Processing techniques not up to the mark	Reduction in the weight of sheet	Identify the specification, sample testing,
	Unfavourable quality grid	E2	Human error	Alleviated cellulose sheet quality because of during processing higher chemical content	Data accuracy
Shipping	Incorrect unloading location instruction	E3	Human error	Multiple handling	By establishing coordination between unloading & inventory workers
	Sheet becomes wet	E4	Rainy weather or moist climate at the delivery time	Affects the weight of the sheet	Cover properly with tarpaulins
	Truck number is not correct	E5	By the inability of suppliers to provide the truck	Latency for the fulfilment of supplies	Identify the standard of the minimum number of the trucks will be used, deliveries deadline

Table 5. (Continued).

Activity	Failure mode	Notation	Causes	Impact	Control
Compliance	Test results of the sample from the supplier and the actual sheet test by the company as a whole	E6	Spoof by supplier that holds a good sample but in reality, sheet is of low-quality wood	Getting cellulose sheet with low quality and incurred financial losses	Quality checks of the cellulose sheet prior to unloading cellulose sheet into the truck
Inspection	Take specimen for error finding	E7	Shortage of tools and knowledge about the correct specimen	Wrongly identified quality of cellulose sheet	Updation of tools that can be used by the whole team and train the team for reducing the error while inspecting activities
Stockpiles	Limited storage area	E8	Storage techniques are less precise for sheets	Limited capacity of company	Improved by engineered practices for pile up
	Nasty Drainage stockpile	E9	Influx of water does not drain	The water in sheets increases	Periodic maintenance of warehouses
	Sheets spilled during loading	E10	Overburdened carrying capacity	The road becomes sludgy by the sheets	Loading techniques improved

By the identification of the failure modes, the next process is weighing which conducted by expert. In this study, the researcher determines the expert who came from the procurement department of ABC company. Expert with his experience and will examine the severity occurrence and detection on the failure mode that has been identified in failure modes table. After that, the expert will examine the RPN and the Fuzzy RPN with the help of MATLAB software.

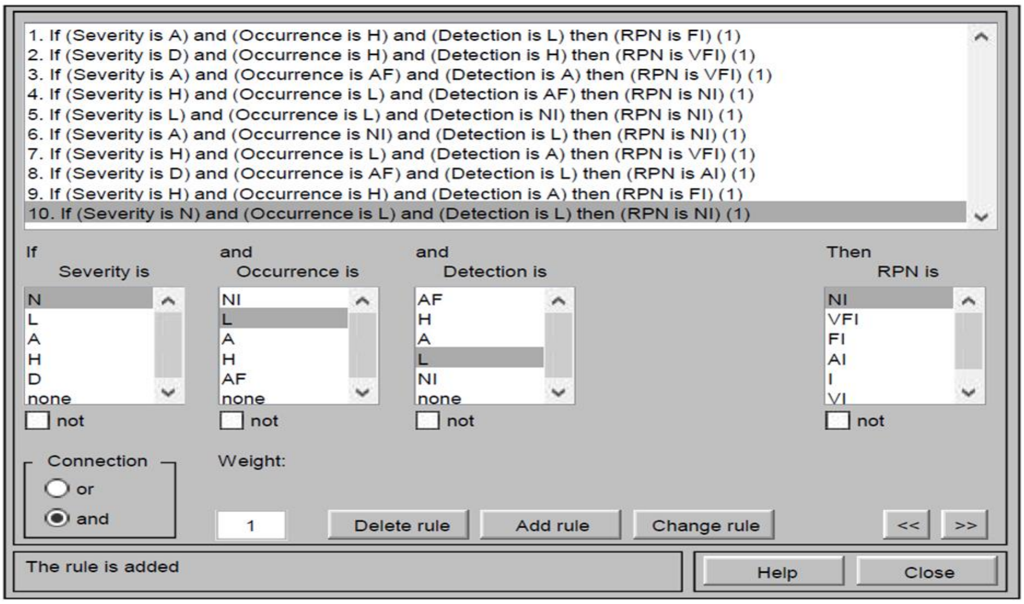

Fig. 6. Fuzzy rules in MATLAB software.

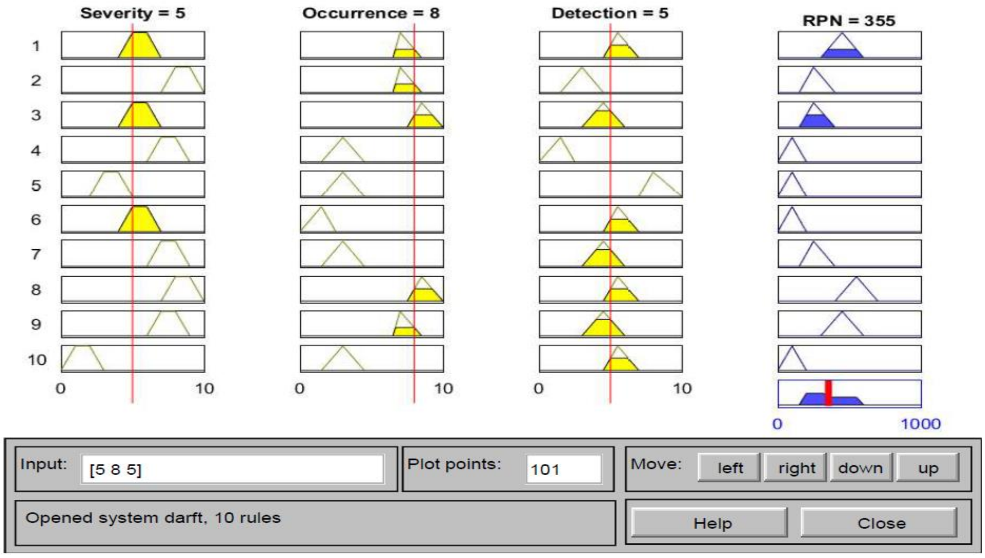

Fig. 7. RPN's input and output illustration.

Table 6. Comparison between FMEA and Fuzzy FMEA.

Failure Mode	Severity (S)	Occurrence (O)	Detection (D)	RPN	(FUZZY) RPN	RPN Ranking	(Fuzzy) RPN Ranking
E1	7	8	7	392	500	1^{st}	4^{th}
E2	8	8	5	192	510	5^{th}	2^{nd}
E3	5	8	5	200	355	4^{th}	5^{th}
E4	9	4	2	72	473	8^{th}	3^{rd}
E5	3	3	8	72	100	8^{th}	4^{th}
E6	4	2	5	40	500	9^{th}	4^{th}
E7	8	3	6	144	500	6^{th}	4^{th}
E8	8	9	5	360	550	2^{nd}	1^{st}
E9	7	7	6	294	500	3^{rd}	4^{th}
E10	3	4	7	84	500	7^{th}	4^{th}

All the above values are taken with help of interview and these values are analysed by the MATLAB software. The results of the assessment based on the table can be illustrated as the comparable results of 10 different types of fundamental RPN and Fuzzy RPN failures of which dominant most seen below in *Table 7*.

Table 7. Ranks for FMEA and fuzzy FMEA.

Rank	Potential Failure (RPN)	Potential Failure (Fuzzy RPN)
1^{st}	Cellulose pulp sheet Spoilage(E1)	Limited storage area(E8)
2^{nd}	Limited storage area(E8)	Unfavourable quality grid(E2)
3^{rd}	Nasty Drainage stockpile(E9)	Sheet becomes wet(E4)

Juxtaposing the results for the traditional FMEA and with the Fuzzy FMEA, the disparities between them are clearly mentioned in *Table 7*. The failure modes E4 and E5 have the same RPN of 72 and have same priority. But the fuzzy FMEA RPN in those cases are different and it would be advantageous for stabilize priority on those components. Considering the failure modes E4 and E5 where their RPN is 72. The value of Severity (S), Occurrence (O) and Detection (D) ratings are 9, 4, 2 and 3, 3, 8 for the E4 and E5 respectively. Notwithstanding the RPN for both failure modes are same and the risk levels are subsequently different. The ranks of E4 and E5 in fuzzy environment are 3 and 4 and the failure mode E4 has greater RPN than E5. Hence, the traditional method FMEA may differ the results. In addition, the ranking presented by the proposed system doesn't segregate the failure modes which has proximate ratings. If the both failure modes bear the same value and have proximate ratings, it will give same RPN to the both components. Nevertheless, the traditional FMEA methods creates the resulting different RPN.

The analysis of the outcomes produced by the traditional FMEA and Fuzzy FMEA methods show much accurate and reasonable results of the ranking which can be accomplish by adopting Fuzzy FMEA. Other finding can be done in the same manner. In addition, the Fuzzy FMEA can also be updated or amended when more information of a product or process is available. So, we can say that the proposed evaluation method can be continuously elevated.

5 | Conclusion

In this study, a failure mode and effect analysis based on the fuzzy logic approach is put forth and a model of the risk evaluation system for expert is developed. The analysis of a cellulose pulp sheet is presents to demonstrate the fuzzy FMEA. The cellulose sheet spoilage is the primary failure as per the classical FMEA approach, the results reflect in the fuzzy logic in FMEA as limited storage area. we identified that fuzzy logic environment gives more satisfactory results due to linguistic function. The subjective discretion was stated in the natural form which was sometimes vague, imperfect and tottered. In applying FMEA by assigning the Severity, Occurrence and Detection rating system in natural form produced and insubstantial and puddled impressions. As per the results, the RPN developed by these three ratings overlooked the proportional importance amongst the parameters and resulted in misunderstanding. The usage of linguistic terms permits the experts to confer a more reasonable and meaningful information for three parameters. Fuzzy based rules allow experts to create the more realistic and logical rule bases. By applying the fuzzy set and the membership functions, the inaccurate information is improved to show the real scenarios. By applying fuzzy IF – THEN, the collected rules from the experts, expert's intellect and experience are incorporated in the risk assessment tools. It is more handy to differentiate the risk representations among the same RPN. Although by constructing the knowledge and estimates are prevented efficiently. Furthermore, the information of each and every failure is updated by the experts. The proposed model for assessment is continuously improved. The major disadvantage of the tradition FMEA is the various combinations of three parameter ratings that produce an identical value for RPN. Notwithstanding, the risk represents a thoroughly differences. In this paper, fuzzy rules-based assessment was executed for the case study to meditate the difficulties grown up in conducting the traditional FMEA technique.

Future research intends to the introduction of the Multi- Criteria Decision Making (MCDM) process along with LDFS analysis with some more data sets. We look forward to that our results of research will be beneficial for researchers in the field of industrial raw material losses, reduction of wastage and many manufacturing industries losses.

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Application of the Fuzzy Inference System to Evaluate the Quality of Air Textured Warp Yarn

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Abstract

It has become one of the indispensable conditions to continuously improve the quality and achieve the quality standards in order to adapt to the increasingly competitive environment in the textile industry. However, the textile production process like many other industrial processes involves the interaction of a large number of variables. For a standard quality production, the relation between raw material properties, process parameters, and environmental factors must be established conclusively. The physical properties of air textured warp yarn that affect the quality of the yarn, construct the strength of the yarn. After the production process, different values of each yarn sample are revealed from the strength tests performed during the quality control process. Six criteria that affect the quality of the yarn and identify the strength of the yarn are defined as a result of strength tests. Those criteria are count, tenacity, elongation shrinkage, Resistance per Kilometer (RKM) and breaking force. The differences between the values of these criteria and linguistic variables cause uncertainty when defining the quality of the yarn. To take into consideration this uncertainty a Fuzzy Inference System (FIS) is developed using six criteria as inputs, 144 rules created, and the linguistic variables of Air Textured Yarn (ATY) samples of a textile manufacturer. The quality level of the products according to the different membership functions are identified with the proposed FIS generated by MATLAB version 2015a and recommendations are made to the manufacturer.

Keywords: Fuzzy ingerence system, Quality evaluation, Air textured warp yarn.

1 | Introduction

Air Textured Yarn (ATY) used as warp yarn which is the one of two component yarns, to turn yarn into a finished fabric and regarded as longitudinal set in a finished woven carpet in carpet weaving industry. Since the ATY forms the length of the carpet it directly affects the quality of the carpet to be produce [1]. Any yarn that does not reflect the quality values cause stretching, shrinkage, elongation, shortening, and irregularity problems in the carpet during or after weaving process. Thus, causes negative feedback from the customers, such as compliments, order cancelation or payment fault. To avoid these defects companies, who use ATY as a raw material, primarily prefer to work with ATY producers that produce high quality yarn groups and provide quality standardization.

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Main specifications of the yarn identified from the customers, such as denier, minimum strength and elasticity required, nodes per unit length, crimp properties and number of filaments. Any yarn does not match with these specifications will be rejected as poor quality. Machine parameters, parent yarn conditions and process parameters affect the quality. Under the same conditions, the properties of ATY can be determined with strength tests [2]. A comparison can be done between the test parameters of the yarn and quality standards values. Making a quality classification about yarn samples according to test parameters can provide to realize that if the yarn sample reflects the quality values and avoiding yarn faults before meet the customer. As a result of test parameters, quality of the yarn can be identified linguistically and uncertainty may occur because of these linguistic variables.

Aim of this study is to develop a fuzzy 'Quality Classification' model for a textile mill in Gaziantep. The company is producing three types of ATY (800 DEN, 1100 DEN and 1350 DEN). The six physical properties of ATY, namely count, tenacity, elongation, shrinkage, breaking force and Resistance per Kilometer (RKM), are the factors directly affect the quality of the yarn. The proposed quality classification model is generated with assigning these properties as inputs of the FIS for three types of the ATY (800 DEN, 1100 DEN and 1350). Different membership functions (triangular, trapezoidal, and z-shaped) are compromised for both input and output parameters. 144 rules are created and operated with if-then statement to relate between input and output parameters.

In the literature there are several fuzzy approaches have been successfully applied in textile industry. Majumdar et al. [3] studied adaptive neuro-fuzzy system to predict the cotton yarn strength from HVI fibre properties. Majumdar and Ghosh [4] introduced a model about the level of ring cotton yarns by translating perception and experience of a spinner into a FIS. Malik and Malik [5] developed an application for the prediction of strength transfer efficiencies of weft and warp yarns, with using adaptive neuro-fuzzy inference system. Amindoust and Saghafinia [6] developed a model for real-life supplier selection problem for a textile company in Malaysia with applying a modular FIS. Vu and Kim [7] studied a complete combination of the wearable application based on a textile sensor and FIS. Sarkar et al. [8] proposed a model to explain the effects of laser parameters on treated fabric parameters and predict other fabric properties with using FIS on MATLAB. However, there is no study about evaluating a quality level about ATY with factors affecting yarn quality, using a FIS. This study involves an extend research about quality of ATY with six factors affecting quality of the yarn, and proposes a fuzzy quality classification model with using these six factors and FIS. A real data set is attained into the literature with this research.

This study is divided into four sections. Firstly, the research takes a look at what is studied, why this topic is studied and the past studies about fuzzy approaches in textile industry at the Introduction section. Secondly it is explained that the method used for this research at the methodology section. In the case study section, there some tables and figures to illustrate the material and the application of case study. The results, recommendations and further attempts are explained at the conclusion section of the study.

2 | Methodology

In this study a FIS approach is developed to to evaluate a quality classification model with data selected from a textile mill in Gaziantep which is producing ATY.

A quality classification model is developed for three types of ATY (800 DEN, 1100 and 1350 DEN) with using FIS for ATY samples to analyze that if the yarn sample reflects the standard quality values and eliminate yarn faults for the company. FIS is a system based on the approach that mapping a set of given input variables to an output variable using fuzzy logic. FIS is presented within the context of fuzzy set theory and one of the useful tolls to solve uncertainty and complicated problems depend on fuzzy logic and close to human thinking [9]. The main structure of a FIS is shown in *Fig. 1*. FIS consists four [10] main components: 1) a fuzzification module that translates crisp inputs into fuzzy values, 2) an

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inference engine which implements a fuzzy reasoning mechanism to attain a fuzzy output, 3) a defuzzification module to translate this latter output to crisp value, and 4) a knowledge base that comprises both fuzzy rules known as the rule base and membership functions known as the database.

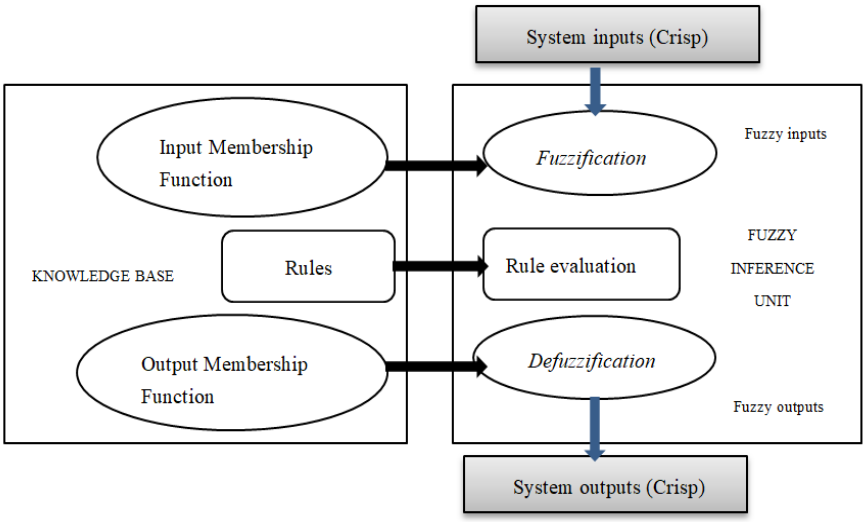

Fig. 1. Structure of FIS [11].

3 | Case Study

This research is motivated from an ATY manufacturer build in Gaziantep in 25.000 m^2 production plant. In a daily production, yarn samples are randomly selected from the production line and subjected to a series of tests to analyze if the values of tested yarn sample reflect the values of standard quality. There are three types of tensile tests applied from quality conrol and R&D department of the company which are tensile strength test, thermo-fisher shrinkage test and thermal resistance test as shown in *Fig. 2*.

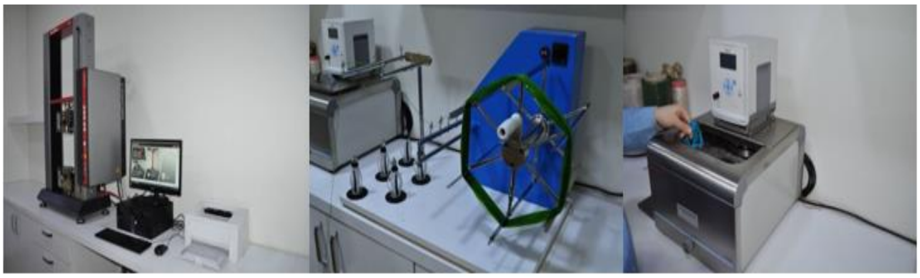

Fig. 2. Tensile test procedure.

The six physical properties of ATY; namely count, tenacity, elongation, shrinkage, breaking force and RKM, are the factors that directly affect yarn quality and construct the yarn strength. *Table 1* shows the terminology of yarn quality criteria. Value of physical properties of the yarn can be analyzed with tensile tests applied. Those six properties also have quantitave mesurable variables with test results values, these variables can easily adapt and subjected to the method used for this research.

Table 1. Yarn quality criteria.

Criteria	Terminology	Expressed as
Count	Count is expressed as the length per unit.	Denier
Tenacity	Tenacity is the breaking strength per denier.	Gram/Denier
Elongation	The ratio of extension of a specimen to its initial length.	Percentage
Shrinkage	The decrease in the length of a specimen caused by a specified treatment.	Percentage
Breaking force	The maximum force applied to a test specimen.	Kilogram
RKM	RKM is the abbreviation of RKM.	Percentage

Data used in this research is validated from the quality control department of the company. In an annual production period, between the months of January 2019 and January 2020, 663 yarn samples of 800 DEN ATY, 655 yarn samples of 1100 DEN ATY and 272 yarn samples of 1350 DEN ATY are subjected to tensile tests. *Table 2* is the brief view of test results of 800 DEN ATY samples (first and last five samples). Each ATY sample has been applied to tensile strength test, thermo-fisher shrinkage test and heat resistance test respectively. Values of tests results for each yarn sample can be either same or different with the same raw material properties, process parameters and environmental factors. Depending on these values, the quality of the yarn can be interpreted linguistically. However, uncertainty can be occurred due to the linguistic variables when defining the quality of ATY. To deal with these problems a quality classification model is developed with Mamdani FIS on MATLAB version 2015a and *Fig. 3* represents the flow chart of the model.

Table 2. Test results of 800 DEN ATY samples.

Sample	Count	Tenacity	Elongation	Shrinkage	Breaking Force	RKM
S1	787.00	4.00	11.00	8.90	3.23	36.52
S2	820.00	4.60	14.50	8.80	3.71	40.37
S3	800.00	4.40	12.50	9.50	3.54	39.45
S4	823.00	4.40	13.10	9.10	3.54	38.31
S5	827.00	4.30	12.50	9.70	3.43	36.94
.	.	.	.	.	.	.
S659	829.00	4.20	13.30	7.20	3.39	36.50
S660	822.00	4.20	13.00	7.50	3.37	36.50
S661	831.00	3.90	11.90	8.20	3.10	33.22
S662	827.00	3.80	11.00	8.20	3.05	32.88
S663	830.00	4.20	12.40	8.00	3.36	36.00

Count, tancity, elongation, shrinkage, breaking force and RKM are the six physical properties of ATY and factors directly affecting yarn quality. These physical properties are used as the input parameters of FIS evaluated. A MATLAB based coding is developed to execute the proposed fuzzy model of quality classification. *Fig. 4* shows the main structure of the model for 800 DEN ATY.

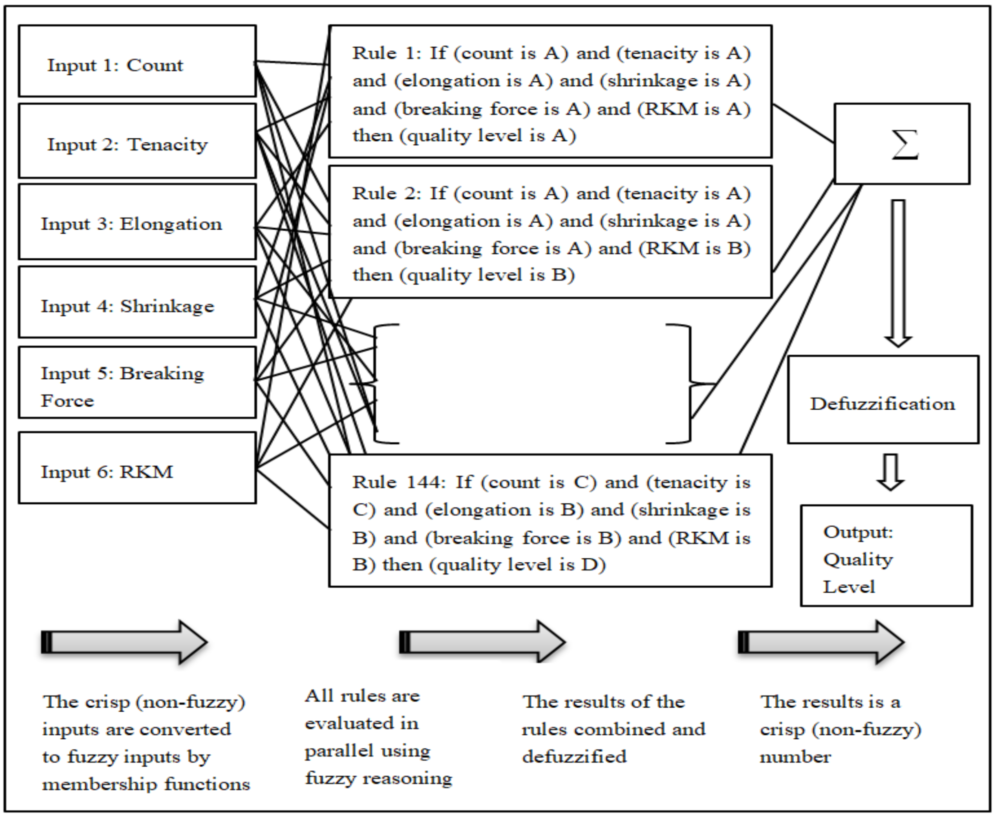

Fig. 3. Flow chart of proposed model.

Three linguistic fuzzy sets; namely best (A), average (B), non-acceptable (C), are generated for the input parameters of count and tenacity. For ‘800 DEN ATY Quality Classification’ model two linguistic fuzzy sets; namely acceptable (A) and non-acceptable (B), are chosen for the input parameters of elongation, shrinkage, breaking force and RKM. Three forms of membership functions (triangular, trapezoidal, z-shaped) are used for the input parameters of count and tenacity. One form membership function (trapezoidal) is used for the input parameters of elongation, shrinkage, breaking force and RKM.

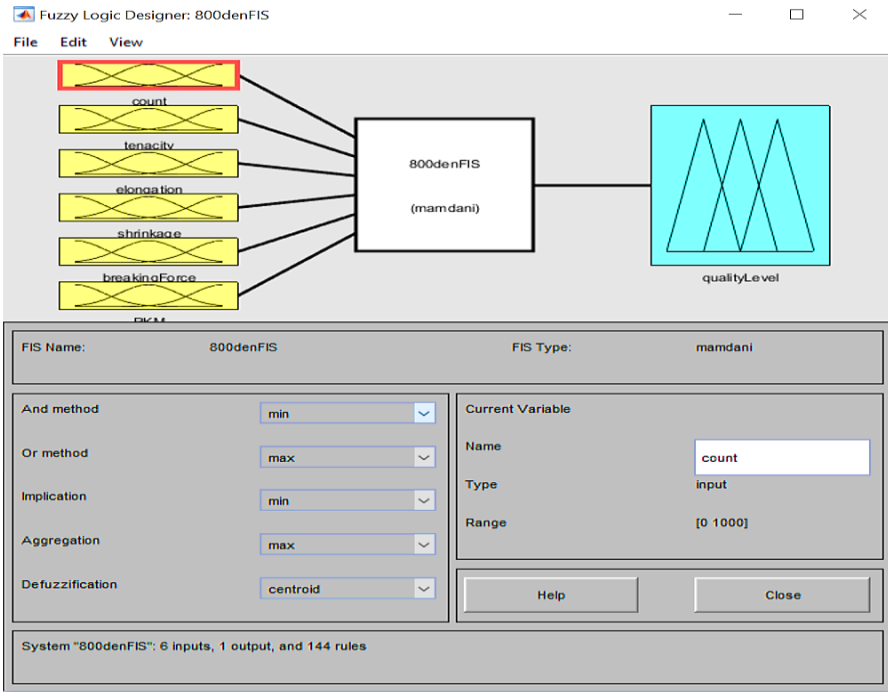

Fig. 4. The main structure of the ‘800 DEN ATY quality classification’ model.

As an output parameter of quality classification; for linguistic fuzzy fuzzy sets, namely best (A), average (B), low (C) and non-acceptable (D), are chosen and one form of membership function (triangular) is used. The ranges of membership functions for both input and output parameters are generated according the quality standards values of 800 DEN ATY as shown in *Table 3*. Membership functions for both input and output parameters are consulted with Fuzzy Logic Toolbox of MATLAB. Types of membership functions and linguistic variables for input variables of ‘800 DEN ATY Quality Classification’ model is shown in *Table 4*. *Fig. 5* represents the membership functions applied for input parameter of count; *Fig. 6* represents the membership functions applied for input parameter of tenacity and *Fig. 7* represents the membership functions applied for the output parameter of quality level.

Table 3. Quality standards of 800 DEN ATY.

Input	Range
Count	800 +- %5
Tenacity	4.00 +- %10
Elongation	>= 10.50
Shrinkage	<10
Breaking force	2.90<= breaking force >= 3.60
RKM	32<= RKM >=38

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Table 4. Membership functions and linguistic variables of ‘800 DEN ATY ‘Quality Classification’ model.

		Linguistic Variable	Ranges	Membeship Function
Input 1	Count	A (Best level)	= 800	Triangular
		B (Avarage level)	760 <= Count <= 840	Trapezodial
		C (Low level)	Count < 760 and Count > 840	Z-shaped
Input 2	Tenacity	A (Best level)	= 4.00	Triangular
		B (Avarage level)	3.60 <= Tenacity <= 4.40	Trapezodial
		C (Low level)	Tenacity < 3.60 and Tenacity > 4.40	Z-shaped
Input 3	Elongation	A (Acceptable level)	=> 10.50	Trapezodial
		B (Non-acceptable Level)	< 10.50	Trapezodial
Input 4	Shrinkage	A (Acceptable level)	< 10.00	Trapezodial
		B (Non-acceptable Level)	=> 10.50	Trapezodial
Input 5	Breaking force	A (Acceptable level)	<=2.90 BF>= 3.60	Trapezodial
		B (Non-acceptable Level)	BF < 2.90 and BF > 3.60	Z-shaped
Input 6	RKM	A (Acceptable level)	<=32 RKM >= 38	Trapezodial
		B (Non-acceptable Level)	BF < 32 and RKM > 38	Z-shaped

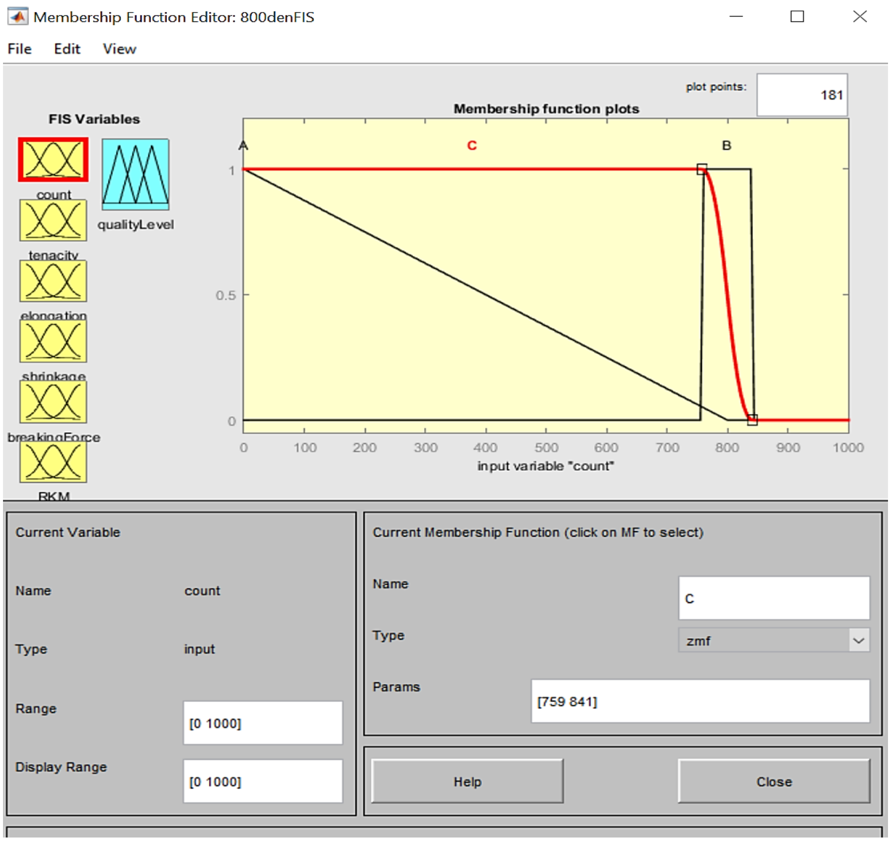

Fig. 5. Membership function of ‘Count’ for 800 DEN ATY.

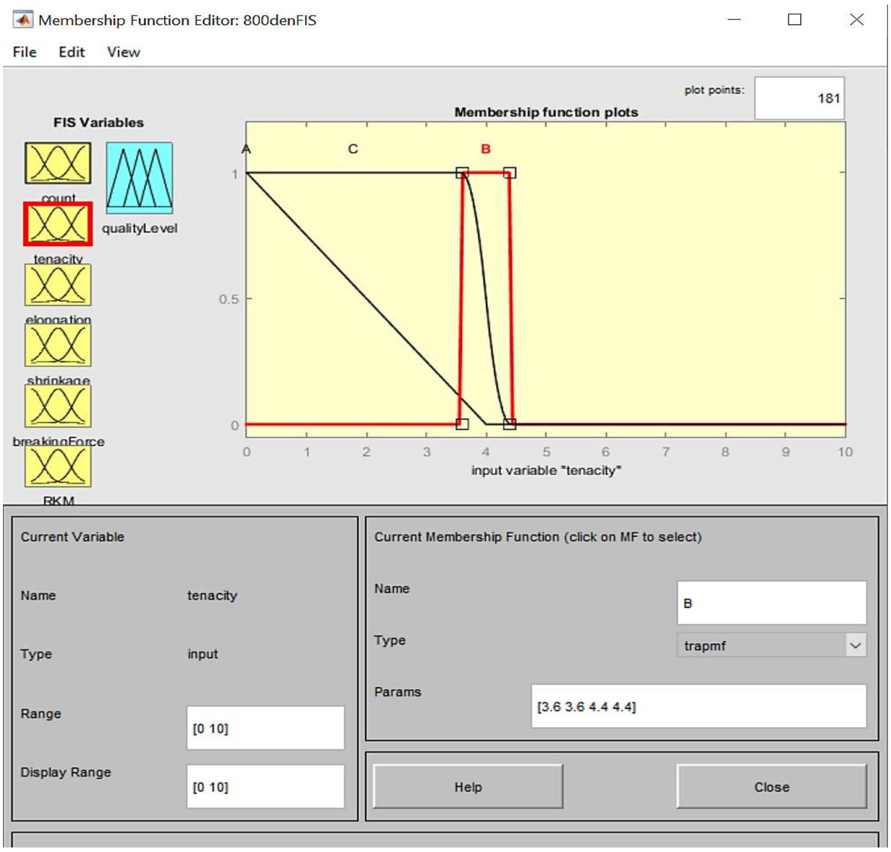

Fig. 6. Membership function of 'Tenacity' for 800 DEN ATY.

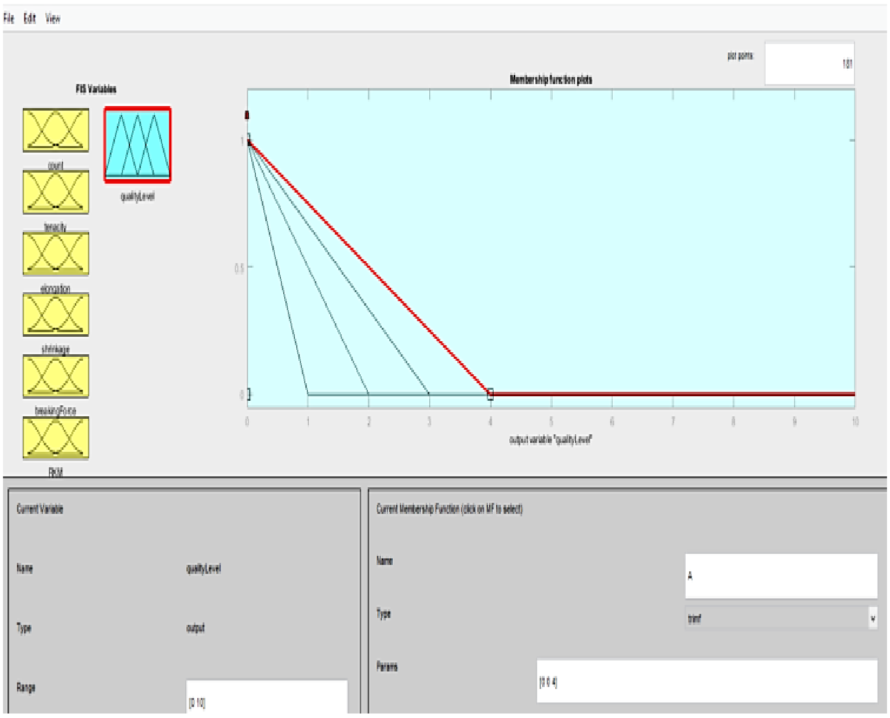

Fig. 7. Membership function of 'Quality Level' for 800 DEN ATY.

Following total 144 rules are created for fuzzification with the help of expert knowledge of the quality control manager of the company. As an example of rule base if (count is B) and (tenacity is A) and (elongation is A) and (shrinkage is A) and (breaking force is A) and (RKM is A) then (output is A). A Mamdani max-min inference approach is applied for combination of fuzzy sets into a single fuzzy set. Finally centroid defuzzification method is used to convert the output into non-fuzzy crisp numeric value. *Fig. 8* represents the rule viewer of the FIS. The rule viewer is the interface that shows the change in output parameter as a result of changes in input parameters. The decision makers can take the final decision by this interface to select optimum input parameters.

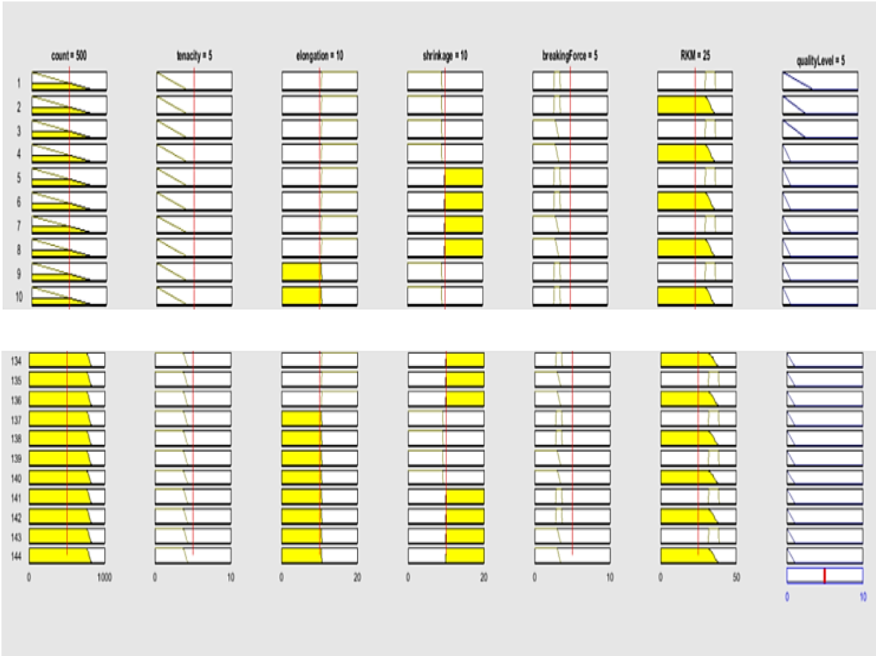

Fig. 8. Rule viewer of 800 DEN ATY quality classification model.

For '1100 DEN ATY Quality Classification' application, four linguistic fuzzy sets, namely best (A), average (B), low (C) and non-acceptable (D) are evaluated for the output parameter of quality classification. One form membership function (triangular) is develoed for the output parameter. The ranges of membership fuctions for both input and output parameters are generated according to quality standards of 1100 DEN ATY as shown in *Table 5* and test results of 1100 DEN ATY samples (first and last five samples) shown in *Table 6*. *Table 7* represents the linguistic variables, types of memberhip functions developed for input variables of quality classification model of 1100 DEN ATY samples. *Fig. 9* shows the membership functions applied for the input parameter of 'Elongation' and *Fig. 10* shows the membership function applied for the input parameter of 'RKM' which are consulted with Fuzzy Logic Toolbox of MATLAB.

Table 5. Quality standards of 1100 DEN ATY.

Input	Range
Count	1100 +- %5
Tenacity	4.00 +- %10
Elongation	>= 12.00
Shrinkage	<9
Breaking force	3.90<= breaking force >= 4.65
RKM	31<= RKM >=38

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Table 6. Test results of 1100 DEN ATY samples.

Sample	Count	Tenacity	Elongation	Shrinkage	Breaking Force	RKM
S1	1083.00	4.20	13.00	8.00	4.65	38.27
S2	1079.00	4.10	13.00	8.10	4.53	37.43
S3	1091.00	4.10	12.50	7.50	4.48	36.56
S4	1078.00	4.10	12.30	7.80	4.46	36.87
S5	1077.00	3.90	11.20	9.30	4.27	35.31
.	.	.	.	.	.	.
S651	1110.00	3.80	12.40	7.10	4.16	33.75
S652	1109.00	3.70	13.00	6.80	4.12	33.14
S653	1100.00	3.80	12.70	6.80	4.19	33.95
S654	1100.00	3.80	12.30	6.20	4.21	34.31
S655	1101.00	3.90	13.50	6.90	4.30	34.50

Table 7. Membership functions and linguistic variables for '1100 DEN ATY 'quality classification' model.

		Linguistic Variable	Ranges	Membeship Function
Input 1	Count	A (Best level)	= 1100	Triangular
		B (Avarage level)	1045 <= Count <= 1155	Trapezodial
		C (Low level)	Count < 1045 and Count > 1155	Z-shaped
Input 2	Tenacity	A (Best level)	= 4.00	Triangular
		B (Avarage level)	3.60 <= Tenacity <= 4.40	Trapezodial
		C (Low level)	Tenacity < 3.60 and Tenacity > 4.40	Z-shaped
Input 3	Elongation	A (Acceptable level)	=> 12.00	Trapezodial
		B (Non-acceptable Level)	< 12.00	Trapezodial
Input 4	Shrinkage	A (Acceptable level)	< 9.00	Trapezodial
		B (Non-acceptable Level)	=> 9.00	Trapezodial
Input 5	Breaking force	A (Acceptable level)	<=3.90 BF>= 4.65	Trapezodial
		B (Non-acceptable Level)	BF < 3.90 and BF > 4.65	Z-shaped
Input 6	RKM	A (Acceptable level)	<=31 RKM >= 38	Trapezodial
		B (Non-acceptable Level)	BF < 31 and RKM > 38	Z-shaped

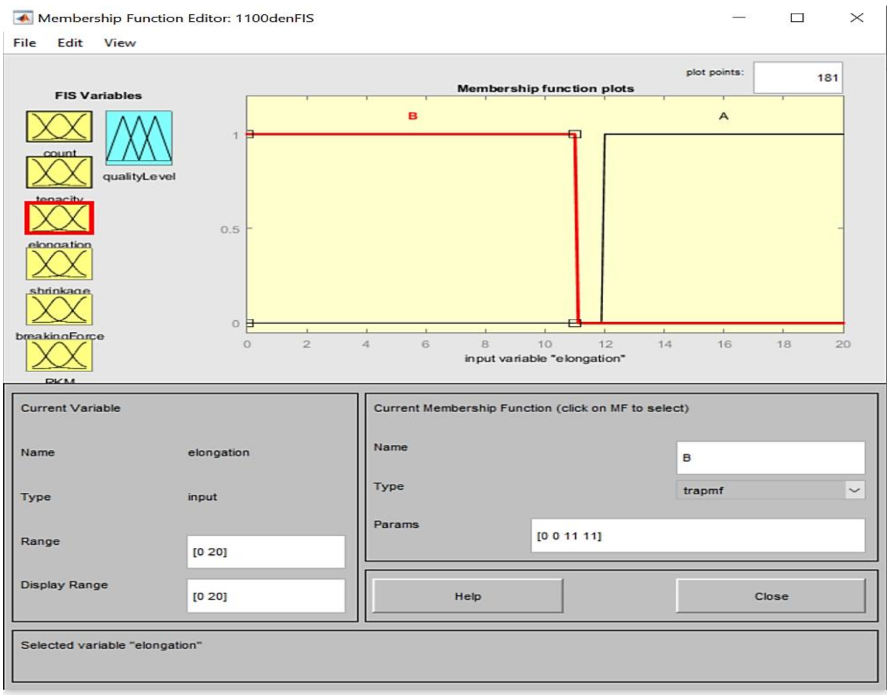

Fig. 9. Membership function of 'Elongation' for 1100 DEN ATY.

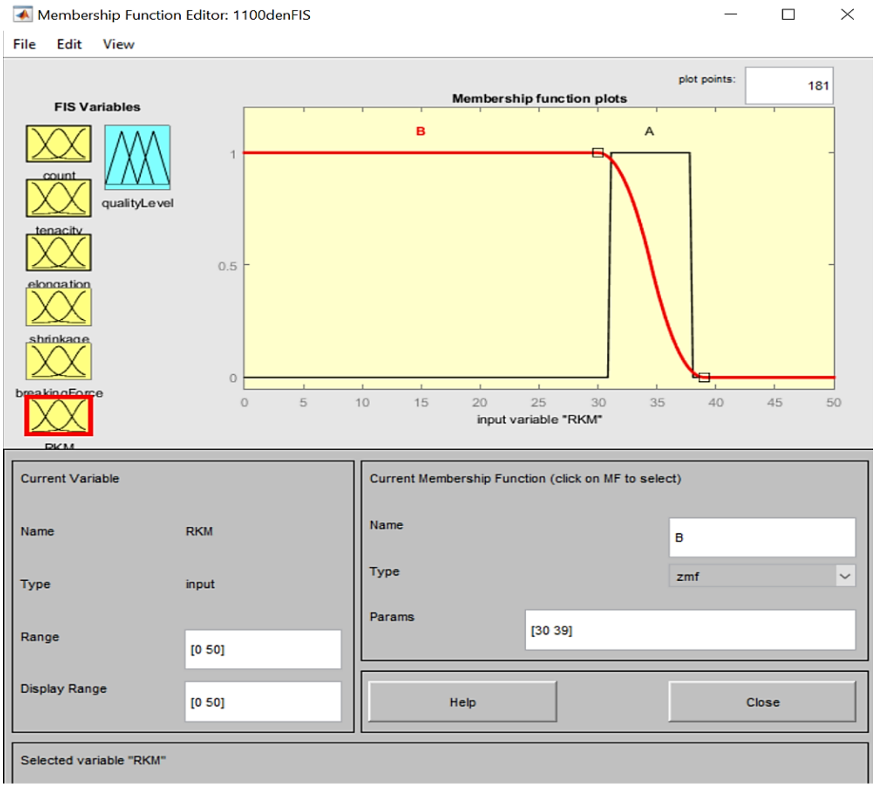

Fig. 10. Membership function of 'RKM' for 1100 DEN ATY.

For '1100 DEN Quality Classifaction' model total 144 rules are created for the fuzzification step of the model. *Fig. 11* represents the rule viewer of 1100 DEN ATY Quality Classification model. A Mamdani max-min inference approach is applied for combination of fuzzy sets into a single fuzzy set. Finally centroid defuzzification method is used to convert the output into non-fuzzy crisp numeric value.

Fig. 11. Rule viewer of 1100 DEN ATY quality classification model.

Quality classification model is also developed for 1350 DEN ATY samples. Four linguistic fuzzy sets, namely best (A), average (B), low (C) and non-acceptable (D) are evaluated for the output parameter of '1350 DEN ATY Quality Classification' model. One form membership function (triangular) is develoed for the output parameter. The ranges of membership fuctions for both input and output parameters are generated according to quality standards of 1350 DEN ATY as shown in *Table 8*. Test results of 1350

DEN ATY samples (first and last five samples) are shown in *Table 9*. *Table 10* represents the linguistic variables, types of memberhip functions developed for input variables of quality classification model of 1100 DEN ATY samples. *Fig. 12* shows the membership functions applied for the input parameter of 'Shrinkage' and *Fig. 13* shows the membership function applied for the input parameter of 'Breaking Force' which are consulted with Fuzzy Logic Toolbox of MATLAB.

Table 8. Quality standards of 1350 DEN ATY.

Input	Range
Count	1350 +- %5
Tenacity	4.20 +- %10
Elongation	>= 12.50
Shrinkage	<9
Breaking force	5.00<= breaking force >= 6.50
RKM	30<= RKM >=38

Table 9. Test results of 1350 DEN ATY samples.

Sample	Count	Tenacity	Elongation	Shrinkage	Breaking Force	RKM
S1	1340.00	4.80	13.20	8.80	6.49	43.25
S2	1335.00	4.50	13.10	7.50	6.13	40.43
S3	1337.00	4.50	13.20	6.10	6.13	40.29
S4	1339.00	4.50	13.40	6.00	6.13	40.29
S5	1344.00	4.40	13.60	8.90	5.95	38.77
.	.	.	.	.	.	.
S268	1363.00	4.30	13.90	7.70	5.74	37.49
S269	1373.00	4.20	12.40	8.10	5.62	36.46
S270	1376.00	4.20	13.50	7.80	5.67	36.68
S271	1370.00	4.30	13.90	7.30	5.74	37.36
S272	1360.00	4.20	13.10	8.00	5.63	36.87

Table 10. Membership functions and linguistic variables of '1300 DEN ATY 'Quality Classification' model.

		Linguistic Variable	Ranges	Membeship Function
Input 1	Count	A (Best level)	= 1350	Triangular
		B (Avarage level)	1282 <= Count <= 1418	Trapezodial
		C (Low level)	Count < 1282 and Count > 1418	Z-shaped
Input 2	Tenacity	A (Best level)	= 4.20	Triangular
		B (Avarage level)	3.78 <= Tenacity <= 4.62	Trapezodial
		C (Low level)	Tenacity < 3.78 and Tenacity > 4.62	Z-shaped
Input 3	Elongation	A (Acceptable level)	=> 12.50	Trapezodial
		B (Non-acceptable Level)	< 12.50	Trapezodial
Input 4	Shrinkage	A (Acceptable level)	< 9.00	Trapezodial
		B (Non-acceptable Level)	=> 9.00	Trapezodial
Input 5	Breaking force	A (Acceptable level)	<=5.00 BF>= 6.50	Trapezodial
		B (Non-acceptable Level)	BF < 5.00 and BF > 6.50	Z-shaped
Input 6	RKM	A (Acceptable level)	<=30 RKM >= 38	Trapezodial
		B (Non-acceptable Level)	BF < 30 and RKM > 38	Z-shaped

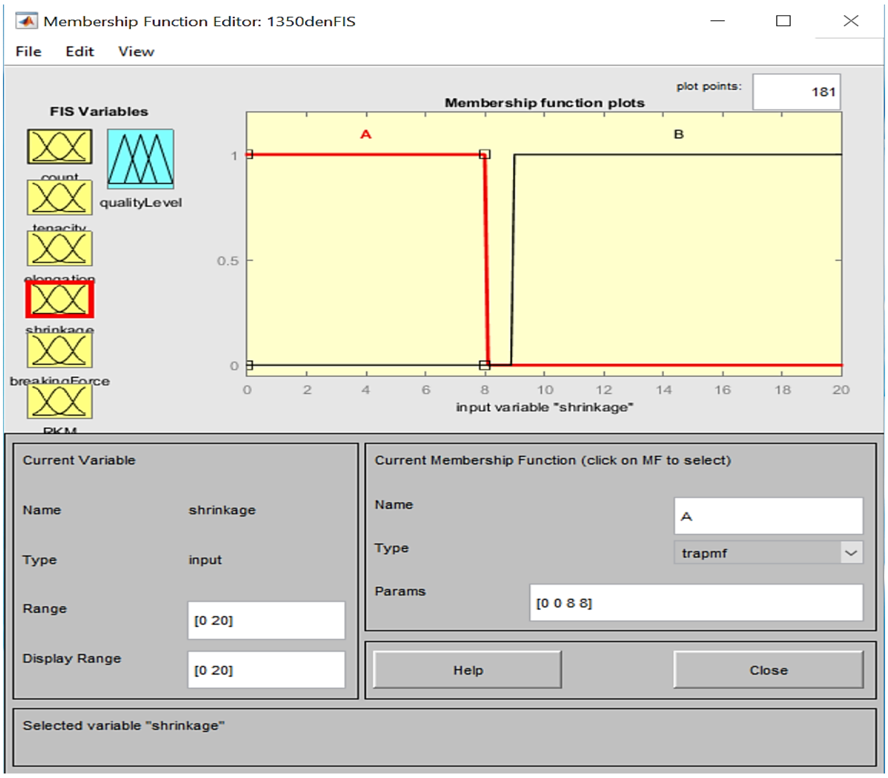

Fig. 12. Membership function of 'Shrinkage' for 1350 DEN ATY.

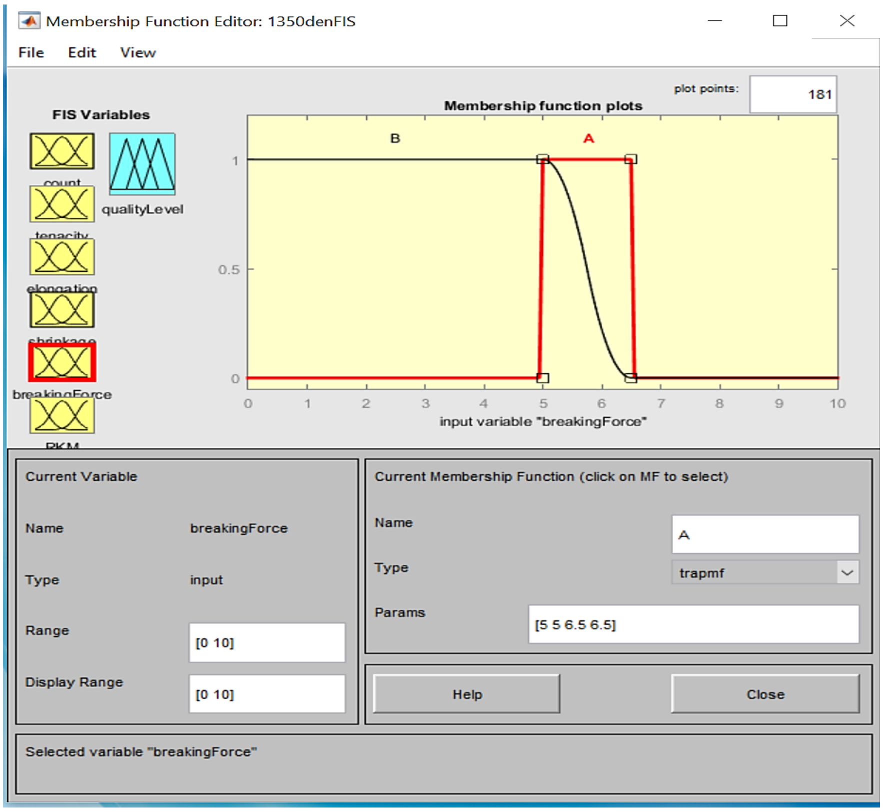

Fig. 13. Membership function of 'Breaking F orce' for 1350 DEN ATY.

Total 144 rules are consulted for the fuzzification of '1350 DEN Quality Classifaction' model. *Fig. 14* represents the rule viewer of '1350 DEN ATY Quality Classification' model. A Mamdani max-min inference approach is re-applied for the combination of fuzzy sets into a single fuzzy set. At the final part of the application, centroid defuzzification method is used to convert the output into non-fuzzy crisp numeric value.

Fig. 14. Rule viewer of 1350 DEN ATY quality classification model.

4 | Conclusion

A FIS is proposed to model the quality level of ATY. The model is evaluated by using factors affecting quality of ATY as inputs into a FIS. The developed fuzzy rules give a very good understanding about the interaction between the factors. Different membership functions are used for input parameters and one form membership function is used for output parameter. The ranges of membership functions are compromised according the quality standards of the yarn. The fuzzy 'Quality Classification' model is implemented for three types of ATY samples (800 DEN, 1100 DEN and 1350 DEN). The Mamdani inference engine is used for the model. As a result of the model; for 800 DEN ATY, it is seen that two of yarn samples are at the best level (A), 382 of yarn samples are at the average level (B), 54 of yarn samples are at the low level (C), 225 of yarn samples are at the non-acceptable level (D) of the quality classification. When the model is practiced for 1100 DEN ATY, it is seen that 606 of yarn samples are at the average level (B), seven of yarn samples are at the low level (C), 42 of yarn samples are at the non-acceptable level of quality classification model. As a result of '1350 DEN ATY Quality Classification' application 188 of yarn samples are at the average level (B), 40 of yarn samples are at the low level (C), 44 of yarn samples are at the non-acceptable level (D) of the model. The test results have been interpreted linguistically from the quality control of the problem and do not coincide with the results of the fuzzy model. As an example, without a 'Qulity Classification' model the quality control manager determined 13 of 800 DEN yarn samples as faulted yarn, but with the proposed model 225 of yarn samples are determined at the non-acceptable level of the model. It is seen that faulted yarns are not often realized before thet-y reach the customers. A quality classification model can help the company to estimate the yarn on which level is it. Therefore, yarn faults can be eliminated before they reach the customer. Quality control manager can take the final decision with the help of the model. Thus, there will be improvements on quality control process and with these impovements there may be decreases in costs. Also, all yarns that are ready for sale are sold at the same price. As an example, a yarn group with has very good values according to the quality standard values and a yarn group with average values are priced at the same selling price. A better pricing strategy can be done with the help of the quality classification model. The system is quite easy to develop and it can be modified easily any type of product which has any different denier. This study has potential limitations. Physical properties of the yarn that affect the quality of the yarn used as criteria of FIS evaluated but environmental conditions (temperature and humidity) are not considered as affecting factors while preparing this study. Three membership functions (triangular, trapezoidal and z-shaped) are used for both input and output parameters of the fuzzy model, other types of membership functions can be developed while constructing the model. Recommendations for future studies are evaluating the model with Sugeno inference engine to compare the results and developing a decision support system for the company

Conflicts of Interest

All co-authors have seen and agree with the contents of the manuscript and there is no financial interest to report. We certify that the submission is original work and is not under review at any other publication.

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Modeling Fuzzimetric Cognition of Technical Analysis Decisions: Reducing Emotional Trading

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Abstract

Stock traders' forecasting strategies are mainly dependent on Technical Analysis (TA) indicators. However, some traders would follow their intuition and emotional aspects when trading instead of following the mathematically solid forecasting techniques of TA(s). The objective of this paper is to help traders to rationalize their choices by generating the maximum and minimum tolerances of possible prices (termed in this paper as "fuzzy spectrum") and hence reducing their "emotional" trading decisions. This would be an important aspect towards avoiding an undesired outcome. Fuzzy logic has been used in this paper to identify such tolerances based on the most popular TA(s). Fuzzification of these TA(s) was used via a modular approach of fuzzy logic and by adopting "fuzzimetric sets" described in this paper to achieve the "fuzzy spectrum" of forecasted price tolerances when buying and selling decisions. Experimental results show the success of developing the "fuzzy spectrum" based on the "fuzzy" tolerances discovered from the TA(s) outputs. As a result, this paper contributes towards a better "rationalized" decision making when it comes to buying and selling stocks in this kind of industry.

Keywords: Cognitive modelling, Fuzzy system, Technical analysis, Trading systems, Stock trading optimization, Fuzzimetric sets.

1 | Introduction

The Stock trading is usually based on technical trading rules to identify the buying/selling signals based on either short or long history of stock prices [53]. The future prices are difficult to predict, as the information gathered at any moment in time may reflect the market efficiency rather than the actual price [1] and [2]. However, current prices may also represent the information overreaction as discussed by Kahneman and Tversky [3]. Investors may overreact to available market information as well as overreact to private information, due to emotional factors about specific shares and securities [4]. In a similar fashion and more recently Ahmad and Shah [5] investigated the influences of overconfidence in stock exchange decisions where a theoretical framework of behavioral finance was adopted as the basis of their research.

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Intelligent techniques to support such decision-making scenarios gathered high interest among researchers. For example, Chopra and Sharma [6] reviewed AI implementation techniques of AI to the stock market forecasting where it was concluded that AI techniques can be a successful methodology of financial market analysis. Also, emotional and psychological factors were studied in Khayamim et al. [7], where fuzzy logic-based reasoning is used to simulate the portfolio optimization model, ignoring the emotional and psychological aspects. Hence, the main reason for inventing the technical trading rules originally was investigated by Park and Irwin [8] is to achieve the maximum profit that can be generated from trading in stocks. All of these technical indicators are of two main types: trend indicator and mean-reverting indicators (also termed as counter-trend), where each type might be suitable for a specific stock chart behavior.

Based on fuzzy logic, Gradojevic and Gençay [9] suggested a mechanism of reducing trading uncertainty, where two problems were addressed. These problems are the market timing and the order size. Along the same vein, Escobar et al. [10], proposed a technical fuzzy indicator that incorporates subjective features simulating the human decision making which uses a comparison between traditional Relative Strength Index (RSI), Moving Average Convergence Divergence (MACD) as opposed to the fuzzy-built multi-agent indicator obtaining the behavior and profit as outputs. Social network-based prediction of short-term stock trading was proposed by Cremonesi et al. [11] where semantic sentiment analysis was used as a mechanism for inspecting Twitter posts. In an attempt to reduce the scope of fuzzy boundaries, Hao et al. [12] studied the merge of information from online news to predict the price of stock price index where fuzzy sets were used to identify the outliers in such decision making. Dong and Ma [13] also, studied the optimal number of and length of fuzzy intervals were multiple relevant factors taken into consideration of a stock index forecasting model.

As such, this paper contributes by two folds. The first is to introduce one of the fuzzy variants termed as "Fuzzimetric sets" and the second is to implement the mechanism of fuzzimetric sets into the implementation of stock trade decision making where maximum and minimum possible tolerances can be identified as an attempt to reduce emotional trading. This has been accomplished by utilizing the characteristics of the most popular Technical Analysis (TA) indicators with the associated trading decision-making. Unlike other research on fuzzy implementation to trading systems, this paper renders some of the most relevant parameters of popular indicators fuzzify and then feeds them into a modular approach of the fuzzy system. The defuzzified outputs are combined into the final fuzzy output after inferring the relative outputs from these indicators. Furthermore, and as part of the characteristics of Fuzzimetric sets, the proposed system uses certain mutations of fuzzy sets to achieve low and high tolerances for each one of the sets that would help the investor in discovering the related price range expected for the security. A combination of these mutation sets provides a "De-fuzzified spectrum" of possible outputs that can act as a decision support system for traders.

The remaining parts of this paper are organized into 4 more sections. Section 2 related work to cognitive and emotional trading discussed as the main motivation of this research. Section 3 provides a brief review of the Fuzzimetric sets which is the adopted variant of fuzzy sets in this research paper. In Section 4 a short review of the most popular TAs and trading strategies where these TAs will be fuzzified as part of the proposed system. Section 5 describes the fuzzy inference mechanism adopted in this research whereas Section 6 describes the data collected and the back-test experimental results. Section 7 provides the conclusion and future work.

2 | Cognitive Decisions of Stock Traders and Emotional Trading

The most viable characteristic of stock traders is that they are of a risk-taking behavior which is dependent on the cultural, cognitive, and personality of the individual [54]. TA is the technique that most investors use to rationally conclude the stock trading investment decisions. There are many types of technical indicators which are defined as two main categories: Trend-following indicators (like MACD) and "Revert to mean" indicators (like RSI). Different indicators would require different rules

(Strategy) of selling and buying stocks. It would be very easy to make mistakes when watching charts, therefore wrong rules might be implemented due to the lack of comprehensive "cognition" of the scenario in place.

On top of possible flaws in the wrong decision when concluding the trade, emotional trading conforms to another major loss/biasedness when conducting the trade. Investors in such an environment do not always think rationally by following the TA rules, but rather irrationally by following their feelings and emotions about certain stocks [14]. Vaščák [15] proposed "fuzzy cognitive maps" as a method to overcome the limitations of rule-based systems by injecting fuzziness to such systems to reduce the complexity of dynamic systems. Adding to this ambiguity, traders' behavior might include a pattern to trade weekly, daily or even hourly. Different rules/indicators would be suitable to a specific trading scenario and the behavior of traders is not always certain, as studied by Richards and Willows [16]; moreover, the effect of such sentiment on the volatility of the market was studied by Rupande [17].

In order to rigorously model the behavior of stock traders, two main driving forces would contribute towards their investment decision-making. *Fig. 1* shows the rational and irrational driving forces where the need for a cognitive development model represents the intersection between the human-trader cognition and the cognitive developed analytics based on the rules generated from technical indicators. Such a "Decision support system" would be a necessity in such a scenario where it can aid the trader in the appropriate decision to take. Ototsky and Manenkov [18] were able to recognize this fact and introduced the concept of "Cognitive Centres" to emphasize the cognitive technology used in management modeling practices by integrating cognitive and information technologies. In order to combine emotions with cognitive architecture, Marco et al. [19] proposed a unified emotional-cognitive-affective architecture to be integrated with intelligent agents to influence and modify the behavior of the agent in real-time, to achieve a more realistic and believable interaction with the user. 'Emotion' management in stock trading organizations can also be a factor towards reducing emotional-driven decision making. Kouatli [20] introduces a Framework Architecture for Managing Emotions (FAME) as a method of controlling emotional intelligence within organizations. A classification of "emotion-based account was studied by Duxbury et al. [21] where a conceptual analysis of how emotions influence the financial market behavior was proposed to buy and sell in asset markets. A similar study of the relationship of social media emotions and stock market crash was studied by Ge et al. [22] where a cognition-based framework of "Emotion-Cognition-Market" has been adopted. Yuan [23] also identified the relationship between cognitive biases and decision-making behavior in financial markets.

To be able to categorize the different stocks, Sun et al. [24] for example, proposed a bi-clustering trading pattern of stock investments styles. Maciel and Ballini [25] proposed a fuzzy-rule-based model to forecast high and low stock prices. A generic framework for Cognitive Analytics Management (CAM) was proposed by Osman and Anouze [26] which was later used to investigate and study the governmental e-services from users' perspective [27]. Cabrera-Paniagua and Rubilar-Torrealba [28] proposed an Artificial Autonomous System (AAS) for the stock market domain where the personality profiles of the individual were considered as one of the proposed "big five" models towards decisions in financial investments. Kareem [29] also studied the personality issue and the emotional and cognitive influence towards decision making in the Iranian stock market where fuzzy analysis was used to provide priority weighting. Kouatli and Arayssi [30] introduced the idea of reducing emotional trading and proposed to build a fuzzy-based model where traders can use. This paper explores and experiments on the proposed strategy of implementing such a model where traders can use it to view the minimum and maximum tolerances and consequently achieve a "fuzzy spectrum" where traders can use as a guideline instead of just following intuition (irrational decisions) and accordingly reducing emotional trading.

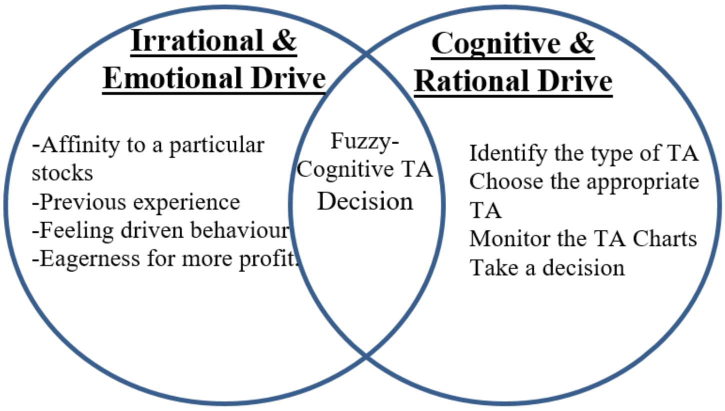


Fig. 1. Stock trader driving forces of investment decisions and the need for fuzzy cognitive decision support system.

3 | The Background of Fuzzimetric Sets

In *Fig.1*, the fuzzy logic started by Zadeh [31] as a mechanism of decision making to deal with uncertainty. It was based on an extension of set theory, where instead of a crisp description of a member belonging (or not) to a set, a member can have partial membership in a specific set. Since the introduction of the traditional fuzzy set theory, other extensions of the theory emerged like type-2 fuzzy sets, intuitionistic fuzzy sets, and hesitant fuzzy sets. Recently, fuzzimetric sets, a newly defined extension of fuzzy sets also emerged which will be briefly reviewed in this paper where more details can be found in Kouatli [32]-[34]. Fuzzy logic was utilized by different researchers to model a cognitive systems. For example, García-Vico et al. [35] used fuzzy logic Fuzzy Rule-Based System (FRBS) to extract patterns in big data to improve decision making process patterns in big data environments. Intuitionistic fuzzy sets was also utilized by Liu et al. [36] to propose a new decision making method. In order to enhance the decision process of stock trading, this paper utilizes the concept of “Fuzzimetric sets” to identify the possible stock price spectrum based on maximum and minimum possible tolerances of fuzzimetric sets where the principle briefly reviewed in the next paragraph(s). This principle of Fuzzimetric sets in the universe of discourse and can be defined as Positive Zero (P0), Positive Small (PS), Positive Medium (PM) and Positive Big (PB) (*Fig. 2. a*). These fuzzy variables can be defined as:

$$PO = \int_{0}^{\pi/2} |\sin(\pi/2 - x)|. \tag{1}$$

$$PS = \int_{0}^{\pi} |\sin(x)|. \tag{2}$$

$$PM = \int_{\pi/2}^{3\pi/2} |\sin(\pi/2 - x)|. \tag{3}$$

$$PB = \int_{\pi}^{3\pi/2} |\sin(x)|. \tag{4}$$

Assuming sinusoidal function, then these representations can be defined in an analogy to trigonometric functions with an exception of taking the absolute values only. Hence, based on the definition of fuzzimetric arcs [42], the concept defines a mechanism of selection, mutation and cross-over fuzzy set shape and hence affecting the overall decision making de-fuzzified value. For example, triangular, trapezoidal etc., can be achieved by a simple mutation of fuzzy sets using a genetic operator:

$$\mu = \frac{\text{ARCSIN (Fuzzy Variable)}}{T} \tag{5}$$

$$= 1 \text{ for } \mu > 1.$$

The fuzzy variable is any of PO, PS, PM or PB, and the T parameter is the shape alternation factor (mutation factor) with the most active range of $0 < T < 2700$. Mutations of these fuzzy variables can thus be identified as:

$$\text{Mutated-PO} = \int_0^{\pi/2} \frac{\text{ARCSIN}(|\sin(\pi/2 - x)|)}{T_P}. \tag{6}$$

$$\text{Mutated-PS} = \int_0^{\pi} \frac{\text{ARCSIN}(|\sin(x)|)}{T_{PS}}. \tag{7}$$

$$\text{Mutated-PM} = \int_{\pi/2}^{3\pi/2} \frac{\text{ARCSIN}(|\sin(\pi/2 - x)}{T_{PM}}. \tag{8}$$

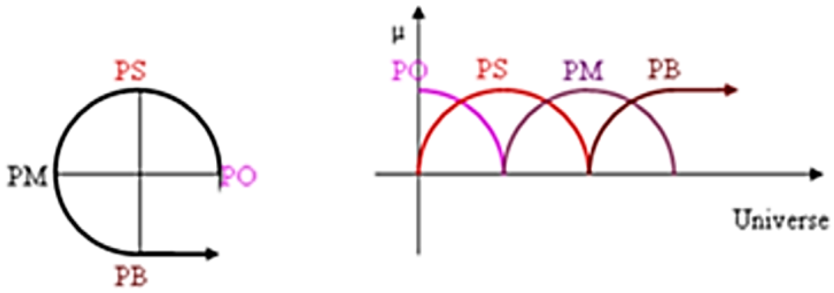
$$\text{Mutated-PB} = \int_{\pi}^{3\pi/2} \frac{\text{ARCSIN}(|\sin(x)|)}{T_{PB}}. \tag{9}$$

Altering the value of mutation factor "T" allows us to mutate the fuzzy variables where examples are shown in *Figs. 2(b)* and *(c)*. More details of fuzzy sets and utilization of this concept to the decision-making process in a manufacturing environment can be found in Kouatli [37], where a robotic example was taken as a vehicle to a step-by-step explanation of inference using fuzzy sets. Formal extended definition of the concept of fuzzimetric arcs with its extensions of mutation and crossover can be found in Kouatli [32] where the formal definition of "fuzzimetric sets" was characterized as a platform combining both types of fuzzy sets. Fuzziness control of such sets can be found in Kouatli [34] with an example of application to employee evaluation system can be found in Kouatli [33].

4 | Technical Analysis, Trading Rules and Strategy

TA uses the technical indicators formulae derived from high, low or close prices collected over a certain period of time. Traders use a graphical format of these indicators to view and analyze the current situation with stock or indices. However, to interpret such graphical charts, the user needs to be able to understand these indicators and to be able to take an action (buy/sell) accordingly. There are so many factors to look at before any decision can be made, and accordingly a cognitive decision support system in trading analysis based on multiple indicators would help towards achieving the right decisions. More detailed information about most popular indicators can be found in Kouatli and Yunis [38]. However, for readers' convenience, these most popular indications will be reviewed briefly in the following sub-sections.

Technical indicators are usually of two types: trend-following charts/indicators and "revert-to-the-mean" charts/indicators. In both types, a system would be necessary to justify the total possible tolerances given a certain situation, and accordingly stopping the traders from irrational speculation of trading and hence reduce the "emotional trading"



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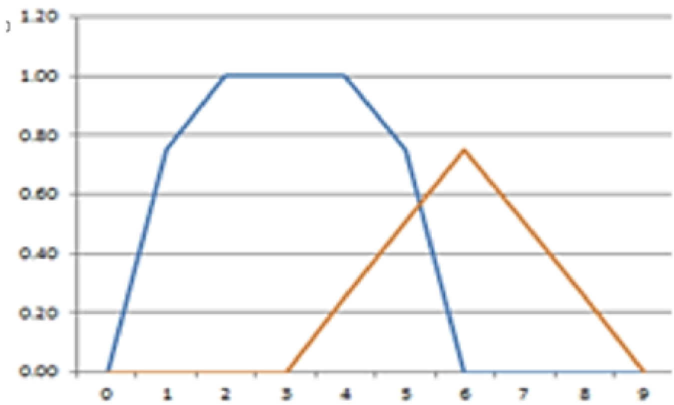

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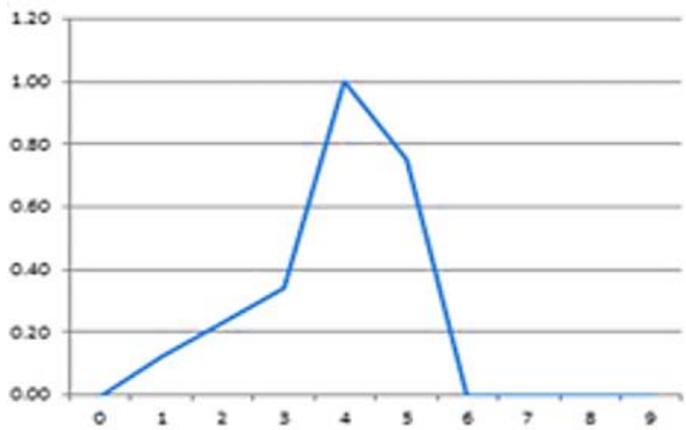

c.

Fig. 2. Evolution of fuzzimetric sets: a. positive fuzzimetric arc with the spread of different fuzzy variable on the universe of discourse; b. example of generated fuzzy set shapes using mutation factor T. Ps: Tps=40, PM: Tpm=120; c. examples of mutated and crossed-over ps fuzzy set. left-half Tps=270, Right-half Tps=40.

4.1 | Trend-Following Indicators

As its name imply, trend-following indicators "follow" a trend to indicate if it is falling or rising where they are based on moving-average (SMA or Exponential Moving Averages (EMA)) with lower and upper bounds. SMA calculates the average price over a fixed number of periods (long and/or short period of time). If the stock price is exceptionally volatile, then a moving average will help to smooth the data and hence filters out any random noise and offers a smoother perspective on the price (EMA). On top of moving average, traders uses a momentum indicators to help them predicting the change in a pattern. Example of these types of indicators are Bollinger Bands (BB) and MACD.

4.1.1 | Bollinger bands (BB)

BB-Trend Indicator is a relatively new type of indicator proposed by Bollinger [55], which measures the strength as well as the direction. It can be found by differential values (periods 20 to 50) of lower and upper bands using the following formula:

Lower-BBTrend = |(lowerBB(20) - lowerBB(50)|,
Upper-BBTrend = |(upperBB(20) - upperBB(50)|,
BBTrend = (lower BBTrend – upper BBTrend) / middle BB(20).

Where BBTrend >0, indicates that the trend is bullish and BBTrend <0 zero, indicates that the trend is bearish where the actual value represents the momentum of the trend.

b% Indicator: is an oscillator variable used with BB. %b plots the stock's closing price as a percentage of the upper and lower bands. Where this range is measured from zero (the lower band) to 1 (the upper band). The objective of %b is to indicate how close the stock's current price is to the bands (e.g. UB=100 & price = 80, then %b=0.8). This is useful to identify when a price jumps a band determining divergences and trend changes.

4.1.2 | Moving average convergence divergence (MACD)

Based on two EMA, MACD is considered to be a trend-following indicator as well as short-term momentum indicator. Crossover-Buy Signal (also called Golden Cross): is when the short term Moving Average MACD-Fast jumps above the long-term MACD-slow which means that the downward trend is ending and a new uptrend is expected to start and hence, such “Golden Cross” suggests a buying signal.

I. Golden Cross: MACD-Fast (t) > MACD-Slow (t) & MACD-Fast (t-1) < MACD-Slow (t-1).
II. Crossover-Sell Signal: (also called Dead cross) is when the long-term MACD-Slow moves above the short term MACD-fast which means that the price is recently moving downwards relative to previous periods which also suggests a selling signal. Hence: Dead Cross (Sell signal) MACD-Fast (t) < MACD-Slow (t) & MACD-Fast (t-1) > MACD-Slow (t-1).

4.2 | Mean-Reversion Indicators

As its name imply, “Mean-Revert” type of indicator (or Oscillators) can be used to conclude a sell/buy decision as the price is usually always reverted back to the mean. Traders looks at prices drifting away from the mean to check the unsustainability in the trend using SMA or EMA with a minimum of 50-periods average. Mean-reversion usage is less popular than trend-following indicators, and it requires more trading experience than trend-following where emotional and psychological factors can be effective elements towards wrong trading decisions. Most popular indicator of “mean-reverting” are RSI and “momentum oscillator”.

4.2.1 | Relative strength index (RSI)

Comparison between up and down days of share prices compromises the functionality of RSI (defaulted to 14-periods for short term trading) with a scale of 0-100 where a value of more than 70 considered being bullish and the value below 30 indicate bearish trend. RSI is also considered to be one type of momentum oscillator with a center line of 50. Above 50, RSI indicates that the momentum is upwards, where traders may consider buying; conversely, RSI value below 50 indicates that there is bearish signal started.

RSI formula is dependent on the amount of gaining over loosing for certain period of time (ideally 14). This is defined by:

$$RSI=100-100/(1+RS).$$

Where RS is Relative Strength in that specific period (e.g. 14) and RS= (Average Gain)/ (Average Loss).

4.2.2 | Momentum oscillators (stochastic indicator)

Stochastic indication measure the strength of a trend with the amount of fast and strong price is going up or down. Above 80% is considered strong uptrend and below 20% is a strong downtrend. Stochastic indicator can be integrated with MACD to provide better trading signals.

%K= 100*(closing-low)/ (high-low) for 14-period.

An average of the last 3 slower stochastic values is usually termed as signal line D% where the following rules are used as trading guidance and which is illustrated in the example of *Fig. 4*.

Trading rules with stochastic Indicator

If %K >80 → overbought stocks (it might fall again).

If %K<20 → oversold stocks (might bounce back upwards).

If %K <80- and %K > %D →Buy signal.

If %K < %D → sell signal.

If %K >90 & started to fall again → sell before %K hits 80.

5 | Fuzzy Inference Mechanisms and Modular Structures

5.1 | Brief History About Fuzzy Inferences

After the initial introduction of Zadeh's [31] theory of fuzzy logic, Mamdani and Assilian [39] utilized the theory to build an inference system based on fuzzy logic to control a steam engine using linguistic control rules in a form of: "If A AND B then C". The objective was to simulate the experienced human operator in controlling such task where the input to the system was the fuzzified value of the crisp input and the output of the system was also a fuzzy set. Sugeno [40] used a similar technique to Mamdani with a main difference in the output where the final output was de-fuzzified to a crisp value by using averaging technique. A similar technique but with modular approach was proposed by Kouatli [37] with an example of robotic manipulator in controlling tasks. The first adaptation of the fuzzy set membership was proposed by Sugeno in order to achieve the desired results of the system. Sugeno proposed the same type of rules as Mamdani with few exceptions that the inputs and the output of each one of the rules can also be a function instead of being a value. Moreover, the weight of each rule can be measured by using the "AND" operator to conclude the strength of each rule. As the process of "decision making" is also "fuzzy" in nature, then the same concept can be implemented in management science and other industry fields where decision has to be taken under uncertainty.

Hence fuzzy system heuristics are built as rule-sets, describing the system model. Rules are usually in the form of (IF A … THEN B) where "A" and "B" are fuzzy variables. For a Single-Input-Single-Output (SISO) system, this would be straight forward and fuzzy inference can be conducted on the fuzzy variables representing the input and output respectively. Most real world problems are in the form of a multivariable structure composed of Multi-Input-Multi-Output (MIMO) systems. In this case, problems may arise in achieving a complete and consistent construction of all possibilities relevant to the output of the system. Rules in this case are of the form (IF A1& A2…. &An THEN B1 & B2 &...Bn). This situation results in additional complexity in knowledge discovery and accurate heuristic modeling the system. Special algorithms are necessary in this case to tune the system as well as to detect

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and remove irrelevant rules. For example, Babuska [41] studied rule compression and selection in the goal of maintaining completeness and consistency of the system. Instead of rule compression, a simplified modular structure was a also proposed by Kouatli [37] and [42] where the fuzzy system can be defined as three main components: The fuzzification component, the knowledge component and the Inference/de-fuzzification component and where the input/output relationships defined by creating sub-rule-sets each of which describe the relationship between one of the inputs and one of the outputs.

Many researchers adopted the principle of modular approach when inferring the output of fuzzy systems. For example, Carrera and Mayorga [43] proposed the use of modular fuzzy inference structure to conclude a decision in supply chain environment where there is a need to optimize the right cost at the right time with the best quality from the right supplier. Junior et al. [44] presents a supplier selection decision method based on fuzzy inference modeling the human reasoning which is an advantage when compared to approaches that combine fuzzy set theory with multi-criteria decision making methods. Amindoust et al. [45] used fuzzy inference mechanism to conclude the sustainability criteria for a given set of suppliers. Lin and Hsu [46] used modular approach for fuzzy inference to handle the impreciseness and uncertainty found in storing image selection process. An example of combination of intelligent techniques like fuzzy logic and genetic algorithm can be seen in Melin et al. [47] where a genetic optimization approach of modular neural-fuzzy integration. This methodology was applied to human recognition problem where the proposed algorithm was able to adjust the number of membership function/rules as well as the variation on the fuzzy set type of logic (type-1 or type-2). In a similar hybrid type of intelligent techniques and to avoid complexity of MIMO system generated by maintaining the complete and consistent rules, Kouatli [48] proposed a modular approach in an analogy of biological structure of genes and chromosomes where each gene represents one rule connecting any one of the inputs with any one with the outputs. Based on this, a chromosome can be defined in terms of pre-fixed number of rules representing the four fuzzy variables (P0, PS, PM and PB). These "chromosomes" can then be constructed using "Input Importance Factor" ε. The value of this ε can be either normalized by the knowledge and intuition of the domain expert or can be deduced using AHP technique proposed by Saaty [49]. Full details of this technique with the mathematical details and definitions can be found in Kouatli [34]. This paper describes the implementation of this approach of modular structure of biological resemblance of intelligent algorithm where a customized strategic stock trading support system has been built to find an optimized threshold where a decision of buying/selling has to take place. The proposed system is based on fuzzifying the existing most popular technical indicators (surveyed in Section 2) in order to achieve better performance of stock trading.

5.2 | The Proposed Fuzzy Inference Methodology and Strategy of Decision Making

Fuzzy system heuristics are built as rule-sets describing the system model. Rules are usually in the form of (IF A … THEN B) (concluded rulesets in Section 4 for buying and selling are examples) where A and B are fuzzy variables. For Single Input, Single Output (SISO) system, fuzzy inferences could be conducted on the fuzzy variables representing the input and output, respectively. However, most decision-making problems are multivariable in Multi-Input, Multi-Output (MIMO) systems. In this case, problems may arise in achieving complete and consistent construction of all possibilities relevant to the output of the system. MIMO rulesets in this case are of the form (IF A1& A2…. & An THEN B1 & B2 &...Bn). This adds complexity in knowledge discovery and accurate heuristic modeling of the system (Kouatli [37]).

Special algorithms are necessary in this case to tune the system as well as to detect and remove irrelevant rules. For example, Kouatli [42] studied rule compression and selection with the goal of maintaining completeness and consistency of the system. Instead of rule compression, a simplified modular structure was also proposed by Kouatli [33] and Carrera and Mayorga [43], where the fuzzy inference can be modular in mature. In the described model, a modular approach is proposed to combine the rules described in Section 4 where equal "input importance factor" is the weighted factor to combine the rules and assumed to be equal to all rules. The final inferred output in this case is governed by the equation:

Where

Yj is the de fuzzified output using averaging method due to the effect of input Xi.

εij=importance level for input i to the output j.

($x_i \circ R_{ij}$) is the compositional rule of inference between a specific input Xi and the relation Rij.

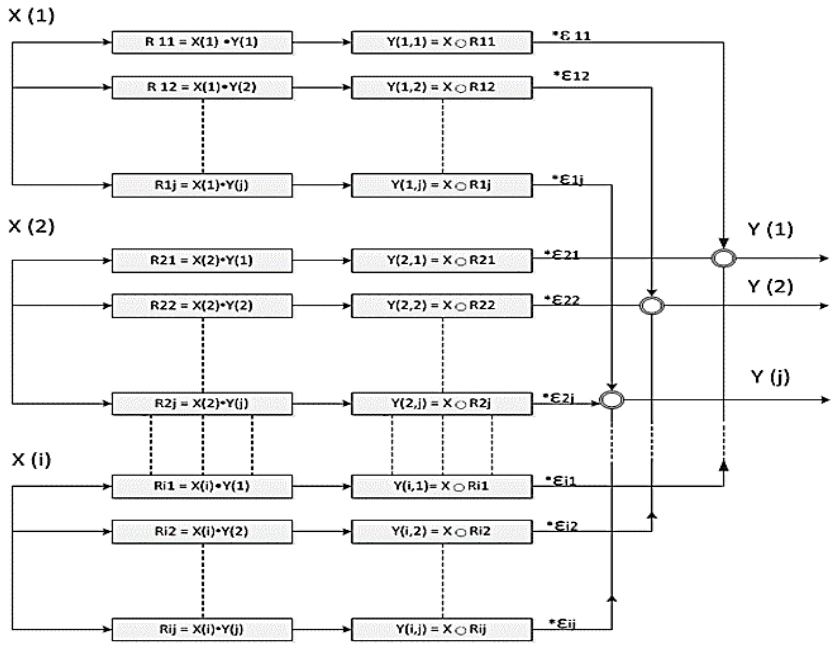
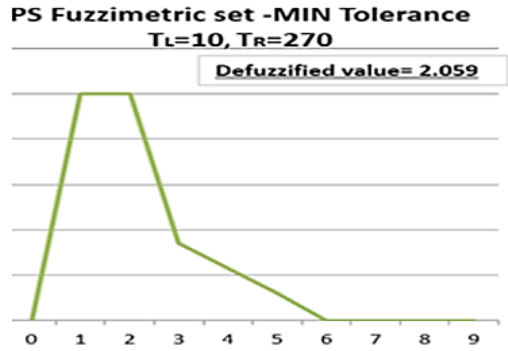


Fig. 3. Modular approach of fuzzy inference.

5.3 | Fuzzimetric Sets Mutations and De-Fuzzified Values

By changing the mutation factor T for a specific interval, would alter the shape of the fuzzimetric set. For example if the fuzzy interval for PS is between {1,6}, in a scale of {1,10}, then by altering the mutation factor T would move the centroid of the mutated Fuzzimetric set from a minimum (towards level 1) or maximum (toward level 3). To be able to achieve this for any Fuzzimetric set, the mutation factor is applied per half of the set (left or right, and accordingly two mutation factors applied for each of the left and right halves of the set (TL and TR). This concept provide a mechanism of finding the minimum and maximum possible tolerances for fuzzimetric sets. *Fig. 3* illustrate this concept by three possibilities of small fuzzy set with an interval of {1-6}. *Fig. 1.a* shows a skewed trapezoidal fuzzimetric set biased towards the lower end of the interval by setting the mutation factor TL=10 and TR=270. Similarly the other extreme (*Fig. 4. C*) shows a skewed trapezoidal fuzzimetric set biased towards the higher end of the interval by setting the mutation factor TL=270 and TR=10. Obviously, if it is required to have a set as a medium (average of 1-6), then the optimum de-fuzzified value would be 3 achieved by setting TL=90 and TR=90 (*Fig. 4. b*).



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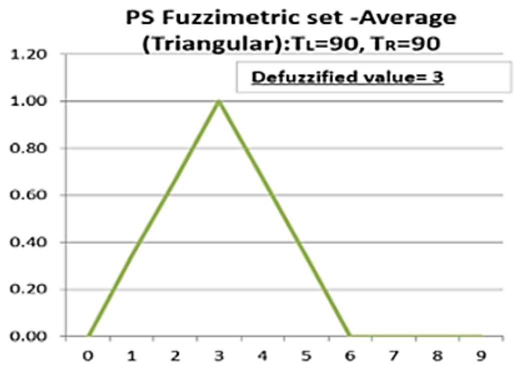

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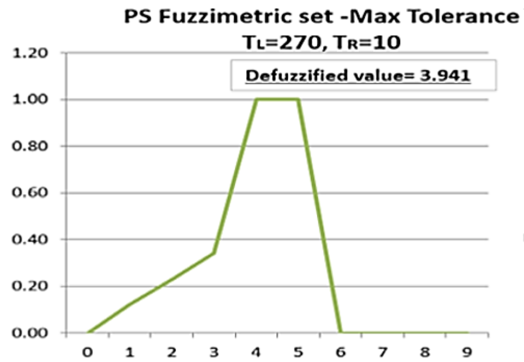

c.

Fig. 4. Variation of fuzzimetric set "small" within a specific interval: a. SMALL fuzzimetric set - min tolerance; b. SMALL fuzzimetric set - average tolerance; c. SMALL fuzzimetric set – max tolerance.

5.4 | Strategy of Finding Minimum and Maximum Possible Tolerances for TA

As can be seen from *Fig. 4*, the average non-biased, un-mutated set with mutation factor =90 would result a triangular shaped set using this mutation factor would generate a minimum and maximum tolerances of the set as explained in *Fig. 4*. Based on this concept and bearing in mind that the universe of discourse is composed of four fuzzimetric sets, then, it would be required to find the combination of tolerances as an outcome to find the minimum and maximum tolerance. Accordingly, the following steps can be adopted:

Step 1. The relative "universe of discourse" approximate scale needs to be identified: to be able to do this, a previous historical data would be necessary. For example, the previous 50 period of trading. Assuming a daily trade, then this would represent the minimum and maximum price within this "testing/observing" period.

Step 2. Scaling the maximum and minimum on the "Fuzzimetric Arc, then fuzzy sets PO, PS, PM and PB can be defined.

Step 3. Considering the PO and PS to be the minimal possible-buying price as opposed to PM and PB as the larger possible selling price, then these fuzzy sets can be mutated to be the minimum or the maximum for min and max tolerances of each of these sets.

Step 4. Following the rule-sets of buying and selling concluded in section four (which are based on the fuzzification of these most popular technical indicators), and assuming that all technical indicators have the same weight (during experimentation scenario, it is not necessary to be equal), then the final combined decision is dependent on that combination. Hence, as an oiutcome of Step 3, and to search for all mutations of the "fuzzimetric sets, then, five main possibilities can be generated:

Triangular fuzzy sets (average to all sets: fuzzimetric set without any mutation),

Triangular sets. Right TPO =90, Left TPS=90, Right TPS=90, Left TPM=90, Right TPM=90, Left TPB=90.

MINMIN (mutated fuzzimetric sets where the centroid is towards the minimum),

MINMIN sets. Right TPO =10, Left TPS=270, Right TPS=10, Left TPM=270, Right TPM=10, Left TPB=270.

MINMAX (mutated P0 and PS to minimum and mutating PM and PB to the maximum possible tolerance),

MINMAX sets. Right TPO =10, Left TPS=270, Right TPS=10, Left TPM=10, Right TPM=270, Left TPB=10.

MAXMIN (mutating P0 and PS to maximum and mutating PM and PB to the minimum possible tolerance),

MAXMIN sets. Right TPO =270, Left TPS=10, Right TPS=270, Left TPM=270, Right TPM=10, Left TPB=270.

MAXMAX (Mutating all four fuzzimetric sets to the maximum tolerance),

MAXMAX sets. Right TPO =270, Left TPS=10, Right TPS=270, Left TPM=10, Right TPM=270, Left TPB=10.

Step 5. Record the minimum and maximum possible price based on the search from Step 4(a-e) and accordingly, these would represent the minimum possible (buying) price and the maximum possible (selling) price based on the fuzzification of most popular technical indicators as explained in Section 4, *Fig. 5* shows the graphic representation of the chosen different variations of fuzzimetric sets as explained in Step 4 which is the basis of such decision support system to help traders in confirming an automated decision proposed by the system and accordingly reduce the possibility of emotional trading.

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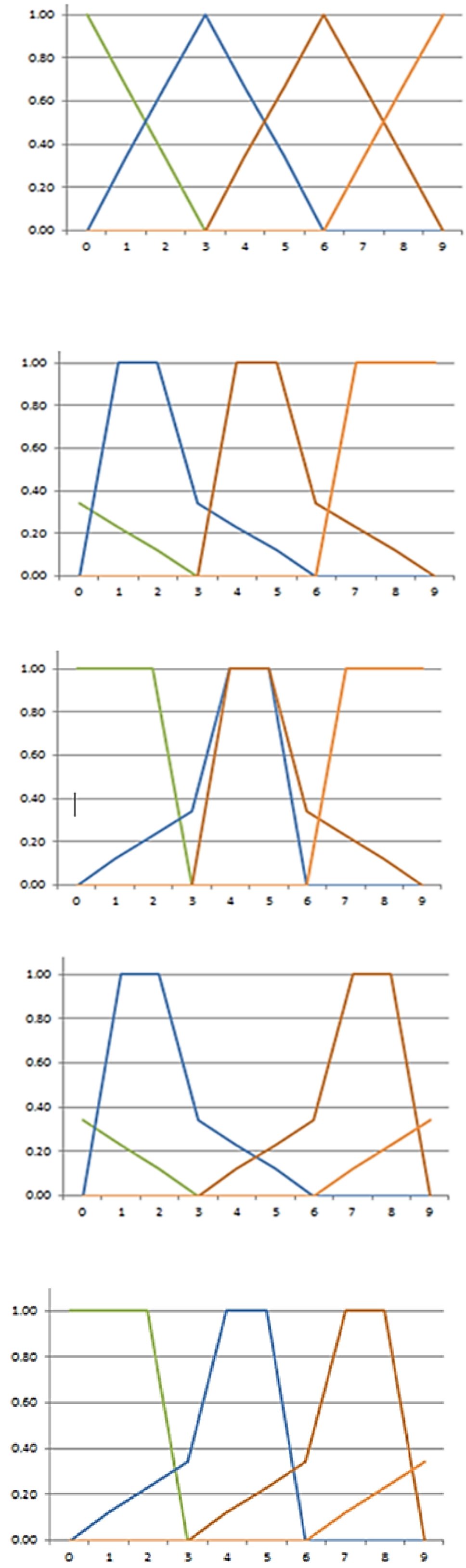

a.

b.

c.

d.

e.

Fig. 5. Different tolerance varieties of mutated fuzzimetric sets: a. triangular; b. minimize tolerance for all sets (MINMIN); c. minimize PO & PS, maximize PM & PB tolerances; d. maximize PO & PS, minimize PM & PB tolerance; e. maximize tolerance for all sets (MAXMAX).

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6 | Data and Methodology Used with the Back Test Experimental Results

Most experienced traders may use a strategy of combining two or more indicators complementing each other (e.g. a trend-following indicator combined with mean-revert indicator) before a final decision. This is usually a common practice when it comes to the development of intelligent systems. Pounder et al. [50], for example, introduced a hybrid architectures concept of a framework to artificial general intelligence termed as Cognitive Function Synthesis (CFS) to investigate the consciousness and imagination. Mella [51] proposed a combinatory system theory by constructing a formal model that explains a vast group of phenomena produced by the cybernetic behavior of the collectivity producing self-organizing synchronization. Lepskiy [52] also investigated the relationship between scientific rationality and cybernetics when developing systems. In case of stock trading example, cybernetic strategy is dependent on the combination of many other “fuzzy” factors than just combining both types of indicators. For example, the volume volatility, the effect of large emotional trading to the current prices, the emotional behavior of favoring one type of indicator to the other and the ultimate uncertainty that is usually embedded in such type of indicators. To avoid such dilemma, fuzzy logic can be used with a “linguistic” types of variables rather than an absolute variables which might be prone to errors. The concept of fuzzimetric sets described in Section 2 will be used to implement the “fuzzy variables” to the most popular indicators described in Section 3. In order to conduct fuzzy inference related to the best strategy, criteria need to be specified with respect to a known interval where fuzzy variables can then be defined as PO, PS, PM or PB (as described in Section 3), rather than crisp values.

Using Yahoo! finance, data for year 2016 was downloaded for a mixture of indices some of which are value-weighted and others are price–weighted. These are: SP500, DowJones 30, Nasdaq, Nikkie220, Russell2000 and SP500. Where a comparison between different indicators with respect to the proposed fuzzy indicator is compared in terms of total annual profit with total transactions executed. From the previous section, it can be noticed that most of the indicators and strategies can be combined together and some of them can complement each other. In order to conduct fuzzy inference related to best strategy, criteria needs to be specified with respect to a known intervals where fuzzy variables can then be defined as PO, PS, PM or PB, rather than crisp values. The chosen criteria out of the well-known most popular indicators (described in the review section) are listed as follows with the relevant fuzzy trading rules:

Buying Fuzzy rules:

BB Related: IF Price is PO AND %b is PO AND Bandwidth is PM OR PB.

MACD Related: IF Price is PO or PS AND Golden-Cross = OK.

RSI Related: IF Price is PO or PS and RSI is PO.

Stochastic Related: IF Price is PO and %K is PB AND %K > %D.

Selling Fuzzy rules:

BB Related: IF Price is PM OR PB AND %b is PB AND Bandwidth is PM OR PB.

MACD Related: IF Price is PM or PB AND Dead-Cross = OK.

RSI Related: IF Price is PB And RSI is PB.

Stochastic Related: IF Price is PB and %K is PM or PB AND %K < %D.

The strategy adopted for analysis using fuzzimetric arcs was defined into 5 main possibilities representing five main tolerances as defined in the previous section where crossover and mutation can be utilized to achieve these tolerances of buying and selling. These are:

Triangular sets. Right TPO =90, Left TPS=90, Right TPS=90, Left TPM=90, Right TPM=90, Left TPB=90.

MINMIN sets. Right TPO =10, Left TPS=270, Right TPS=10, Left TPM=270, Right TPM=10, Left TPB=2700.

MINMAX sets. Right TPO =10, Left TPS=270, Right TPS=10, Left TPM=10, Right TPM=270, Left TPB=10.

MAXMIN sets. Right TPO =270, Left TPS=10, Right TPS=270, Left TPM=270, Right TPM=10, Left TPB=270.

MAXMAX sets. Right TPO =270, Left TPS=10, Right TPS=270, Left TPM=10, Right TPM=270, Left TPB=10.

The triangular sets represents the non-biased adoption of the indicators towards buying and selling while the MINMIN sets represents the strategy of minimum tolerance of buying with minimum tolerance of selling which indicated the strategy of a trader who wait for minimum price and sell as soon as the price is acceptable (and profitable) minimum tolerance. Similarly, the MINMAX represents the minimum tolerance of buying and maximum tolerance of selling and so on. *Fig. 3* shows the graphical representation of these different definitions of fuzzimetric sets.

Back test conducted on the chosen indices for the 2016 annual trading where the data was cleansed to ensure that the first transaction is a "Buy" and the last transaction is a "Sell" in that specific year. Assuming a capital of around $50000 was utilized as the initial trading budget at the beginning of year 2016, where the actual investment is dependent on the number of shares (real whole numbers of shares disallowing fraction of a share) that can be bought within the budget limit.

Fig. 6 shows the experimental results of the back test using the prototyped system where the initial investment with total profit for year 2016 in monetary values as well as percentage value are shown together with the comparative results of traditional as well as the defined fuzzimetric sets described in previous section where the final results shows the maximum profit were obvious using the fuzzified principle of the most known traditional indicators. It should be noted that the system was tuned to the Triangular sets where non-biased tolerances was considered for buying as well as selling transactions representing the expectation of actual traders' behaviours. However, the maximum profit does not have to be associated with the Triangular sets. As the table shows the maximum profit for Dow Jones and S&P proved to be the MINMIN tolerance being the best choice for maximum profit while the MAXMAX tolerance shows the minimum profit achieved in case of S&P 500. Nikkie 220 using Triangular fuzzy sets shows the highest profitability with highest possible spectrum as opposed of minimum volatility with minimum profitability in S&P500 using MINMIN fuzzy sets.

			Most Popular Technical Indicators				Fuzzified Technical Indicators with Type-2 sets					
			BB	MACD	RSI	St0	Triangular	MINMIN	MINMAX	MAXMIN	MAXMAX	Fuzzy Spectrum
Value-Weighted Indicies		Year 2016 % Profit	8.23%	2.74%	14.40%	0.53%	7.18%	18.74%	18.74%	7.18%	7.18%	7.18% -18.74%
	DowJones	Year 2016 Profit	2884.1	977.9	4967.92	259.26	3397.74	6462.88	6462.88	3397.74	3397.74	
		Investment	35025.42	35670.84	34493.19	49195.71	47300.22	34493.19	34493.19	47300.22	47300.22	
		No. of Shares	2	2	2	3	3	2	2	3	3	
		Profit per share	1442.05	488.95	2483.96	86.42	1132.58	3231.44	3231.44	1132.58	1132.58	
		No. of Transaction	10	18	6	2	2	6	6	2	2	
		Year 2016 % Profit	13.35%	1.02%	14.19%	-0.12%	3.96%	14.40%	13.26%	7.53%	3.14%	3.96% - 14.40%
	S&P 500	Year 2016 Profit	6634.08	507.36	6870	-60	1917.84	7004.64	6516.24	3655.92	1519.92	
		Investment	49682.75	49571.1	48416.16	48091.75	48481.86	48659.1	49135.96	48541.2	48481.86	
		No. of Shares	24	24	24	25	24	24	24	24	24	
		Profit per share	276.42	21.14	286.25	-2.4	79.91	291.86	271.51	152.33	63.33	
		No. of Transaction	18	16	6	2	8	8	12	6	8	
		Year 2016 % Profit	10.14%	14.22%	18.14%	13.95%	18.36%	14.99%	12.81%	6.51%	12.11%	6.51% - 18.36%
	Russell 2000	Year 2016 Profit	5032.28	7034.37	8893.28	6880.28	9049.5	7484.58	6396.28	3254.99	5984.44	
		Investment	49640.07	49459.33	49013.51	49311.53	49285.24	49921.57	49931.46	49987.61	49437.23	
		No. of Shares	44	43	44	44	45	43	44	41	44	
		Profit per share	114.37	163.59	202.12	156.37	201.1	174.06	145.37	79.39	136.01	
		No. of Transaction	12	20	6	6	8	6	10	2	6	
Price-Weighted Indicies		Year 2016 % Profit	13.49%	-1.26%	12.73%	2.90%	14.65%	6.29%	3.73%	5.53%	4.30%	3.73% - 14.65%
	NASDAQ	Year 2016 Profit	6738.4	-627.7	6147.4	1360.5	7013.5	3095.2	1684.98	2542	1977.9	
		Investment	49948.59	49847.77	48278.5	46870.75	47868.93	49208.18	45168.59	45986.65	45986.65	
		No. of Shares	10	10	10	10	10	10	9	10	10	
		Profit per share	673.84	-62.77	614.74	136.05	701.35	309.52	187.22	254.2	197.79	
		No. of Transaction	18	18	8	4	8	8	12	4	4	
		Year 2016 % Profit	20.61%	16.21%	9.32%	12.93%	21.08%	17.09%	10.14%	18.51%	19.27%	10.14% - 21.08%
	Nikkie	Year 2016 Profit	10291.89	7952.19	4589.94	6340.14	10461.12	5740.28	3386.08	9229.08	9558.42	
		Investment	49938.12	49048.18	49255.37	49047.46	49619.55	33587.6	33401.73	49868.09	49599.46	
		No. of Shares	3	3	3	3	3	2	2	3	3	
		Profit per share	3430.63	2650.73	1529.98	2113.38	3487.04	2870.14	1693.04	3076.36	3186.14	
		No. of Transaction	18	20	6	6	14	12	16	10	12	

Fig. 6. Back-test comparative results of the output of technical analysis trading decision.

7 | Conclusion

Prediction of the stock market behavior is not an easy task. Traditional technical indicators can provide a good sign of buy/sell signals. However, due to uncertainty of such trade, traders follow their intuition and emotional rather than the solid predictive outcome of the TAs. In an attempt to reduce emotional trading, this paper demonstrated the use of fuzzy logic to identify the maximum and minimum tolerances when buying or selling stocks and shares. Fuzzimetric sets was used to fuzzify most technical indicators where mutations of the sets provides the maximum and possible tolerances generating a "fuzzy spectrum" showing the level of the daily price variation , identifying the level of uncertainty of technical indicators and hence reducing emotional trade. The proposed trading rules was deduced from the most popular technical indcators: MACD, BB, RSI and momentum indicators. Backtests was conducted to five main indicies. These are" Dow-Jones, S&P 500, Russel 2000, NASDAK, and Nikkie to whow the development of "fuzzy spectrum" as indication of level of uncertainty in shares and stocks.

7.1 | Further Research

The modular fuzzy inference mechanism proposed in Section 5 uses a weighting factor per indicator sub-decision (MACD, RSI…) item as part of this system. Some of these technical indicators are "Trend-following" and others are designed for "revert to mean". Indices in stock market can follow either/or any one of these behaviors. Hence this paper regulate the "input importance weighting factor" εi (described in Section 5) depending on the type of the indicator. Further research would be required to automate this feature where – before running the fuzzy inference - the system should recognize the type of most suitable indicator and evaluate the appropriate ε.

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A Hybrid Intelligent Parameter Tuning Approach for COVID-19 Time Series Modeling and Prediction

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Abstract

A novel hybrid intelligent approach for tuning the parameters of Interval Type-2 Intuitionistic Fuzzy Logic System (IT2IFLS) is introduced for the modeling and prediction of coronavirus disease 2019 (COVID-19) time series. COVID-19 is known to be a virus caused by Severe Acute Respiratory Syndrome Coronavirus 2 (SARSCoV-2) with a huge negative impact on human, work and world economy. Globally, more than 100 million people have been infected with over two million deaths and it is not certain when the pandemic will end. Predicting the trend of the COVID-19 therefore becomes an important and challenging task. Many approaches ranging from statistical approaches to machine learning methods have been formulated and applied for the prediction of the disease. In this work, the sliding mode control learning algorithm is used to adjust the parameters of the antecedent parts of IT2IFLS system while the gradient descent backpropagation is adopted to tune the consequent parameters in a hybrid manner. The results of the hybrid intelligent learning model are compared with results of single learning models using sliding mode control and gradient descent algorithms and found to provide good performance in terms of Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE) especially in noisy environments. The type-2 hybrid model also outperforms its type-1 counterparts in the different problem instances.

Keywords: Interval type-2 intuitionistic fuzzy set, Gradient descent algorithm, Sliding mode control algorithm, Intuitionistic fuzzy index.

1 | Introduction

Coronavirus disease 2019, popularly known as COVID-19, is an illness caused by a novel coronavirus called Severe Acute Respiratory Syndrome Coronavirus 2 (SARSCoV-2). This novel coronavirus, designated as 2019-nCoV emerged in Wuhan, China in December, 2019. On January 30, 2020, the World Health Organization (WHO) declared the COVID-19 outbreak a global health emergency and on March 11, 2020, it was declared a global pandemic. As of April 13, 2020, COVID-19 has been recognized in 196 countries with a total of 1,876,707 laboratory confirmed cases, 435,591 recovered and 116,789 death cases [1].

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As of 5th February 2021, the number of confirmed cases worldwide had risen to 104,165,066 with over 2,265,354 deaths as per WHO COVID-19 Dashboard. COVID-19 has therefore become a huge challenge the world over; affecting peoples' lives and work. Many countries were locked-down due to the devastating effects of COVID-19 with the aim of slowing down the spread of the virus. It behooves therefore on researchers to provide effective and reliable prediction models to COVID-19 pandemic. So far, many datamining methodologies have been applied for the prediction of COVID-19 pandemic. For instance, Muhammed et al. [2] recently developed data mining models for the prediction of COVID-19 infected patients recovery using epidemiological data set of COVID-19 patients of South Korea. The decision tree, support vector machine, naïve Bayes, logistic regression, random forest, and K-nearest neighbour algorithms were applied. Ayyoubzadeh et al. [3] predicted the incidence of COVID-19 in Iran. Linear Regression and Long Short-Term Memory (LSTM) models were used with data obtained from the Google Trends website to estimate the number of positive COVID-19 cases. Their models were evaluated using 10-fold cross validation with Root Mean Squared Error (RMSE) as the performance metric. Ardabili et al. [4] have presented a comparative analysis of machine learning and soft computing models to predict the COVID-19 outbreak as an alternative to Susceptible-Infected-Recovered (SIR) and Susceptible Exposed Infections Removed (SEIR) models. Agbelusi and Olayemi [5] developed a predictive model for the mortality rate of patients infected with COVID-19 in Nigeria using data mining techniques available in Waikato Environment for Knowledge Analysis (WEKA). Martin et al. [6] proposed a COVID-19 diagnostic model based on plithogenic cognitive maps. Matta and Saraf [7] developed a machine learning model to predict whether a patient is suffering from COVID-19 or not.

The COVID-19 time series data are highly complex, nonlinear and uncertain [8]. The COVID-19 data therefore present itself as a challenging, yet interesting prediction problem. With the levels of uncertainties in COVID-19 data, methodologies that can handle these uncertainties must be employed. In the literature, fuzzy logic comes in handy as a concept that can adequately model these uncertainties [9]. So far, type-1 Fuzzy Sets (FSs) have been adopted for the prediction of COVID-19 time series. For instance, Dhiman and Sharma [10] presented fuzzy logic inference for identification and prevention of COVID-19. Al-Qaness et al. [11] utilized Adaptive Neuro-Fuzzy Inference System (ANFIS) with parameters tuned with Flower Pollination (FPA) and Salp Swarm Algorithm (SSA) to predict confirmed cases of COVID-19 in China. Van Tinh [12] applied fuzzy time series model with particle swarm optimization for COVID-19 prediction. Fong et al. [13] used a hybridized deep learning and fuzzy rule induction for the analysis of COVID-19. Fatima et al. [14] used Internet of Things (IoT) coupled with fuzzy inference system for monitoring COVID-19 while Verma et al. [15] adopted arima and fuzzy time series models for COVID-19 prediction. Arora et al. [16] proposed a fuzzy based COVID-19 decision making system using individual's symptoms and parameters. The proposed system provided good results with 97.2% accuracy. Ly [17] presented a study on the prediction of COVID-19 in the United Kingdom using ANFIS.

However, type-1 fuzzy sets may not solve the uncertainty problem completely because once the Membership Function (MF) of type-1 FS is specified, the uncertainty disappears leaving a precise value. An Intuitionistic Fuzzy Set (IFS) introduced by Atanassov [18] provides some flexibility about fuzzy sets thus allowing domain experts to express more uncertainty about a fuzzy set by defining separate MFs and Non-Membership Functions (NMFs) for an element in a set. Authors have adopted the concept of IFS in uncertainty modelling of COVID-19 data. For instance, Kozae et al. [19], adopted intuitionistic fuzzy distance to determine those infected with COVID-19. In Traneva and Tranev [20], multilayered intuitionistic fuzzy intercriteria analysis is conducted on some key disease indicators to determine the mortality from COVID-19 in European Union. The same authors in Traneva and Tranev [21], proposed a two-way intuitionistic fuzzy analysis of variance for evaluating the spread of COVID-19 cases in Europe. Eyo et al. [22] utilised rule-based Intuitionistic Fuzzy Logic System (IFLS) for the analysis of COVID-19 time series in Nigeria. Similar to type-1 FS, the IFS of type-1 could not handle uncertainty well. Atanassov and Gargov [23] extended IFS to Interval-Valued IFS (IVIFS) where upper membership and upper non-membership of the set add up to 1. In Eyoh et al. [24], a rule-based IT2IFLS is proposed. However, for IT2IFS, upper membership and lower non-membership lie between 0 and 1, Similarly, lower membership and upper non-membership also lie within the boundary of 0 and 1 which makes IT2IFS different from

IVIFS. According to Eyoh et al. [25], IT2IFS can be used to capture concepts not possible with IVIFS. This presents IVIFS as a special case of IT2IFS [26] and [27]. The IT2IFSs have been applied successfully to solve many real-world problems such as clustering [27], regression problems [28], [29] and [30], transportation problems [31] and [32], risk assessment [33], load forecasting [34], resource allocation [35], time series forecasting [9] and [36], system identification problems [25] and [37] and water management [38] to mention but a few. Other studies involving IT2IFS include Singh and Garg [39], where different types of distances between T2IFSs are proposed. In Dan et al. [40], certain properties of IT2IFS are presented. Li et al. [41] introduced grey relational bidirectional projection method based on trapezoidal type-2 intuitionistic fuzzy numbers while Demiralp and Haçat [42] put forward an ordering method of c-control charts with IT2IFS.

1.1 | Research Gap and Motivation

The IT2IFS has gained widespread attention in recent years with significant and promising results in diverse problem domains and characteristics. Many algorithms have also been adopted for the optimization of the parameters of IT2IFLS for solving many application problems. These algorithms include Gradient Descent (GD) [9], [28], [29] and [30] where IT2IFLS is used for regression and time series predictions. Luo et al. [36] presented an evolving Recurrent Interval Type-2 Intuitionistic Fuzzy Neural Network (eRIT2IFNN) with the parameters of the model adjusted using Extended Kalman Filter (EKF) for online learning and time series prediction. The decoupled version of EKF is applied to tune the parameters of IT2IFLS in Eyoh et al. [44] and [45]. Recently, Sliding Mode Control (SMC) learning algorithm is used to learn the parameters of IT2IFLS [37] and [46] with application to system identification problems. A hybrid model of GD and EKF has been reported in the literature for training IT2IFLS for identification problems [25]. However, both approaches are derivative-based which are computationally intensive because they involve computing the partial derivatives of the parameters and convergence may be slow [25] and [43]. To reduce the computational cost and speed up convergence, we are motivated to integrate both derivative (GD) and derivative-free (SMC) approaches for adjusting the parameters of IT2IFLS. The rationale behind this approach is that the antecedent parameters of the model are highly nonlinear and for optimization problems, computing the gradient of the cost function in each step for nonlinear parameters is difficult and chain rule must be used [47]. Also, convergence of nonlinear parameters may sometimes be very slow leading to non-convergence of the solution [47]. Therefore, using SMC, a gradient-free parameter tuning approach is more appropriate for the non-linear antecedent parameters. On the other hand, using GD approach for tuning the consequent parameters is more practical as the parameters are linear. Moreover, the two algorithms of SMC and GD are based upon well-established mathematical background for training FLSs [48].

1.2 | Main Contributions

The contributions of this paper are as follows: 1) the analysis of COVID-19 time series using IT2IFLS, 2) the optimization of the parameters of IT2IFLS with a novel hybrid intelligent learning algorithm of SMC and GD. To the best knowledge of the authors, no work has been done on COVID-19 time series using intuitionistic fuzzy approaches of type-2 and no learning of the parameters of IT2IFLS has been carried out using SMC and GD in a hybrid manner.

1.3 | Paper Organization

The rest of the paper is organized as follows: Section 2 provides some basic definitions for IFS. Section 3 describes the IT2IFLS. In Section 4, the methodologies for the realization of the hybrid learning apparatus are given and in Section 5, the update rules are derived. Section 6 provides a brief description of the COVID-19 datasets and the experimental set-up for the study while performance evaluation is carried out in Section 7. The conclusion is drawn in Section 8.

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2 | Preliminaries

2.1 | Type-1 Intuitionistic Fuzzy Set

Definition 1. [18]. An intuitionistic fuzzy set is composed of MF and NMF and defined as: $A^* = (x: \mu_{A^*}(x)$; $\nu_{A^*}(x) \mid x \in X)$. It satisfies the condition that: $0 \le \mu_{A^*}(x) + \nu_{A^*}(x) \le 1$.

There exists another component called the hesitation index, π, such that $\pi(x) = 1 - (\mu_{A^*}(x) + \nu_{A^*}(x))$. Obviously, $0 \le \pi(x) \le 1$. Authors in [49], [50] and [51] have proposed methods for the formulation of MFs and NMFs of IFS. The Gaussian intuitionistic MFs and NMFs defined in [49] are adopted for this study and are expressed as in *Eqs. (1)* and *(2)*:

$$\mu_{ik}(x_i) = \exp\left(-\frac{(x_i - c_{ik})^2}{2\sigma^2}\right) - \pi. \quad (1)$$

$$\nu_{ik}(x_i) = 1 - \exp\left(-\frac{(x_i - c_{ik})^2}{2\sigma^2}\right). \quad (2)$$

Where x is the input, c is the center and σ is the standard deviation. Unlike the traditional FS, the IFS is not necessarily a complementary set.

2.2 | Interval Type-2 Intuitionistic Fuzzy Set

Definition 2. [24]. An IT2IFS is characterized by fuzzy MFs and fuzzy NMFs defined as

$$A^* = (\bar{\mu}_{\tilde{A}^*}(x), \underline{\mu}_{\tilde{A}^*}(x), \bar{\nu}_{\tilde{A}^*}(x), \underline{\nu}_{\tilde{A}^*}(x)).$$

Where $\underline{\mu}_{\tilde{A}^*}(x): X \to (0,1)$, $\bar{\mu}_{\tilde{A}^*}(x): X \to (0,1)$, $\underline{\upsilon}_{\tilde{A}^*}(x): X \to (0,1)$, $\bar{\upsilon}_{\tilde{A}^*}(x): X \to (0,1)$ such that $0 \le \bar{\mu}_{\tilde{A}^*}(x) + \underline{\upsilon}_{\tilde{A}^*}(x) \le 1$ and $0 \le \underline{\mu}_{\tilde{A}^*}(x) + \bar{\upsilon}_{\tilde{A}^*}(x) \le 1 \; \forall x \in X$.

The IT2IFS is defined in this study using type-2 Gaussian function described with uncertain standard deviation (see *Fig. 1*) and *Eqs. (3)* to *(6)* describe the upper MF, lower MF, upper NMF and lower NMF respectively.

$$\bar{\mu}_{ik}(x_i) = \exp\left(-\frac{(x_i - c_{ik})^2}{2\bar{\sigma}^2_{2,ik}}\right) - \pi. \quad (3)$$

$$\underline{\mu}_{ik}(x_i) = \exp\left(-\frac{(x_i - c_{ik})^2}{2\bar{\sigma}^2_{1,ik}}\right) - \pi. \quad (4)$$

$$\bar{\nu}_{ik}(x_i) = 1 - \exp\left(-\frac{(x_i - c_{ik})^2}{2\bar{\sigma}^2_{1,ik}}\right). \quad (5)$$

$$\underline{\nu}_{ik}(x_i) = 1 - \exp\left(-\frac{(x_i - c_{ik})^2}{2\bar{\sigma}^2_{2,ik}}\right). \quad (6)$$

Where π is the intuitionistic fuzzy index and lies between 0 and 1. Thus for IT2IFSs, two footprints of uncertainties suffice which are defined as *Eqs. (7)* and *(8)* [25]:

$$\mathrm{FOU}_{\mu}(\tilde{A}^{*}) = \bigcup_{\forall x \in X} \left[\underline{\mu}_{\tilde{A}^{*}}(x), \bar{\mu}_{\tilde{A}^{*}}(x)\right]. \tag{7}$$

$$\mathrm{FOU}_{\nu}(\tilde{A}^{*}) = \bigcup_{\forall x \in X} \left[\underline{\nu}_{\tilde{A}^{*}}(x), \bar{\nu}_{\tilde{A}^{*}}(x)\right]. \tag{8}$$

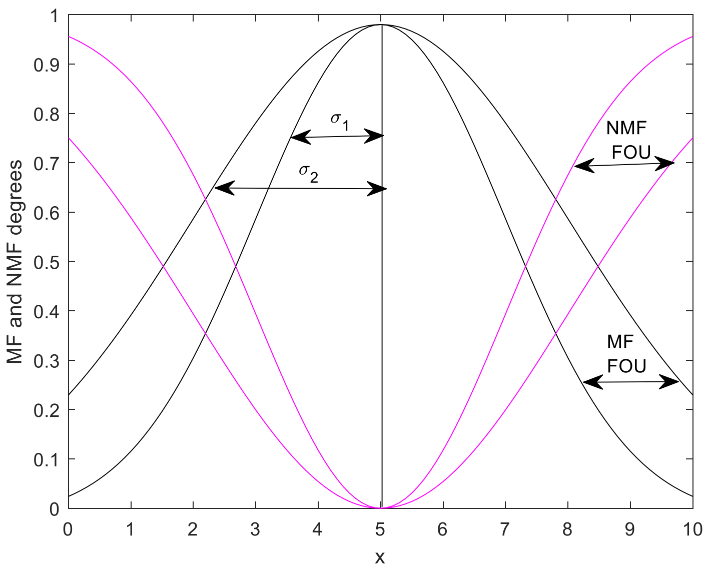

Fig. 1. Interval type-2 intuitionistic fuzzy set with uncertain standard deviation [30].

Eqs. (8) and *(9)* represent FOUs for MFs and NMFs respectively. Any system that utilizes IT2IFS either in the antecedent and/or consequent is called an IT2IFLS.

3 | Interval Type-2 Intuitionistic Fuzzy Logic System

Shown in *Fig. 2* is the block diagram of IT2IFLS comprising the intuitionistic-fuzzifier, rule base, inference engine, type-reducer and defuzzifier while *Fig. 3* shows the structure of the IT2IFLS with five layers.

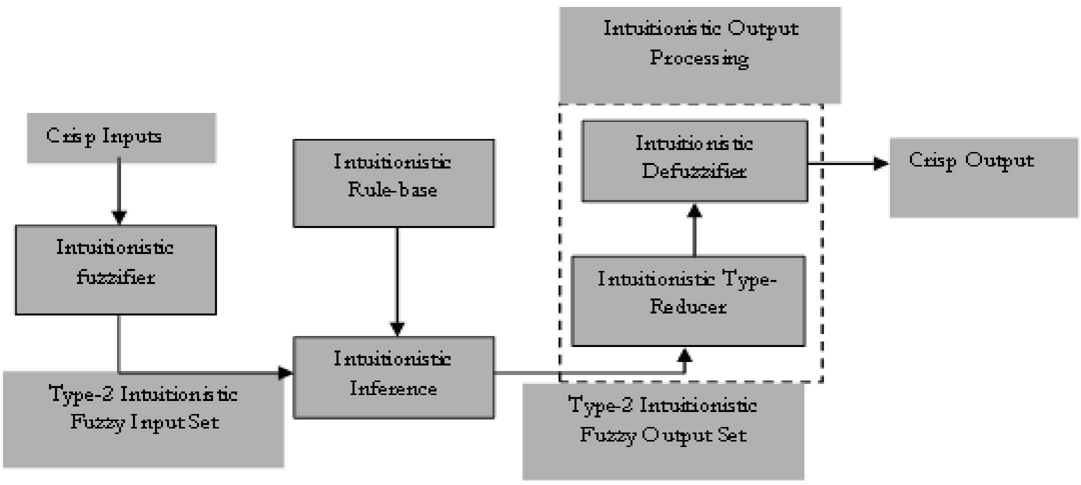

Fig. 2. Block diagram of IT2IFLS [30].

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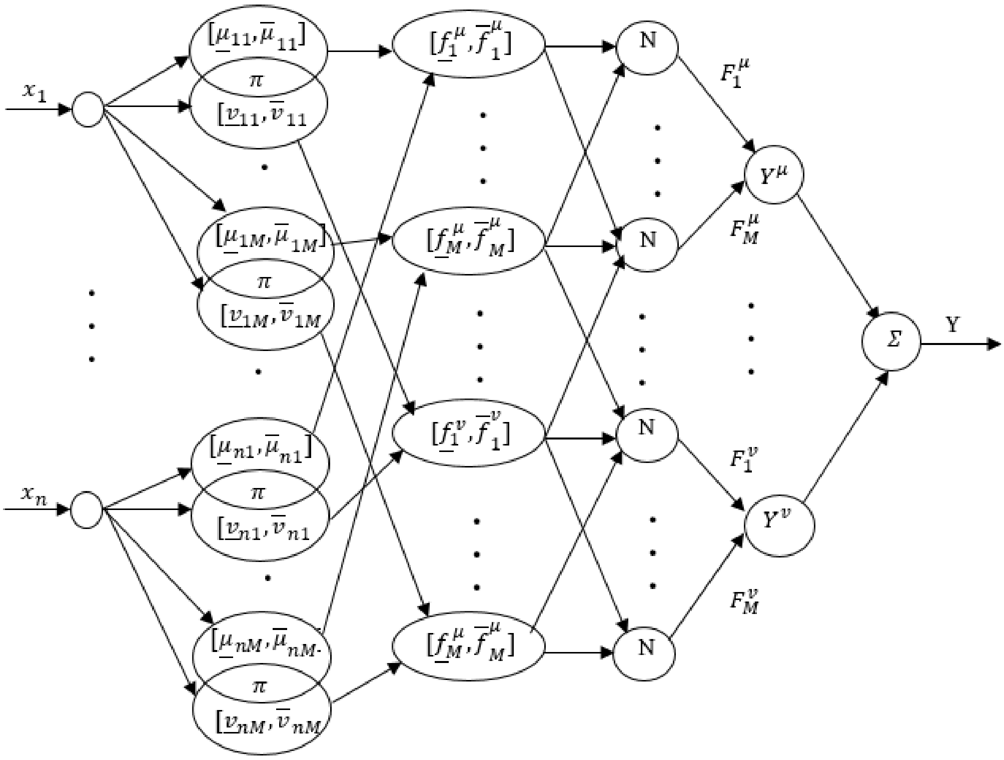

Fig. 3. Structure of IT2IFLS [52].

3.1 | Fuzzification

The external inputs are propagated forward and the intuitionistic fuzzifier accepts these crisp input vectors, translates them into intuitionistic MFs and NMFs. The intersection of the MF and NMF in *Fig. 3* represents the intuitionistic fuzzy index (π). The Gaussian function is adopted for the definition of the IT2IFS because it is differentiable at all points, thus, making it quite suitable for these optimization problems.

3.2 | Rules

The IT2IFLS rule structure can be expressed as in *Eq. (9)*. The rule representation of IT2IFLS is similar to the classical IT2FLS. However, for IT2IFLS, the IT2IFS are utilized.

$$R_k : \text{if } x_i \text{ is } \tilde{A}_{ik}^{*} \text{ and } \dots \text{ and } x_n \text{ is } \tilde{A}_{nk}^{*} \text{ then } y_k = \sum_{i=1}^{n} w_{ik} x_i + b_k. \tag{9}$$

Where $\tilde{A}_{ik}^{*}, \dots, \tilde{A}_{nk}^{*}$ are IT2IFSs and y_k is the output of the kth rule. IT2IFLS uses IT2IFSs in the rule base.

The rule in *Eq. (9)* may further be decomposed into two parts, one for MF and the other for NMF as shown in *Eqs. (10)* and *(11)* respectively. For the MFs, the rule in *Eq. (9)* becomes:

$$R_k^{\mu} : \text{if } x_i \text{ is } \tilde{A}_{ik}^{*\mu} \text{ and ... and } x_n \text{ is } \tilde{A}_{nk}^{*\mu} \text{ then } y_k^{\mu} = \sum_{i=1}^{n} w_{ik}^{\mu} x_i + b_k^{\mu}. \tag{10}$$

For NMFs, the rule becomes:

$$R_k^{v} : \text{if } x_i \text{ is } \tilde{A}_{ik}^{*v} \text{ and } \dots \text{ and } x_n \text{ is } \tilde{A}_{nk}^{*v} \text{ then } y_k^{v} = \sum_{i=1}^{n} w_{ik}^{v} x_i + b_k^{v}. \tag{11}$$

where y_k^{μ} is the MF output and y_k^{v} is the NMF outputs of the k^{th} rule, w and b are the consequent parameters.

The inferencing procedure for IT2IFLS can be approached either as Mamdani or TSK. In this work, the TSK inferencing known as A2-CO is adopted. A2-CO inferencing requires that the antecedents (A) of the rule be IT2IFSs while the consequents (C) of the rule be linear functions of the inputs. With the TSK-inferencing, the computationally intensive type-reduction is circumvented and the output is directly computed.

3.3 | Defuzzification

The final output of IT2IFLS is computed as a weighted average of both MF and NMF's outputs [44]. Thus, the computationally complicated type-reduction procedure is by-passed to directly compute the outputs of IT2IFLS. The final output of IT2IFLS is defined as follows [24]:

$$y=(1-\beta)\sum_{k=1}^{M}\tilde{f}_k^{\mu}y_k^{\mu}+\beta\sum_{k=1}^{M}\tilde{f}_k^{v}y_k^{v},$$

where

$$\tilde{f}_k^{\mu}=\frac{\left(\underline{f}_k^{\mu}+\overline{f}_k^{\mu}\right)}{\sum_{k=1}^{M}\underline{f}_k^{\mu}+\sum_{k=1}^{M}\overline{f}_k^{\mu}}, \tag{12}$$

$$\tilde{f}_k^{v}=\frac{\left(\underline{f}_k^{v}+\overline{f}_k^{v}\right)}{\sum_{k=1}^{M}\underline{f}_k^{v}+\sum_{k=1}^{M}\overline{f}_k^{v}}.$$

Where $\tilde{f}_k^{\mu}$ and $\tilde{f}_k^{v}$ are normalized firing signals for MFs and NMFs respectively and utilises the "prod" t-norm to specify the firing strength such that:

$$\underline{f}_k^{\mu}(x)=\prod_{i=1}^{n}\underline{\mu}_{\tilde{A}_{ik}^{*}}(x_i),$$

$$\overline{f}_k^{\mu}(x)=\prod_{i=1}^{n}\overline{\mu}_{\tilde{A}_{ik}^{*}}(x_i),$$

$$\underline{f}_k^{v}(x)=\prod_{i=1}^{n}\underline{v}_{\tilde{A}_{ik}^{*}}(x_i),$$

$$\overline{f}_k^{v}(x)=\prod_{i=1}^{n}\overline{v}_{\tilde{A}_{ik}^{*}}(x_i).$$

where $\left(\underline{f}_k^{\mu},\overline{f}_k^{\mu}\right)$ and $\left(\underline{f}_k^{v},\overline{f}_k^{v}\right)$ are the firing strengths for MF and NMFs respectively, $\prod$ is the "prod" operator, y_k^{μ} and y_k^{v} are the outputs of the k[th] rule of IT2IFLS, with β as the user defined parameter such that $0\leq\beta\leq 1$. The value of β influences the contribution of MF and NMF values to the final output. It follows that:

$$y=\begin{cases}\text{MF only} & \text{if } \beta=0\\ \text{NMF only} & \text{if } \beta=1\\ \text{MF and NMF} & \text{if } 0<\beta<1\end{cases}.$$

This implies that the MF alone contributes to the final output if the value of β is 0 and NMF alone contributes to the final output if β is 1. When the value of β lies between 0 and 1, then both MFs and NMFs simultaneously contribute to the final output of IT2IFLS.

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4 | Hybrid Learning Methodology

In this research, a novel hybrid learning approach for the optimization of IT2IFLS parameters is introduced. The ensuing algorithm is used for evaluating COVID-19 time series in five countries. Here, the consequent parameters are tuned using the GD backpropagation and the antecedent parameters are tuned with SMC learning methodology.

4.1 | Sliding Mode Control Learning Algorithm

Although the SMC can be applied to both linear and nonlinear systems, in this study, it is applied to tune the non-linear parameters. The error which is the difference between the actual output and the output of the IT2IFLS can be defined as in *Eq. (13)*.

$$e(t)=y^a(t)-y(t).$$

The zero value of the error coordinate can be specified as a time varying sliding surface [48] defined as:

$$S(e(t))=e(t)=y^a(t)-y(t)=0.$$

Which guarantees a system in a sliding mode to be on the sliding surface such that the predicted output using the IT2IFLS will follow the actual output signal for all time $t>t_h$, where th is the hitting time for $e(t)=0$.

Definition 3. A sliding motion will be on a sliding manifold $S(e(t))=e(t)=0$ after a time th if the condition $S(t)\dot{S}(t)=e(t)\dot{e}(t)<0$ is valid $\forall t$ in some nontrivial semi open subinterval of time of the form $[t,t_h)\subset(-\infty,t_h)$ [53] and [54].

4.2 | Gradient Descent Learning Algorithm

To adjust the consequent parameters (weight and bias) of the IT2IFLS, GD algorithm is executed. For a single output, the cost function is expressed as:

$$E=\frac{1}{2}(y^a-y)^2.$$

Where y^a is the actual output and y is the IT2IFLS output. The generic GD update rule is as follows:

$$\theta_{ik}(t+1)=\theta_{ik}(t)-\gamma\frac{\partial E}{\partial\theta_{ik}}.$$

Where θ is the generic parameter to be updated and γ is the learning rate.

5 | Parameter Update

In this study, the parameter update is achieved using two different methods, namely SMC and GD in a hybrid manner. The non-linear antecedent parameters namely center c, lower standard deviation σ1 and upper standard deviation σ2 are updated using SMC learning algorithm while the linear parameters (weight (w) and bias (b)) are updated using GD.

The update rules for the consequent parameters (w and b) using GD are as follows [29]:

$$w_{ik}(t+1) = w_{ik}(t) - \gamma \frac{\partial E}{\partial w_{ik}}. \tag{14}$$

$$b_k(t+1) = b_k(t) - \gamma \frac{\partial E}{\partial b_k}. \tag{15}$$

Where γ must be carefully chosen as a large value may lead to instability. On the other hand, small value may lead to slow learning. The derivatives in *Eqs. (14)* and *(15)* are computed as follows:

$$\frac{\partial E}{\partial w_{ik}} = \frac{\partial E}{\partial y}\frac{\partial y}{\partial y_k}\frac{\partial y_k}{\partial w_{ik}} = \sum_{k=1}^{M}\frac{\partial E}{\partial y}\left[\frac{\partial y}{\partial y_k^{\mu}}\frac{\partial y_k^{\mu}}{\partial w_{ik}} + \frac{\partial y}{\partial y_k^{v}}\frac{\partial y_k^{v}}{\partial w_{ik}}\right] = (y^{a}(t) - y(t)) * \left[(1-\beta)\left(\frac{\underline{f}_k^{\mu}}{\sum_{k=1}^{M}\underline{f}_k^{\mu} + \sum_{k=1}^{M}\overline{f}_k^{\mu}} + \frac{\overline{f}_k^{\mu}}{\sum_{k=1}^{M}\underline{f}_k^{\mu} + \sum_{k=1}^{M}\overline{f}_k^{\mu}}\right) + \beta\left(\frac{\underline{f}_k^{v}}{\sum_{k=1}^{M}\underline{f}_k^{v} + \sum_{k=1}^{M}\overline{f}_k^{v}} + \frac{\overline{f}_k^{v}}{\sum_{k=1}^{M}\underline{f}_k^{v} + \sum_{k=1}^{M}\overline{f}_k^{v}}\right)\right] * x_i,$$

$$\frac{\partial E}{\partial b_k} = \frac{\partial E}{\partial y}\frac{\partial y}{\partial y_k}\frac{\partial y_k}{\partial b_k} = \sum_{k=1}^{M}\frac{\partial E}{\partial y}\left[\frac{\partial y}{\partial y_k^{\mu}}\frac{\partial y_k^{\mu}}{\partial b_k} + \frac{\partial y}{\partial y_k^{v}}\frac{\partial y_k^{v}}{\partial b_k}\right] = (y^{a}(t) - y(t)) * \left[(1-\beta)\left(\frac{\underline{f}_k^{\mu}}{\sum_{k=1}^{M}\underline{f}_k^{\mu} + \sum_{k=1}^{M}\overline{f}_k^{\mu}} + \frac{\overline{f}_k^{\mu}}{\sum_{k=1}^{M}\underline{f}_k^{\mu} + \sum_{k=1}^{M}\overline{f}_k^{\mu}}\right) + \beta\left(\frac{\underline{f}_k^{v}}{\sum_{k=1}^{M}\underline{f}_k^{v} + \sum_{k=1}^{M}\overline{f}_k^{v}} + \frac{\overline{f}_k^{v}}{\sum_{k=1}^{M}\underline{f}_k^{v} + \sum_{k=1}^{M}\overline{f}_k^{v}}\right)\right] * 1.$$

The user-defined parameter, β, update also follows the GD procedure as defined in [29]

$$\beta(t+1) = \beta_{ik}(t) - \gamma \frac{\partial E}{\partial \beta_{ik}}. \tag{16}$$

The update rules using SMC for antecedent parameters are as follows [54]:

$$\dot{c}_{ik} = \dot{x}_i + (x_i - c_{ik})\alpha_1 \operatorname{sgn}(e). \tag{17}$$

$$\dot{\underline{\sigma}}_{ik}^{\mu} = -\left(\underline{\sigma}_{ik} + \frac{(\underline{\sigma}_{ik})^3}{(x_i - c_{ik})^2}\right)\alpha_1 \operatorname{sgn}(e). \tag{18}$$

$$\dot{\overline{\sigma}}_{ik}^{\mu} = -\left(\overline{\sigma}_{ik} + \frac{(\overline{\sigma}_{ik})^3}{(x_i - c_{ik})^2}\right)\alpha_1 \operatorname{sgn}(e). \tag{19}$$

For the NMFs, the lower NMF standard deviation is updated using the value of the upper MF standard deviation while the upper NMF standard deviation is updated using the value of the lower MF standard deviation [37] and [55], that is:

$$\dot{\underline{\sigma}}_{ik}^{\nu} = -\left(\overline{\sigma}_{ik} + \frac{(\overline{\sigma}_{ik})^3}{(x_i - c_{ik})^2}\right)\alpha_1 \operatorname{sgn}(e). \tag{20}$$

$$\dot{\underline{\sigma}}_{ik}^{\nu} = -\left(\overline{\sigma}_{ik} + \frac{(\overline{\sigma}_{ik})^3}{(x_i - c_{ik})^2}\right)\alpha_1 \operatorname{sgn}(e). \tag{21}$$

Algorithm 1. Hybrid IT2IFLS-SMC+GD learning approach.

Input: training data (x_i, y_i^a).

(1) Initialize weight (w), bias (b), center (c), standard deviation (σ), hesitation index (π), user defined parameter (β) and learning rate (γ).
(2) Initialize training epoch to unity.
(3) Initialize training data, (x_1, y_1^a).
(4) Propagate the training data through the IT2IFLS.
(5) Tune the weights, bias and the user defined parameter of the hybrid model using *Eqs. (14) - (16)* respectively.
(6) Compute the output of IT2IFLS using *Eq. (12)*.
(7) Compute the model error using *Eq. (13)*.

(8) Tune the center of IT2IFLS using *Eq. (17)*.
(9) Tune the standard deviation of MF using *Eqs. (18)* and *(19)*.
(10) Tune the standard deviation of NMF using *Eqs. (20)* and *(21)*.
(11) Pick the next training data. If training data ≤ total number of training data, go to step 4 else increment training epoch by 1.
(12) If maximum epoch is reached END; else.
(13) Go to step 4.
Output: Prediction error.

6 | Data Preprocessing and Experimental Setup

The COVID-19 dataset for the analysis is downloaded from Kaggle https://www.kaggle.com/antgoldbloom/covid19-data-from-john-hopkins-university obtained from the COVID-19 data repository of Center for Systems Science and Engineering (CSSE) at Johns Hopkins University (JHU). This work adopts COVID-19 confirmed cases for five of the most affected regions globally as at November 6, 2020. The most affected regions and their COVID-19 confirmed cases are USA (10,903,889), India (8,814,579), Brazil (5,848,959), Russia (1,887,836) and France (1,867,721) respectively. The corresponding death cases for each region are also adopted for the study. The data is made available in both raw and convenient forms and covers the period of January 23, 2020 to November 6, 2020.

This work uses the convenient form representation of the COVID-19 data and each set consists of 297 instances. Shown in *Fig. 4* is the trend of the COVID-19 confirmed cases for USA, India, Brazil, Russia and France while *Fig. 5* represents the associated death cases from COVID-19 for the five countries. For experimental analysis, each COVID-19 dataset is scaled to lie between 0 and 1 using the min-max normalization. As artificial neural network forms an integral part of the IT2IFLS, normalizing the data aids in smooth learning process and improves network performance in terms of prediction accuracy. Each of the COVID-19 data is modeled as a time series using the input-output generating vector: [y(t) y(t-1) y(t-2); y(t+1)], where y(t+1) is the one-day ahead output to be predicted. For each of the COVID-19 time series case, 10 simulation runs are conducted with terminating condition set to 100 epochs. The initial user-defined parameter is selected as 0.5 while the learning rate is set at 0.01.

7 | Performance Evaluation

The performances of IT2IFLS-SMC+GD are measured using two of the metrics for prediction problems namely: RMSE and Mean Absolute Error (MAE) defined as:

$$\mathrm{RMSE} = \sqrt{\frac{1}{\mathrm{T}}\sum_{i=1}^{\mathrm{T}} (y^{a} - y)^{2}}.$$

$$\mathrm{MAE} = \frac{1}{\mathrm{T}}\sum_{i=1}^{\mathrm{T}} |y^{a} - y|.$$

Where T is the total number of instances, y^a denote the actual output and y is the predicted output of IT2IFLS. For all experiments, three IT2IFSs are used in the antecedents. To aid comparison, the individual learning algorithms are also used to tune the parameters of the IT2IFLS. That is, IT2IFLS-GD (SMC) where GD (SMC) is used to tune the consequent and antecedent parameters respectively. The first sets of experiments are conducted to evaluate the performance of IT2IFLS-SMC+GD on COVID-19 confirmed cases in the five selected regions. In the second experiments, the confirmed cases are combined to obtain a 297 by 1485 data samples and then modelled as time series.

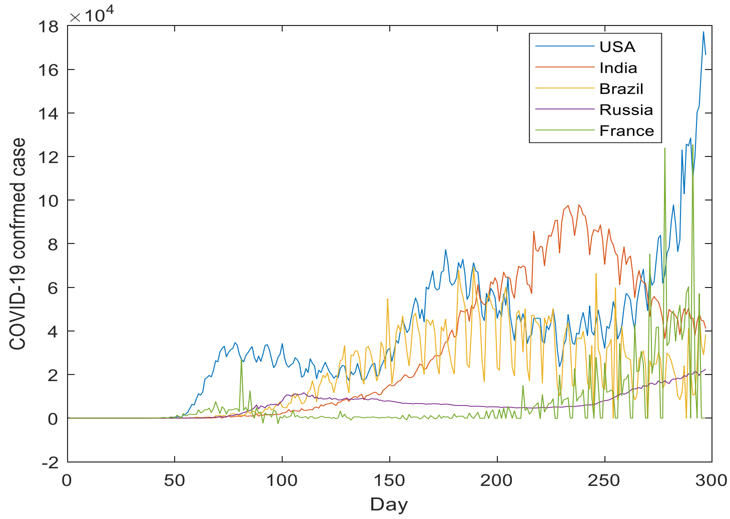

Fig. 4. COVID-19 confirmed case.

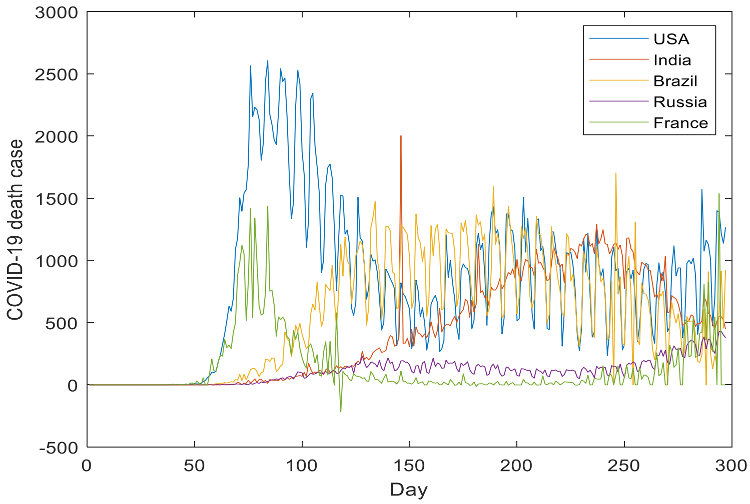

Fig. 5. COVID-19 death case.

Table 1 shows the performance of IT2IFLS-SMC+GD against IT2IFLS-GD and IT2IFLS-SMC on confirmed COVID-19 cases in the selected regions. As shown in *Table 1*, the accuracies of IT2IFLS-SMC+GD and IT2IFLS-SMC are quite similar. Howbeit, the hybrid model provides better performance than IT2IFLS-SMC in most of the cases. The GD-based learning model suffers some loss in performance in the confirmed cases for all regions except for Brazil in the noise-free dataset. For the combined dataset, the hybrid model outperforms both GD and SMC trained IT2IFLS with reduced RMSE and MAE.

Next some noise is injected into the COVID-19 confirmed cases data for the five countries and in the combined confirmed cases. Shown in *Figs. 6* and *Fig. 7* are the data points for COVID-19 combined noise free and noisy confirmed cases respectively. The aim is to study the behaviour of IT2IFLS-SMC+GD learning model under noisy condition. Here, an additive white Gaussian noise with SNR = 0dB is added to the COVID-19 confirmed cases in USA, India, Brazil, Russia and France and also in the combined confirmed cases. The SNR=0dB is chosen because it represents a high noise level.

Table 1. Performance comparison of IT2IFLS-SMC+GD with individual learning models on COVID-19 confirmed cases.

Country	Model	RMSE(NF)	MAE(NF)	RMSE(N)	MAE(N)
USA	IT2IFLS-GD	0.1283	0.0338	0.5501	0.2391
	IT2IFLS-SMC	0.0994	0.0250	0.4329	0.1871
	IT2IFLS-SMC+GD	0.0948	0.0237	0.2908	0.1260
	IT2IFLS-GD	0.0762	0.0311	0.6456	0.2837
INDIA	IT2IFLS-SMC	0.0740	0.0302	0.5618	0.2455
	IT2IFLS-SMC+GD	0.0739	0.0303	0.3413	0.1495
	IT2IFLS-GD	0.0720	0.0288	0.5799	0.2555
BRAZIL	IT2IFLS-SMC	0.0910	0.0373	0.5549	0.2482
	IT2IFLS-SMC+GD	0.0885	0.0367	0.5412	0.2367
	IT2IFLS-GD	0.1760	0.0631	0.5920	0.2590
RUSSIA	IT2IFLS-SMC	0.1520	0.0525	0.5529	0.2459
	IT2IFLS-SMC+GD	0.1531	0.0529	0.3524	0.1543
	IT2IFLS-GD	0.0380	0.0118	0.5231	0.2457
FRANCE	IT2IFLS-SMC	0.0318	0.0099	0.5288	0.2291
	IT2IFLS-SMC+GD	0.0313	0.0096	0.2431	0.1040
	IT2IFLS-GD	0.0013	0.0007	0.5709	0.2555
Combined confirmed case	IT2IFLS-SMC	0.00096	0.00052	0.5488	0.2417
	IT2IFLS-SMC+GD	0.00084	0.00045	0.5205	0.2295

*NF = Noise Free COVID-19 data, N = Noisy COVID-19 data

Also shown in *Table 1* are the performances of the three models of IT2IFLS-GD, IT2IFLS-SMC and IT2IFLS-SMC+GD under noisy condition (SNR=0dB). The benefit of IT2IFLS-SMC+GD is apparent in *Table 1*, as noise is injected into the COVID-19 time series data. As shown in *Table 1*, with additive noise, the new hybrid IT2IFLS-SMC+GD provides significant performance improvements over the individual learning models of GD and SMC in terms of RMSE and MAE.

Based on these results, the authors conjectured that in the presence of noise and uncertainties in a system, the proposed hybrid model may stand as the best choice compared to IT2IFLS-GD and IT2IFLS-SMC.

Nevertheless, for noise-free data under investigation, IT2IFLS-SMC may suffice because of its computational simplicity. Finally, an experiment is conducted to show the performance difference between type-1 IFLS-SMC+GD and IT2IFLS-SMC+GD. This is done using COVID-19 death cases of the five selected countries and then the combined death cases of the five regions, under noise-free and noisy conditions.

Table 2 shows the prediction performance of IT2IFLS-SMC+GD against IFLS-SMC+GD. As shown in *Table 2*, the accuracy of IT2IFLS-SMC+GD is better than IFLS trained with the same hybrid algorithm on the individual region's COVID-19 death cases except for France, where the errors are very close on the noise free dataset. *Fig. 8* is an instance of the real COVID-19 death cases and the predicted death cases using IFLS-SMC+GD and IT2IFLS-SMC+GD. The inset figure within *Fig. 8* clearly shows that the prediction accuracy of IT2IFLS-SMC+GD is better compared to its type-1 counterparts as it follows the actual data more closely; using an instance of USA. However, in the combined noise free COVID-19 death cases, the performance of IFLS is comparable with that of IT2IFLS (see *Table 2*), an indication that type-1 IFLSs do model uncertainty in some cases better and that any "T2FLSs must be used when needed" [56], whether classical or intuitionistic.

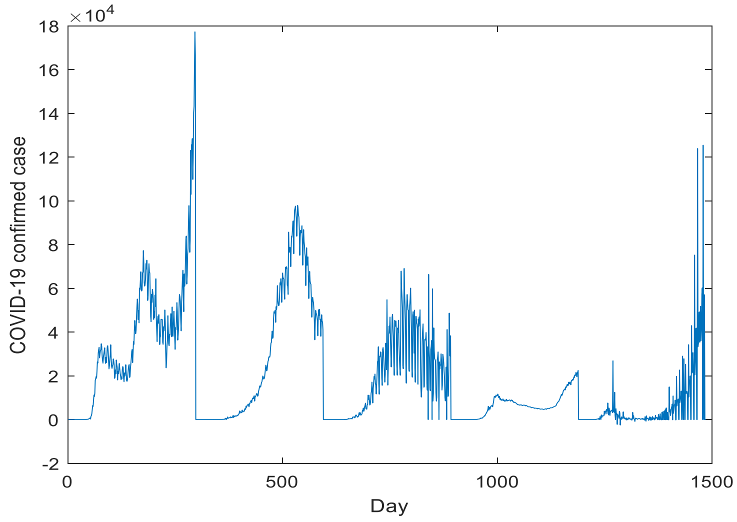

Fig. 6. Noise-free combined confirmed cases.

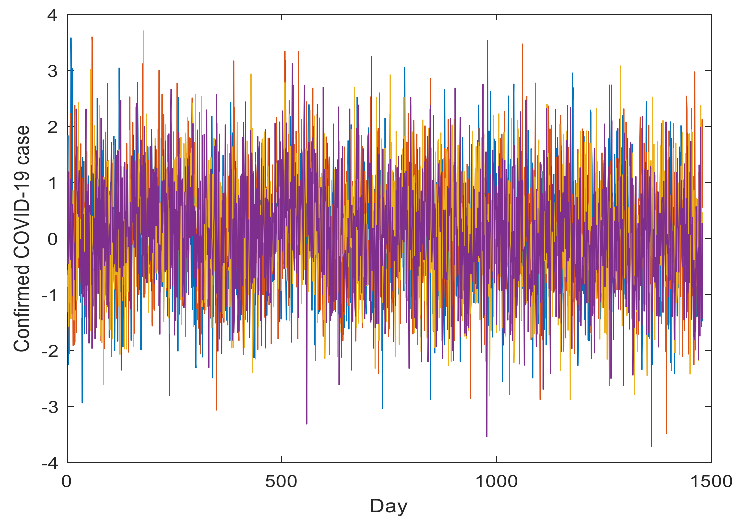

Fig. 7. Noisy combined confirmed cases.

Table 2. Performance comparison of IFLS-SMC+GD and IT2IFLS-SMC+GD on COVID-19 death cases.

Country	Model	RMSE(NF)	MAE(NF)	RMSE(N)	MAE(N)
USA	IFLS-SMC+GD	0.0366	0.0169	0.5200	0.2316
	IT2IFLS-SMC+GD	0.0229	0.0103	0.5122	0.2258
INDIA	IFLS-SMC+GD	0.0342	0.0152	0.5844	0.2593
	IT2IFLS-SMC+GD	0.0325	0.0148	0.5608	0.2465
BRAZIL	IFLS-SMC+GD	0.0862	0.0332	0.5398	0.2366
	IT2IFLS-SMC+GD	0.0619	0.0243	0.5269	0.2325
RUSSIA	IFLS-SMC+GD	0.1442	0.0497	0.5899	0.2579
	IT2IFLS-SMC+GD	0.1380	0.0362	0.5797	0.2564
FRANCE	IFLS-SMC+GD	0.0333	0.0096	0.5621	0.2486
	IT2IFLS-SMC+GD	0.0323	0.0096	0.5388	0.2347
Combined death case	IFLS-SMC+GD	0.00925	0.00399	0.54098	0.23668
	IT2IFLS-SMC+GD	0.01038	0.00401	0.5196	0.22038

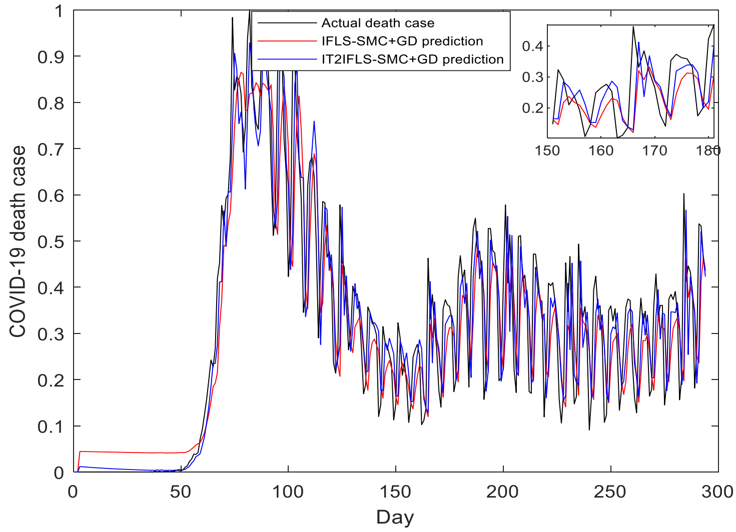

Fig. 8. IFLS-SMC+GD and IT2IFLS-SMC+GD (instance of USA).

8 | Conclusion

In this paper, a hybrid learning model comprising GD backpropagation and SMC algorithm for optimizing the parameters of IT2IFLS is introduced for the first time. The SMC learning algorithm is applied to tune the antecedent parameters due to non-linearity of these parameters while GD is used to tune the linear parameters in the consequent parts. As shown from experimental analyses, the IT2IFLS-SMC+GD learning model provides better prediction accuracies especially in noisy environment where uncertainty abounds. Analyses also reveal that although the prediction performance of IT2IFLS-SMC+GD is better than IT2IFLS-SMC especially for high level of noise, their prediction accuracies are similar for noise-free data. Thus, for noisy data such as noisy COVID-19 pandemic cases, the novel IT2IFLS hybrid learning model stands as a better prediction model compared to those of the individual models such as IT2IFLS-SMC and IT2IFLS-GD. The intuitionistic type-2 hybrid model is also shown to outperform its type-1 counterpart in many cases and clearly shows significant performance in the noisy COVID-19 instances. Overall, IT2IFLS-SMC+GD gives significant improvement when there is noise in the system compared to IT2IFLS-SMC, IT2IFLS-GD and its type-1 counterpart. In the future, we intend to adopt the general IT2IFLS and other learning models such as particle swarm optimization and simulated annealing for the analysis of the COVID-19 pandemic data.

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Conflicts of Interest

All co-authors have seen and agree with the contents of this manuscript and there is no financial interest to report. We certify that the submission is original work and is not under review at any other publication.

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How Warehouse Location Decisions Changed in Medical Sector after Pandemic? A Fuzzy Comparative Study

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Abstract

In the decision theory, there are many useful tools for operations in logistics and Supply Chain Management (SCM). One of the vital trivets of logistics operations is warehouse management which is also one of the parts of a supply chain. Deciding on the location of a warehouse has a critical issue especially during an outbreak. In this study, we aimed that to figure out differences between the perceived importance of the considered criteria in the decision process regarding warehouse location in the medical sector in terms of the changing dynamics after the Covid-19 pandemic. Pursuing this goal, the results of a preliminary study which was resulted from the gathered data of a decision-making group including industry professionals before the pandemic outbreak were accepted as an anchor to obtain a comparison with the current state. To construct a proper representation of the post-Covid state, a similar methodology was used, and similar decision-makers data were collected with the preliminary study in the identification of the importance figures and causal relationships between criteria. According to comparative results of pre-and post-Covid studies, it is found that there are significant changes in the perceived role of adjacency to target markets and customs criteria in medical warehouse location decisions. It is obvious that the results will shed light on medical sector professionals' decision process while adapting to the current pandemic conditions.

Keywords: Supply chain management, Logistics, Warehouse, Pythagorean fuzzy sets, Fuzzy DEMATEL, MCDM, Covid-19.

1 | Introduction

The medical sector has been one of the most influenced industries from the Covid-19 (the disease caused by Severe Acute Respiratory Syndrome Coronavirus (SARSCOV-2)) outbreak which was originated in the last month of 2019 and became a pandemic that has influenced the entire global operations.

As for the other sectors such as e-commerce, fast-moving consumer goods, tourism, construction, healthcare, automotive, oil, aviation, etc. had been influenced directly [1]. Besides, the supply chain of healthcare is the most affected one among those. Additionally, the location of the pharmaceutical warehouses, wholesalers, retailers, and hospitals was becoming more important because of the conjectural change after the pandemic outbreak.

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From the customers' perspective, medical firms' supply chain visibility, transparency, and agility became more and more vital due to the changes in the world. Fast delivery of pharmaceutical goods and materials has been a vital issue for citizens during that era [2] and [3].

In this connection, customer demands started to increase its priority which has not been faced before. And the health sector leads the change of this perception for the human being. Those changes have been observed in logistics operations as well. Especially the vital parts of SCM, namely, warehousing and inventory management have started a special period during the pandemic.

As the outbreak spreads, the popularity of electronic commerce and logistics services has increased so much. On the contrary, the industries such as entertainment, manufacturing, and tourism have lost their potential growth.The priority for all goods and services in e-commerce, especially for warehouse management in the medical sector has been raised. Particularly, for the disinfectant materials and medical protective masks, there has been a sudden increase in demand.

In line with the changing dynamics in the medical sector operations after the pandemic outbreak, it is expected that there should be a change in the importance levels of considered criteria in the decision-making process of the medical warehouse location as well. In this paper, the authors are aimed to reveal potential changes in the assessment process and also shifts in the perceived importance of medical warehouse location selection criteria.

Amid 2019, a previous study was performed by the authors before the pandemic outbreak, which includes the evaluations of the criteria that have an influence on the decisions of medical warehouse locations by the professionals of the medical sector [4]. Since Fuzzy Sets (FS) have limitations in handling uncertainty and vagueness, in that research, authors used the Pythagorean Fuzzy (PF) based-Decision MAking Trial and Evaluation Laboratory (DEMATEL) technique [5] and [6] to find out the criteria' importance and the causal relationships between criteria [7] and [8].

This study, it is aimed that to figure out differences between the perceived importance of the considered criteria in the decision process regarding warehouse location selection in the medical sector in terms of the changing sector dynamics after the Covid-19 pandemic. Tuncalı Yaman and Akkartal [4] following the aim, the results of the previous study were accepted as a clear picture of the pre-Covid 19 phase in terms of understanding the decision process of the decision-makers in the medical sector, and a similar design was made to figure out the changing patterns in this process if any. This study, it is aimed that to figure out differences between the perceived importance of the considered criteria in the decision process regarding warehouse location selection in the medical sector in terms of the changing sector dynamics after the Covid-19 pandemic.

Therefore, evaluations were made by the same medical sector professionals against the same criteria which play a role in selecting the location of the medical sector warehouse. Further, the same DEMATEL method was implemented to figure out the current causal relationships and criteria importance to clarify the standing situation. In this manner, previously obtained results were able to be compared with the status quo. By using the aforementioned design of the previous study, the goal of the study which is about understanding the changing perceptions in deciding the location of the medical warehouse has been achieved. The results of this study are expected to pioneer the academicians and researchers about the dynamics of decision regarding the change of the decision process regarding the selection of the location of a medical warehouse after the Covid-19 pandemic.

In the following section, the warehouse location selection problem is emphasized, and the problem statement has been declared for the medical sector. A literature review has been given in the third section. A detailed section regarding the used method has been included in the fourth part. And the fifth section includes the results of the empirical study which compares the pre and post era of the outbreak. In conclusion, comparative results and the possible future studies are discussed.

2 | Warehouse Location Selection Problem

This study aims to investigate the changes in perceived importance and switching patterns between the causal relationships of warehouse location selection criteria determined in the previous study and the current study that was performed after the Covid-19 period. Plus, warehouse location selection criteria, which have a great contribution in ensuring product flow and sustainability in the supply chain, were determined as follows before the Covid-19 period [4].

- *Adjacency to target markets.*
- *Adjacency to terminals, ports, and customs.*
- *Adjacency to the pharmaceutical production facilities.*
- *The warehouse site decision should be made by considering the capacity estimation and location criteria.*
- *Adjacency of qualified employees.*
- *Infrastructure of the site. (Electricity, water, internet sewage, natural gas, etc.).*
- *Climate conditions of the site.*
- *Ground properties of the site (impact of construction on excavation cost).*
- *Leasing cost of the site.*
- *Traffic congestion of the site.*

However, as of the pandemic period, the need for drugs and drug production suddenly increased. As a result, the perceived importance criteria for companies to decide on the choice of warehouse location have also changed. Most of the virus remained in the economy. This change showed itself in the way of transforming it into an agile turn that can respond to very momentary needs. They are expected to be more transparent, flexible, and healthy to the increasing demand. As of this, it is so preferred in warehouse location preferences.

With the building having the effects of meeting customer requests and needs instantly, the criteria for choosing a warehouse location were redefined in a way to maximize customer satisfaction. These criteria include:

I. Selection of suitable land to increase the storage capacity depending on the increasing demand intensity.
II. Choosing the materials used in the warehouse from materials suitable for recycling to prevent waste.
III. Choosing sites with a wider technological infrastructure so that customers can follow their products to provide a transparent warehouse management approach.
IV. Selection of sites that will enable vehicles to perform loading and unloading more easily to meet the increased instantaneous demand due to pandemic conditions.

Since the main objective of the study was defined as obtaining a comparative result between two phases, criticism of the newly appeared criteria could be a salient aspect of another study.

3 | Literature Review

It is seemed that the factors affecting the selection of the medical warehouse location are not focused on the literature, especially after the pandemic period. However, the importance of the location of the warehouse has increased especially in the drug supply chain. Additionally, there are no new criteria to replace, determined before the pandemic period in the selection of the storage location and this creates gaps in the literature.

Even though academic research papers are not focused on the factors affecting the selection of the medical warehouse location in the literature, especially after the pandemic period, in line with the changing dynamics after the pandemic, the importance of the location of the warehouse has increased especially in the medical supply chain to perform responsive operations. Additionally, customer orientation strategies are getting to be more significant for the medical industry. To avoid these risks, supply chain and warehouse management issues are getting more indispensable. Alike manufacturing firms, medical sector players are obligated for competitive warehousing strategies to select suitable warehouses for the effective supply chain management. Considering increased competitiveness in the market scenario and shifted demand uncertainty after the pandemic, the quantity of the product in the workflow of the warehouse is becoming more crucial occasionally medical firms struggle to find a space for the whole lot in warehouses [9]. At that point, the location of the warehouse, especially in the pharmaceutical industry, also plays an important role to prevent delays and interruptions. In its nature, services provided by medical warehouses have a direct influence on the services offered in medical firms as suppliers and in pharmacies where transportation is offered to customers. Warehousing must be achieved in such a proper way that problems can be avoided, namely, delays in progress regarding the procurement of a drug from the pharmaceutical firm and transportation of it to the patient through pharmacies. That is also an emphasis on how the location of the warehouse plays a key role in distribution activities.

As well as the need for immediate access to medical equipment such as medical masks and disinfectants, which the medical team needs at any time, has increased more during the pandemic period, as well as supplies such as medicines and vaccines. The organization of the warehouse which helps to fulfill the demand of a specific trade region has an important role for successful retailing. Logisticians must take into consideration so many factors such as avoiding the probable discrepancies in the future for a good warehouse quality, suitable political rules and regulations regional functionality regarding relevant consumptions, good traffic conditions possessing effective accessibility, and relatively low rental costs [10].

Considering the studies in the literature focusing on the problem of warehouse location selection, it seemed that Ashrafzadeh et al. [11] have emphasized that the location of a warehouse is generally one of the most important and strategic decisions in the optimization of logistic systems. In another example, Ansari and Smith [12], suggested a clustering method based on the gravity model for warehouses in which there is more than one pick per trip. According to the results, their proposed method improved the performance. Kostikov et al. [13] provided an approach to find the optimal location selection solution based on the model of the Modified Steiner-Weber Problem with restrictive conditions. Their method was found relevant for the central warehouse's optimal location, and it was diminished distribution costs from the central warehouse to sub warehouses/branches located in individual EU countries. Taş [14], studied the evaluation of criteria for the selection of pre-disposal temporary landfill sites for medical products in Turkey. Eight criteria were determined as important in the selection process of these sites. The weights of the criteria were calculated by one of the fuzzy Multi-Criteria Decision-Making (MCDM) methods called the fuzzy Pivot Pairwise Relative Criteria Importance Assessment (PIPRECIA) method. In [15], a pharmaceutical warehouse location implementation was performed.

Further, the need for warehouses as a result of the pandemic process of the regulations brought many questions. Are the warehouse and distribution centers at a level that can meet the needs of the health sector as the economies close? In the health facilities whose capacities are not fulfilled regarding displaying and forecasting the demand, transferring prescription services to pharmacies can help to reduce the quantity of the pharmaceuticals whose expiry dates are approaching [16]. Although many criteria should be considered in the selection of the warehouse location after the Covid process, storage, one of the most important parts of the past before the Covid period, is of vital importance in many sectors, including the medical sector. Parallel to this aspect, in their study, [17] has concluded that it is important to determine the appropriate warehouse location to improve the efficiency of physical distribution and minimize the total cost. For this reason, it is a priority condition that the success factors and methods to be used in the selection of the warehouse location is determined correctly. In addition, all other sectors, including the medical sector, need to design an efficient and efficient warehouse to gain a competitive advantage and to ensure sustainability, especially in terms of efficiency [18] in their study have concluded that, designing of the logistics system, regardless of the industry, is a complicated task. Incorrect decisions in this matter result in the ineffective implementation of transport and storage processes. Thus, it may lead to loss of competitiveness on the market and other serious consequences, including the bankruptcy of an enterprise or group of enterprises. Several health recordings and related files generated by clinical diagnosis equipment are constructed daily. These valuable data are embedded in various medical information. Relevant documents are saved daily generated by many health records and clinical diagnosis equipment. These crucial data are saved in various medical information systems such as HIS, PACS, RIS. Data required to make a better medical decision are hidden in heterogeneous health systems which are not integrated properly. Consequently, it is important for these medical records' integration of one warehouse [19]. Simultaneously, it becomes evident that the warehouses and distribution centers should be distributed daily to pharmacies and health institutions to fulfill the need for thermotics, especially in the supply of medical equipment and medicine. As well, the warehouse managers need to control their inventory to satisfy the market requirements. In his study, the author declared that "The objective is to develop a capacity and warehouse management plan that satisfies the expected market demands with the lowest possible cost" [20]. Besides, Khan et. al. [20] proposed solution measures that help managers to develop action plans for early recovery from COVID-19 disruption for the medical supply chain. Their study identifies the impacts and helps to formulate for mitigating those impacts in various functional levels. Another up-to-date study relating new supply chain processes has been illustrated with the new pattern health care supply chain under Covid-19 in China's Belt and Road Initiative (CBRI) countries. The results of the study represented a novel healthcare supply chain developing process during the pandemic in CBRI [21].

4 | Methodology

DEMATEL is one of the most well-known and effective methods used to determine the causal relationship between the criteria that will form the basis of evaluation for any multi-criteria decision problem [22]. Since an attempt on revealing latent causal relationships is seemed to be a sagacious approach for the interpretation of the decision-making process, in this context, the DEMATEL method was used to determine whether there is a potential relationship among the ten criteria determined as a result of both the review conducted in the current literature in the context of the warehouse location selection problem and feedbacks of sector professionals. In addition, since this study aims to determine the change in the importance levels of the criteria related to the warehouse location selection problem in the same sector before Covid-19, it would be appropriate not to make any changes in the method followed in the reference study [4] and comparable results can be obtained without any bias due to the method changes. Like most MCDM techniques, in literature, fuzzy extensions of DEMATEL are implemented, and it was stated that PF extension of the DEMATEL was found superior compared to traditional approaches when crisp numbers have some extent of limitations in handling vagueness and uncertainty [23]. Prior to detailed and comparative results of PF DEMATEL some preliminaries of the technique along with both fundamental definitions of fuzzy sets and algebraic operations of PF Sets (PFS) are detailed below [7].

4.1 | Pythagorean Fuzzy Sets

Lotfi Zadeh [23] proposed the fuzzy set theory. Since then, fuzzy measurements of vague human behaviors, intentions, evaluations, judgments, are found more realistic and precise. According to Zadeh's definition [23], $X = \{x_1, x_2, \dots, x_n,\}$ is the universal set and form of any FS on X is $F = \left\{< x, \mu_F(x) > | x\epsilon X\right\}$ where $\mu_F: X \to [0,1]$ for all $x\epsilon X$. The degree of the membership of x in F is denoted by $\mu_F(x)$.

Claiming to confront vagueness and uncertainty better, Atanassov introduced Intuitionistic Fuzzy Sets (IFS) in 1986 [24]. By definition, $X = \{x_1, x_2, \dots, x_n,\}$ is the universal set and form of any IF on X is $I = \left\{< x, \mu_I(x), \nu_I(x) > | x\epsilon X\right\}$ where $\mu_I: X \to [0,1]$ and $\nu_I: X \to [0,1]$ for all $x\epsilon X$ under the condition of $0 \leq \mu_I(x) + \upsilon_I(x) \leq 1$. The degree of the membership of x in I is denoted by $\mu_I(x)$ and the degree of the non-membership of x in I is denoted by $\nu_I(x)$. Notation of an IF value is is $I = \langle \mu_I, \nu_I \rangle$.

Finally, in 2013, Yager [25] announced a new extension of fuzzy sets called Pythagorean Fuzzy Sets with the following definition [26]: $X = \{x_1, x_2, \dots, x_n,\}$ is the universal set and form of any PFS on X is $P =$

$\{< x, \mu_P(x), \nu_P(x) > | x\epsilon X\}$ where $\mu_P: X \to [0,1]$ and $\nu_P: X \to [0,1]$ for all $x\epsilon X$ under the condition of $0 \leq \mu_P(x)^2 + \upsilon_P(x)^2 \leq 1$. The degree of the membership of x in I is denoted by $\mu_P(x)$ and the degree of the non-membership of x in I is denoted by $\nu_P(x)$. The notation of a PF Value (PFV) is $P = \langle \mu_P, \nu_P \rangle$ where P stands for PFV. The degree of indeterminacy of x to P is given by, $\pi_p(x) = \sqrt{1 - \mu_p^2 - \upsilon_p^2}$ where $\mu_P, \nu_P \epsilon[0,1]$ and $\mu_P{}^2 + \upsilon_P{}^2 \leq 1$. Algebraic operations of PFS are detailed below [27].

If $K = (\mu_K, \upsilon_K)$ and $L = (\mu_L, \upsilon_L)$ are two PFVs where $\mu_K, \nu_K \epsilon[0,1]$ and $\mu_L, \nu_L \epsilon[0,1]$, arithmetic operations over two PFVs are,

$$\mathrm{K} \cup \mathrm{L} = \mathrm{P}\left(\max\{\mu_\mathrm{K}, \mu_\mathrm{L}\}, \min\{\mathrm{v}_\mathrm{K}, \mathrm{v}_\mathrm{L}\}\right). \tag{1}$$

$$\mathrm{K} \cap \mathrm{L} = \mathrm{P}\left(\min\{\mu_\mathrm{K}, \mu_\mathrm{L}\}, \max\{\mathrm{v}_\mathrm{K}, \mathrm{v}_\mathrm{L}\}\right). \tag{2}$$

$$\mathrm{K}^\mathrm{C} = \mathrm{P}(\mathrm{v}_\mathrm{K}, \mu_\mathrm{K}). \tag{3}$$

$$\mathrm{K} = \mathrm{L} \Longleftrightarrow \mu_\mathrm{K} = \mu_\mathrm{L}, \mathrm{v}_\mathrm{K} = \mathrm{v}_\mathrm{L}. \tag{4}$$

$$\mathrm{K} \subset \mathrm{L} \Longleftrightarrow \mu_\mathrm{K} \leq \mu_\mathrm{L}, \mathrm{v}_\mathrm{K} \geq \mathrm{v}_\mathrm{L}. \tag{5}$$

$$\mathrm{K} \oplus \mathrm{L} = \mathrm{P}\left(\sqrt{\mu_\mathrm{K}^2 + \mu_\mathrm{L}^2 - \mu_\mathrm{K}^2\mu_\mathrm{L}^2}, \mathrm{v}_\mathrm{K}\mathrm{v}_\mathrm{L}\right). \tag{6}$$

$$\mathrm{K} \otimes \mathrm{L} = \mathrm{P}\left(\mu_\mathrm{K}\mu_\mathrm{L}, \sqrt{\mathrm{v}_\mathrm{K}^2 + \mathrm{v}_\mathrm{L}^2 - \mathrm{v}_\mathrm{K}^2\mathrm{v}_\mathrm{L}^2}\right). \tag{7}$$

$$\lambda\mathrm{K} = \mathrm{P}\left(\sqrt{1 - \left(1 - \mu_\mathrm{K}^2\right)^\lambda}, (\mathrm{v}_\mathrm{K})^\lambda\right), \lambda > 0. \tag{8}$$

$$\lambda(\mathrm{K} \oplus \mathrm{L}) = \lambda\mathrm{K} \oplus \lambda\mathrm{L}, \lambda > 0. \tag{9}$$

$$\mathrm{K}^\lambda = \mathrm{P}\left((\mu_\mathrm{K})^\lambda, \sqrt{1 - \left(1 - \mathrm{v}_\mathrm{K}^2\right)^\lambda}\right), \lambda > 0. \tag{10}$$

Nowadays, there are many theoretical studies on PFSs in the literature, just like the new approaches regarding other fuzzy numbers [28]. For instance, both papers [29] and [30] introduced a modified PF correlation measure. In [31], generalized triparametric correlation coefficient for pythagorean fuzzy sets was discussed in terms of being a useful tool for solving multi criteria decision making problems. The following section is focused on the calculation steps of the Pythagorean Fuzzy DEMATEL method.

4.2 | Pythagorean Fuzzy DEMATEL

In DEMATEL methodology, to reveal casual relationships between criteria, pair-wise comparisons are made with the linguistic variable "influence" with the help of a seven-point rating scale which is constructed with seven terms. In practice, experts make their evaluations with correspıning linguistic terms of crisp numbers. PFVs of each linguistic terms are appeared in *Table 1* [5].

Where the decision problem includes n criteria, the initial direct relation matrix $Z = [z_{ij}]_{n\times n}$ which is constituted from pair-wise comparisons of criteria by linguistic terms, is constructed as an $n \times n$ matrix. In practice, the total number of Zs is equal to the total number of experts. Elements of the matrix (z_{ij}) are PFVs, and each indicates the degree to which criterion i, affects the criterion j, where $(i, j = 1,2, \dots, n)$. *Eq. (11)* indicates the initial direct relation matrix of the kth expert.

$$Z_k = \left[z_{ij}\right]_{n\times n} = \begin{array}{c} Cr_1 \\ \vdots \\ Cr_n \end{array}\begin{bmatrix} Cr_1 & & Cr_n \\ \langle 0,0\rangle & \cdots & \mu_{k_1 n}, v_{k_1 n} \\ \vdots & \ddots & \vdots \\ \langle \mu_{k_n 1}, v_{k_n 1}\rangle & \cdots & \langle 0,0\rangle \end{bmatrix}. \tag{11}$$

Table 1. Rating scale of influence factor.

Degree of "Influence"	Rating Scale Crisp Number	PFV
Very low	0	⟨0,0⟩
Low	1	⟨0.1,0.9⟩
Medium low	2	⟨0.2,0.9⟩
Medium	3	⟨0.4,0.6⟩
Medium high	4	⟨0.5,0.7⟩
High	5	⟨0.7,0.2⟩
Very high	6	⟨0.9,0.1⟩

The weighted initial direct relation matrix $W = \left[w_{ij}\right]_{n\times n}$ is calculated by *Eq. (8)* which indicates the multiplication of the initial relationship matrix $Z = \left[z_{ij}\right]_{n\times n}$ by expert weight λ_k (for expert k, weight is denoted by λ_k). If there are k experts, $\sum_{i=0}^{k} \lambda_i = 1, \lambda_i > 0$ where $(i = 1,2, \dots, k)$. *Eq. (12)* illustrates the weighted initial direct relation matrix of the kth expert.

$$W_k = \left[w_{ij}\right]_{n\times n} = \begin{array}{c} Cr_1 \\ \vdots \\ Cr_n \end{array}\begin{bmatrix} Cr_1 & & Cr_n \\ \lambda_k\langle 0,0\rangle & \cdots & \lambda_k z_{1j} \\ \vdots & \ddots & \vdots \\ \lambda_k z_{i1} & \cdots & \lambda_k\langle 0,0\rangle \end{bmatrix}. \tag{12}$$

To aggregate weighted initial direct relation matrices of all experts, the addition operator in *Eq. (6)* is used. The summation of all weighted initial direct relation matrices gives the total aggregated matrix C. For illustration purposes, the notation of an example aggregated matrix of the first two experts is given below.

$$C_{1,2} = W_1 \oplus W_2 = \begin{array}{c} Cr_1 \\ \vdots \\ Cr_n \end{array}\begin{bmatrix} Cr_1 & & Cr_n \\ \lambda_1\langle 0,0\rangle \oplus \lambda_2\langle 0,0\rangle & \cdots & \left(\lambda_k z_{1j}\right)_1 \oplus \left(\lambda_k z_{1j}\right)_2 \\ \vdots & \ddots & \vdots \\ (\lambda_k z_{i1})_1 \oplus (\lambda_k z_{i1})_2 & \cdots & \lambda_1\langle 0,0\rangle \oplus \lambda_2\langle 0,0\rangle \end{bmatrix}. \tag{13}$$

Defuzzification of the total aggregated matrix creates the crisp valued total average matrix $A = \left[a_{ij}\right]_{n\times n}$ where $a_{ij} \in [-1,1]$. As the defuzzification function, the following score function is used.

$$a_{ij} = \mu^2_{C_{ij}} - v^2_{C_{ij}}. \tag{14}$$

$$A = \left[a_{ij}\right]_{n\times n} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}. \tag{15}$$

The normalized total average matrix $N = \left[n_{ij}\right]_{n\times n}$ where $0 \le n_{ij} \le 1$ is found by *Eq. (16)* with the use of a normalization factor.

$$N = \left[n_{ij}\right]_{n\times n} = s.A, \tag{16}$$

where $s = \frac{1}{max\left(max_{1\leq i\leq n}\sum_{i=1}^{n} a_{ij}, max_{1\leq j\leq n}\sum_{j=1}^{n} a_{ij}\right)}$ $\quad i, j = 1,2, \ldots, n.$

Based on the normalized total average matrix N, the total relation matrix $T = \left[t_{ij}\right]_{n\times n}$ is obtained by following *Eq. (17)*.

$$T = N(I - N)^{-1}, \tag{17}$$

where I is the identity matrix.

Causal relationships could be identified by sum of rows (c) and sum of columns (r) of the total relation matrix T.

$$c = \sum_{i=1}^{n} t_{ij}. \tag{18}$$

$$r = \sum_{j=1}^{n} t_{ij}. \tag{19}$$

The last step of the process is illustrating the causal relationships of the criteria with casual diagram. Summation and subtraction of c and d are used in plotting the diagram where the horizontal axis indicates $(c + r)$. A positive $(c - r)$ value addresses that the criterion is under the "cause" category. Otherwise, the criterion is evaluated in the "effect" group.

5 | Results

According to the results of the pre-Covid phase study [4] that was conducted in mid-2019, cause and effect groups of criteria were detected by performing previously detailed steps of the PF-DEMATEL algorithm. In line with the objective of this paper which is to uncover the implicit changes in the causal relationship among criteria of warehouse location selection problem in the medical sector, the same methodology was followed with the contemporary evaluations of the same experts after the pandemic, and cause and effect groups of the same criteria were constituted and compared with the results of the previous study that was conducted before the pandemic.

This section covers detailed results of the current study are detailed in steps of PF-DEMATEL and the comparison between two consecutive studies.

To reveal causal relationships among previously defined criteria that influence warehouse location selection for medical sector companies were evaluated by the same six experts who were consulted. Also, appointed weigh scores, based on the experiences of the experts, were kept the same.

Table 2. Expert weights.

Experts	λ_k
E1	0.23
E2	0.23
E3	0.22
E4	0.12
E5	0.10
E6	0.10

Computer administered telephone interviews were realized with the experts and as judgments, linguistic terms-based pair-wise comparisons of criteria were collected. For each expert, pair-wise comparisons in the linguistic terms of influences between criteria were transformed into 10×10 personal initial direct

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relation matrices. For six experts, six different initial direct relation matrices were created by *Eq. (11)*. *Table 3* presents the initial direct relation matrix $Z_{k1} = \left[z_{ij}\right]_{10\times10}$ of Expert #1. The z_{ij} values which are the elements of the initial direct relation matrix imply the degree to which the criterion i affects the criterion j.

Table 3. Initial direct relation matrix of expert #1.

Criteria		C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
C1	μ	0	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
	v	0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
C2	μ	0.7	0	0.4	0.5	0.5	0.5	0.7	0.5	0.5	0.7
	v	0.2	0	0.6	0.7	0.7	0.7	0.2	0.7	0.7	0.2
C3	μ	0.5	0.5	0	0.9	0.5	0.4	0.7	0.7	0.5	0.7
	v	0.7	0.7	0	0.1	0.7	0.6	0.2	0.2	0.7	0.2
C4	μ	0.5	0.5	0.5	0	0.9	0.5	0.7	0.5	0.9	0.5
	v	0.7	0.7	0.7	0	0.1	0.7	0.2	0.7	0.1	0.7
C5	μ	0.5	0.7	0.5	0.5	0	0.7	0.7	0.5	0.9	0.5
	v	0.7	0.2	0.7	0.7	0	0.2	0.2	0.7	0.1	0.7
C6	μ	0.5	0.7	0.5	0.4	0.7	0	0.1	0.5	0.4	0.5
	v	0.7	0.2	0.7	0.6	0.2	0	0.9	0.7	0.6	0.7
C7	μ	0.9	0.5	0.9	0.5	0.7	0.5	0	0.4	0.7	0.7
	v	0.1	0.7	0.1	0.7	0.2	0.7	0	0.6	0.2	0.2
C8	μ	0.4	0.5	0.5	0.5	0.4	0.7	0.7	0	0.5	0.5
	v	0.6	0.7	0.7	0.7	0.6	0.2	0.2	0	0.7	0.7
C9	μ	0.1	0.7	0.7	0.5	0.5	0.7	0.7	0.5	0	0.7
	v	0.9	0.2	0.2	0.7	0.7	0.2	0.2	0.7	0	0.2
C10	μ	0.5	0.5	0.7	0.7	0.5	0.5	0.7	0.7	0.7	0
	v	0.7	0.7	0.2	0.2	0.7	0.7	0.2	0.2	0.2	0

Elements of W_k for each expert were provided by *Eq. (12)*, getting the product of expert weights (λ_k) (see *Table 2*) by the elements of related initial direct relation matrix (W_k). *Table 4* illustrates the weighted initial direct relation matrix $W_{k1} = \left[w_{ij}\right]_{10\times10}$ of Expert #1. For all provided matrices, the numbers are rounded to 2 decimals.

Table 4. Weighted initial direct relation matrix of expert #1.

Criteria		C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
C1	μ	0.00	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57
	v	0.00	0.59	0.59	0.59	0.59	0.59	0.59	0.59	0.59	0.59
C2	μ	0.38	0.00	0.20	0.25	0.25	0.25	0.38	0.25	0.25	0.38
	v	0.69	0.00	0.89	0.92	0.92	0.92	0.69	0.92	0.92	0.69
C3	μ	0.25	0.25	0.00	0.57	0.25	0.20	0.38	0.38	0.25	0.38
	v	0.92	0.92	0.00	0.59	0.92	0.89	0.69	0.69	0.92	0.69
C4	μ	0.25	0.25	0.25	0.00	0.57	0.25	0.38	0.25	0.57	0.25
	v	0.92	0.92	0.92	0.00	0.59	0.92	0.69	0.92	0.59	0.92
C5	μ	0.25	0.38	0.25	0.25	0.00	0.38	0.38	0.25	0.57	0.25
	v	0.92	0.69	0.92	0.92	0.00	0.69	0.69	0.92	0.59	0.92
C6	μ	0.25	0.38	0.25	0.20	0.38	0.00	0.05	0.25	0.20	0.25
	v	0.92	0.69	0.92	0.89	0.69	0.00	0.98	0.92	0.89	0.92
C7	μ	0.57	0.25	0.57	0.25	0.38	0.25	0.00	0.20	0.38	0.38
	v	0.59	0.92	0.59	0.92	0.69	0.92	0.00	0.89	0.69	0.69
C8	μ	0.20	0.25	0.25	0.25	0.20	0.38	0.38	0.00	0.25	0.25
	v	0.89	0.92	0.92	0.92	0.89	0.69	0.69	0.00	0.92	0.92
C9	μ	0.05	0.38	0.38	0.25	0.25	0.38	0.38	0.25	0.00	0.38
	v	0.98	0.69	0.69	0.92	0.92	0.69	0.69	0.92	0.00	0.69
C10	μ	0.25	0.25	0.38	0.38	0.25	0.25	0.38	0.38	0.38	0.00
	v	0.92	0.92	0.69	0.69	0.92	0.92	0.69	0.69	0.69	0.00

The total aggregated matrix (C), which originally provides the summation of all weighted initial direct relation matrices of experts, was created by the addition operator in *Eq. (6)*. In *Table 5*, the total aggregated matrix is given below.

Table 5. Total aggregated matrix.

Criteria		C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
C1	μ	0.00	0.43	0.43	0.43	0.43	0.43	0.43	0.43	0.43	0.43
	v	0.00	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
C2	μ	0.28	0.00	0.23	0.24	0.22	0.20	0.27	0.18	0.29	0.34
	v	0.40	0.00	0.43	0.39	0.51	0.64	0.00	0.73	0.37	0.34
C3	μ	0.22	0.30	0.00	0.30	0.24	0.29	0.32	0.23	0.33	0.21
	v	0.53	0.58	0.00	0.25	0.40	0.31	0.00	0.39	0.27	0.52
C4	μ	0.20	0.20	0.23	0.00	0.29	0.21	0.24	0.29	0.32	0.20
	v	0.51	0.58	0.48	0.00	0.33	0.52	0.00	0.40	0.34	0.61
C5	μ	0.25	0.21	0.17	0.22	0.00	0.30	0.28	0.23	0.33	0.25
	v	0.41	0.55	0.67	0.55	0.00	0.32	0.00	0.47	0.23	0.39
C6	μ	0.19	0.23	0.22	0.20	0.22	0.00	0.26	0.32	0.29	0.22
	v	0.00	0.00	0.52	0.50	0.52	0.00	0.62	0.00	0.37	0.52
C7	μ	0.31	0.23	0.30	0.19	0.24	0.24	0.00	0.17	0.24	0.28
	v	0.44	0.53	0.46	0.00	0.33	0.50	0.00	0.69	0.41	0.39
C8	μ	0.22	0.21	0.29	0.25	0.29	0.32	0.24	0.00	0.33	0.16
	v	0.51	0.52	0.47	0.00	0.42	0.00	0.00	0.00	0.44	0.64
C9	μ	0.28	0.25	0.24	0.29	0.19	0.25	0.31	0.19	0.00	0.21
	v	0.42	0.29	0.40	0.36	0.00	0.40	0.00	0.00	0.00	0.54
C10	μ	0.23	0.18	0.25	0.27	0.20	0.24	0.31	0.23	0.24	0.00
	v	0.62	0.70	0.39	0.00	0.62	0.48	0.32	0.46	0.34	0.00

following step, defuzzification of the total aggregated matrix was realized by the score function (see *Eq. (14)*). The final total average matrix (A) with crisp values whose notation is given in *Eq. (15)*, is presented in *Table 6*.

Table 6. Total average matrix.

Criteria	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
C1	0.00	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17
C2	-0.08	0.00	-0.13	-0.10	-0.21	-0.37	0.08	-0.50	-0.05	0.00
C3	-0.24	-0.25	0.00	0.03	-0.10	-0.01	0.10	-0.10	0.04	-0.23
C4	-0.22	-0.30	-0.17	0.00	-0.02	-0.23	0.06	-0.07	-0.01	-0.34
C5	-0.10	-0.26	-0.41	-0.26	0.00	-0.01	0.08	-0.17	0.06	-0.09
C6	0.04	0.05	-0.22	-0.21	-0.22	0.00	-0.31	0.10	-0.05	-0.23
C7	-0.10	-0.23	-0.12	0.03	-0.05	-0.19	0.00	-0.44	-0.11	-0.07
C8	-0.21	-0.23	-0.13	0.06	-0.09	0.10	0.06	0.00	-0.09	-0.39
C9	-0.10	-0.02	-0.10	-0.05	0.04	-0.10	0.10	0.04	0.00	-0.25
C10	-0.34	-0.46	-0.09	0.07	-0.35	-0.17	0.00	-0.16	-0.06	0.00

For normalization purposes, the normalization facor (s) was used and, the normalized total average matrix $N = \left[n_{ij}\right]_{10\times 10}$ was calculated by *Eq. (16)*. The final elements of N are provided in *Table 7*.

Table 7. Normalized total average matrix.

Criteria	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
C1	0.00	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53
C2	-0.24	0.00	-0.40	-0.30	-0.64	-1.12	0.23	-1.52	-0.16	0.01
C3	-0.72	-0.76	0.00	0.09	-0.31	-0.03	0.30	-0.29	0.12	-0.70
C4	-0.66	-0.91	-0.53	0.00	-0.07	-0.70	0.18	-0.22	-0.03	-1.03
C5	-0.31	-0.79	-1.26	-0.78	0.00	-0.03	0.24	-0.51	0.18	-0.28
C6	0.11	0.16	-0.66	-0.63	-0.68	0.00	-0.94	0.30	-0.16	-0.69
C7	-0.29	-0.70	-0.37	0.11	-0.15	-0.58	0.00	-1.34	-0.33	-0.23
C8	-0.64	-0.69	-0.40	0.19	-0.28	0.31	0.18	0.00	-0.26	-1.18
C9	-0.29	-0.06	-0.31	-0.15	0.11	-0.31	0.29	0.11	0.00	-0.75
C10	-1.02	-1.39	-0.27	0.22	-1.06	-0.52	-0.01	-0.49	-0.18	0.00

The total relation matrix $T = \left[t_{ij}\right]_{n\times n}$, whose column and row sums are used in the provision of c and r figures, was calculated by *Eq. (17)*. In *Table 8*, along with elements of T, c and r values are detailed. In the calculation of c and d, *Eq. (18)* and *Eq. (19)* were used respectively.

Table 8. Total relation matrix.

Criteria	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	c
C1	-0.19	-0.11	-0.39	0.09	0.38	0.10	0.38	-0.02	0.30	0.14	0.69
C2	-0.57	-1.42	0.29	0.39	-0.03	-0.78	0.32	-0.76	-0.09	0.54	-2.11
C3	-0.07	-0.01	-0.02	-0.25	0.13	0.09	0.17	-0.29	0.19	-0.41	-0.47
C4	0.17	-0.21	-0.99	-0.41	0.67	-0.10	0.14	0.08	0.09	-0.14	-0.69
C5	0.13	0.13	-0.65	-0.51	-0.51	0.34	-0.35	0.27	-0.02	0.45	-0.72
C6	0.03	0.63	0.53	0.03	-0.64	-0.59	-0.27	-0.27	-0.03	-0.07	-0.66
C7	0.02	-0.56	-0.24	0.08	0.15	0.08	-0.29	-0.18	-0.13	0.15	-0.93
C8	0.16	0.41	-0.29	-0.09	0.04	0.10	0.06	-0.34	-0.15	-0.38	-0.48
C9	-0.02	-0.22	-0.53	-0.24	0.45	-0.33	0.39	-0.30	0.02	0.22	-0.58
C10	-0.21	0.03	0.17	0.18	-0.52	0.38	-0.45	0.76	-0.29	-1.10	-1.06
d	-0.55	-1.32	-2.12	-0.74	0.11	-0.71	0.11	-1.05	-0.12	-0.61	

To identify causal relationships between criteria, $(c + r)$ and $(c - r)$ values of each criterion were gathered. For interpretation, one should consider that if a $(c - r)$ is positive, then the belonging criteria will be in the cause category. Otherwise, then the criterion should be evaluated in the effect category. According to the results of the Pre-Covid phase study [4], there were five criteria in the cause group, namely "Proximity to the ports and customs.", "Proximity to the pharmaceutical production centers.", "Proximity of qualified workforce.", "Location decision of a warehouse must be submitted together with capacity and demand estimation." and "Ground properties of the location (impact of construction on excavation cost)". One can expect that criteria under the cause category have an influence on the criteria in the effect group. The rest of the criteria were classified in the effect group ("Proximity to target markets. (Hospitals, pharmacies)", "Infrastructure of the area (electricity, water, sewage, transportation, natural gas, etc.)", "Climate of the location.", "Leasing cost of the location.", and "Traffic density of location").

When we observed the results of the current study, the criterion "Proximity to target markets. (Hospitals, pharmacies)" has moved to the effect group with a higher importance value whereas "Proximity to the ports and customs" has moved to the effect group. Due to comparative purposes, results of both studies, including $(c + r)$ and $(c - r)$ values, are given in *Table 9*.

Table 9. (c + r) and (c − r) values by criterion.

Criteria	Pre-Covid (c + r)	Pre-Covid (c − r)	Post-Covid (c + r)		Post-Covid (c − r)
C1	Proximity to target markets.	-0.84	-0.14	0.14	1.24
C2	Proximity to the ports and customs.	-2.91	0.18	-3.43	-0.79
C3	Proximity to the pharm. production centers.	-1.94	1.22	-2.59	1.66
C4	Proximity of qualified workforce.	-2.46	0.09	-1.44	0.05
C5	Infrastructure of the area.	-1.51	-0.40	-0.61	-0.83
C6	Capacity and demand estimation.	-1.61	0.13	-1.37	0.05
C7	Climate of the location.	-1.22	-0.75	-0.82	-1.04
C8	Ground properties of the location.	-2.49	0.73	-1.53	0.57
C9	Leasing cost of the location.	-1.05	-0.50	-0.70	-0.46
C10	Traffic density of location.	-1.10	-0.58	-1.67	-0.45

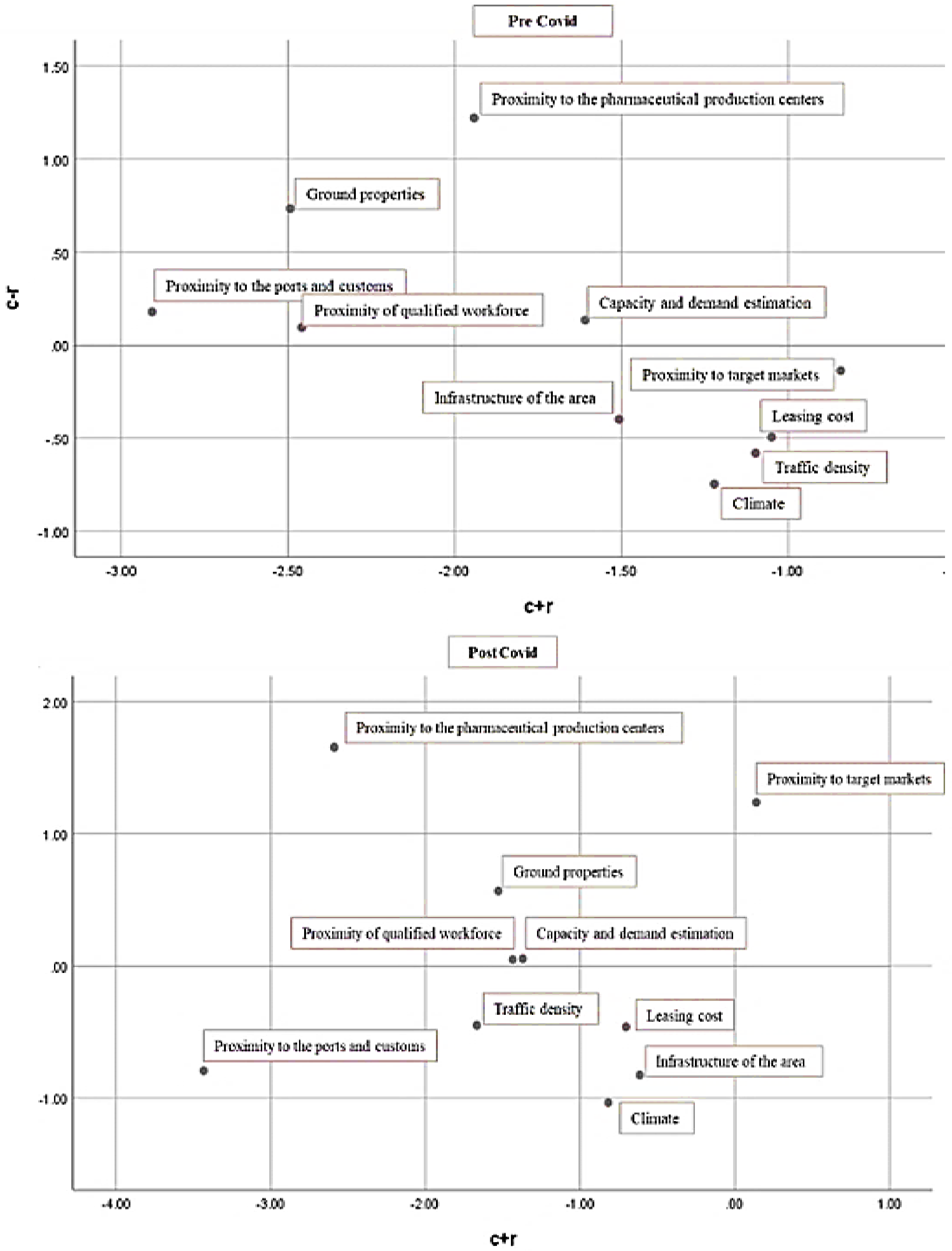

Fig. 1. Causal diagrams.

By plotting related $(c - r)$ and $(c + r)$ values of each criterion where the horizontal axis values are $(c + r)$s, a causal diagram was also obtained. By pursuing a comparative assessment, causal diagrams of both studies are illustrated in *Fig. 1*. The aforementioned changes in the characteristics of the criteria between the two phases could be easily observed via the causal diagrams. For the interpretation of the diagram, one could note that the criterion "Proximity to target markets. (Hospitals, pharmacies)" was on the negative quadrant in the pre-Covid phase diagram and moved to the positive one in the post-Covid phase plot.

6 | Conclusion and Discussion

Since the beginning of the Covid-19 pandemic, studies continue to be carried out to observe the effects in many different areas [1]. In this study, the authors carried out a similar study with [4], which was realized at the end of 2019, to determine the factors that affect the selection of warehouse location in the medical sector through the PF-DEMATEL method. The main purpose of this subsequent study, which was realized at the end of 2020, is to determine how the warehouse location decisions the medical sector was affected by the changing situations after Covid-19. As discussed in the introduction, along with e-commerce, consumer products, and logistics, the medical sector can be counted as one of the most affected sectors after the pandemic. The fact that the medical industry is in a direct and natural relationship with human health, the unexpected differentiation of customer behaviors and the incredible shift in both customers' and stakeholders' expectations under pandemic conditions, finally the pursuit of both responsiveness and profitability under these circumstances are undoubtedly reflected in the perceptions of industry professionals. Considering that the warehouse location of any medical firm directly affects the effectiveness of logistics operations and the responsiveness level of the organization, we investigated that how the changing dynamics of the Covid-19 pandemic are altered the medical sector professionals' judgments and potential actions.

In this study, we aimed to create a comparative inference between pre-Covid and post-Covid eras in terms of the perceptions of decision-makers towards important criteria in the medical warehouse location decisions. With the help of the proposed study design and the PF-DEMATEL method-based approach, causal relationships between criteria were identified by analyzing decision-makers' evaluations on predefined criteria set. Data collection of this study is realized after pandemic outbreak then these results are accepted as the indicators of the post-Covid phase. There were the results of the previous study, which covers causal relationships between the same criteria, were held at the beginning of 2019 already at hand, thus the results of that research were considered as the benchmark that reflects the pre-Covid phase's perceptions. According to the results of the Pre-Covid phase study [4], there were five criteria in the cause group, namely "Proximity to the ports and customs.", "Proximity to the pharmaceutical production centers.", "Proximity of qualified workforce.", "Location decision of a warehouse must be submitted together with capacity and demand estimation." and "Ground properties of the location (impact of construction on excavation cost)". The rest of the evaluated criteria were classified in the effect group ("Proximity to target markets. (Hospitals, pharmacies)", "Infrastructure of the area (electricity, water, sewage, transportation, natural gas, etc.)", "Climate of the location.", "Leasing cost of the location.", and "Traffic density of location").

When we observed the results of the current post-Covid phase study, the criterion "Proximity to target markets. (Hospitals, pharmacies)" has moved to the effect group with a higher importance value whereas "Proximity to the ports and customs" has moved to the effect group. In the light of the obtained comparative results gathered by sector experts' evaluation of the important criteria in the selection of medical warehouse location via the proposed methodology, it was noted that the criterion is related to the proximity of the warehouse location to the target markets is gained higher importance and clustered in the cause group in contradistinction to the pre-Covid phase study. Plus, the criterion that is related to proximity to the ports and customs has moved to the effect group. In this case, the priority of the decision-makers in the medical sector became to provide faster service by being close to the target market compared to the market conditions before the pandemic. However, their perception regarding being close to the customs by considering the operations related to exports and imports has been changed and the criterion is accepted as an affected one by the causal criteria. It is clear that the results should be evaluated as an effort for sector professionals to adapt to the pandemic conditions and also can be accepted as the essential offering of the article.

It is recommended that the aforementioned changes in the perspectives of the medical industry, which plays the most crucial role in the organization of medical supply and vaccines especially after the pandemic, should be considered in future studies. It is recommended that approaches providing comparative results for future studies should be used to assess the impact of the pandemic more objectively in different sectors and/or different decision problems. By considering that there is no similar study performed in the tackled problem here, different MCDM methods can be applied to assess the superiority of the different approaches.

Conflicts of Interest

All co-authors have seen and agreed with the contents of the manuscript and there is no financial interest to report. We certify that the submission is original work and is not under review at any other publication.

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Introduction to Plithogenic Sociogram with Preference Representations by Plithogenic Number

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Abstract

The theory of plithogeny is gaining momentum in recent times as it generalizes the concepts of fuzzy, intuitionistic, neutrosophy and other extended representations of fuzzy sets. The relativity of the comprehensive and accommodative nature of plithogenic sets in dealing with attributes shall be applied to handle the decision–making problems in the field of sociology. This paper introduces the concepts of Plithogenic Sociogram (PS) and Plithogenic Number (PN) where the former is the integration of plithogeny to the sociometric technique of sociogram and the latter is the generalization of fuzzy, intuitionistic and neutrosophic numbers that shall be used in representations of preferences in group dynamics. This research work outlines the conceptual development of these two newly proposed concepts and discusses the merits of the existing theory of similar kind with suitable substantiation. The plithogenic sociogram model encompassing the attributive preferences with plithogenic number representation is also developed to explicate how it can be materialized in the real social field. A conjectural illustration is put forth to analyze the efficiency and the feasibility of the proposed plithogenic sociogram model and its function in decision-making. This paper also throws light on generalized plithogenic number, dominant attribute constrained plithogenic number and combined dominant attribute constrained plithogenic number together with its operations and suitable illustrations.

Keywords: Plithogeny, Plithogenic sociogram, Attributes, Preferences generalized plithogenic number, Dominant attribute constrained plithogenic number, Combined dominant attribute constrained plithogenic number.

1 | Introduction

The social relationship is the resultant of the social interaction between persons and the longevity of their relationship depends on the alikeness in thoughts, behaviour and sometimes the influence of one's attribute over another. The formation of social groups for carrying out group activities is sometimes deliberate but quite natural in any social setting ranging from small schools, organizations to mammoth industries. Should we concern about the strength of the interrelationship between the members of the group? Will making the bond strong between the members benefit the group? The answer is certainly yes, because the extent of functioning as a group with common objectives and the success in goal attainment depends on the coordination and cooperativeness of the members of the group.

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Hence, the study of interpersonal relationships in a group, preferably a social group has greater significance in group dynamics. Sociogram developed by Jacob Levy Moreno and it is one of the sociometric techniques that is widely used in the quantitative study on interpersonal relationship [1]. This technique is used to determine the structure of interrelationship in a group setting by determining the order of preferences of the members of the group to work with through a questionnaire. The preferential positions of the members determine the most influential and isolated people of the group and as the result, the decision-makers or the group coordinators can work on enhancing interpersonal relationship and make other alternatives for improving the group efficiency.

Conventional sociogram characterized by crisp preferential ordering, matrix and graphical representations finds several applications in a various social setting. The uncertainty in the order of preferences led to the development of fuzzy sociogram with fuzzy matrix and fuzzy graphical representations and it has made the researchers explore its applicability in determining the interrelationship between the members [2] and [3]. The decision-making environment is characterized not only by uncertainty but also indeterminacy, to handle such circumstances, Abdel-Basset et al. [4] and Smarandache [5] introduced neutrosophic sets which consist of truth values, indeterminacy values and falsity values. Neutrosophic sets are used in decision-making on green supply chain management [6], decision support systems and in many other. Gómez et al. [7] extended fuzzy sociogram to neutrosophic sociogram to incorporate the notion of the existence of indeterminacy in relationships. The preferential ordering is certainly influenced by the indeterminacy that occurs when the members are not sure of certain attributes of others and also they may not sure of their compatibility or suitability to perform a particular task. A hypothetical example was used to illustrate the applicability of the neutrosophic sociogram model to group analysis. On profound analysis over the transition from conventional or the classical sociogram to neutrosophic sociogram, the order of preferences or the preferential ordering is influenced by certainty in the case of classical, uncertainty in the case of fuzzy and indeterminacy in the case of neutrosophic. This fact has led the authors to investigate the factors that influence preferential ordering as it is the deciding factor of the nature of the sociogram. This is the origin of the plithogenic sociogram which encompasses the attributive preferential ordering, i.e order of preference based on the attributives of the members. Before making the order of preferences, in the sociograms of earlier kinds, the activities (such as quiz program, team-based tasks) that require group work are stated first and the members express their preference for working with others, but in the realm, the choice of choosing or giving preference to the members to get involved in activity also depends on the attributes possessed by the members that are essential to make partnership to take part in any particular activity and many times these attributes may be an essential requisite to take part in the activity or the activities may itself demand the same. In such circumstances, the preferential ordering will be characterized not just by stating the members preferred alone but it also carries the additional information on why the members are being preferred and naturally it brings the attributes of the members and the extent to which the members possess in the perception of the choice-maker, i.e the person who makes the preference. The making of choice in preferring a person depending on the attributes has led to the development of plithogenic sociogram and on exploring will certainly yield better results.

Plithogeny is the recently evolved philosophy that deals with the evolution of entities and their attributes. Smarandache [8] introduced plithogenic sets that are widely applied in decision making on sustainability [9], medical decision system model [10] and supply chain management [11]. Plithogenic sets are used in decision making as it is highly embedded with wide-ranging generalization approaches in accommodating crisp, fuzzy, intuitionistic, neutrosophic sets and the other kinds of extended sets. The preferential ordering assumes either crisp, fuzzy or neutrosophic values, but if the preferential ordering presumes linguistic representation then the linguistic variable requires to be quantified using either fuzzy, intuitionistic or neutrosophic numbers. To make such kind of representations more comprehensive, the notion of plithogenic number shall be used. This research work intends to investigate and unveil the plithogenic sociogram with plithogenic number representing the preferential ordering.

The paper is structured into the following sections, Section 2 introduces plithogenic number and discusses their nature; Section 3 describes plithogenic sociogram and its utility in decision making and the last section concludes the work.

2 | Plithogenic Number

Zadeh [12] introduced Fuzzy numbers and their arithmetic operations to characterize uncertainty. A fuzzy number is a fuzzy set if it is a normal fuzzy set with bounded support and alpha cut being a closed interval for every alpha belonging to [0,1]. The fuzzy numbers are the special kind of fuzzy sets used to quantify linguistic variables and it is applied to represent quantities that are uncertain in nature, for instance, the costs parameters, demand are represented as fuzzy numbers. Stefanini et al. [13] and [14] discussed fuzzy numbers, fuzzy arithmetic. Dison Ebinesar [15] presented the different kinds of fuzzy numbers and their properties. Mallak and Bedo [16] described special kinds of fuzzy numbers. Grzegorzewski and Stefanini [17] illustrated the applications of fuzzy numbers. Thus, fuzzy numbers are the simple form of representing uncertainty and are extended to intuitionistic fuzzy numbers which are the next higher or extended form that are extensively applied in decision-making models. Atanassov [18] introduced the concept of intuitionistic sets. Intuitionistic fuzzy numbers are characterized by membership and non-membership values. Mahapatra and Roy [19] briefed the applications of an intuitionistic fuzzy number. Seikh et al. [20] presented the various kinds of intuitionistic fuzzy numbers. Researchers have discussed the different ordering techniques of IFN [21]-[23]. Smarandache [8] extended Intuitionistic sets to neutrosophic sets and discussed the arithmetic operations of neutrosophic numbers. Neutrosophic numbers are the extended or the higher forms of representing uncertainty. Gahlot and Saraswat [24] described single-valued neutrosophic number, Sun et al. [25] elaborated interval-valued neutrosophic number, Karaaslan [26] explored Gaussian neutrosophic number, Chakraborty et al. [27] discussed the applications of Cylindrical neutrosophic single-valued number in networking, decision making. Researchers like Saini et al. [28], El-Hefenawy et al. [29] stated the applications of neutrosophic number in various fields of decision making [30]. Neutrosophic numbers are the extended forms of intuitionistic and fuzzy numbers and neutrosophic numbers can be stated as higher forms or super forms of fuzzy numbers. The defuzzification techniques of the extended higher/super forms of fuzzy numbers to its next sub forms of fuzzy numbers are also discussed by Radhika et al. [31], Mert [32], İrfan and Öztürk [33], and many others. The above discussed forms of fuzzy numbers ranging from simple to higher versions shall be generalized into plithogenic number.

Classical plithogenic set is characterized by (P, a, V, d, c), where P is a set, a is the attribute, V is the set of attribute values, d is the degree of appurtenance stating the extent of elements belonging to P satisfying the attribute values and c is the contradiction degree. In this work, the plithogenic set is newly characterized as (P, A, V A, d, c), where A is a system of attributes and V A is the set of all possible attribute values corresponding to each attribute a in A. The classical characterization is with respect to a single attribute and this newly proposed pertains to the system of attributes. To define plithogenic number, the attributes should also be considered and the plithogenic number can also be differentiated into plithogenic fuzzy number, plithogenic intuitionistic fuzzy number, plithogenic neutrosophic number based on the degree of appurtenance

Let U be a universe of discourse, and a non-empty set M included in U.

Let x be a generic element from M.

Let's consider the attributes $A_1, A_2, \ldots, A_n$, for $n \geq 1$.

The attribute A_1 has the attribute values $A_{11}, A_{12}, \ldots, A_{1m1}$, where $m_1 \geq 1$.

The attribute A_2 has the attribute values $A_{21}, A_{22}, \ldots, A_{2m2}$, where $m_2 \geq 1$.

The attribute An has the attribute values A_{n1}, A_{n2}, ...,A_{nm} where m, n≥ 1.

The plithogenic fuzzy number will be of the form

M = {x($A_{11}(t_{11})$, $A_{12}(t_{12})$, ..., $A_{1m1}(t_{1m1})$; $A_{21}(t_{21})$, $A_{22}(t_{22})$, ..., $A_{2m2}(t_{2m2})$; ... $A_{n1}(t_{n1})$, $A_{n2}(t_{n2})$, $A_{nm}(t_{nm})$; with x in U}, where t_{11} is the degree of appurtenance of element x to the set M with respect to the attribute value A_{11}; t_{12} is the degree of appurtenance of element x to the set M with respect to the attribute value A_{12} etc.

The plithogenic intuitionistic fuzzy number will be of the form

M = {x($A_{11}(t_{11}, f_{11})$, $A_{12}(t_{12}, f_{12})$, ..., $A_{1}m_{1}(t_{1}m_{1}, f_{1}m_{1})$; $A_{21}(t_{21}, f_{21})$, $A_{22}(t_{22}, f_{22})$, ..., $A_{2m2}(t_{2m2}, f_{2m2})$; ... $A_{n1}(t_{n1}, f_{n1})$, $A_{n2}(t_{n2}, f_{n2})$, $A_{nm}(t_{nm}, f_{nm})$; with x in U}, where t_{11} is the degree of appurtenance of element x to the set M with respect to the attribute value A_{11} and f_{11} is the degree of non-appurtenance of element x to the set M with respect to the attribute value A_{11}; t_{12} is the degree of appurtenance of element x to the set M with respect to the attribute value $A_{12}$and f_{12} is the degree of non-appurtenance of element x to the set M with respect to the attribute value A_{12} etc.

The neutrosophic plithogenic set:

M = {x($A_{11}(t_{11}, i_{11}, f_{11})$, $A_{12}(t_{12}, i_{12}, f_{12})$, ..., $A_{1m1}(t_{1m1}, i_{1m1}, f_{1m1})$; $A_{21}(t_{21}, i_{21}, f_{21})$, $A_{22}(t_{22}, i_{22}, f_{22})$, ..., $A_{2m2}(t_{2m2}, i_{2m2}, f_{2m2})$; ... $A_{n1}(t_{n1}, i_{n1}, f_{n1})$, $A_{n2}(t_{n2}, i_{n2}, f_{n2})$, $A_{nm}(t_{nm}, i_{nm}, f_{nm})$; with x in U}, where t_{11} is the degree of appurtenance of element x to the set M with respect to the attribute value A_{11},i_{11} is the degree of indeterminacy of element x to the set M with respect to the attribute value $A_{11}$and f_{11} is the degree of non-appurtenance of element x to the set M with respect to the attribute value A_{11}; t_{12} is the degree of appurtenance of element x to the set M with respect to the attribute value A_{12},i_{12} is the degree of indeterminacy of element x to the set M with respect to the attribute value $A_{12}$and f_{12} is the degree of non-appurtenance of element x to the set M with respect to the attribute value A_{12}; etc.

Example. Let U = { a, b, c, d, e, f}, M = { b, c,e}, A = { a_1,a_2,a_3}, V_{a1} = { A_{11},A_{12},A_{13}}, Va_2 = { A_{21},A_{22}} Va_3 = {A_{31},A_{32},A_{33},A_{34}}.

The plithogenic number with fuzzy degree of appurtenance to all the attribute values will be of the form P={b(A11(0.2), A_{12}(0.5), A_{13}(0.6), A_{21}(0.7), A_{22}(0.6), A_{31}(0.5), A_{32}(0.4), A_{33}(0.8), A_{34}(0.9)), c(A11(0.3), A_{12}(0.5), A_{13}(0.6), A_{21}(0.5), A_{22}(0.8), A_{31}(0.9), A_{32}(0.7), A_{33}(0.5), A_{34}(0.6))}This plithogenic number may be termed as generalized plithogenic fuzzy number as it encompasses all the attribute values. From the values of intuitionistic and neutrosophic degrees of appurtenance to all the attribute values the generalized plithogenic intuitionistic and generalized plithogenic neutrosophic numbers can be defined.

2.1 | Dominant Attribute Constrained Plithogenic Number

This section also proposes the concept of dominant attribute constrained plithogenic number and it shall be defined by considering only the dominant attribute values.

Let U = { a, b, c, d, e, f}, M = { b, c,e}, A = { a_1,a_2,a_3}, V_{a1} = { A_{11},A_{12},A_{13}}, Va_2 = {A_{21},A_{22}}, Va_3 = {A_{31},A_{32},A_{33},A_{34}}.

Contradiction Degree		0	1/3	2/3	0	½	0	¼	2/4	3/4
Attribute Values		A_{11}	A_{12}	A_{13}	A_{21}	A_{22}	A_{31}	A_{32}	A_{33}	A_{34}
Fuzzy Degree of Appurtenance	b	0.2	0.5	0.6	0.7	0.6	0.5	0.4	0.8	0.9
	c	0.3	0.5	0.6	0.5	0.8	0.9	0.7	0.5	0.6
	e	0.5	0.6	0.4	0.7	0.5	0.6	0.8	0.4	0.5
Intuitionistic Degree of Appurtenance	b	(0.7,0.2)	(0.8,0.1)	(0.4,0.6)	(0.7,0.3)	(0.6,0.2)	(0.7,0.1)	(0.8,0.2)	(0.6,0.3)	(0.5,0.3)
	c	(0.8,0.1)	(0.7,0.3)	(0.5,03)	(0.6,0.2)	(0.7,0.3)	(0.6,0.3)	(0.7,0.2)	(0.8,0.1)	(0.6,0.2)
	e	(0.5,0.3)	(0.8,01)	(0.7,0.2)	(0.8,0.2)	(0.8,0.1)	(0.6,0.2)	(0.7,0.2)	(0.5,0.3)	(0.6,0.3)
Neutrosophic Degree of Appurtenance	b	(0.7,0.2,0.3)	(0.8,0.1,0.3)	(0.6,0.4,0.5)	(0.7,0.2,0.3)	(0.6,0.2,0.3)	(0.5,0.2,0.4)	(0.5,0.1,0.3)	(0.7,0.2,0.3)	(0.7,0.2,0.2)
	c	(0.5,0.1,0.3)	(0.7,0.2,0.3	(0.5,0.1,0.3)	(0.5,0.2,0.4)	(0.7,0.2,0.3)	(0.6,0.4,0.2)	(0.4,0.1,0.3)	(0.7,0.2,0.2)	(0.8,0.1,0.3)
	e	0.8,0.1,0.3)	(0.5,0.1,0.3)	(0.6,0.4,0.5)	(0.7,0.2,0.3)	(0.5,0.1,0.3)	(0.7,0.2,0.3)	(0.6,0.4,0.5)	(0.5,0.1,0.3)	(0.7,0.2,0.2)

In this example, the attribute values A_{11}, A_{21}, A_{31} are considered to be dominant and the plithogenic number considering the values of degree of appurtenance corresponding only to the dominant attribute values are called as Dominant Attribute Constrained Plithogenic Number**.**

Let P1 = {b (A_{11}(0.5), A_{21}(0.7), A_{31}(0.8)) , c (A_{11}(0.4), A_{21}(0.5), A_{31}(0.6)), b (A_{11}(0.4), A_{21}(0.6), A_{31}(0.7))} and P2 = {b (A_{11}(0.6), A_{21}(0.5), A_{31}(0.3)), c (A_{11}(0.5), A_{21}(0.2), A_{31}(0.5)), b (A_{11}(0.5), A_{21}(0.6), A_{31}(0.8))}where P1 and P2 are the Dominant Attribute Constrained plithogenic fuzzy numbers with fuzzy degree of appurtenance with respect to the dominant attribute values.

The union of two Dominant Attribute Constrained plithogenic fuzzy numbers is P1U_FP2 is defined as max {a1($A_\alpha(t_\alpha)$, $A_\beta(t_\beta)$, ..., $A_\lambda(t_\lambda)$), a2 ($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)),……am($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ))},

Where A_α , A_β, …. A_λ are the dominant attribute values and t_α , t_β, …. t_λ are the respective fuzzy degree of appurtenance with respective to each elements of M.

P1U_FP2={b (A_{11}(0.6), A_{21}(0.7),A_{31}(0.8)) , c (A_{11}(0.5), A_{21}(0.5),A_{31}(0.6)), b (A_{11}(0.5), A_{21}(0.6),A_{31}(0.8))}.

The intersection of two Dominant Attribute Constrained plithogenic fuzzy numbers is P1 $\cap_F$P2 is defined as min {a1($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)), a2 ($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)),……am($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ))}.

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P1 $\cap_F$P2= {b (A_{11}(0.5), A_{21}(0.5),A_{31}(0.3)) , c (A_{11}(0.4), A_{21}(0.2),A_{31}(0.5)), b (A_{11}(0.4), A_{21}(0.6),A_{31}(0.7))}.

Let P1 = {b (A_{11}(0.7,0.2), A_{21}(0.8,0.1), A_{31}(0.7,0.1)) , c (A_{11}(0.7,0.3), A_{21}(0.4,0.6), A_{31}(0.5,03)), e (A_{11}(0.6,0.2), A_{21}(0.5,0.3), A_{31}(0.7,0.2)} and P2 = { b (A_{11}(0.6,0.3), A_{21} (0.5,0.3), A_{31}(0.6,0.3)) , c (A_{11}(0.7,0.1), A_{21}(0.5,03), A_{31}(0.7,0.3)), e (A_{11}(0.5,0.3), A_{21}(0.6,0.3), A_{31} (0.8,0.2))} where P1 and P2 are the Dominant Attribute Constrained plithogenic intuitionistic fuzzy numbers with intuitionistic fuzzy degree of appurtenance with respect to the dominant attribute values.

The union of two Dominant Attribute Constrained plithogenic intuitionistic fuzzy numbers is P1U_{IF}P2 is defined as (max {a1($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)), a2 ($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)),……am($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)), min {a1($A_\alpha(f_\alpha)$, A_β (f_β), ..., A_λ (f_λ)), a2 ($A_\alpha(f_\alpha)$, A_β (f_β), ..., A_λ (f_λ)),……am($A_\alpha(f_\alpha)$, A_β (f_β), ..., A_λ (f_λ))}.

P1U_{IF}P2 ={b (A_{11}(0.7,0.2), A_{21}(0.8,0.1), A_{31}(0.7,0.1)) , c (A_{11}(0.7,0.1), A_{21}(0.5,0.3), A_{31}(0.7,03)), e (A_{11}(0.6,0.2), A_{21}(0.6,0.3), A_{31}(0.8,0.2)}.

The intersection of two Dominant Attribute Constrained plithogenic intuitionistic fuzzy numbers is P1$\cap_{IF}$P2 is defined as (min{a1($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)), a2 ($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)),……am($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)), max {a1($A_\alpha(f_\alpha)$, A_β (f_β), ..., A_λ (f_λ)), a2 ($A_\alpha(f_\alpha)$, $A_\beta(f_\beta)$, ..., A_λ (f_λ)),……am($A_\alpha(f_\alpha)$, A_β (f_β), ..., A_λ (f_λ))}.

P1$\cap_{IF}$P2 = {b (A_{11}(0.6,0.3), A_{21}(0.5,0.3), A_{31}(0.6,0.3)) , c (A_{11}(0.7,0.3), A_{21} (0.4,0.6), A_{31}(0.5,03)), e (A_{11}(0.5,0.3), A_{21}(0.5,0.3), A_{31}(0.7,0.2)}.

Let P1 = {b (A_{11}(0.7,0.2,0.3), A_{21}(0.8,0.1,0.3), A_{31}(0.6,0.4,0.5)), c (A_{11}(0.6,0.4,0.2), A_{21}(0.5,0.1,0.3), A_{31} (0.7,0.2,0.2)), e (A_{11}(0.6,0.2,0.1), A_{21}(0.5,0.1,0.3), A_{31}(0.7,0.2,0.3)} and P2 = { b (A_{11}(0.6,0.2,0.3) , A_{21} (0.5,0.2,0.4), A_{31}(0.6,0.4,0.2)),c (A_{11}(0.7,0.2,0.3), A_{21}(0.5,0.2,0.4), A_{31}(0.7,0.2,0.3)), e (A_{11}(0.6,0.4,0.2), A_{21}(0.7,0.2,0.3), A_{31}(0.8,0.1,0.3))} where P1 and P2 are the Dominant Attribute Constrained plithogenic neutrosophic numbers with neutrosophic degree of appurtenance with respect to the dominant attribute values.

The union of two Dominant Attribute Constrained plithogenic neutrosophic numbers is P1U_NP2 is defined as (max {a1($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)), a2 ($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)),……am($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)), max {a1($A_\alpha(I_\alpha)$, A_β (I_β), ..., A_λ (I_λ)), a2 ($A_\alpha(I_\alpha)$, A_β (I_β), ..., A_λ (I_λ)),……am($A_\alpha(I_\alpha)$, A_β (I_β), ..., A_λ (I_λ)),min {a1($A_\alpha(f_\alpha)$, A_β (f_β), ..., A_λ (f_λ)), a2 ($A_\alpha(f_\alpha)$, A_β (f_β), ..., A_λ (f_λ)),……am($A_\alpha(f_\alpha)$, A_β (f_β), ..., A_λ (f_λ))}).

P1U_NP2 = = { b (A_{11}(0.7,0.2,0.3) , A_{21} (0.8,0.2,0.3), A_{31}(0.6,0.4,0.2)),c (A_{11}(0.7,0.4,0.2), A_{21}(0.5,0.2,0.3), A_{31}(0.7,0.2,0.2)), e (A_{11}(0.6,0.4,0.1), A_{21}(0.7,0.2,0.3), A_{31}(0.8,0.2,0.3))}.

The intersection of two Dominant Attribute Constrained plithogenic neutrosophic numbers is P1$\cap_N$P2 is defined as (min {a1($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)), a2 ($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)),……am($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)), max {a1($A_\alpha(I_\alpha)$, A_β (I_β), ..., A_λ (I_λ)), a2 ($A_\alpha(I_\alpha)$, A_β (I_β), ..., A_λ (I_λ)),……am($A_\alpha(I_\alpha)$, A_β (I_β), ..., A_λ (I_λ)),max {a1($A_\alpha(f_\alpha)$, A_β (f_β), ..., A_λ (f_λ)), a2 ($A_\alpha(f_\alpha)$, A_β (f_β), ..., A_λ (f_λ)),……am($A_\alpha(f_\alpha)$, A_β (f_β), ..., A_λ (f_λ))}).

P1$\cap_N$P2 = {b (A_{11}(0.6,0.2,0.3), A_{21}(0.5,0.1,0.4) , A_{31}(0.6,0.4,0.5)), c (A_{11}(0.6,0.4,0.3), A_{21}(0.5,0.2,0.4), A_{31}(0.7,0.2,0.3)), e (A_{11}(0.6,0.4,0.2), A_{21}(0.5,0.2,0.3), A_{31}(0.7,0.2,0.3)}.

2.2 | Combined Dominant Attribute Constrained Plithogenic Number

In Combined Dominant Attribute Constrained Plithogenic Number, the attribute values possess combined degree of appurtenance of the attribute values. For instance

P1 = {b (A_{11}(0.7,0.2), A_{21}(0.8,0.1),A_{31} (0.7,0.1)), c (A_{11}(0.5), A_{21} (0.5),A_{31} (0.3),e (A_{11}(0.6,0.4,0.2), A_{21}(0.5,0.2,0.3), A_{31}(0.7,0.2,0.3)}. In this plithogenic representation, the element b has intuitionistic degree of appurtenance with respect to the attribute values, the element c has fuzzy degree of appurtenance with respect to the attribute values and the element e has neutrosophic degree of appurtenance with respect to the attribute values.

On other hand the combined plithogenic number can also be represented as P1 = {b (A_{11}(0.7,0.2), A_{21} (0.8), A_{31}(0.7,0.1,0.1)) , c (A_{11} (0.5), A_{21}(0.7,0.2) , A_{31} (0.3)),e (A_{11}(0.6,0.4,0.2), A_{21}(0.5,0.2), A_{31} (0.7)} in which the element b has the combination of intuitionistic, fuzzy and neutrosophic degree of appurtenance with respect to the dominant attribute values and the other elements c and e also have a combination of degree of appurtenance.

The union and intersection of combined plithogenic numbers shall be computed after converting the combined degrees of appurtenance into a same degree of appurtenance using 2.1, 2.2 or 2.3

Method I. (Imprecision Membership): Any neutrosophic fuzzy set N_A = (T_A, I_A, F_A) including neutrosophic fuzzy values are transformed into intuitionistic fuzzy values or vague values as η(A) = (T_A, f_A) where f_A is estimated the formula stated below which is called as Impression membership method [34].

$$f_A = \begin{cases} F_A + \frac{[1-F_A-I_A][1-F_A]}{[F_A+I_A]} & \text{if } F_A = 0 \\ F_A + \frac{[1-F_A-I_A][F_A]}{[F_A+I_A]} & \text{if } 0 < F_A \le 0.5. \\ F_A + [1-F_A-I_A]\left[0.5 + \frac{F_A-0.5}{F_A+I_A}\right] & \text{if } 0.5 < F_A \le 1 \end{cases}$$

Method II. (Defuzzification): After Method I (median membership), intuitionistic (vague), fuzzy values of the form η(A)= (T_A, f_A) are transformed into fuzzy set including fuzzy values

as<Δ(A)> = <$\frac{T_A}{[T_A+f_A]}$> [34].

The score function of the intuitionistic set of the form (μ_A, ϑ_A) is μ_A-ϑ_A [34].

Let P1 = {b (A_{11}(0.7,0.2), A_{21}(0.8), A_{31}(0.7,0.1,0.1)) , c (A_{11} (0.5), A_{21}(0.7,0.2) , A_{31} (0.3)),e (A_{11}(0.6,0.4,0.2), A_{21}(0.5,0.2), A_{31}(0.7)} and P2 = {b (A_{11}(0.7), A_{21}(0.5,0.2), A_{31}(0.6)), c (A_{11}(0.5,0.2), A_{21}(0.8), A_{31}(0.2)), e (A_{11}(0.6,0.4), A_{21}(0.5,0.2,0.1), A_{31}(0.5)} be two combined plithogenic number with different degrees of appurtenance and it can be converted to plithogenic number with same degree of appurtenance using the above methods I and II. The modified plithogenic numbers are

P_1'= {b (A_{11}(0.5), A_{21}(0.8), A_{31}(0.58)), c (A_{11}(0.5), A_{21}(0.5), A_{31}(0.3)),e (A_{11}(0.64), A_{21}(0.3), A_{31}(0.7))} and P_2' = {b (A_{11}(0.7), A_{21}(0.3), A_{31}(0.6)), c (A_{11}(0.3), A_{21}(0.8) , A_{31}(0.2)),e (A_{11}(0.2), A_{21}(0.6), A_{31}(0.5)}.

$P_1' \cup P_2'$ = {b (A_{11} (0.7), A_{21} (0.8), A_{31} (0.6)), c (A_{11} (0.5), A_{21} (0.8) , A_{31} (0.3)),e (A_{11} (0.64), A_{21} (0.6), A_{31} (0.7)}.

$P_1' \cap P_2'$= {b (A_{11}(0.5), A_{21}(0.3), A_{31}(0.58)), c (A_{11}(0.3), A_{21}(0.5) , A_{31}(0.2)),e (A_{11}(0.2), A_{21}(0.6), A_{31}(0.5)}.

3 | Plithogenic Sociogram

In this section, the concept of plithogenic sociogram is discussed with a simple illustration based on the conceptualization of Neutrosophic sociogram developed by Smarandache. A group of members are given a questionnaire to give their choices of preference in partaking as a team with other members based on certain attributives.

Let $S = \{s_1,s_2,s_3,s_4,s_5\}$ be the members interviewed with the following questions. The members are asked to give their preferential choices of teaming with respect to the attributes.

Write your friends with whom you want to work as a team with respect to their

Q_1: Degree of compatibility,

Q_2: Optimistic approaches,

Q_3: Disciplinary Knowledge.

These questions are focusing on the attributive preferential choice making.

The attributes are the degree of compatibility, optimistic approach and disciplinary knowledge. The attribute values of the attributes are as follows

Degree of compatibility = {low (Q_{11}), moderate (Q_{12}), high (Q_{13})}.

Optimistic Approach = {Dispositional (Q_{21}), Unrealistic (Q_{22}), comparative (Q_{23})}.

Disciplinary Knowledge = {Excellent (Q_{31}), good (Q_{32}), average (Q_{33})}.

The preferential choice making of the members with respect to the dominant attributive values say high (Q_{13}), Dispositional (Q_{21}), Excellent (Q_{31}) are presented in the form of Dominant attribute constrained plithogenic number in *Table 1*.

Table 1. Attributive preferential choice-making of the members.

Members	Attributive Preferential Choice-Making
s_1	$\{s_2(Q_{13}(0.5),Q_{21}(0.6),Q_{31}(0.8)),s_4(Q_{13}(0.6),Q_{21}(0.7),Q_{31}(0.8))\}$
s_2	$\{s_1(Q_{13}(0.4),Q_{21}(0.7),Q_{31}(0.6)),s_3(Q_{13}(0.5),Q_{21}(0.6),Q_{31}(0.9)), S_5(Q_{13}(0.3),Q_{21}(0.4),Q_{31}(0.6))\}$
s_3	$\{s_2(Q_{13}(0.5),Q_{21}(0.6),Q_{31}(0.7)),s_4(Q_{13}(0.4),Q_{21}(0.2),Q_{31}(0.5))\}$
s_4	$\{s_1(Q_{13}(0.7),Q_{21}(0.8),Q_{31}(0.6)),s_3(Q_{13}(0.7),Q_{21}(0.5),Q_{31}(0.3))\}$
s_5	$\{s_2(Q_{13}(0.5),Q_{21}(0.6),Q_{31}(0.7)),s_4(Q_{13}(0.5),Q_{21}(0.6),Q_{31}(0.6))\}$

S_1 prefers S_2 with the plithogenic fuzzy degree of appurtenance of 0.5 to high degree of compatibility, 0.6 to dispositional optimistic approach and 0.8 to excellent disciplinary knowledge and similarly the preference to S_4 can also be comprehended with the help of fuzzy degree of appurtenance. The approach of plithogenic sociogram is based on the methodology of neutrosophic sociogram.

The evaluation matrix Mk = (mgh), where mgh assumes the degree of appurtenance (in this case, it is fuzzy) of the member sg selecting sh with respect to the dominant attribute values and when g=h mgh = 0. In neutrosophic sociogram the elements of the evaluation matrix assumes either 0 or 1 based on the number of times a member selects another.

The evaluation matrix M_1 for the dominant attribute value Q_{13} is

	s_1	s_2	s_3	s_4	s_5
s_1	0	0.5	0	0.6	0
s_2	0.4	0	0.5	0	0.3
s_3	0	0.5	0	0.4	0
s_4	0.7	0	0.7	0	0
s_5	0	0.5	0	0.5	0

The evaluation matrix M_2 for the dominant attribute value Q_{21} is

	s_1	s_2	s_3	s_4	s_5
s_1	0	0.6	0	0.7	0
s_2	0.7	0	0.6	0	0.4
s_3	0	0.6	0	0.2	0
s_4	0.8	0	0.5	0	0
s_5	0	0.6	0	0.6	0

The evaluation matrix M_3 for the dominant attribute value Q_{31} is

	s_1	s_2	s_3	s_4	s_5
s_1	0	0.8	0	0.8	0
s_2	0.6	0	0.9	0	0.6
s_3	0	0.7	0	0.5	0
s_4	0.6	0	0.3	0	0
s_5	0	0.7	0	0.6	0

In neutrosophic sociogram each question was given weightage but here in plithogenic sociogram the dominant attributes are given weightage. By considering the weights of the dominant attributes values, the final weighted evaluation matrix is determined by assigning the weights as 0.5, 0.25 and 0.25 to the dominant attribute values high (Q_{13}), Dispositional (Q_{21}) and Excellent (Q_{31}) respectively.

	s_1	s_2	s_3	s_4	s_5
s_1	0	0.56	0	0.69	0
s_2	0.56	0	0.6	0	0.47
s_3	0	0.6	0	0.45	0
s_4	0.69	0	0.45	0	0
s_5	0	0.47	0	0	0

	s_1	s_2	s_3	s_4	s_5
s_1	0	0.6	0	0.675	0
s_2	0.525	0	0.625	0	0.4
s_3	0	0.575	0	0.375	0
s_4	0.7	0	0.55	0	0
s_5	0	0.575	0	0.55	0

The fuzzy amicable degree t_{gh} is calculated by using the formula $\frac{2}{t_{gh}} = \frac{1}{f_{gh}} + \frac{1}{f_{hg}}$, where f_{gh} represents the compatibility existing between the members g and h which means the member g prefers h and it is vice-versa for f_{hg}.

The final scores of the members $s_g(i = 1,2,..5)$ of the group, F (s_g) is determined by $\frac{\Sigma_h t_{gh}}{\Sigma_g \Sigma_h t_{gh}}$.

Table 2. Preferential scores of the members.

s_1	0.225632
s_2	0.294224
s_3	0.189531
s_4	0.205776
s_5	0.084838

Based on the scores as in *Table 2*, it is very vivid that the member s_2 has the maximum score and it represents the significance of the member s_2 in the group and his influencing attributes have made s_2 more preferable, on other hand, the member s_5 has the least score and it shows that the member is not much preferred as the attributes of s_5 may not seems to be influential. This preferential ranking is based on considering plithogenic fuzzy degree of appurtenance. Plithogenic intuitionistic fuzzy, plithogenic

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neutrosophic degrees of appurtenance and the concept of combined plithogenic shall also be used to represent the attributive preferential choice making.

3.1 | Plithogenic Sociogram in Decision-Making

The approach of plithogenic sociogram shall also be used in decision-making on the alternatives that satisfy the criteria. Let A be the set of alternative methods of food processing say

A= $\{A_1,A_2,A_3,A_4,A_5\}$ and C be the set of criteria or the attributives with attributive values.

C = $\{C_1,C_2,C_3\}$,

C = { cost efficiency, energy efficiency, quality conservation}.

The attribute values are

Cost efficiency ={highly economic (C_{11}), moderately economic (C_{12}), lowly economic (C_{13})},

Energy efficiency = { above 90% (C_{21}), above 70% (C_{22}), above 50% (C_{23})},

Quality conservation = {very good (C_{31}), good (C_{32}), average (C_{33})}.

The comparative attributive preferential choice making over compatibility of the alternatives from expert's point of view with respect to the dominant attribute values highly economic (C_{11}), above 90% (C_{21}) and very good (C_{31}) is presented in the *Table 3*.

Table 3. Alternatives and its compatibility comparison.

Alternatives	Comparative Attributive Preferential Choice-Making over Compatibility Expert-I	Expert-II
A_1	$\{A_3(C_{11}(0.4),C_{21}(0.6),C_{31}(0.8), A_4(C_{11}(0.6),C_{21}(0.6),C_{31}(0.7))\}$	$\{A_2(C_{11}(0.6),C_{21}(0.6),C_{31}(0.8)),A_4(C_{11}(0.7),C_{21}(0.8),C_{31}(0.7))\}$
A_2	$\{A_1(C_{11}(0.5),C_{21}(0.8),C_{31}(0.7)), A_3(C_{11}(0.7),C_{21}(0.5),C_{31}(0.8)),A_4(C_{11}(0.8), C_{21}(0.6),C_{31}(0.7))\}$	$\{A_1(C_{11}(0.6),C_{21}(0.6),C_{31}(0.7)), A_3(C_{11}(0.8),C_{21}(0.6),C_{31}(0.8)),A_5(C_{11}(0.9), C_{21}(0.6),C_{31}(0.7))\}$
A_3	$\{A_4(C_{11}(0.5),C_{21}(0.7),C_{31}(0.9)), A_5(C_{11}(0.6),C_{21}(0.7),C_{31}(0.8))\}$	$\{A_1(C_{11}(0.6),C_{21}(0.7),C_{31}(0.8)), A_2(C_{11}(0.7),C_{21}(0.5),C_{31}(0.8)\}$
A_4	$\{A_2(C_{11}(0.6),C_{21}(0.8),C_{31}(0.8)), A_3(C_{11}(0.6),C_{21}(0.5),C_{31}(0.7))\}$	$\{A_1(C_{11}(0.6),C_{21}(0.7),C_{31}(0.7)), A_3(C_{11}(0.5),C_{21}(0.6),C_{31}(0.8))\}$
A_5	$\{A_3(C_{11}(0.7),C_{21}(0.6),C_{31}(0.7)), A_4(C_{11}(0.5),C_{21}(0.6),C_{31}(0.6))\}$	$\{A_2(C_{11}(0.8),C_{21}(0.6),C_{31}(0.5)), A_4(C_{11}(0.5),C_{21}(0.7),C_{31}(0.8))\}$

With respect to the dominant attribute values, the alternative A_1 is compatible in comparison with the alternatives A_3 and A_4, according to the viewpoint of Expert I and compatible in comparison with the alternatives A_2 and A_4 according to the viewpoint of Expert II.

The weights of the dominant attributes values are considered and the final weighted evaluation matrix is determined by assigning the weights as 0.5, 0.25 and 0.25 to the dominant attribute values highly economic (C_{11}), above 90% (C_{21}) and very good (C_{31}), respectively.

	A_1	A_2	A_3	A_4	A_5
A_1	0	0.325	0.275	0.675	0
A_2	0.625	0	0.7125	0.3625	0.3875
A_3	0.3375	0.3375	0	0.325	0.3
A_4	0.325	0.35	0.6	0	0
A_5	0	0.3375	0.3375	0.5875	0

The amicable degree is presented as in the below

	A_1	A_2	A_3	A_4	A_5
A_1	0	0.428	0.3031	0.439	0
A_2	0.428	0	0.458	0.356	0.361
A_3	0.3031	0.458	0	0.422	0.3176
A_4	0.439	0.356	0.422	0	0
A_5	0	0.361	0.3176	0	0

The score values of the alternatives are presented in *Table 4*.

Table 4. Score values of alternatives.

A_1	0.189662
A_2	0.259831
A_3	0.243249
A_4	0.197264
A_5	0.109994

The alternative A_2 is the most preferred method of food processing based on the satisfaction of the dominant attribute values and in comparison with other alternatives. This plithogenic sociogram is used to determine the most influential member in the group based on the attributives and the most preferred alternative in decision-making.

4 | Conclusion

This paper introduces the concept of generalized plithogenic number, dominant attribute constrained plithogenic number, combined dominant attribute constrained plithogenic number and its utility in plithogenic sociogram. On comparing the proposed plithogenic sociogram with neutrosophic sociogram the former approach is more comprehensive in nature. In neutrosophic sociogram, the questions were deterministic and indeterminate in nature, in the sense, the members are asked to make the selection of their choice with whom they are very sure to take part in a quiz or study and also they are not sure of teaming up for the group activities. The calculation was done separately by considering members of deterministic teaming and later together with the deterministic and indeterminate teaming. Finally, based on the neutrosophic amicable degree, the opportunity of enhancing the relationship between the members, leadership index and potential leadership index was discussed. But in the neutrosophic sociogram, the reasons for preferring and hesitance were not much explored which are very significant to enhance the relationship in future. The calculation of the numerical ranges representing the extent of the relationship shall become more meaningful if the attributes are considered. This is the origin of the plithogenic sociogram in which the choice of the members are based on the attributes and the degree of appurtenance states the nature of their preference. The qualitative nature of the members plays a vital role in decision making on the choice of the members preferred. The score values of the members indicate their preference and significance in the group. The members with the least score can be subjected to counselling and made exposed to other kinds of training programs to enhance their attributes of group dynamics. Thus in the plithogenic sociogram with dominant attribute constrained plithogenic number representing the degree of appurtenance, the attributive preferential choice-making appears to be more realistic and pragmatic in nature. This works on the principle of identifying the attribute deficiency of the members and finds the possibilities of enhancing it to improve the efficiency of teamwork. On enriching the attributes of the members then all the members of the group shall team up with each other without any constraints. The proposed concept shall be extended and employed in decision-making and the illustrations of plithogenic sociogram and plithogenic sociogram in decision making shall be discussed under intuitionistic or neutrosophic degrees of appurtenance.

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