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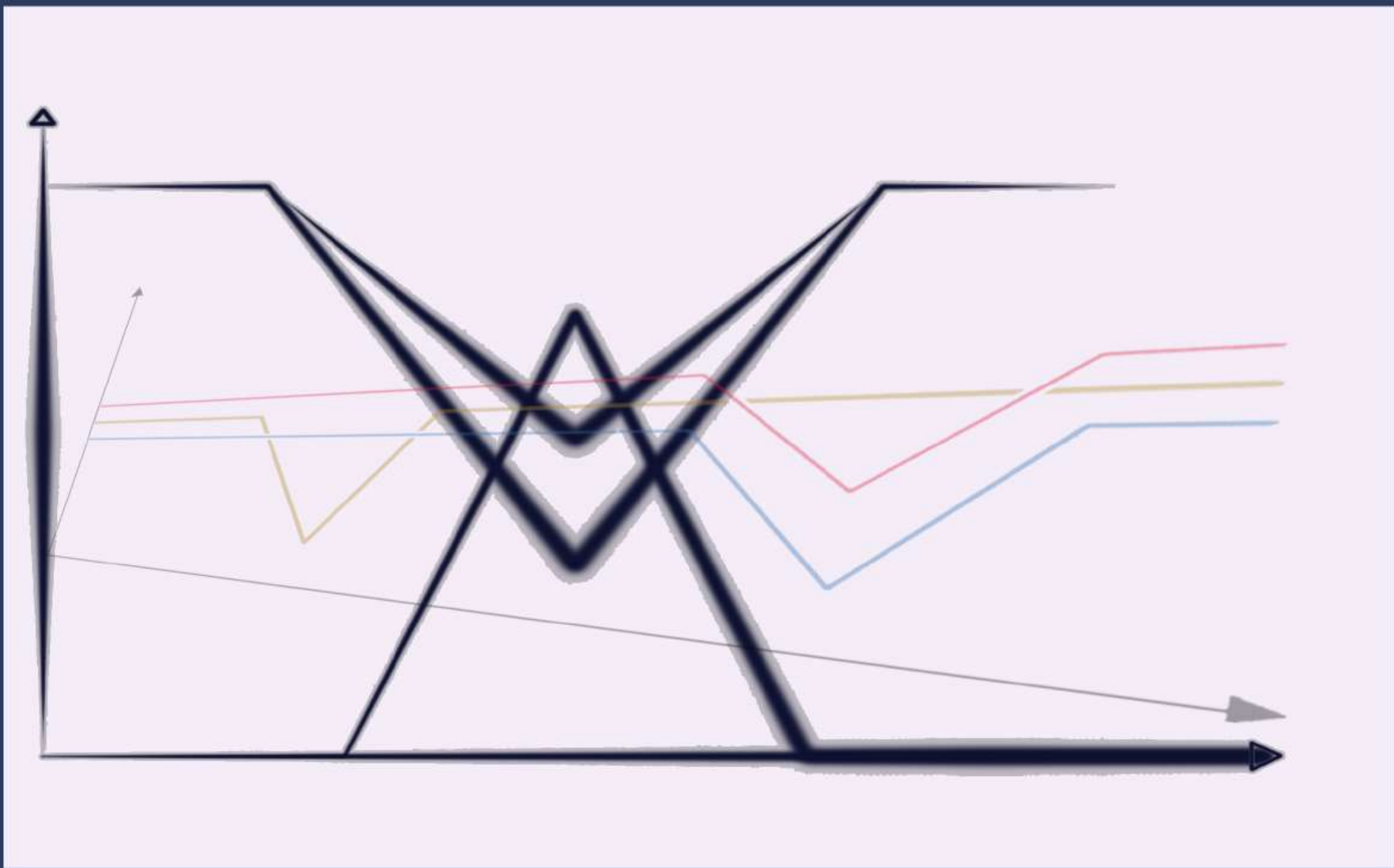

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Prediction of Human Behavior with TOPSIS

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Abstract

The present paper proposes a new application of the prediction of human behavior using TOPSIS as an appropriate tool for data optimization. Our hypothesis was that the analysis of the candidates with this method was influenced by the change of their behavior. We found that the behavior change could occur in more than one time span when the behavior of two candidates changed simultaneously. One of the advantages of this study is that the pattern of the behavior change with time is predicted with this method. Another advantage is that the modifications in the TOPSIS algorithm have made the predictions independent from the need of changing the fuzzy membership degrees of the candidates. This is the first time that these modifications in this technique with a new application including the numerical analysis of cognitive date are reported. Our results can be used in cognitive science, experimental psychology, cognitive informatics and artificial intelligence.

Keywords: TOPSIS analysis, MCDMA, Behavior change, Classification, Artificial intelligence.

1 | Introduction

Making decision is an important aspect for data optimization that includes some issues such as prediction of the best alternative and ranking of candidates. Multiple Criteria Decision-Making Algorithms (MCDMA) are the decision-making methods that involve quantitative and qualitative factors. Several MCDMAs have been used during recent years in order to choose the probable optimal alternatives. In these methods, the ratings of alternatives versus criteria, and the importance weights of all criteria, are assessed in linguistic values represented by different numbers [1]-[3]. TOPSIS is a multi-criteria approach that was developed by Hwang and Yoon in order to determine solutions from a finite set of alternatives [4]-[5]. In a study, this method was used for the prediction of human behavior according to the analysis of body posture and movement [6]. The prediction of behavior resistance to organizational change is another aspect of human behavior that has been analyzed with TOPSIS [7].

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1.1 | Contents of This Paper

This paper includes the presentation of the TOPSIS algorithm, two modifications that we have made in its algorithm and our results with the unmodified and modified algorithms for the prediction of human behavior in different time spans. Our current work is presented in four sections. The Section 1 presented here concerns the introduction of the paper. The Section 2 includes the preliminaries. We have performed two modifications in the TOPSIS algorithm. The Section 3 of this work presents our results with the unmodified and modified algorithms and discussion on the obtained results. The Section 4 includes the conclusion of the current work.

1.2 | Motivations and Research Outline

Several motivations have initiated the research and analysis work in the current paper. It was interesting to determine which kinds of cognitive factors or criteria that affect the human behavior were not analyzed with TOPSIS, yet. Secondly, as no previous study on the chosen criteria with TOPSIS was carried out, it was interesting to perform the presented analysis on these alternatives and criteria in different time spans. Thirdly, we were interested to perform some modifications in this algorithm to make it autonomous for changing the fuzzy membership degrees of candidates. Finally, we were interested to get the numerical results in order to determine the rankings of alternatives according to the data, their weights and criteria types in the prediction of human behavior in different time spans.

The objectives of the current study have been to use TOPSIS for the prediction of human behavior in different time spans and determine how the modifications in the algorithm of this method can help make it independent of any need of change of data and fuzzy membership degrees of candidates so that the algorithm could make these changes itself without the need of human involvement for these changes.

Human behavior is an important aspect of personality that can be characterized by a change in behavior patterns over a short time. The modelling of temporal behavior is done over a time series, where time explains the behavior of the relevant variable [8]-[9]. The prediction of human characteristics with TOPSIS has been focused on the analysis of social aspects of human life such as energy consumption, stock price development, determination of urban surface temperatures, etc. [10]-[12]. The human characteristics can be analyzed according to their differences based on the values of their fuzzy degrees of membership [13]-[14].

Acting successfully is complex because the events on which we react are not always fully predictable and itis required to know which warning signals precede critical events. Learning helps to anticipate the onset of critical events in order to adjust behavior to react adequately. Thus, to understand how humans excel in different skills, we need to understand their behavior is adapted in time [15]. Time is an integral part of cognitive phenomena. It is built into most behavioral and cognitive processes [16]. Change over time is the basic foundation of developmental cognition. However, few theoretical models of human behavior focus almost exclusively on the passage of time [16]. Although the events in which human behavior is involved are not fully predictable, the change in the human characteristics having impact on them can be analyzed. Therefore, there is a real need in predicting the change of human behavior with time.

Some previous studies have been made on the prediction of human behavior with TOPSIS concerning other cognitive aspects. A study has been made on the ergonomic behavior of humans, which required their body movement in order to select the best work shift group in the industry using this algorithm. This study showed that a significant percentage of the worker's behavior were unergonomic [17]. Another study has been made with an enhanced fuzzy delphi method in forecasting and decision-making in order to transform the subjective data to quasi-objective ones. The authors have explained that a forecasted success requires various steps of operations including the critical path method that is more efficient for forecasting in such cases [18]. In another study, a fuzzy multi-objective assignment was proposed with the advantage of dealing situations realistically. The authors investigated the stability of their method without differentiability that corresponds to the obtained solution [19]. These methods can be considered when the data for the prediction of human behavior are investigated.

1.3 | Contributions of This Paper

The prediction of human behavior based on the change of human characteristics in different time spans is a novel application of TOPSIS, which can improve the applications of this approach in cognitive informatics and artificial intelligence. To our knowledge, this is the first time that this application is reported for the prediction of human behavior with different time spans. Our results in this paper show that we have achieved our objectives with the two modifications of TOPSIS that we present here for the prediction of human behavior.

This prediction with this method can be applied to several fields such as cognitive science, artificial intelligence, cognitive informatics, etc. The combination of our results reported in this paper with those of other researchers can help improve these fields.

2 | Preliminaries

2.1 | TOPSIS Method

A version of TOPSIS in python was developed by Chakravorty in 2016, with the code available on the GitHub website (https://github.com/Glitchfix/TOPSIS-Python/blob/master/topsis.py) that we used for our analysis.

Apart from positive characteristics that have positive influence in the output of human work, negative characteristics should also be considered for a temporal prediction of human behavior. To perform this numerical analysis with TOPSIS, the first and second groups of characteristics are considered as profit criteria and cost criteria, respectively.

As in TOPSIS, the calculations are made on the fuzzy degrees of membership of the entry data, it is necessary to present the definitions of these concepts of the fuzzy logic.

Definition 1. Fuzzy degree of membership. The fuzzy degree of membership is a parameter that determines the membership of an element to a set. This parameter is represented with μ. For example, $\mu=0$ and $\mu=1$ mean that the considered element does not belong or belongs to the set. According to the description of this parameter, the values between 0 and 1 can also be considered for the degree of membership that determine some percentage of membership to the element to the set [14].

Definition 2. Universe of speech. The universe of speech is a set of references that is used for the determination of the values of the elements of a set. In other words, all the values that the degrees of membership can have are found in the universe of speech [14].

Definition 3. Fuzzy set. A fuzzy set is a set that contains some elements according to an extensional definition, the properties of elements or the function of their membership. The elements are attributed to the fuzzy set according to their degrees of membership [14].

In TOPSIS, the members or elements of the fuzzy set are the alternatives and their properties are the criteria. This method includes the steps below [20]-[21]:

I. Create a normalized decision matrix.

The normalized R decision matrix is created according to the formula below:

$$r_{ij} = X_{ij} / \sum \sqrt{i = X_{ij}^2}.$$

II. Create a weighted normalized decision matrix.

This matrix is created according to the formula below:

$$v_{ij} = w_i . x_{ij}.$$

III. Determine the positive ideal solution A^+ and the negative ideal solution A^-.

The positive ideal solution A+ is the maximum value of the data and the negative ideal solution A- is the minimum value of the data for profit criteria, whereas the positive ideal solution A+ is the minimum value of the data and the negative ideal solution A- is the maximum value of the data for cost criteria, respectively.

IV. Calculate the separation distance from the positive ideal solution S^+ and the other distance from the negative ideal solution S^- for each alternative.

These distances are calculated according to the formulas below:

$$D^*_j = \sqrt{\Sigma_{i=1}^{n}} \left(v_{ij} - v_i^*\right)^2, \quad j = 1, \ldots, J.$$

$$D^-_j = \sqrt{\Sigma_{i=1}^{n}} \left(v_{ij} - v_i^-\right)^2, \quad j = 1, \ldots, J.$$

V. Calculate the similarity coefficients according to the proximity relative to the positive ideal solution.

The similarity coefficients for alternatives are calculated according to the formula below:

$$C^*_j = D^-_j / D^+_j + D^-_j, \quad j = 1,\ldots,J.$$

This formula is used in TOPSIS for performing the ranking of alternatives according to their preference order. The alternative has a good performance if it has large value of closeness coefficient (C_j^*). The alternative with the greatest relative closeness to the ideal solution is the best one [21].

2.2 | Modifications of TOPSIS

We performed two modifications in the TOPSIS code in Python in order to make it capable of giving the same outputs without needing to modify the average values of the fuzzy membership degrees of candidates 1 and 5.

Modification 1. We added these lines to the first step of the code:

evaluation matrix[row_size-5][column_size-1] = evaluation matrix[row_size-5][column_size-1] + 1.0

evaluation matrix[row_size-5][column_size-2] = evaluation matrix[row_size-5][column_size-2] + 1.0

This modification added the value of 1.0 to the fuzzy membership degrees of the first candidate in the two last columns of the entry matrix. Therefore, this increased the value of the cost criteria of this candidate. The same output was obtained as the one that we observed for the behavior change of candidate 1 for the time span of Y-25-Y30 according to *Table 2*.

Modification 2. We added these lines to the first step of the code in another analysis:

evaluation matrix[row_size-5][column_size-1] = evaluation matrix[row_size-5][column_size-1] +1.0

evaluation matrix[row_size-5][column_size-2] = evaluation matrix[row_size-5][column_size-2] +1.0

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evaluation matrix[row_size-1][column_size-1] = evaluation matrix[row_size-1][column_size-1] -0.3

evaluation matrix[row_size-1][column_size-2] = evaluation matrix[row_size-1][column_size-2] - 0.6

This modification added the value of 1.0 to the fuzzy membership degrees of the first candidate in the two last columns of the entry matrix. Therefore, this increased the value of the cost criteria of this candidate. Moreover, it subtracted the values of 0.3 and 0.6 from the fuzzy membership degrees of the last candidate in the last column and in the before last column, respectively. Therefore, this reduced the values of the cost criteria of this candidate.

The same output was obtained as the one that we observed for the behavior change of candidates 1 and 5 for the time span of Y-25-Y30 according to *Tables 2* and *3*.

3 | Results and Discussion

Choosing the best behavior change is required for the prediction of human behavior according to an analyzable pattern. The characteristics that are analyzed in the example in this paper include the profit and cost criteria that can change with time. Therefore, this study will discuss the method that is expected to help determine the best candidate according to his characteristic change.

3.1 | Results with Unmodified TOPSIS

TOPSIS is the method that is used in making this best selection of human candidates. This study includes these steps using TOPSIS method:

I. Determine the TOPSIS criteria and candidates in the decision matrix.

In this study, the human characteristics as the criteria for the selection of human candidates are shown in *Table 1*. The information about five candidates, C-1, C-2, C-3, C-4 and C-5, with their different characteristics during 30 years in different time spans is presented in the table. The first four characteristics, creative, attentive, perseverant and productive, have a positive effect on the output of the candidates' work as they increase the efficiency of their work. Therefore, these characteristics are considered as profit criteria.

The two other ones, lazy and unmotivated, have a negative effect on this output as they reduce the efficiency of their work. So, they are considered as cost criteria. The values of fuzzy membership degrees of the candidates' characteristics that vary between 0.0 and 1.0 are indicated in the *Table 1*.

Table 1. Fuzzy membership degrees of the characteristics of five candidates in the first time span in the decision matrix.

Candidates/Criteria	Creative	Attentive	Perseverant	Productive	Lazy	Unmotivated
C-1	1.0,1.0,1.0	1.0,1.0,1.0	1.0,1.0,1.0	1.0,1.0,1.0	0.0,0.0,0.0	0.0,0.0,0.0
C-2	1.0,1.0,1.0	1.0,1.0,1.0	1.0,1.0,1.0	0.6,0.7,0.8	0.8,0.9,1.0	0.1,0.2,0.3
C-3	1.0,1.0,1.0	0.5,0.6,0.7	0.8,0.9,1.0	0.8,0.9,1.0	0.1,0.2,0.3	0.3,0.4,0.5
C-4	1.0,1.0,1.0	0.7,0.8,0.9	1.0,1.0,1.0	0.7,0.8,0.9	0.4,0.5,0.6	0.4,0.5,0.6
C-5	0.6,0.7,0.8	0.7,0.8,0.9	0.3,0.4,0.5	0.5,0.6,0.7	0.5,0.6,0.7	0.2,0.3,0.4

In order to determine the effect of the change in the behavior of candidate 1 according to the change of his characteristics when the behaviors of the other candidates do not change and their characteristics do not reveal any change too, we considered different types of changes in the values of the degrees of membership of candidate 1 with no change in those of other candidates.

Table 2 shows the changes in the values of fuzzy membership degrees for candidate 1 during several time spans. The first, second, third, fourth, fifth and sixth time spans include 5 years, 2 years, 3 years, 5 years, 4 years and 6 years, respectively.

We considered that the behaviors of candidates 1 and 6 changes with time.

We consider that the behavior of candidate 5 changes in the last time span.

Table 3 shows the change in the values of fuzzy membership degrees for candidate 5 during the first and last time spans.

Table 2. The change in the values of fuzzy membership degrees for candidate 1 during 30 years in several time spans.

Time Spans/ Characteristics	Creative	Attentive	Perseverant	Productive	Lazy	Unmotivated
Y1-Y5	1.0,1.0,1.0	1.0,1.0,1.0	1.0,1.0,1.0	1.0,1.0,1.0	0.0,0.0,0.0	0.0,0.0,0.0
Y6-Y7	1.0,1.0,1.0	1.0,1.0,1.0	1.0,1.0,1.0	0.6,0.7,0.8	0.1,0.2,0.3	0.1,0.2,0.3
Y11-Y13	0.6,0.7,0.8	0.7,0.8,0.9	0.3,0.4,0.5	0.5,0.6,0.7	0.1,0.2,0.3	0.1,0.2,0.3
Y16-Y20	0.6,0.7,0.8	0.7,0.8,0.9	0.3,0.4,0.5	0.5,0.6,0.7	0.2,0.3,0.4	0.2,0.3,0.4
Y21-Y24	0.6,0.7,0.8	0.7,0.8,0.9	0.3,0.4,0.5	0.5,0.6,0.7	0.5,0.6,0.7	0.5,0.6,0.7
Y25-Y30	1.0,1.0,1.0	1.0,1.0,1.0	1.0,1.0,1.0	1.0,1.0,1.0	1.0,1.0,1.0	1.0,1.0,1.0

Table 3. The change in the values of fuzzy membership degrees for candidate 5 during the first and last time spans.

Time Spans/ Characteristics	Creative	Attentive	Perseverant	Productive	Lazy	Unmotivated
Y1-Y5	0.6,0.7,0.8	0.7,0.8,0.9	0.3,0.4,0.5	0.5,0.6,0.7	0.5,0.6,0.7	0.2,0.3,0.4
Y25-Y30	0.6,0.7,0.8	0.7,0.8,0.9	0.3,0.4,0.5	0.5,0.6,0.7	0.0,0.0,0.0	0.0,0.0,0.0

Tables 4, 5 and *6* represent the mean values of the fuzzy data presented in *Tables 1, 2* and *3*, respectively.

Table 4. Mean values of fuzzy membership degrees of the characteristics of five candidates in the first time span in the decision matrix.

Candidates/Criteria	Creative	Attentive	Perseverant	Productive	Lazy	Unmotivated
C-1	1.0	1.0	1.0	1.0	0.0	0.0
C-2	1.0	1.0	1.0	0.7	0.0	0.2
C-3	1.0	0.6	0.9	0.9	0.2	0.4
C-4	1.0	0.8	1.0	0.8	0.5	0.5
C-5	0.7	0.8	0.4	0.6	0.6	0.3

Table 5. The change in the mean values of fuzzy membership degrees for candidate 1 during 30 years in several time spans.

Time Spans/ Characteristics	Creative	Attentive	Perseverant	Productive	Lazy	Unmotivated
Y1-Y5	1.0	1.0	1.0	1.0	0.0	0.0
Y6-Y7	1.0	1.0	1.0	1.0	0.2	0.2
Y11-Y13	0.7	0.8	0.4	0.6	0.2	0.2
Y16-Y20	0.7	0.8	0.4	0.6	0.3	0.3
Y21-Y24	0.7	0.8	0.4	0.6	0.6	0.6
Y25-Y30	1.0	1.0	1.0	1.0	1.0	1.0

Table 6. The change in the mean values of fuzzy membership degrees for candidate 5 during the first and last time spans.

Time Spans/ Characteristics	Creative	Attentive	Perseverant	Productive	Lazy	Unmotivated
Y1-Y5	0.7	0.8	0.4	0.6	0.6	0.3
Y25-Y30	0.7	0.8	0.4	0.6	0.0	0.0

II. Determine the weights of each alternative for each criterion.

Table 7 shows the weights of alternatives for each criterion.

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Table 7. Weights of alternatives for each criterion.

Alternatives/Values	Creative	Attentive	Perseverant	Productive	Lazy	Unmotivated
C1-C5	0.5	0.5	0.5	0.5	0.5	0.5

The same weight values are chosen for different criteria in order to make them have the same influence on human behavior.

III. Determine the criteria matrix.

The next step is the determination of the criteria matrix. *Table 8* shows the criteria matrix indicating True for the profit criteria and false for the cost criteria, respectively.

Table 8. Criteria matrix.

Alternatives/Values	Creative	Attentive	Perseverant	Productive	Lazy	Unmotivated
C1-C5	True	True	True	True	False	False

The steps below correspond to the results obtained in our software with TOPSIS for the first time span, which concerns the first five years in *Table 2*.

IV. Normalization of fuzzy membership degrees and weights.

The vector normalization of the fuzzy membership degrees of the characteristics of five candidates as well as the normalization of their weights is followed by their multiplication, which results in the weighted normalization matrix. *Tables 9* and *10* show the results of the normalized decision matrix and weighted normalized decision matrix, respectively.

Table 9. Results of the normalized decision matrix.

Candidates/Criteria	Creative	Attentive	Perseverant	Productive	Lazy	Unmotivated
C-1	0.47192918	0.52414242	0.55048188	0.55048188	0.00000000	0.00000000
C-2	0.47192918	0.52414242	0.38533732	0.38533732	0.00000000	0.00000000
C-3	0.47192918	0.31448545	0.49543369	0.49543369	0.24806947	0.54433105
C-4	0.47192918	0.41931393	0.440.38551	0.44038551	0.62017367	0.68041382
C-5	0.33035042	0.41931393	0.33028913	0.33028913	0.74420841	0.40824829

Table 10. Results of the weighted normalized decision matrix.

Candidates/Criteria	Creative	Attentive	Perseverant	Productive	Lazy	Unmotivated
C-1	0.07865486	0.08735707	0.0836476	0.09174698	0.00000000	0.00000000
C-2	0.07865486	0.08735707	0.0836476	0.06422289	0.00000000	0.04536092
C-3	0.07865486	0.05241424	0.07528284	0.08257228	0.04134491	0.09072184
C-4	0.07865486	0.06988566	0.0836476	0.07339758	0.10336228	0.1134023
C-5	0.0550584	0.06988566	0.03345904	0.05504819	0.12403473	0.06804138

V. Determine the best alternative and the worst alternative.

Table 11 shows the results of the best alternative and the worst alternative.

Table 11. The results of the best alternative (A^+) and the worst alternative (A^-).

Candidates/Criteria	Creative	Attentive	Perseverant	Productive	Lazy	Unmotivated
A^+	0.07865486	0.08735707	0.0836476	0.09174698	0.00000000	0.00000000
A^-	0.0550584	0.055241424	0.03345904	0.05504819	0.12403473	0.1134023

VI. Determine the distances from the best alternative (d_i^*) and the worst alternative (d_i^-).

Table 12 shows the results of the distances from the best alternative (d_i^*) and the worst alternative (d_i^-) for the candidates.

VII. Determine the similarity coefficients of the candidates.

Table 13 shows the similarity coefficients and the rankings of the candidates according to the worst similarity for different time spans.

Table 12. The results of distances from the best alternative (di*) and the worst alternative (di-) for the candidates.

Candidates	di*	di-
C-1	0.00000000	0.18408744
C-2	0.05305835	0.15618933
C-3	0.10637199	0.10205689
C-4	0.15551782	0.06438156
C-5	0.15729584	0.04860929

As expected and shown in *Table 13*, the fuzzy membership degrees of the cost criteria were increased in the time span of Y6-Y7 in comparison with those of the first time span of Y1-Y5 for the first candidate, it lost its rank and was ranked in the second place. The reduction of the fuzzy membership degrees of the profit criteria for candidate 1 in the time span of Y11-Y13 made it lose again its place in the ranking and it was ranked in the third place. Moreover, the small increase in the fuzzy membership degrees of the cost criteria for candidate 1 in the time span of Y16-Y20 did not change its rank in comparison with the one in the time span of Y11-Y13. As expected, the increase of the fuzzy membership degrees of the cost criteria for candidate 1 made it lose its place in the ranking and it was ranked in the last place.

Then, we analyzed the maximum values of the fuzzy membership degrees for all the criteria of candidate 1. The maximum values of the cost criteria for candidate 1 made it be ranked in the last place in the time span of Y21-Y25. The behavior change of candidate 1 in these time spans in comparison with no behavior change of the other candidates confirmed the impacts of profit and cost criteria in their rankings.

We were also interested to analyze the behavior change of candidate 5 in order to observe the improvement of its place in the ranking. *Table 14* shows the similarity coefficients (CC_i) and the ranking of the candidates according to the worst similarity for the time span of Y25-Y30 with the behavior change of candidates 1 and 5.

The decrease in the fuzzy membership degrees of the cost criteria for candidate 5, made this candidate be ranked in the second place as candidate 1 kept its membership values as in the last analysis.

The change in the rankings of the candidates according to their behavior change revealed the importance of the improvement of the profit criteria and the reduction of the cost criteria. As the candidates lost their appropriate characteristics and increased their inappropriate characteristics, they lost their place in the rankings.

Table 13. The similarity coefficients (CCi) and the ranking of the candidates according to the worst similarity for different time spans.

Time Spans/ Candidates	CC_i Y1-Y5	Ranking	CC_i Y6-Y7	Ranking	CC_i Y11-Y13	Ranking
C-1	1.00000000	1	0.76138319	2	0.5624454	2
C-2	0.74643277	2	0.84690711	1	0.88363263	3
C-3	0.48964851	3	0.58800964	3	0.59477793	1
C-4	0.29277736	4	0.34374949	4	0.36702603	4
C-5	0.23607616	5	0.25136886	5	0.25100868	5
Time Spans/ Candidates	**CC_i Y16-Y20**	**Ranking**	**CC_i Y21-Y24**	**Ranking**	**CC_i Y25-Y30**	**Ranking**
C-1	0.43940569	2	0.11530851	2	0.30840767	2
C-2	0.88070026	3	0.87406436	3	0.86800327	3
C-3	0.59582695	1	0.61226684	4	0.7304002	4
C-4	0.37472539	4	0.41694505	5	0.58087955	5
C-5	0.2493377	5	0.31089593	1	0.50952421	1

Table 14. The similarity coefficients (CCi) and the ranking of the candidates according to the worst similarity for the time span of Y25-Y30 (with the behavior change of candidates 1 and 5).

Candidates	CC_i	Ranking
C-1	0.27135808	2
C-2	0.83347546	5
C-3	0.67909256	3
C-4	0.53124602	4
C-5	0.74649548	1

3.2 | Results with Modified TOPSIS

Table 15 shows the similarity coefficients (CCi) and the ranking of the candidates according to the worst similarity with the first and second modifications of TOPSIS in the two left and two right columns, respectively.

Table 15. The similarity coefficients (CCi) and the ranking of the candidates according to the worst similarity with the first and second modifications of TOPSIS in the two left and two right columns, respectively.

Candidates	CC_i	Ranking	CC_i	Ranking
C-1	0.3084767	2	0.27135808	2
C-2	0.86800327	3	0.83347546	5
C-3	0.7304002	4	0.67909256	3
C-4	0.58087955	5	0.53124602	4
C-5	0.50952421	1	0.74649548	1

The results in this table also showed that the first modification of TOPSIS using the fuzzy membership degrees of the first time span of Y1-Y5 for candidate 1 gave the same results as the ones that we obtained for this candidate in the time span of Y25-Y30 and showed in *Table 13*. Moreover, the first modification in TOPSIS was efficient in order to not need any change in the fuzzy membership degrees of candidate 1. Therefore, this modification was efficient for changing these values without any change in the data.

The second modification of TOPSIS using the fuzzy membership degrees of the time span of Y1-Y5 for both candidates 1 and 5 gave the same results as the ones presented in *Table 13*. The second modification in TOPSIS was efficient in order to not need any change in the fuzzy membership degrees of candidates 1 and 5. Therefore, this modification was also efficient for changing these values without needing to do any change in the data.

3.3 | Results with Sensitive Analysis

We performed the sensitive analysis on the data that we analyzed with the unmodified TOPSIS presented above in two steps. First, we analyzed the impact of the modification of data in the evaluation matrix on the ranking of candidates. Then, we determined how the ranking of the candidates could change with the changes of the weight values and those of the types of criteria.

Table 16 shows the new evaluation matrix. In this table, the data in the last column for the third candidate were changed from 0.4 to 1.0 in comparison with the weights presented in *Table 4*.

Table 16. The new evaluation matrix for the first sensitive analysis.

Candidates/Criteria	Creative	Attentive	Perseverant	Productive	Lazy	Unmotivated
C-1	1.0	1.0	1.0	1.0	0.0	0.0
C-2	1.0	1.0	1.0	0.7	0.0	0.2
C-3	1.0	0.6	0.9	0.9	0.2	1.0
C-4	1.0	0.8	1.0	0.8	0.5	0.5
C-5	0.7	0.8	0.4	0.6	0.6	0.3

Table 17 shows the ranking of candidates with TOPSIS after this modification.

Table 17. The ranking of candidates with TOPSIS for the first sensitive analysis.

Candidates	CC_i	Ranking
C-1	0.79628707	2
C-2	0.86554283	1
C-3	0.43913286	4
C-4	0.4595051	3
C-5	0.41647469	5

Table 18 shows the new matrix of weights. In this table, the weights of the first four columns were changed from 0.5 to 0.1 in comparison with the weights presented in *Table 7*.

Table 18. The matrix of weights for the second sensitive analysis.

Alternatives/Values	Creative	Attentive	Perseverant	Productive	Lazy	Unmotivated
C1-C5	0.1	0.1	0.1	0.1	0.5	0.5

Table 19 shows the new matrix of criteria.

Table 19. The matrix of criteria for the second sensitive analysis.

Alternatives/Values	Creative	Attentive	Perseverant	Productive	Lazy	Unmotivated
C1-C5	True	True	False	False	False	False

In this table, the false values have been attributed to two criteria, perseverant and productive, which indicates that they are considered as cost criteria as their excess can make humans lose time and reduce their output.

Table 20 shows the ranking of candidates with TOPSIS after these modifications.

The results that we obtained with the sensitive analysis showed that the change in a data presented in the evaluation matrix or those in the values of weights or types of criteria could affect the output of TOPSIS. As expected, the rankings changed with these modifications. These results confirmed the robustness of this method that we used in this study.

Candidates	CC_i	Ranking

Table 20. The ranking of candidates with TOPSIS for the second sensitive analysis.

C-1	0.93118765	1
C-2	0.75311467	2
C-3	0.46100335	3
C-4	0.12411629	5
C-5	0.24986587	4

Figs. 1 and *2* show the flowcharts of the unmodified and modified TOPSIS that we used in this study.

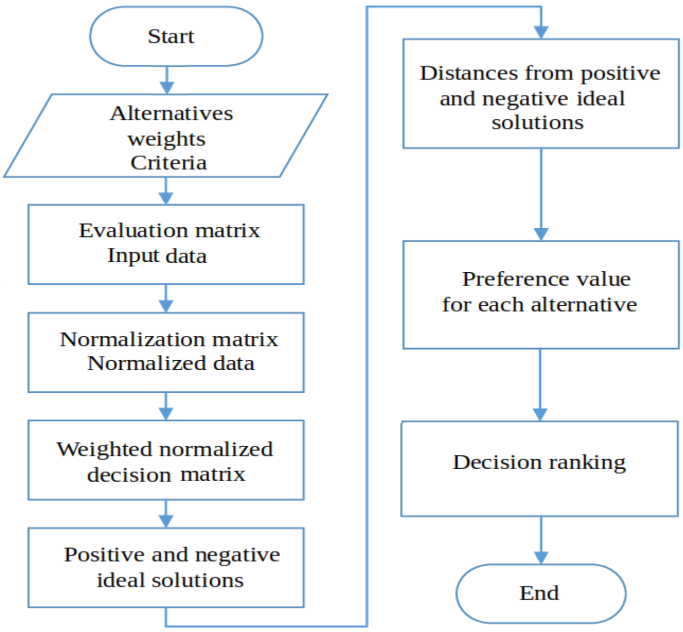

Fig. 1. Flowchart of the unmodified TOPSIS.

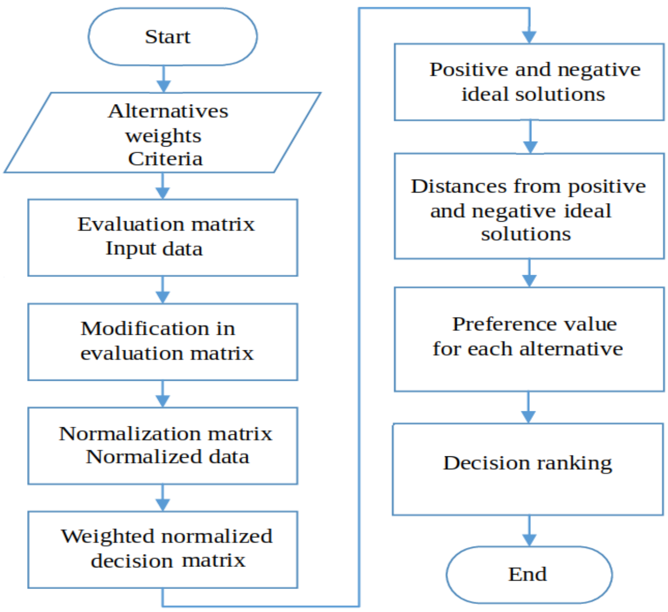

Fig. 2. Flowchart of the modified TOPSIS.

The comparison of *Figs. 1* and *2* show the difference between the unmodified and modified TOPSIS algorithms that appears in the first step of the preparation of the evaluation matrix including the entry data, which are the degrees of membership of the criteria for the alternatives, and before the second steps for their normalization by the preparation of the normalized matrix. In fact, the additional code lines that we presented in the preliminaries for the modified TOPSIS are applied in the first step of TOPSIS as shown in *Fig. 2*.

Table 21 presents the comparative information about TOPSIS and other decision-making methods.

Table 21. Comparison of TOPSIS with other decision-making methods.

Method	Characteristics		
	Calculation Level	Features	Output Results
VIKOR	Medium	Considers the importance of the optimal distance to the best and worst case in calculating the distances of the options.	The closer the coefficients to zero, the more important
TOPSIS	Medium	Considers the shortest distance from the ideal answer and the farthest distance from the counter-ideal answer for the selected option.	The closer the coefficients to one, the more important
Electre	Complex	Non-rank method parallel comparison of criterions and elimination of defeated criterions.	The more wins and losses, the more priority
SAW	Easy	Scoring method Ranking after weighting the used variables.	The closer the coefficients to one, the more important

The comparison of TOPSIS with other decision-making methods in *Table 21* shows that the computational level of this first method is medium. Moreover, it is a ranking method in which the ranking of alternatives is based on the calculation of the distances from positive and negative optimal solutions and not the weights.

The prediction of human behavior with TOPSIS that we used in this study has important merits and limitations. One of the merits of this method is that it is able to determine the best alternative according to the ranking. In fact, it determines the distances of each candidate from its best and worst ideal solutions to obtain the best alternative. This is the specificity of this method as no other decision making method determines the best alternative according to the calculations of these distances. Another advantage of TOPSIS is that it is sensitive to the changes of the entry data, their weights or criteria types. This helps determine the effect of any of these changes on the output of the algorithm. However, the limitations of this method should also be considered. One of its limitations is that without the determination of the types of criteria it is not possible to perform the numerical analysis for the prediction of human behavior with this method. Therefore, first it is necessary to determine which characteristics are profit and cost criteria and then indicate them with the terms true and false in the matrix of criteria, respectively. Another limitation is that the addition or modification of alternatives can change their ranking in the output of TOPSIS. In the current work, we first determined the types of criteria as profit and cost criteria and then we performed the numerical analysis with this algorithm in order to overcome the first limitation of this technique. As the addition of further alternatives or their modification was not required, we did not encounter the second limitation of this method. The analysis results presented in this paper can have important perspectives in cognitive science [22]-[27]. Buying behavior [28]-[34], competitiveness [35]-[38], performance [39]-[44], user activity [45]-[46], satisfaction [47]-[50] and conflicting objectives such as quality, cost, and delivery time [51]-[57], are various commercial issues that can be analyzed with TOPSIS. The human characteristics that we analyzed in this paper can have an impact on these aspects of human behavior. Our results can also be used in other fields such as nanotechnology, microelectronics, pure sciences and engineering [58]-[62]. TOPSIS can be used for the prediction of the performance of materials [63]-[68] according to their characteristics. The tendency of humans to use these materials can be predicted with this approach. Our results can also be used in robotics for the development of robots that could distinguish and analyze the cognitive disorder of humans [69]-[71].

It has been shown that emotion and perception have important impacts in human behavior [72]-[78]. The statistical analysis of data with TOPSIS has been used in several studies [79]-[85]. We will investigate these aspects of human behavior with this method in our future work.

4 | Conclusion

This paper presented the analysis results of applying TOPSIS in the prediction of human behavior for data optimization. The change in the fuzzy membership degrees of the candidates' characteristics revealed the influence of the profit and cost criteria on the rankings. In light of the results obtained in this paper, it can be concluded that the modifications on TOPSIS can make an appropriate prediction of human behavior without needing the modification of data for different time spans. The changes in the human characteristics have been taken into account in order to determine which one of them has an influence on the rankings. Ranking comparisons using unmodified and modified TOPSIS methods were made in order to validate the proposed analysis. From the results, this study has shown the efficacy of this method in the prediction of human behavior. For our future work, we aim to determine the pattern of the human behavior change according to the analyzed time spans for the characterization of human cognition. For this purpose, we will investigate the factors that influence the human cognition such as emotion, reason, imagination, etc. and the prediction of their effects with TOPSIS. This future work will help us obtain supplementary results to those that we presented in this paper. Moreover, we will investigate the stability of the human behavior according to these cognitive factors. This second work will lead to a better understanding of the predictions of their stability will TOPSIS in relation with these last ones.

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Intuitive Multiple Centroid Defuzzification of Intuitionistic Z-Numbers

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Abstract

In fuzzy decision-making, incomplete information always leads to uncertain and partially reliable judgements. The emergence of fuzzy set theory helps decision-makers in handling uncertainty and vagueness when making judgements. Intuitionistic Fuzzy Numbers (IFN) measure the degree of uncertainty better than classical fuzzy numbers, while Z-numbers help to highlight the reliability of the judgements. Combining these two fuzzy numbers produces Intuitionistic Z-Numbers (IZN). Both restriction and reliability components are characterized by the membership and non-membership functions, exhibiting a degree of uncertainties that arise due to the lack of information when decision-makers are making preferences. Decision information in the form of IZN needs to be defuzzified during the decision-making process before the final preferences can be determined. This paper proposes an Intuitive Multiple Centroid (IMC) defuzzification of IZN. A novel Multi-Criteria Decision-Making (MCDM) model based on IZN is developed. The proposed MCDM model is implemented in a supplier selection problem for an automobile manufacturing company. An arithmetic averaging operator is used to aggregate the preferences of all decision-makers, and a ranking function based on centroid is used to rank the alternatives. The IZN play the role of representing the uncertainty of decision-makers, which finally determine the ranking of alternatives.

Keywords: Defuzzification, Intuitionistic Z-numbers, Intuitive multiple centroid, Ranking function.

1 | Introduction

Decision-making is a cognitive process in selecting the preferred alternatives by gathering information and making an assessment based on the obtained information. The nature of decision-making always deals with the uncertain and partially reliable judgement due to incomplete and vague information [1]. Vague information is not well-defined [1], and this fact may lead to uncertainty of judgements due to the incompetency of decision-makers, psychological biases of opinions and the complexity of alternatives [2]. In fact, decision-making under uncertainty is a tough task faced by many decision-makers [3].

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The classical decision-making methods used crisp numbers 0 or 1 to describe whether a statement is either false or true, respectively. For instance, the importance of a matter is determined by the linguistic terms "important" and "not important", in which the level of importance between these two linguistic terms could not be exactly measured. The emergence of the fuzzy set theory by Zadeh [4] improved the decision-making methods, in which the truth of a statement is justified by a membership function taking any number in the interval [0,1]. From the previous example, the level of importance of a matter can now be measured by many linguistic terms such as "extremely not important", "not important", "moderately important", "important", and "extremely important". Using the knowledge of the fuzzy set, each of these linguistic terms can be represented by a membership value which takes any number in the interval [0,1]. Hence, the vagueness of the opinion on the importance of such a thing can be catered. Moreover, Bellman and Zadeh [5] implemented the fuzzy set theory in a decision-making application.

Additionally, Zadeh [6] and [7] further extended the concept of the fuzzy set to define a fuzzy subset of the real number line whose maximum membership values are clustered around a mean value. The arithmetic properties of fuzzy numbers were further studied by Dubois and Prade [8]. Some commonly used shapes of fuzzy numbers are triangular and trapezoidal. Fuzzy numbers have a better capability of handling vagueness than the classical fuzzy set. Making use of the concept of fuzzy numbers, Chen and Hwang [9] developed fuzzy Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) based on trapezoidal fuzzy numbers. Apart from that, Chang [10] replaced crisp values with fuzzy triangular numbers to construct pairwise comparison matrices in fuzzy Analytical Hierarchy Process (AHP). Many other Multi-Criteria Decision-Making (MCDM) methods were developed based on fuzzy numbers. However, the classical fuzzy set has its limitation since it does not consider the non-membership function and the hesitation degree [11].

Instead of considering the membership function alone, Atanassov [12] generalized the fuzzy set into an Intuitionistic Fuzzy Set (IFS) by defining the non-membership function. The membership and non-membership values represent the degrees of belongingness and non-belongingness to the fuzzy set, respectively. The IFS has better flexibility in handling uncertainty as compared to the classical fuzzy set [13]. In fact, the IFS is very powerful when human evaluations are needed to solve problems with incomplete information [14]. The IFS has also become an important tool for researchers [15], and it has been implemented in many real-world applications such as pattern recognition, time series forecasting and MCDM. Among the applications of IFSs in MCDM are intuitionistic fuzzy TOPSIS [16], intuitionistic fuzzy preference relations [17] and triangular intuitionistic fuzzy AHP [18].

Zadeh [19] introduced a new type of fuzzy number called a Z-number. The Z-number consists of two components, namely the restriction and reliability components. The first component denotes the degree of values that a variable can take, while the second component measures the reliability of the first component. For simplicity, both components can be considered as trapezoidal fuzzy numbers [19]. For example, the statement "the book is very thick, very surely" is a form of Z-statement in which the first statement determines the thickness of the book. In contrast, the second component indicates the degree of sureness and certainty, which measures the level of reliability of the first component. The application of Z-number in many fields has highlighted its strength in describing the preferences of decision-makers or experts.

Many MCDM methods have been developed by implementing Z-numbers to describe the decision information. Among the early years of this development, Kang et al. [20] proposed a defuzzification method of Z-numbers based on the fuzzy expectation theory. The proposed method was implemented in a vehicle selection problem [21]. Also, Ku Khalif et al. [22] improved the defuzzification method using an Intuitive Multiple Centroid (IMC) approach. Using this method, the trapezoidal fuzzy numbers are partitioned into three areas, and the sub-centroid of each area is calculated. Then, a conversion method of Z-numbers into regular fuzzy numbers was proposed. The proposed defuzzification method was applied in a staff recruitment problem.

Recently, Sari and Kahraman [23] proposed a combination of the Intuitionistic Fuzzy Number (IFN) and Z-number, namely Intuitionistic fuzzy Z-Number (IZN). In the IZN, both restriction and reliability components are characterized by the membership and non-membership functions. Hence the uncertainty and reliability of the preferences are handled. A defuzzification of IZN was given in [23], mimicking the concept of the accuracy function of IFN. Considering the applicability of IMC in [22], this paper aims to propose a defuzzification method for IZN via the IMC approach. The conversion method of IZN into regular IFN is proposed once the IMC index of the membership and non-membership functions of IZN is obtained. An MCDM problem on supplier selection is adopted to illustrate the proposed defuzzification method. The proposed IMC approach has the advantage of representing the fuzzy number together with the height of the membership function. Therefore, converting IZN into IFN results in easy implementation for the application of the MCDM. In fact, the IMC approach considers both the membership and non-membership functions of IFN. Hence the uncertainty in the decision-making process is handled.

This paper is organized as follows: Section 1 presents the introduction. Section 2 reviews some preliminaries related to IFN, Z-numbers and IZN. The methodology in developing the MCDM method is proposed in Section 3. Section 4 illustrates an MCDM problem, and Section 5 concludes the paper.

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2 | Preliminaries

This section reviews some preliminaries related to IFS, Z-numbers and IZN. The IFS is a generalization of the classical fuzzy set, defined as follows [12]:

Definition 1. An IFS is a subset of the universe of discourse, U, written in the form of:

$$I=\{x,\mu_I(x),v_I(x)\,|\,x\in U\}. \tag{1}$$

Where $\mu_I : x \to [0,1]$ and $v_I : x \to [0,1]$ are the membership and non-membership functions of the element x, respectively.

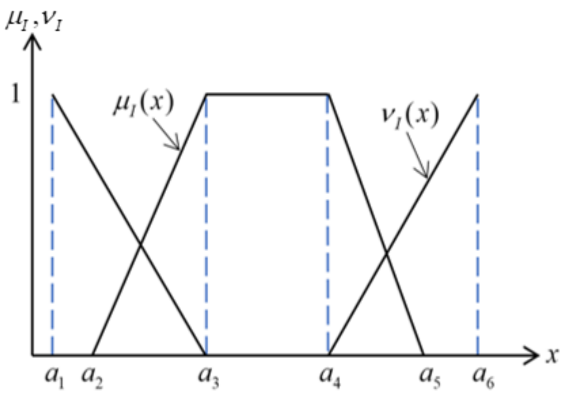

Fig. 1. A trapezoidal IFN.

An extension of the IFS is an IFN. A trapezoidal IFN (TrIFN), $A_I=(a_2,a_3,a_4,a_5;a_1,a_3,a_4,a_6)$ as illustrated in *Fig. 1,* is defined below.

Definition 2. A TrIFN $A_I=(a_2,a_3,a_4,a_5;a_1,a_3,a_4,a_6)$ is characterized by the following membership function.

$$\mu_{A_I}=\begin{cases}\dfrac{x-a_2}{a_3-a_2} & , \ x\in[a_2,a_3]\\ 1 & , \ x\in[a_3,a_4]\\ \dfrac{a_5-x}{a_5-a_4} & , \ x\in[a_4,a_5]\\ 0 & , \ \text{otherwise}\end{cases}. \quad (2)$$

And non-membership function is expressed by

$$\nu_{A_I}=\begin{cases}\dfrac{a_3-x}{a_3-a_1} & , \ x\in[a_1,a_3]\\ 0 & , \ x\in[a_3,a_4]\\ \dfrac{x-a_4}{a_5-a_4} & , \ x\in[a_4,a_6]\\ 1 & , \ \text{otherwise.}\end{cases}. \quad (3)$$

Next, the definition of the Z-number is reviewed.

Definition 3. A Z-number, $Z=(A,R)$, consists of two components A and R. A is the fuzzy constraint on the values that a variable can take while R it is a measure of the reliability of A. For simplicity, both components of the Z-number can be represented by trapezoidal fuzzy numbers [19], as shown in *Fig. 2*.

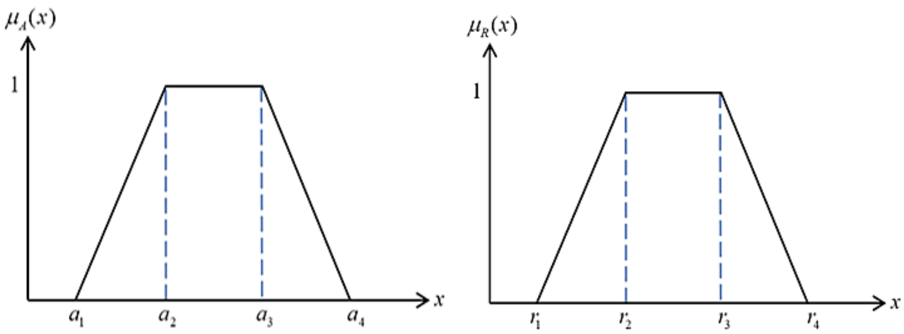

Fig. 2. Representation of a Z-number for trapezoidal fuzzy numbers.

Making use of Z-numbers in solving a MCDM problem, Ku Khalif et al. [22] proposed an IMC defuzzification approach. In their method, the trapezoidal plane is divided into three parts, where the sub-centroid of each area is found. Then, the sub-centroids are connected to form a triangular plane, and the IMC index is calculated. The IMC index of a trapezoidal fuzzy number $A=(a_1,a_2,a_3,a_4;h)$ is given as follows:

$$\mathrm{IMC}(x,y)=\left(\frac{2(a_1+a_4)+7(a_2+a_3)}{18},\frac{7h}{18}\right). \quad (4)$$

Using the obtained IMC, the Z-number $Z=(A,R)$ is converted into a regular fuzzy number using the following three steps:

Step 1. Convert R into a crisp number σ.

$$\sigma=\frac{2\left(r_1+r_4\right)+7\left(r_2+r_3\right)}{18}. \tag{5}$$

Step 2. Add σ into A.

$$Z^{\sigma}=\left\{\left\langle x,\mu_{R^{\sigma}}(x)\right\rangle \mid \mu_{R^{\sigma}}(x)=\sigma\mu_R(x), x\in[0,1]\right\}. \tag{6}$$

Step 3. Convert Z^{σ} into a regular fuzzy number.

$$Z'=\left\{\left\langle x,\mu_{Z'}(x)\right\rangle \mid \mu_{Z'}(x)=\mu_A\left(\sqrt{\sigma x}\right), x\in[0,1]\right\}. \tag{7}$$

Combining the IFN and Z-number, Sari and Kahraman [23] defined the IZN $Z_I=\left(A_I,R_I\right)$, in which the components A_I and R_I are represented by trapezoidal and triangular fuzzy numbers, respectively, as shown in *Fig. 3*

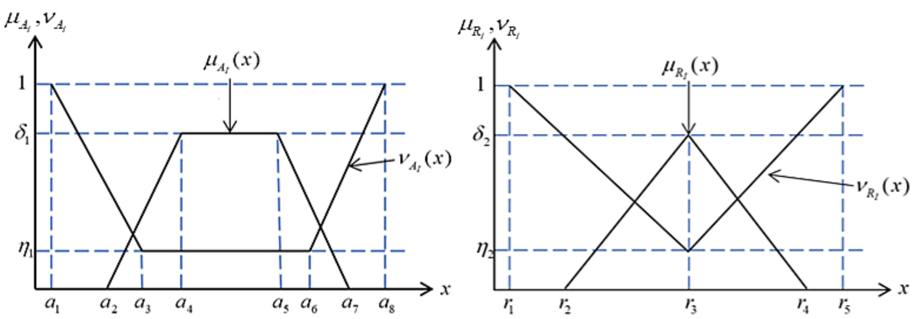

Fig. 3. An Intuitionistic Z-Number (IZN) [14].

In this paper, both components of IZN are represented by trapezoidal fuzzy numbers, following Zadeh's suggestion in [19]. Let $Z_I=\left(A_I,R_I\right)$, where $A_I=\left(a_2,a_3,a_4,a_5,\delta_1;a_1,a_3,a_4,a_6,\eta_1\right)$ and $R_I=\left(r_2,r_3,r_4,r_5,\delta_2;r_1,r_3,r_4,r_6,\eta_2\right)$. Then, A_I and R_I are characterized by the membership and non-membership functions defined in *Eq. (8)* and *Eq. (9)*, respectively.

$$\mu_{A_I}=\begin{cases}\dfrac{x-a_2}{a_3-a_2}\delta_1, & x\in\left[a_2,a_3\right]\\ \delta_1, & x\in\left[a_3,a_4\right]\\ \dfrac{a_5-x}{a_5-a_4}\delta_1, & x\in\left[a_4,a_5\right]\\ 0, & \text{otherwise}\end{cases},\quad \nu_{A_I}=\begin{cases}\dfrac{\eta_1-1}{a_3-a_1}x+\dfrac{a_3-\eta_1 a_1}{a_3-a_1}, & x\in\left[a_1,a_3\right]\\ \eta_1, & x\in\left[a_3,a_4\right]\\ \dfrac{1-\eta_1}{a_6-a_4}x+\dfrac{\eta_1 a_6-a_4}{a_6-a_4}, & x\in\left[a_4,a_6\right]\\ 1, & \text{otherwise}\end{cases}. \tag{8}$$

$$\mu_{R_I}=\begin{cases}\dfrac{x-r_2}{r_3-r_2}\delta_2, & x\in\left[r_2,r_3\right]\\ \delta_2, & x\in\left[r_3,r_4\right]\\ \dfrac{r_5-x}{r_5-r_4}\delta_2, & x\in\left[r_4,r_5\right]\\ 0, & \text{otherwise}\end{cases},\quad \nu_{R_I}=\begin{cases}\dfrac{\eta_2-1}{r_3-r_1}x+\dfrac{r_3-\eta_2 r_1}{r_3-r_1}, & x\in\left[r_1,r_3\right]\\ \eta_2, & x\in\left[r_3,r_4\right]\\ \dfrac{1-\eta_2}{r_6-r_4}x+\dfrac{\eta_2 r_6-r_4}{r_6-r_4}, & x\in\left[r_4,r_6\right]\\ 1, & \text{otherwise.}\end{cases}. \tag{9}$$

3 | Proposed Methodology

This section proposes a methodology used in developing the MCDM model. The methodology consists of three phases, as shown in *Fig. 4*.

PHASE 1: Identification of IMC index for non-membership function of IZN

↓

PHASE 2: Conversion of IZN into regular IFN

↓

PHASE 3: Development of MCM model based on IZN

Fig. 4. Proposed methodology.

The IMCs defuzzification of IZN is proposed, considering both the membership and non-membership functions. The IMC defuzzification for the membership function of IZN is given by *Eq. (4)*. Hence, the main contribution of this paper is the development of the IMC defuzzification for the non-membership function of IZN.

3.1 | IMC Index of Non-Membership Function of IZN

Before the conversion method of IZN into a regular fuzzy number could be proposed, the IMC index of the non-membership function should be determined. For this purpose, consider a trapezoidal plane, then the IMC index can be developed using the following steps:

Step 1. Divide the trapezoidal plane into three areas. Hence, the sub-centroid for each area can be calculated using *Eq. (10)* to *(12)*.

$$\alpha(x,y)=\left(r_1+\frac{2}{3}(r_3-r_1),\eta_2+\frac{2}{3}(1-\eta_2)\right). \tag{10}$$

$$\beta(x,y)=\left(\frac{r_3+r_4}{2},\eta_2+\frac{1}{2}(1-\eta_2)\right). \tag{11}$$

$$\gamma(x,y)=\left(r_4+\frac{1}{3}(r_6-r_4),\eta_2+\frac{2}{3}(1-\eta_2)\right). \tag{12}$$

Step 2. Connect the sub-centroids α, β and γ. Hence, a triangular plane is formed, as shown in *Fig. 5*.

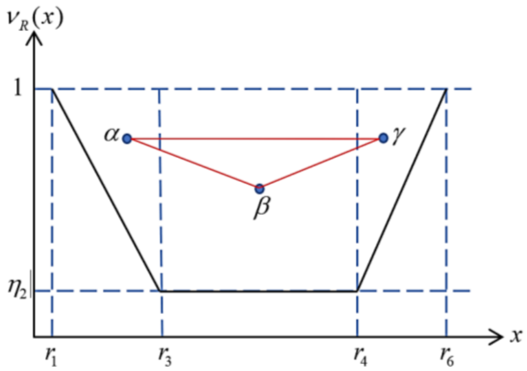

Fig. 5. The triangular plane is formed by connecting the sub-centroids of three areas of non-membership function.

Step 3. The centroid of the triangular plane is calculated by averaging the x- and y-ordinates of the sub-centroids α, β and γ. The IMC index is obtained as follows:

$$\mathrm{IMC}(x,y)=\left(\frac{\alpha(x)+\beta(x)+\gamma(x)}{3},\frac{\alpha(y)+\beta(y)+\gamma(y)}{3}\right)=\left(\frac{2(r_1+r_6)+7(r_3+r_4)}{18},\frac{\eta_2+11}{18}\right). \tag{13}$$

3.2 | Conversion of IZN into Regular IFNs

Using the IMC indices for the membership and non-membership functions as obtained in *Eq. (4)* and *Eq. (13)*, respectively, the reliability components are converted into a crisp number for the purpose of converting the IZN into regular IFN. The conversion of the IZN, $Z_I=(A_I,R_I)$ follows three steps below:

Step 1. Consider *Eq. (4)* and *Eq. (13)*, convert R_I into a crisp number σ.

$$\sigma=\frac{2(r_2+r_5)+7(r_3+r_4)}{18}\times\frac{7\delta_2}{18}+\frac{2(r_1+r_6)+7(r_3+r_4)}{18}\times\frac{\eta_2+11}{18}. \tag{14}$$

Step 2. Add the weight σ to A_I.

$$Z_I^{\sigma}=\left\{\left\langle x,\mu_{R^{\sigma}}(x),\nu_{R^{\sigma}}(x)\right\rangle \mid \mu_{R^{\sigma}}(x)=\sigma\mu_{R_I}(x),\nu_{R^{\sigma}}(x)=\sigma\nu_{R_I}(x),x\in[0,1]\right\}. \tag{15}$$

Theorem 1. $E_{A_I^{\sigma}}(x)=\sigma E_{A_I}(x)$, $x\in X$ subject to $\mu_{A_I^{\sigma}}(x)=\sigma\mu_{A_I}(x)$ and $\nu_{A_I^{\sigma}}(x)=\sigma\nu_{A_I}(x)$, $x\in X$.

Proof.

$$\begin{aligned}E_{A_I^{\sigma}}(x)=\Bigg(&a_2,a_3,a_4,a_5;\frac{2(r_2+r_5)+7(r_3+r_4)}{18}\times\frac{7\delta_2}{18}+\frac{2(r_1+r_6)+7(r_3+r_4)}{18}\times\frac{\eta_2+11}{18};\\&a_1,a_3,a_4,a_6;\frac{2(r_2+r_5)+7(r_3+r_4)}{18}\times\frac{7\delta_2}{18}+\frac{2(r_1+r_6)+7(r_3+r_4)}{18}\times\frac{\eta_2+11}{18}\Bigg)\\=&\left(a_2,a_3,a_4,a_5;\sigma;a_1,a_3,a_4,a_6;\sigma\right)\\=&\sigma E_{A_I}(x).\end{aligned}$$

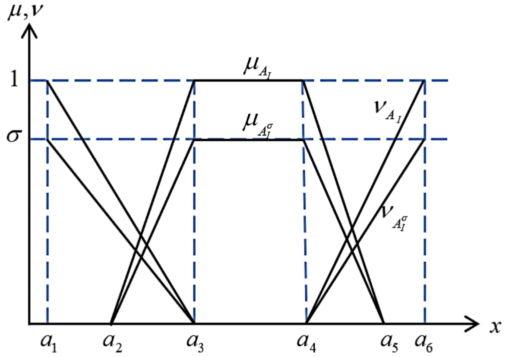

Fig. 6. Membership and non-membership functions of the weighted IZN.

Step 3. Convert Z_I^{σ} into a regular IFN.

$$Z' = \left\{ \left\langle x, \mu_{Z'}(x), \nu_{Z'}(x) \right\rangle \mid \mu_{Z'}(x) = \mu_{A_I}\left(\sqrt{\sigma}x\right), \nu_{Z'}(x) = \nu_{A_I}\left(\sqrt{\sigma}x\right), x \in [0,1] \right\}. \tag{16}$$

Theorem 2. $E_{Z'}(x) = \sigma E_{A_I}(x)$, $x \in \sqrt{\sigma}X$ subject to $\mu_{Z'}(x) = \mu_{A_I}\left(\sqrt{\sigma}x\right)$ and $\nu_{Z'}(x) = \nu_{A_I}\left(\sqrt{\sigma}x\right)$, $x \in \sqrt{\sigma}X$.

Proof.

$$E_{Z'}(x) = \left(a_2, a_3, a_4, a_5; \sqrt{\frac{2(r_2+r_5)+7(r_3+r_4)}{18} \times \frac{7\delta_2}{18} + \frac{2(r_1+r_6)+7(r_3+r_4)}{18} \times \frac{\eta_2+11}{18}}; \right.$$

$$\left. a_1, a_3, a_4, a_6; \sqrt{\frac{2(r_2+r_5)+7(r_3+r_4)}{18} \times \frac{7\delta_2}{18} + \frac{2(r_1+r_6)+7(r_3+r_4)}{18} \times \frac{\eta_2+110}{18}} \right)$$

$$= \left(a_2, a_3, a_4, a_5; \sqrt{\sigma}; a_1, a_3, a_4, a_6; \sqrt{\sigma} \right)$$

$$= \sigma E_{A_I}(x).$$

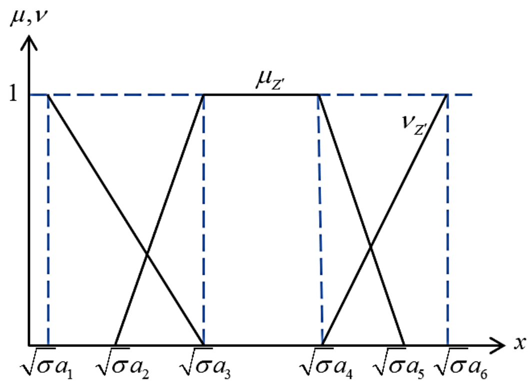

Fig. 7. The IFN converted from IZN.

Theorem 3. $E_{Z'}(x) = E_{A_I}(x)$.

Proof. From *Theorem 1* and *Theorem 2*, $E_{A_I^{\sigma}}(x) = \sigma E_{A_I}(x)$ and $E_{Z'}(x) = \sigma E_{A_I}(x)$. Therefore, $E_{Z'}(x) = E_{A_I}(x)$.

3.3 | IZN-Based Decision-Making Model

The advantages of IZN in handling uncertainty and vagueness, as discussed in the Introduction section, can be further highlighted in the implementation for solving an MCDM problem. In the previous sub-section, a method of converting the IZN into regular IFN was proposed. Hence, using the proposed defuzzification method, a novel MCDM model can be developed. In this paper, the arithmetic averaging operator is used to aggregate the decision-maker's preferences in the form of IZN. Furthermore, a ranking function based on centroid is used to rank the alternatives. Finally, the MCDM method based on IZN is given as follows:

Step 1. The decision-makers preferences in linguistic variables are converted into trapezoidal IZN. For this model, the linguistic terms of the restriction and reliability components are given in *Table 1* and *Table 2*, respectively.

Table 1. IZN corresponds to linguistic terms for the restriction component.

Linguistic Terms	Trapezoidal IZN
Very Low (VL)	⟨(0.0,0.0,0.0,0.0;1),(0.0,0.0,0.0,0.0;0)⟩
Low (L)	⟨(0.0,0.1,0.2,0.3;1),(0.0,0.1,0.2,0.3;0)⟩
Medium Low (ML)	⟨(0.1,0.2,0.3,0.4;1),(0.0,0.2,0.3,0.5;0)⟩
Medium (M)	⟨(0.3,0.4,0.5,0.6;1),(0.2,0.4,0.5,0.7;0)⟩
Medium High (MH)	⟨(0.5,0.6,0.7,0.8;1),(0.4,0.6,0.7,0.9;0)⟩
High (H)	⟨(0.7,0.8,0.9,1.0;1),(0.7,0.8,0.9,1.0;0)⟩
Very High (VH)	⟨(1.0,1.0,1.0,1.0;1),(1.0,1.0,1.0,1.0;0)⟩

Table 2. IZN corresponds to linguistic terms for the reliability component.

Linguistic Terms	Trapezoidal IZN
Not Sure (NS)	⟨(0.0,0.0,0.0,0.1;1),(0.0,0.0,0.0,0.1;0)⟩
Not Very Sure (NVS)	⟨(0.1,0.2,0.4,0.5;1),(0.1,0.2,0.4,0.5;0)⟩
Sure (S)	⟨(0.5,0.6,0.8,0.9;1),(0.5,0.6,0.8,0.9;0)⟩
Very Sure (VS)	⟨(0.9,1.0,1.0,1.0;1),(0.9,1.0,1.0,1.0;0)⟩

Step 2. Decision-makers' preferences in form of trapezoidal IZN are converted into the trapezoidal IFN using the defuzzification method proposed in Section 3.2.

Step 3. If there is more than one decision-maker involved, then all the decision-makers preferences are aggregated using the arithmetic averaging operator. Let n be the number of decision-makers, and then the arithmetic averaging operator is given in *Eq. (17)*.

$$\frac{1}{n}\sum_{k=1}^{n}\left(a_{k2},a_{k3},a_{k4},a_{k5};a_{k1},a_{k2},a_{k3},a_{k6}\right)$$
$$=\left(\frac{\sum_{k=1}^{n}a_{k2}}{n},\frac{\sum_{k=1}^{n}a_{k3}}{n},\frac{\sum_{k=1}^{n}a_{k4}}{n},\frac{\sum_{k=1}^{n}a_{k5}}{n};\frac{\sum_{k=1}^{n}a_{k1}}{n},\frac{\sum_{k=1}^{n}a_{k2}}{n},\frac{\sum_{k=1}^{n}a_{k3}}{n},\frac{\sum_{k=1}^{n}a_{k6}}{n}\right). \tag{17}$$

Step 4. All criteria are aggregated using the arithmetic averaging operator for each alternative (in each row). If there are p criteria, then the aggregated trapezoidal IFN is given by,

$$\frac{1}{p}\sum_{k=1}^{p}\left(a_{k2},a_{k3},a_{k4},a_{k5};a_{k1},a_{k2},a_{k3},a_{k6}\right)$$
$$=\left(\frac{\sum_{k=1}^{p}a_{k2}}{p},\frac{\sum_{k=1}^{p}a_{k3}}{p},\frac{\sum_{k=1}^{p}a_{k4}}{p},\frac{\sum_{k=1}^{p}a_{k5}}{p};\frac{\sum_{k=1}^{p}a_{k1}}{p},\frac{\sum_{k=1}^{p}a_{k2}}{p},\frac{\sum_{k=1}^{p}a_{k3}}{p},\frac{\sum_{k=1}^{p}a_{k6}}{p}\right). \tag{18}$$

Step 5. The ranking of each alternative is calculated. For this purpose, the ranking function for trapezoidal IFN based on centroid [24] is used. Let $I=\left(a_2,a_3,a_4,a_5;a_1,a_3,a_4,a_6\right)$ be a trapezoidal IFN, and then its ranking is given by,

$$R(I)=\sqrt{\frac{1}{2}\left[\tilde{x}_{\mu}(I)-\tilde{y}_{\mu}(I)\right]^2+\frac{1}{2}\left[\tilde{x}_{v}(I)-\tilde{y}_{v}(I)\right]^2}. \tag{19}$$

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Where $\tilde{x}_{\mu}(I)=\frac{1}{3}\left(\frac{a_4^2+a_5^2-a_2^2-a_3^2-a_2a_3+a_4a_5}{a_4+a_5-a_2-a_3}\right)$, $\tilde{x}_{\nu}(I)=\frac{1}{3}\left(\frac{2a_6^2-2a_1^2+2a_3^2+2a_4^2+a_1a_3-a_4a_6}{a_4+a_6-a_1-a_3}\right)$,

$\tilde{y}_{\mu}(I)=\frac{1}{3}\left(\frac{a_2+2a_3-2a_4-a_5}{a_2+a_3-a_4-a_5}\right)$ and $\tilde{y}_{\nu}(I)=\frac{1}{3}\left(\frac{2a_1+a_3-a_4-2a_6}{a_1+a_3-a_4-a_6}\right)$.

4 | Supplier Selection Problem

A supplier selection problem is adopted from [25] to illustrate the implementation and advantages of IZNs in MCDM. The considered problem aims to select the best supplier in an automobile manufacturing company with five criteria: quality (C1), cost (C2), technological capability (C3), partnership (C4), and on-time delivery (C5). Six suppliers are considered as alternatives.

Step 1. Three decision-makers (experts) were requested to give preferences on all the criteria for each alternative. First, their preferences are presented in *Tables 3* to *5*. Then, the linguistic preferences are converted into trapezoidal IZN according to *Tables 1* to *2*.

Table 3. Linguistic preferences of Expert 1 [16].

Suppliers	C1	C2	C3	C4	C5
A1	(VL, NVS)	(ML, VS)	(MH, S)	(H, NVS)	(MH, NS)
A2	(L, S)	(MH, NVS)	(M, NS)	(H, VS)	(M, NVS)
A3	(MH, NVS)	(ML, S)	(H, VS)	(M, NVS)	(VL, NVS)
A4	(H, S)	(H, NS)	(MH, VS)	(ML, NS)	(VH, S)
A5	(M, NVS)	(M, VS)	(MH, NS)	(H, NS)	(VH, S)
A6	(ML, NS)	(ML, NS)	(H, S)	(H, NVS)	(L, S)

Table 4. Linguistic preferences of Expert 2 [16].

Suppliers	C1	C2	C3	C4	C5
A1	(M, NVS)	(ML, NS)	(MH, S)	(MH, NVS)	(H, VS)
A2	(MH, S)	(M, S)	(MH, NVS)	(H, NS)	(M, VS)
A3	(MH, NS)	(M, VS)	(H, NVS)	(M, NVS)	(MH, NVS)
A4	(MH, NVS)	(L, NVS)	(VH, NS)	(M, NS)	(ML, NS)
A5	(MH, VS)	(VH, NS)	(M, VS)	(H, NS)	(VH, NS)
A6	(H, NS)	(H, NVS)	(ML, NVS)	(H, S)	(MH, VS)

Table 5. Linguistic preferences of Expert 3 [16].

Suppliers	C1	C2	C3	C4	C5
A1	(MH, S)	(M, S)	(H, NVS)	(M, NVS)	(M, S)
A2	(H, NVS)	(MH, NVS)	(H, NS)	(M, NS)	(ML, NVS)
A3	(M, VS)	(ML, NS)	(H, S)	(MH, S)	(MH, NS)
A4	(MH, VS)	(VL, NS)	(VH, S)	(MH, S)	(M, NS)
A5	(H, NVS)	(VH, NVS)	(ML, NS)	(H, VS)	(VH, NVS)
A6	(MH, S)	(H, NS)	(ML, NS)	(VH, NVS)	(H, NS)

Step 2. Using the proposed IMC defuzzification of IZN as presented in Section 3.2, trapezoidal IZN is converted into trapezoidal IFN. *Table 6* shows the trapezoidal IFN obtained by defuzzifying Expert 1's preferences in the form of IZN. Then, the rest of the experts' preferences are defuzzified analogously.

Table 6. Converted IFN of preferences for Expert 1.

	C1	C2	C3	C4	C5
A1	(0.000,0.000,0.000,0.000;0.000,0.000,0.000,0.000)	(0.099,0.199,0.298,0.398;0.000,0.199,0.298,0.497)	(0.418,0.502,0.586,0.669;0.335,0.502,0.586,0.753)	(0.383,0.438,0.493,0.548;0.383,0.438,0.493,0.548)	(0.000,0.000,0.000,0.000;0.000,0.000,0.000,0.000)
A2	(0.000,0.084,0.167,0.251;0.000,0.084,0.167,0.251)	(0.274,0.329,0.383,0.438;0.219,0.329,0.383,0.493)	(0.000,0.000,0.000,0.000;0.000,0.000,0.000,0.000)	(0.696,0.796,0.895,0.994;0.696,0.796,0.895,0.994)	(0.274,0.329,0.383,0.438;0.219,0.329,0.383,0.493)
A3	(0.274,0.329,0.383,0.438;0.219,0.329,0.383,0.493)	(0.084,0.167,0.251,0.335;0.000,0.167,0.251,0.418)	(0.696,0.796,0.895,0.994;0.696,0.796,0.895,0.994)	(0.164,0.219,0.274,0.329;0.110,0.219,0.274,0.383)	(0.000,0.000,0.000,0.000;0.000,0.000,0.000,0.000)
A4	(0.586,0.669,0.753,0.837;0.586,0.669,0.753,0.837)	(0.000,0.000,0.000,0.000;0.000,0.000,0.000,0.000)	(0.497,0.597,0.696,0.796;0.398,0.597,0.696,0.895)	(0.000,0.000,0.000,0.000;0.000,0.000,0.000,0.000)	(0.837,0.837,0.837,0.837;0.837,0.837,0.837,0.837)
A5	(0.164,0.219,0.274,0.329;0.110,0.219,0.274,0.383)	(0.298,0.398,0.497,0.597;0.199,0.398,0.497,0.696)	(0.000,0.000,0.000,0.000;0.000,0.000,0.000,0.000)	(0.000,0.000,0.000,0.000;0.000,0.000,0.000,0.000)	(0.837,0.837,0.837,0.837;0.837,0.837,0.837,0.837)
A6	(0.000,0.000,0.000,0.000;0.000,0.000,0.000,0.000)	(0.000,0.000,0.000,0.000;0.000,0.000,0.000,0.000)	(0.586,0.669,0.753,0.837;0.586,0.669,0.753,0.837)	(0.383,0.438,0.493,0.548;0.383,0.438,0.493,0.548)	(0.000,0.084,0.167,0.251;0.000,0.084,0.167,0.251)

Step 3. Once the decision-maker's preferences have been converted into trapezoidal IFN, aggregation can be done to combine the preferences of all involved experts (3 experts). The aggregated trapezoidal IFN representing all the three experts are presented in *Table 7*.

Table 7. Aggregated IFN of preferences for all experts.

	C1	C2	C3	C4	C5
A1	(0.194,0.240,0.287,0.333;0.148,0.240,0.287,0.379)	(0.117,0.178,0.239,0.300;0.056,0.178,0.239,0.361)	(0.407,0.481,0.555,0.629;0.351,0.481,0.555,0.685)	(0.274,0.329,0.383,0.438;0.237,0.329,0.383,0.475)	(0.316,0.377,0.438,0.499;0.288,0.377,0.438,0.527)
A2	(0.267,0.341,0.415,0.489;0.239,0.341,0.415,0.517)	(0.266,0.331,0.395,0.459;0.202,0.331,0.395,0.459)	(0.091,0.110,0.128,0.146;0.073,0.110,0.128,0.164)	(0.232,0.265,0.298,0.331;0.232,0.265,0.298,0.331)	(0.209,0.279,0.348,0.418;0.139,0.279,0.348,0.488)
A3	(0.191,0.242,0.294,0.345;0.139,0.242,0.294,0.396)	(0.127,0.188,0.249,0.310;0.066,0.188,0.249,0.371)	(0.555,0.634,0.714,0.793;0.555,0.634,0.714,0.793)	(0.249,0.313,0.378,0.442;0.185,0.313,0.378,0.507)	(0.091,0.110,0.128,0.146;0.073,0.110,0.128,0.164)
A4	(0.452,0.532,0.611,0.690;0.401,0.532,0.611,0.742)	(0.000,0.018,0.037,0.055;0.000,0.018,0.037,0.055)	(0.445,0.478,0.511,0.544;0.411,0.478,0.511,0.577)	(0.139,0.167,0.195,0.223;0.112,0.167,0.195,0.251)	(0.279,0.279,0.279,0.279;0.279,0.279,0.279,0.279)
A5	(0.348,0.418,0.488,0.557;0.297,0.418,0.488,0.609)	(0.282,0.315,0.348,0.381;0.249,0.315,0.348,0.415)	(0.099,0.133,0.166,0.199;0.066,0.133,0.166,0.232)	(0.232,0.265,0.298,0.331;0.232,0.265,0.298,0.331)	(0.461,0.461,0.461,0.461;0.461,0.461,0.461,0.461)
A6	(0.139,0.167,0.195,0.223;0.112,0.167,0.195,0.251)	(0.128,0.146,0.164,0.183;0.128,0.146,0.164,0.183)	(0.213,0.260,0.306,0.351;0.195,0.260,0.306,0.370)	(0.506,0.552,0.598,0.644;0.506,0.552,0.598,0.644)	(0.166,0.227,0.288,0.349;0.133,0.227,0.288,0.382)

Step 4. Based on the aggregated IFN of preferences in *Table 7*, all the criteria preferences are aggregated for each alternative (in each row). Hence, the final aggregation result is shown in *Table 8* below.

Table 8. Final aggregation of experts' preferences for each alternative.

Suppliers	Aggregated IFN
A1	(0.261,0.321,0.380,0.440;0.216,0.321,0.380,0.485)
A2	(0.213,0.265,0.317,0.369;0.177,0.265,0.317,0.405)
A3	(0.243,0.298,0.352,0.407;0.204,0.298,0.352,0.446)
A4	(0.263,0.295,0.326,0.358;0.241,0.295,0.326,0.381)
A5	(0.285,0.318,0.352,0.386;0.261,0.318,0.352,0.410)
A6	(0.230,0.270,0.310,0.350;0.215,0.270,0.310,0.366)

Step 5. Based on the final aggregation result in *Table 8*, the ranking function is evaluated using *Eq. (19)*. The ranking function value for each alternative is listed in *Table 9* below.

Table 9. Final aggregation of experts' preferences for each alternative.

Suppliers	Ranking Function	Ranking Order
A1	0.123685	4
A2	0.090099	6
A3	0.104851	5
A4	0.278681	2
A5	0.338737	1
A6	0.168720	3

From the ranking function evaluated in the final step of the MCDM model, the Supplier A_5 is ranked first while the supplier A_2 is ranked last. The obtained ranking is $A_2 \prec A_3 \prec A_1 \prec A_6 \prec A_4 \prec A_5$, which $A_i \prec A_j$ denotes that the ranking of alternative A_i is lower than A_j. Comparing this ranking order to the one obtained in [25] $A_2 \prec A_1 \prec A_3 \prec A_6 \prec A_5 \prec A_4$, there is a slight change in their order. This paper used IZN to describe the decision information while Z-numbers were used in [25]. IZN is believed to be able to better handle the uncertainty of the judgement as compared to the regular Z-numbers. Hence, this fact has become one of the factors which determine the final ranking order of alternatives.

5 | Conclusion

A defuzzification method of IZN via the IMC method was proposed. The IZN has a better capability of handling uncertainty and vagueness as compared to the classical Z-numbers. This is because the restriction and reliability components of Z-numbers are characterized by both the membership and non-membership functions to highlight the uncertainties which arise due to the lack of information when preferences are made by decision-makers. The IMC method was used to defuzzify IZN into regular IFN due to its applicability and practicality in converting Z-numbers into regular fuzzy numbers, as discussed in the previous literature. A supplier selection problem was implemented to illustrate the proposed defuzzification approach. The limitation of this research is that the defuzzification of IZN may lead to a great loss of information since the preferences of decision-makers, which are initially in the form of IZN, are converted into regular IFN. Hence, a method of solving the MCDM problem without converting the IZN into any other form of fuzzy numbers should be developed in the future. The original decision information should be kept as IZN throughout the decision-making process, and a magnitude-based ranking method is suggested to be used for ranking the alternatives. This will avoid the loss of information. Hence, the advantages of IZN in handling uncertainty can be better highlighted.

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Conflicts of Interest

All co-authors have seen and agree with the contents of the manuscript, and there is no financial interest to report. In addition, the authors certified that the submission is original work and is not under review at any other publication.

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Fermatean Fuzzy Modified Composite Relation and its Application in Pattern Recognition

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Abstract

Fermatean Fuzzy Sets (FFSs) provide an effective way to handle uncertainty and vagueness by expanding the scope of membership and Non-Membership Degrees (NMDs) of Intuitionistic Fuzzy Set (IFS) and Pythagorean Fuzzy Set (PFS), respectively. FFS handles uncertain information more easily in the process of decision making. The concept of composite relation is an operational information measure for decision making. This study establishes Fermatean fuzzy composite relation based on max-average rule to enhance the viability of FFSs in machine learning via soft computing approach. Some numerical illustrations are provided to show the merit of the proposed max-average approach over existing the max-min-max computational process. To demonstrate the application of the approach, we discuss some pattern recognition problems of building materials and mineral fields with the aid of the Fermatean fuzzy modified composite relation and Fermatean fuzzy max-min-max approach to underscore comparative analyses. In recap, the objectives of the paper include: 1) discussion of FFS and its composite relations, 2) numerical demonstration of Fermatean fuzzy composite relations, 3) establishment of a decision application framework under FFS in pattern recognition cases, and 4) comparative analyses to showcase the merit of the new approach of Fermatean fuzzy composite relation. In future, this Fermatean fuzzy modified composite relation could be studied in different environments like picture fuzzy sets, spherical fuzzy sets, and so on.

Keywords: Intuitionistic fuzzy sets, Pythagorean fuzzy sets, Fermatean fuzzy sets, Fermatean fuzzy composite relation, Pattern recognition.

1 | Introduction

Zadeh [1] proposed the concept of fuzzy sets which deal with imprecise and vague information, and also serve as an effective tool to solve decision-making problems. However, fuzzy set considered the Membership Degree (MD) only. Sequel to this setback, Atanassov [2] proposed the Intuitionistic Fuzzy Sets (IFSs) to characterize uncertainty information by incorporating MD and Non-Membership Degree (NMD). The introduction of IFSs received a lot of attention in different fields, such as in medical diagnosis, pattern recognition [3]-[5] etc. But in a case where the sum of MD and NMD is greater than 1, IFS is no longer applicable.

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Yager [6] introduced Pythagorean Fuzzy Sets (PFSs) where the squared sum of its MD and NMD is less than or equal to 1. Since the concept was brought up it has been widely applied in different fields such as service control of domestic air lines [7], investment decision making [8], career placements based on academic performance [9], etc.

Although IFS and PFS facilitate the resolution of fuzzy decision problems, they still have obvious shortcomings, especially in extremely contradictory decision environments. PFS and IFS are unable to handle a situation where the sum of MD and NMD is greater than 1 and the sum of squares is still greater than 1, but the sum of cube is less than or equal to 1. For such cases, Senapati and Yager [10] developed the novel concept called Fermatean Fuzzy Set (FFS), which satisfies the criterion that the sum of the third power of MD and NMD must be less than or equal to 1. With comparison to IFS and PFS, FFS gains a stronger ability to describe uncertain information by expanding the spatial scope of MD and NMD. Based on FFS, Wang et al. [11] developed a hesitant Fermatean fuzzy multicriteria decision-making method using Archimedean Bonferroni mean operators, Senapati and Yager [10] proposed Fermatean fuzzy information weighted aggregation operators, and Liu et al. [12] developed a distance measure method for Fermatean fuzzy linguistic term sets. Furthermore, Liu et al. [13] defined a new concept of a Fermatean fuzzy linguistic set and Senapati and Yager [14] developed some new operations between Fermatean fuzzy Numbers (FFNs).

Sahoo [15] presented a similarity measure for FFSs with group decision-making application. Some score functions on FFSs have been studied and applied in transportation problem and bride selection [16] and [17]. Some uncertain approaches have been studied and applied [18]-[20]. Ejegwa et al. [21] studied composite relation on FFSs based on max-min-max approach with application to medical diagnosis. The Fermatean fuzzy composite relation presented in [21] used the extreme values of MD and NMD of the FFSs. This approach is not reliable because it cannot show the relation between two similar FFSs. Hence, this study seeks to explore Fermatean Fuzzy Max-Min-Max Composite Relation (FFMMMCR), highlight its setback and modify its applications in pattern recognition cases. The specific objectives of the work are to:

Discuss the FFS and its composite relations with some properties of the Fermatean fuzzy composite relation, numerically demonstrate the Fermatean fuzzy composite relations, establish a decision application framework under FFSs in pattern recognition cases, and present comparative analyses to showcase the merit of the new approach of Fermatean fuzzy composite relation.

This paper is presented as follows: Section two presents IFS, PFS, FFSs and their characterizations, Section three discusses Fermatean fuzzy composite relations with some examples, Section four presents the application of the studied Fermatean fuzzy composite relation and the approach in [21] to pattern recognition problems and discusses the results, and Section five summaries the paper with recommendations.

2 | Preliminaries

Definition 1. [2]. IFS A is defined on a non-empty set X as object having the form

$$A= \{\langle x,\alpha_A(x), \beta_A(x)\rangle : x \in X\}.$$

Where the functions

$$\alpha_A: X\rightarrow [0, 1] \text{ and } \beta_A: X\rightarrow [0, 1].$$

Denote MD and NMD of each element x ∈ X to the set A, respectively, and

$$0 \leq \alpha_A(x) + \beta_A(x) \leq 1 \text{ for all } x \in X.$$

Obviously, when $\beta_A(x)=1-\alpha_A(x)$ for all x ∈ X, the set A becomes a fuzzy set. Furthermore, we have

$$\pi_A(x)=1-\alpha_A(x)-\beta_A(x).$$

Called the IFS index or hesitation margin of x in A. The function $\pi_A(x)$ is the degree of indeterminacy of x ∈ X to the IFS A and $\pi_A(x)\in[0,1]$ i.e., $\pi_A: X\to[0,1]$ and $0\leq\pi_A(x)\leq 1$ for every x ∈ X . $\pi_A(x)$ expresses the lack of knowledge of whether x belong to IFS A or not.

Definition 2. [7]. Let A and B be IFSs, then we have

I. $A\subseteq B \Leftrightarrow \alpha_A(x)\leq\alpha_B(x)$ and $\beta_A(x)\geq\beta_B(x)\forall$ x ∈ X.
II. $Ac=\{\langle x, \beta_A(x), \alpha_A(x)\rangle: x\in X\}$.
III. $A\cup B=\{\langle x, \max(\alpha_A(x),\alpha_B(x)), min(\beta_A(x),\beta_B(x))\rangle: x\in X\}$.
IV. $A\cap B=\{\langle x, \min(\alpha_A(x),\alpha_B(x)), max(\beta_A(x),\beta_B(x))\rangle: x\in X\}$.
V. $A\oplus B=\{\langle x, \alpha_A(x)+\alpha_B(x)-\alpha_A(x)\alpha_B(x), \beta_A(x), \beta_B(x)\rangle: x\in X\}$.
VI. $A\otimes B=\{\langle x\alpha_A(x)\alpha_B(x), \beta_A(x)+\beta_B(x)-\beta_A(x)\beta_B(x)\rangle: x\in X\}$.

Definition 3. [7]. PFS defined on a non-empty set X is an object having the form $P=\{\langle x, \alpha_P(x), \beta_P(x)\rangle: x\in X\}$, where the functions α_P: X⟶[0,1] and β_P: X⟶[0,1] denote the degree of membership and the degree of NMD of each element x ∈ X to the set P, respectively, and

$$0\leq\alpha_P^2(x)+\beta_P^2(x)\leq 1.$$

For every x ∈ X.

For any PFS P, the function $\pi_P(x)=\sqrt{1-\alpha_P^2(x)-\beta_P^2(x)}$ is called the degree of indeterminacy of x to P. A Pythagorean fuzzy number of a PFS P is denoted by $P=(\alpha_P,\beta_P)$.

Definition 4. [7]. Given two PFNs $P=(\alpha_P,\beta_P)$ and $Q=(\alpha_Q,\beta_Q)$ where $\alpha_P,\beta_P\in[0,1]$, $\alpha_Q,\beta_Q\in[0,1]$, then some arithmetic operations can be described as follows:

I. $P\cup Q=(max\{\alpha_P,\alpha_Q\}, min\{\beta_P,\beta_Q\})$.
II. $P\cap Q=(min\{\alpha_P,\alpha_Q\}, max\{\beta_P,\beta_Q\})$.
III. $P^c=(\beta_P,\alpha_P)$.

Definition 5. [14]. Let X be a universe of discourse. A FFS F in X is an object having the form

$$F=\{\langle x, \boldsymbol{\alpha_F(x)}, \boldsymbol{\beta_F(x)}\rangle: x\in X\}.$$

Where α_F : X⟶ [0, 1] and β_F : X⟶ [0, 1] and $0\leq\alpha_F^3(x)+\beta_F^3(x)\leq 1$, for all x ∈ X. The numbers $\boldsymbol{\alpha_F}(x)$ and $\boldsymbol{\beta_F}(x)$ represent MD and NMD, respectively of F. For any FFS F, the function $\pi_F(x)=\sqrt[3]{1-\alpha_F^3(x)-\beta_F^3(x)}$ is identified as the degree of indeterminacy of x ∈ X in F.

For convenience, Senapati and Yager [14] called $(\alpha_F(x),\beta_F(x))$ a FFN denoted by $F=(\alpha_F,\beta_F)$.

Definition 6. [14]. Let $F=(\alpha_F, \beta_F)$, $F_1=(\alpha_{F_1},\beta_{F_1})$ and $F_2=(\alpha_{F_2},\beta_{F_2})$ be three FFNs, then their operations are defined as follows:

I. $F_1\cap F_2=(min\{\alpha_{F_1},\alpha_{F_2}\}, max\{\beta_{F_1},\beta_{F_2}\})$.
II. $F_1\cup F_2=(max\{\alpha_{F_1},\alpha_{F_2}\}, min\{\beta_{F_1},\beta_{F_2}\})$.
III. $F^c=(\beta_F,\alpha_F)$.

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Definition 7. [21]. Fermatean Fuzzy Values (FFVs) are describe by $\langle x, y\rangle$ for $x^3 + y^3 \leq 1$ where $x, y \in [0,1]$. FFVs assess the FFS where the constituents x *and* y are taken to mean MD and NMD, respectively.

FFS generalizes IFS/PFS such that $IFS \subset PFS \subset FFS$. For their differences, see *Table 1*.

Table 1. IFS, PFS and FFS.

IFS	PFS	FFS
$0 \leq \alpha + \beta \leq 1$	$0 \leq \alpha^2 + \beta^2 \leq 1$	$0 \leq \alpha^3 + \beta^3 \leq 1$
$\pi = 1 - \alpha - \beta$	$\pi = \sqrt{1 - \alpha^2 - \beta^2}$	$\pi = \sqrt[3]{1 - \alpha^3 - \beta^3}$
$\boldsymbol{\alpha + \beta + \pi = 1}$	$\alpha^2 + \beta^2 + \pi^2 = 1$	$\alpha^3 + \beta^3 + \pi^3 = 1$

3 | Fermatean Fuzzy Composite Relation

Here, we discuss the Fermatean fuzzy max-min-max introduced in [21], and present a new approach that do not use the extreme values adopted in [21].

3.1 | Fermatean Fuzzy Max-Min-Max Composite Relation (FFMMMCR)

The concept of max-min-max composite relation under IFSs and PFSs were presented in [22]-[24] with application to decision-making. To improve on it we explore information measure called FFMMMCR as presented in [21].

Assume X and Y are nonempty sets. Then the Fermatean Fuzzy Relation (FFR), ϕ from X to Y is a FFS of $X x Y$ consisting of MD, α_ϕ and NMD, β_ϕ and represented by $\boldsymbol{\phi}$ **(**$X \to Y$).

Definition 8. Suppose ϕ and ψ are FFRs of $X \times Y$ and $Y \times Z$ denoted by ϕ **(**$X \to Y$) and ψ ($Y \to Z$), respectively. Then FFMMMCR, $\sigma = \phi \circ \psi$ of $X \times Z$ is of the form

$$\sigma = \left\{\langle (x, z), \alpha_\sigma(x, z), \beta_\sigma(x, z)\rangle : (x, z) \in X \times Z\right\},$$

where

$$\alpha_\sigma(x, z) = \max\left\{\min\left(\alpha_\phi(x, y),\ \alpha_\psi(y, z)\right)\right\}.$$
$$\beta_\sigma(x, z) = \min\left\{\max\left(\alpha_\phi(x, y),\ \alpha_\psi(y, z)\right)\right\}.$$

For all $(x, y) \in X \times Y$ and $(y, z) \in Y \times Z$.

From Definition 8, FFMMCR, $\sigma = \phi \circ \psi$ can be computed by

$$\sigma = \alpha_\sigma(x, z) - \beta_\sigma(x, z)\pi_\sigma(x, z), \text{ for all } (x, z) \in X \times Z.$$

3.2 | Modified Fermatean Fuzzy Composite Relation

The Fermatean fuzzy composite relation presented in [21] uses the extreme values, i.e., the maximum of the minimum of MD and the minimum of the maximum of NMD. This approach is not reliable because it cannot show the relation between two similar FFSs. Hence, we modify the approach in [21] based on maximum average approach to resolve the limitation therein.

Definition 9. Suppose $\check{\phi}$ and $\check{\psi}$ are FFRs of $X \times Y$ and $Y \times Z$ denoted by $\check{\phi}$ **(**$X \to Y$) and $\check{\psi}$ ($Y \to Z$), respectively. Then the modified Fermatean fuzzy composite relation, $\check{\sigma} = \check{\phi} \circ \check{\psi}$ of $X \times Z$ is of the form

$$\check{\sigma} = \left\{\langle (x, z), \alpha_{\check{\sigma}}(x, z), \beta_{\check{\sigma}}(x, z)\rangle : (x, z) \in X \times Z\right\},$$

where

$$\alpha_{\breve{\sigma}}(x,z) = \max\left\{\text{average}\left(\alpha_{\breve{\phi}}(x,y),\ \alpha_{\breve{\psi}}(y,z)\right)\right\}.$$
$$\beta_{\breve{\sigma}}(x,z) = \min\left\{\text{average}\left(\alpha_{\breve{\phi}}(x,y),\ \alpha_{\breve{\psi}}(y,z)\right)\right\}.$$

For $0 \le \alpha_{\breve{\sigma}}^3(x,z) + \beta_{\breve{\sigma}}^3(x,z) \le 1$, and certainly, $\pi_{\breve{\sigma}}(x,z) = \sqrt[3]{1-\alpha_{\breve{\sigma}}^3(x,z)-\beta_{\breve{\sigma}}^3(x,z)} \in [0,1]$ for all $(x,y) \in X \times Y$, $(y,z) \in Y \times Z$ and $(x,z) \in X \times Z$.

From Definition 9, modified Fermatean fuzzy composite relation, $\breve{\sigma} = \breve{\phi} \circ \breve{\psi}$ can be computed by

$$\breve{\sigma} = \alpha_{\breve{\sigma}}(x,z) - \beta_{\breve{\sigma}}(x,z)\pi_{\breve{\sigma}}(x,z), \text{ for all } (x,z) \in X \times Z.$$

Definition 10. Given a binary FFR ϕ between X and Y, then ϕ^{-1} between Y and X is defined by

$$\alpha_{\phi^{-1}}(y,x) = \alpha_{\phi}(x,y),\ \beta_{\phi^{-1}}(y,x) = \beta_{\phi}(x,y),\ \forall\ (y,x) \in Y \times X \text{ and } \forall\ (x,y) \in X \times Y.$$

is called the inverse relation of ϕ.

Definition 11. If ϕ and ψ are two FFRs in $X \times Y$, then

I. $\phi \le \psi$ iff $\alpha_{\phi}(x,y) \le \alpha_{\psi}(x,y)$ and $\beta_{\phi}(x,y) \ge \beta_{\psi}(x,y)$ $\forall (x,y) \in X \times Y$.
II. $\phi \preccurlyeq \psi$ iff $\alpha_{\phi}(x,y) \le \alpha_{\psi}(x,y)$ and $\beta_{\phi}(x,y) \le \beta_{\psi}(x,y)$ $\forall (x,y) \in X \times Y$.
III. $\phi \vee \psi = \left\{\langle (x,y), \max(\alpha_{\phi}(x,y),\alpha_{\psi}(x,y)), \min(\beta_{\phi}(x,y),\beta_{\psi}(x,y)) \rangle : (x,y) \in X \times Y\right\}$.
IV. $\phi \wedge \psi = \left\{\langle (x,y), \min(\alpha_{\phi}(x,y),\alpha_{\psi}(x,y)), \max(\beta_{\phi}(x,y),\beta_{\psi}(x,y)) \rangle : (x,y) \in X \times Y\right\}$.
V. $\phi^c = \left\{\langle (x,y), \beta_{\phi}(x,y), \alpha_{\phi}(x,y) \rangle : \in X \times Y\right\}$, $\psi^c = \left\{\langle (x,y), \beta_{\psi}(x,y), \alpha_{\psi}(x,y) \rangle : X \times Y\right\}$.

3.3 | Some Properties of Modified Fermatean Fuzzy Composite Relation

Theorem 1. Suppose ϕ, ψ, φ be three FFRs in X x Y, then

I. $\phi \le \psi \Rightarrow \phi^{-1} \le \psi^{-1}$.
II. $(\phi \vee \psi)^{-1} = \phi^{-1} \vee \psi^{-1}$.
III. $(\phi \wedge \psi)^{-1} = \phi^{-1} \wedge \psi^{-1}$.
IV. $(\phi^{-1})^{-1} = \phi$.
V. $\phi \wedge (\psi \vee \varphi) = (\phi \wedge \psi) \vee (\phi \wedge \varphi)$.
VI. $\phi \vee (\psi \wedge \varphi) = (\phi \vee \psi) \wedge (\phi \vee \varphi)$.
VII. if $\phi \ge \psi$ and $\phi \ge \varphi$, then $\phi \ge \psi \wedge \varphi$.
VIII. if $\phi \le \psi$ and $\phi \le \varphi$, then $\phi \le \psi \vee \varphi$.
IX. if $\phi \wedge \psi \le \phi$ then $\phi \wedge \psi \le \psi$.
X. if $\phi \vee \psi \le \phi$ then $\phi \vee \psi \le \psi$.

Proof.

First we prove (i). Assume $\phi \le \psi$, then $\alpha_{\phi^{-1}}(y,x) = \alpha_{\phi}(x,y) \le \alpha_{\psi}(x,y) = \alpha_{\psi^{-1}}(y,x)$ and

$$\beta_{\phi^{-1}}(y,x) = \beta_{\phi}(x,y) \ \ge\ \beta_{\psi}(x,y) = \beta_{\psi^{-1}}(y,x)\ \forall\ (x,y) \in X \times Y.$$

For the proof of (ii), we have

$$\alpha_{(\phi \vee \psi)^{-1}}(y,x) = \alpha_{(\phi \vee \psi)}(x,y) = \max\{\alpha_{\phi}(x,y), \alpha_{\psi}(x,y)\} =$$
$$\max\{\alpha_{\phi^{-1}}(y,x), \alpha_{\psi^{-1}}(y,x)\} = \alpha_{\phi^{-1} \vee \psi^{-1}}(y,x)\ \forall\ (x,y) \in X \times Y.$$

Similarly,

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$$\beta_{(\phi\vee\psi)^{-1}}(y,x)=\beta_{(\phi\vee\psi)}(x,y)=\min\{\beta_{\phi}(x,y),\beta_{\psi}(x,y)\}=$$
$$\min\{\beta_{\phi^{-1}}(y,x),\beta_{\psi^{-1}}(y,x)\}=\beta_{\phi^{-1}\vee\psi^{-1}}(y,x)\ \forall\ (x,y)\in X\times Y.$$

The proof of (iii) is similar to (ii). Now, we proof (iv) is thus:

$$\alpha_{(\phi^{-1})^{-1}}(y,x)=\alpha_{\phi^{-1}}(x,y)=\alpha_{\phi}(y,x)=\alpha_{\phi}(x,y),\text{ alike } \beta_{(\phi^{-1})^{-1}}(y,x)=.\ \beta_{\phi}(x,y).$$

The proof of (v) is thus:

$$\alpha_{\phi\wedge(\psi\vee\varphi)}(x,y)=\min\{\alpha_{\phi}(x,y),\max\{\psi(x,y),\alpha_{\varphi}(x,y)\}=$$
$$\max\{\min\{\alpha_{\phi}(x,y),\psi(x,y)\},\min\{\alpha_{\phi}(x,y),\alpha_{\varphi}(x,y)\}\}=$$
$$\max\{\alpha_{\phi\wedge\psi}(x,y),\alpha_{\phi\wedge\varphi}(x,y)\}=\ \alpha_{(\phi\wedge\psi)\vee(\phi\wedge\varphi)}(x,y)\ \forall\ (x,y)\in X\times Y.$$

In like manner, $\beta_{\phi\wedge(\psi\vee\varphi)}(x,y)=\beta_{(\phi\wedge\psi)\vee(\phi\wedge\varphi)}(x,y)\ \forall\ (x,y)\in X\times Y.$

The proofs of (vi) to (x) are forthright.

Theorem 2. We have $(\psi\ o\ \phi)^{-1}=\phi^{-1}o\ \psi^{-1}$ if ϕ is a FFR in $(X\times Y)$ and ψ is a FFR in $(Y\times Z)$, respectively.

Proof. For the proof, we have

$$\alpha_{(\psi\ o\ \phi)^{-1}}(z,x)=\alpha_{\psi\ o\ \phi}(x,z)=\max\{\frac{\alpha_{\phi}(x,y)+\alpha_{\psi}(y,z)}{2}\}=\ \max\left\{\frac{\alpha_{\phi^{-1}}(y,x)+\ \alpha_{\psi^{-1}}(z,y)}{2}\right\}=\max$$
$$\left\{\frac{\alpha_{\psi^{-1}}(z,y)+\alpha_{\phi^{-1}}(y,x)}{2}\right\}=\ \alpha_{\phi^{-1}o\ \psi^{-1}}(z,x)\ \forall\ (z,x)\ \in(Z\ \times X).$$

Similarly, we have

$$\beta_{(\psi\ o\ \phi)^{-1}}(z,x)=\beta_{\psi\ o\ \phi}(x,z)=\min\{\frac{\beta_{\phi}(x,y)+\beta_{\psi}(y,z)}{2}\}\ =\ \min\left\{\frac{\beta_{\phi^{-1}}(y,x)+\ \beta_{\psi^{-1}}(z,y)}{2}\right\}=\min$$
$$\left\{\frac{\beta_{\psi^{-1}}(z,y)+\beta_{\phi^{-1}}(y,x)}{2}\right\}=\ \beta_{\phi^{-1}o\ \psi^{-1}}(z,x)\ \forall\ (z,x)\ \in(Z\ \times X).$$

Theorem 3. Let ϕ and ψ be FFRs in $(Y\times Z)$ and φ be FFR in $(X\times Y)$, then

I. $(\phi\vee\psi)\ \mathrm{o}\ \varphi\geq(\phi\ \mathrm{o}\ \varphi)\ \vee\ (\psi\ \mathrm{o}\ \varphi)$.
II. $(\phi\ \wedge\ \psi)o\ \varphi\ \leq\ (\phi\ o\ \varphi)\ \wedge\ (\psi\ o\ \varphi)$.

Proof. From Theorem 1, we have either $\phi\vee\psi\ \geq\phi$ or $\phi\ \vee\psi\ \geq\psi$. Thus, we have $(\phi\vee\psi)\mathrm{o}\ \varphi\geq(\phi\mathrm{o}\ \varphi)$ or $(\phi\ \vee\psi)\mathrm{o}\ \varphi\ \geq(\psi\mathrm{o}\ \varphi)$. Hence, it is clear that

$$(\phi\vee\psi)\ \mathrm{o}\ \varphi\geq(\phi\ \mathrm{o}\ \varphi)\ \vee\ (\psi\ \mathrm{o}\ \varphi),$$

which proves (i).

Similarly, it is either $\phi\wedge\psi\ \geq\phi$ or $\phi\ \wedge\psi\ \geq\psi$, and so $(\phi\wedge\psi)\mathrm{o}\ \varphi\leq(\phi\mathrm{o}\ \varphi)$ or $(\phi\wedge\psi)\mathrm{o}\ \varphi\ \leq(\psi\mathrm{o}\ \varphi)$.

Therefore, $(\phi\wedge\psi)\ \mathrm{o}\ \varphi\leq(\phi\ \mathrm{o}\ \varphi)\ \wedge\ (\psi\ \mathrm{o}\ \varphi)$, which proves (ii).

3.4 | Computation of Composite Relations under IFSs, PFSs and FFSs

We apply the proposed FFMMMCR, σ and the modified Fermatean fuzzy composite relation, $\check{\sigma}$ to compute the composite relation between FFSs $F_1=\{\langle x_1,0.7,0.2\rangle,\langle x_2,0.5,0.6\rangle,\langle x_3,0.5,0.4\rangle\}$ and $F_2=\{\langle x_1,0.8,0.3\rangle,\langle x_2,0.6,0.1\rangle,\langle x_3,0.5,0.2\rangle\}$ in $X=\{x_1,x_2,x_3\}$.

The algorithm for the computation using the new approach of Fermatean fuzzy composite relation includes:

Step 1. Establish a relation between F_1 and F_2 as FFVs.

Step 2. Identify MD and NMD of $\breve{\sigma} = \breve{\phi} \circ \breve{\psi}$ between F_1 and F_2 in X.

Step 3. Calculate $\breve{\sigma} = \breve{\phi} \circ \breve{\psi}$ between F_1 and F_2 in X using the information from Step 2 and the formula of $\breve{\sigma}$.

First, we apply the approach in [21] as follows:

$\alpha_\sigma(x_i, x_j) = \max\{0.7, 0.5, 0.5\} = 0.7$, and $\beta_\sigma(x_i, x_j) = \min\{0.3, 0.6, 0.4\} = 0.3$, where x_i is from F_1 and x_j is from F_2. Now, the composite relation between F_1 and F_2 using σ is:

$$\sigma = 0.7 - (0.3 \times 0.8573) = 0.4428.$$

The same example can also be captured in the frameworks of IFSs and PFSs, since 0.7 and 0.3 can define IFS and PFS. Using intuitionistic fuzzy information, we get:

$$\sigma = 0.7 - (0.3 \times 0) = 0.7.$$

Using Pythagorean fuzzy information, we get:

$$\sigma = 0.7 - (0.3 \times 0.6481) = 0.5056.$$

Next, we use the modified approach as follows:

$\alpha_{\breve{\sigma}}(x_i, x_j) = \max\{0.75, 0.55, 0.5\} = 0.75$, and $\beta_{\breve{\sigma}}(x_i, x_j) = \min\{0.25, 0.35, 0.3\} = 0.25$, where x_i is from F_1 and x_j is from F_2. Now, the composite relation between F_1 and F_2 using $\breve{\sigma}$ is:

$$\breve{\sigma} = 0.75 - (0.25 \times 0.8255) = 0.5436.$$

The same example can also be captured in the frameworks of IFSs and PFSs, since 0.75 and 0.25 can define IFS and PFS. Using intuitionistic fuzzy information, we get:

$$\breve{\sigma} = 0.75 - (0.25 \times 0) = 0.75.$$

Using Pythagorean fuzzy information, we get:

$$\breve{\sigma} = 0.75 - (0.25 \times 0.6124) = 0.5969.$$

The results are contained in *Table 2* for quick comparison.

Table 2. Results of composite relations.

FFCRs	IFS	PFS	FFS
FFMMMCR [21]	0.7	0.5056	0.4428
New Method	0.75	0.5969	0.5436

The information in *Table 2* is represented by *Fig. 1*. *Table 2* and *Fig. 1* clearly indicate that the composite relations in [21] and our approach under IFS gives the best relation between F_1 and F_2, follows by the results under PFS, and the results under FFS is the least; because of the inability of IFS and PFS to reliably curb uncertainties. However, in each of the environments, our new method gives the best measure of composite relation between F_1 and F_2.

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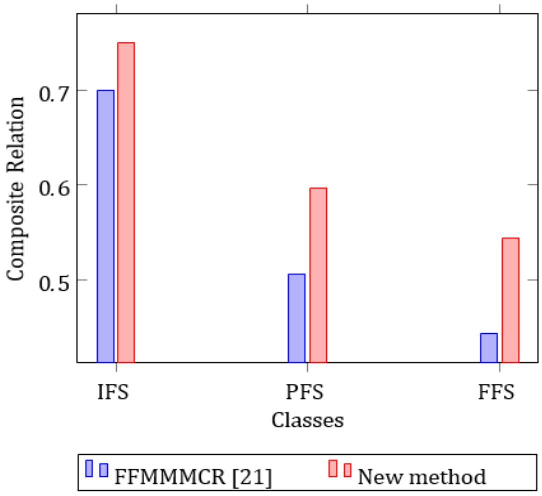

Fig. 1. Graphical representation of composite relations in terms of IFS, PFS, FFS.

4 | Application of Fermatean Fuzzy Composite Relation in Pattern Recognition Cases

Pattern recognition is the process of recognizing patterns by using a machine learning algorithm. Pattern recognition can be defined as the classification of data based on knowledge already gained or on statistical information extracted from patterns or their representation. Due to the presence of uncertainties in the process of pattern recognition and the ability of FFSs to curb uncertainties, it is expedient to carry out pattern recognition under Fermatean fuzzy information. In this section, we present the application of Fermatean fuzzy composite relation in pattern recognition problems based on max-average approach and the approach in [21]. The patterns are adopted from the work of Wang and Xin [25], but presented as Fermatean Fuzzy data. The pattern recognition problems are as follow:

4.1 | Problem 1

Given a pattern recognition problem about the classification of building materials. Four classes of building material, each is represented by FFS M_1, M_2, M_3, M_4 in the space $S = \{s_1, s_2, \ldots, s_{12}\}$ (see *Table 3*). Suppose we have another kind of unknown building material N, our aim is to show which class the unknown pattern N belong to.

Table 3. Pattern of building materials in FFVs.

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}
α_{M1}	0.173	0.102	0.530	0.965	0.420	0.008	0.331	1.000	0.215	0.432	0.750	0.432
β_{M1}	0.524	0.818	0.326	0.008	0.351	0.956	0.512	0.000	0.625	0.534	0.126	0.432
α_{M2}	0.510	0.627	1.000	0.125	0.026	0.732	0.556	0.650	1.000	0.145	0.047	0.760
β_{M2}	0.365	0.125	0.000	0.648	0.823	0.153	0.303	0.267	0.000	0.762	0.923	0.231
α_{M3}	0.495	0.603	0.987	0.073	0.037	0.690	0.147	0.213	0.501	1.000	0.324	0.045
β_{M3}	0.387	0.298	0.006	0.849	0.923	0.268	0.812	0.653	0.284	0.000	0.483	0.912
α_{M4}	1.000	1.000	0.857	0.734	0.021	0.076	0.152	0.113	0.489	1.000	0.386	0.028
β_{M4}	0.000	0.000	0.123	0.158	0.896	0.912	0.712	0.756	0.389	0.000	0.485	0.912
α_N	0.978	0.980	0.798	0.693	0.051	0.123	0.152	0.113	0.494	0.987	0.376	0.012
β_N	0.003	0.012	0.132	0.213	0.876	0.756	0.721	0.732	0.368	0.000	0.423	0.897

By deploying our method using the information in *Table 3*, we get the following results in *Table 4*.

Table 4. Result from problem 1.

FFCR	(M_1, N)	(M_2, N)	(M_3, N)	(M_4, N)
New method	0.781	0.881	0.994	0.994
Ranking	3rd	2nd	1st	1st

From the result in *Table 4*, we can see that the composite relation between M_3 and N, and between M_4 and N are the greatest. Hence the unknown building material N can be classified into either M_3 or M_4, respectively.

For comparison sake, we deploy the method in [21] and our method to the information in *Table 3*, and get results as seen in *Table 5* and *Fig. 2*.

Table 5. Comparison for the FFCRs for problem 1

FFCRs	(M_1, N)	(M_2, N)	(M_3, N)	(M_4, N)
FFMMMCR [21]	0.5078	0.6995	0.987	0.987
New method	0.781	0.881	0.994	0.994

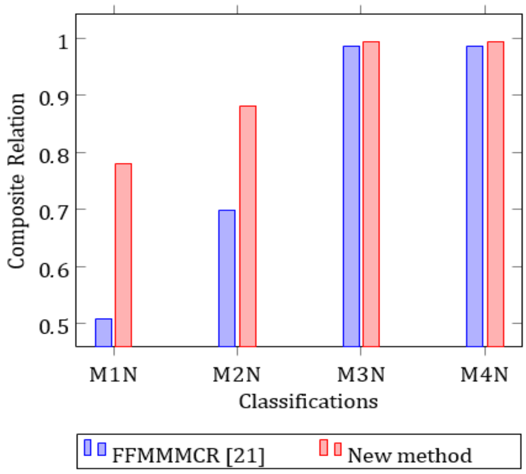

Fig. 2. Graphical representation of FFCRs for problem 1.

From the results, we observe that our new method of finding FFCR is better than the existing FFMMMCR method [21] because our method yields greater values of the composite relation.

Table 6. Patterns of mineral fields in FFVs

	s_1	s_2	s_3	s_4	s_5	s_6
$\alpha_{C_1}, \beta_{C_1}$	0.739, 0.125	0.033, 0.818	0.188, 0.626	0.492, 0.358	0.020, 0.628	0.739, 0.125
$\alpha_{C_2}, \beta_{C_2}$	0.124, 0.665	0.030, 0.825	0.048, 0.800	0.136, 0.648	0.823, 1.000	0.393, 0.653
$\alpha_{C_3}, \beta_{C_3}$	0.449, 0.387	0.662, 0.298	1.000, 0.000	1.000, 0.000	0.000, 0.188	1.000, 0.000
$\alpha_{C_4}, \beta_{C_4}$	0.280, 0.715	0.521, 0.368	0.470, 0.423	0.295, 0.658	0.806, 0.049	0.735, 0.118
$\alpha_{C_5}, \beta_{C_5}$	0.326, 0.452	1.000, 0.000	0.182, 0.725	0.156, 0.765	0.806, 0.049	0.675, 0.263
α_B, β_B	0.629, 0.303	0.524, 0.356	0.210,0. 689	0.218, 0.753	0.069, 0.876	0.658, 0.256

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4.2 | Problem 2

Given five kind of mineral fields, each is featured by the content of six minerals and has one kind of typical hybrid mineral. We can express the five kinds of typical hybrid minerals by five FFSs C_1, C_2, C_3, C_4 in the feature space $S = \{s_1, s_2, \ldots, s_6\}$ (see *Table 6*). Given another kind of hybrid mineral B in S, we find which field this kind of mineral B belongs to using our method and FFMMMCR approach [21].

From the data in *Table 6*, the results in *Table 7* are obtained based on our method.

Table 7. Results from problem 2.

FFCR	(C_1, B)	(C_2, B)	(C_3, B)	(C_4, B)	(C_5, B)
New Method	0.574	0.737	0.723	0.781	0.784
Ranking	5th	3rd	4th	2nd	1st

From *Table 7*, it is clearly shown that the hybrid mineral B could likely be produced by the mineral field C_5 since $(C_5, B) \geq (C_4, B) \geq (C_2, B) \geq (C_3, B) \geq (C_1, B)$.

For the sake of comparison, we apply the method in [21] and our method to the information in *Table 6*, and get results as seen in *Table 8* and *Fig. 3*.

Table 8. Comparison for the FFCRs for problem 2.

FFCRs	(C_1, B)	(C_2, B)	(C_3, B)	(C_4, B)	(C_5, B)
FFMMMCR [21]	0.431	-0.175	0.431	0.431	0.425
New method	0.574	0.737	0.723	0.781	0.784

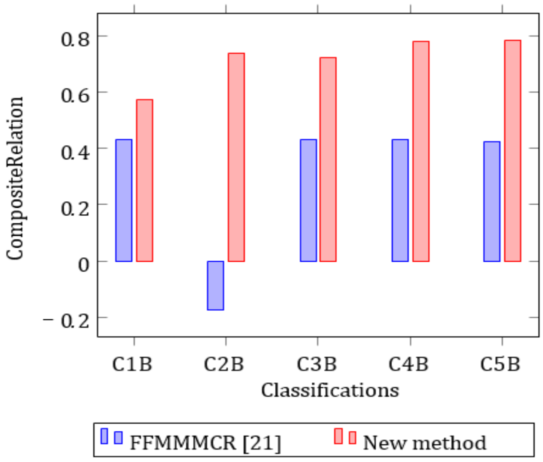

Fig. 3. Graphical representation of FFCRs for problem 2.

From *Table 8* and *Fig. 3*, we see that our new method yields better composite relations compare to the approach in [21], because the existing method is strictly based on extreme values whereas our method is based on max-mean values.

4.3 | Advantages of the New Approach of Fermatean Fuzzy Composite Relation

The method of Fermatean fuzzy composite relation in [21] incorporated the approach of extreme values. But from the knowledge of central tendency, such approach cannot yield a reliable result. This setback informed the present study. The following are some of the advantages of the new method of Fermatean fuzzy composite relation, which include:

- *The new method of Fermatean fuzzy composite relation uses maximum-average approach against the max-min-max approach adopted in [21], to avoid error due to the use of extreme values.*

- *The new method of Fermatean fuzzy composite relation can measure the composite relation between similar FFSs against the Fermatean fuzzy max-min-max approach adopted in [21].*
- *The new method of Fermatean fuzzy composite relation yields better result with regards to accuracy compare to the method in [21].*

5 | Conclusion

FFSs are generalizations of IFSs and PFSs which are capable of handling higher levels of uncertainties more efficient than IFSs and PFSs. In order to better appreciate the application of FFSs, this work studied a new composite relation under FFSs which modified the approach in [21] with better output. In this work, we succinctly differentiated FFSs from other generalized fuzzy sets with some characteristics. Numerical examples on pattern recognition were carried out under FFSs to demonstrate the application of our new method. The values gotten through this process clearly show where each pattern should belong. To demonstrate the merit our new approach, we presented certain comparative analyses between our method and the method in [21], which show that our method is more reliable, efficient and proficient than the approach in [21]. Due to the reliability of FFS as a competent soft computing tool, it can be used in multiple attributes decision making based on the new approach of Fermatean fuzzy composite relation. The limitation of this approach is that, it cannot be used to measure the composite relation between some generalized fuzzy sets like picture fuzzy sets, spherical fuzzy sets, etc. To achieve a reliable result, we recommend that the new approach of Fermatean fuzzy composite relation could be modified to be used in other environments like picture fuzzy sets, spherical fuzzy sets, etc.

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A Hybrid Method for the Assessment of Analogical Reasoning Skills

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Abstract

Much of a person's cognitive activity depends on the ability to reason analogically. Analogical reasoning (AR) compares the similarities between new and past knowledge and uses them to obtain an understanding of the new knowledge. The mechanisms, however, under which the human mind works are not fully investigated and as a result AR is characterized by a degree of fuzziness and uncertainty. Probability theory has been proved sufficient for dealing with the cases of uncertainty due to randomness. During the last 50-60 years, however, various mathematical theories have been introduced for tackling effectively the other forms of uncertainty, including fuzzy sets, intuitionistic fuzzy sets, neutrosophic sets, rough sets, etc. The combination of two or more of those theories gives frequently better results for the solution of the corresponding problems. A hybrid assessment method of AR skills under fuzzy conditions is developed in this work using Grey Numbers (GN) and soft sets as tools, which is illustrated by an application on evaluating student analogical problem solving skills.

Keywords: Soft set, Grey Number, Analogical Reasoning, Assessment under fuzzy conditions.

1 | Introduction

Analogical Reasoning (AR) is a way of reasoning that compares the similarities between new and past knowledge and uses these similarities to obtain an understanding of the new knowledge. Polya, who introduced the use of heuristic strategies for problem solving, suggests: "When you are not sure how to solve a given problem, a good hint would be to look to a similar problem solved in the past and try to exploit its solution for use with the new problem" [1]. In Artificial Intelligence the case-based Reasoning approach includes a range of methods based on AR for retrieving and utilizing the knowledge of past cases, usually stored in a computer's memory, for solving new problems or for producing new knowledge [2] and [3]. As many specialists report, much of the human cognitive activity depends in general on the ability to reason analogically [4]-[6]. Within cognitive science mental processes are simulated in computer programs (e.g. neural networks) and serve as models to support reasoning in new domains.

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It becomes evident therefore, that the assessment of the AR skills is a very important process. The mechanisms, however, under which the human mind works have not fully investigated yet. As a result, human reasoning in general and AR in particular are characterized by a degree of fuzziness and uncertainty. The first of the present authors proposed several methods in the past for assessment under fuzzy conditions including the measurement of a system's uncertainty, the use of the center of gravity defuzzification technique, the use of triangular fuzzy numbers or of GNs, etc. All these methods are reviewed in [7] accompanied by suitable examples. Recently he has also used soft sets for assessment with respect to a finite set of parameters [8].

In this work we introduce a hybrid method for assessing AR skills that uses soft sets and GNs as tools. The rest of the paper is organized as follows: Section 2 contains the elements from the theory of GNs and soft sets that are necessary for understanding the paper. The hybrid assessment method is presented in Section 3, illustrated by an example of assessing student analogical problem solving skills. The article closes with the final conclusion and some hints for future research included in Section 4.

2 | Background

The frequently appearing in real world, in science and in everyday life uncertainty is due to several reasons, like randomness, imprecise data, vague information, etc. Probability theory has been proved sufficient for dealing with the cases of uncertainty due to randomness. During the last 50-60 years, however, various mathematical theories have been introduced for tackling effectively the other forms of uncertainty, including fuzzy sets [9], intuitionistic fuzzy sets [10], neutrosophic sets [11], rough sets [12] and several others [13]. The combination of two or more of those theories gives frequently better results for the solution of the corresponding problems and that is what we are going to attempt here for the assessment of AR skills.

2.1 | Grey Numbers (GNs)

Ju-Long [14] introduced in 1982 the theory of grey systems as a new tool for dealing with the uncertainty created by the use of approximate data. A system is characterized as grey if it lacks information about its structure, operation and/or its behavior. The use of GNs [15] is the tool for performing the necessary calculations in grey systems.

A GN, say A, is an interval estimate [x, y] of a real number, whose exact value within [x, y] is not known. We symbolize then A by writing A ∈ [x, y]. A GN A is frequently accompanied by a whitenization function g: [x, y] → [0, 1], such that, if a ∈ [x, y], then the closer is g(a) to 1, the better a approximates the unknown real value of A. If no whitenization function has been defined, we usually consider as the crisp approximation of A the real number

$$W(A)=\frac{x+y}{2}. \tag{1}$$

The arithmetic operations between GNs are defined with the help of the known arithmetic of the real intervals [16]. In this work we make use only of the addition A+B of GNs and the scalar product of a GN with a positive number, say r, which are defined as follows: if A ∈ [x1, y1] and B ∈ [x2, y2] are GNs, then:

$$A+B\in\left[x_1+y_1,\ x_2+y_2\right]. \tag{2}$$

$$rG_1\in\left[rx_1,\ ry_1\right]. \tag{3}$$

2.2 | Soft Sets

All the theories, mentioned in the beginning of this section, which have been developed for dealing with the several types of the existing uncertainty, have their own difficulties. The reason of these difficulties is usually the inadequacy of the parameterization tools used. In fuzzy set theory, for example, there is a difficulty with the membership function, the definition of which is not unique depending on the observers' personal criteria [17]. To overpass these difficulties Molodtsov [18] introduced the concept of soft set for tackling the uncertainty in a parametric number.

Let U be the set of the discourse, let E be a set of parameters and let f: E → P(U) be a map from E to the power set P(U) of U. Then the soft set on U depending on f and E, symbolized by (f, E), is defined as the set of the ordered pairs

$$(f, E) = \{(e, f(e)): e \in E\}. \tag{4}$$

The set (f, E), being actually a family of subsets of U, is called "soft" because its form depends on the parameters. For general facts on soft sets we refer to [19].

3 | The Hybrid Assessment Method

Several researches ([20]-[24], etc.) have provided detailed models for the AR process which are broadly consistent with problem solving strategies. According to these models the main steps of AR include Representation (R) of the problem under solution (target problem), Search and Retrieval (S-R) of the related past problem (source problem), Mapping (M) of the representations of the target to the source problem for finding the corresponding similarities between them and adaptation (A) of the solution of the source for use with the target problem.

The purpose of the following classroom application was the assessment of AR skills of a class of 20 students. For this, four sets of problems were given to students for solution. Each set included the target problem, the source problem which was a remote analogue to the target problem sharing structure but not surface characteristics with it, a distractor problem sharing surface but not structure characteristics with the target problem and an unrelated to the target problem. The solution procedures of the last three problems were given to students, who were asked to solve the target problem with the help of the given solutions. As an example, we present below one of the sets of those problems [24].

Target problem. A box contains 8 balls numbered from 1 to 8. One makes three successive drawings, putting back the corresponding ball to the box before the next drawing. Find the probability of getting all the balls drawn out of the box different to each other.

Solution. The probability is equal to the quotient of the total number of the ordered samples of 3 objects from 8 (favourable outcomes) to the total number of the corresponding samples with replacement (possible outcomes).

Remote analogue. How many numbers of 2 digits can be formed by using the digits from 1 to 6 and how many of them have their digits different?

Solution's sketch given to students. Find the total number of the ordered samples of 2 objects from 6 with and without replacement respectively.

Distractor problem. A box contains 3 white, 4 blue and 6 black balls. If we draw out 2 balls, what is the probability to be of the same colour?

Solution's sketch given to students. The number of all favourable outcomes is equal to the sum of the total number of combinations of 3, 4 and 6 objects taken 2 at each time respectively, while the number of all possible outcomes is equal to the total number of combinations of 13 objects taken 2 at each time.

Unrelated problem. Find the number of all possible anagrams of the word "SIMPLE". How many of them start with S and how many of them start with E and end with E?

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Solution's sketch given to students. The number of all possible anagrams is equal to the total number 6! of the permutations of 6 objects. The anagrams starting with S are 5! and the anagrams starting with S and ending with E are 4!

The student AR skills were assessed by using the linguistic grades A = excellent, B= very good, C = good, D = fair and F = unsatisfactory (failed). It becomes evident that such kind of assessment involves a degree of fuzziness caused by the existence of the linguistic grades, which are less accurate than the numerical scores.

Let $U = \{S_1, S_2, \ldots, S_{20}\}$ be the set of the students of the class under assessment and let $E = \{A, B, C, D, F\}$ be the set of the linguistic grades (parameters) mentioned before. Then, according to the results of the student assessment, we defined a map $f: E \to P(U)$ as follows: $f(A) = \{S_5, S_{14}, S_{18}\}$, $f(B) = \{S_1, S_3, S_9, S_{12}, S_{15}, S_{17}\}$, $f(C) = \{S_2, S_6, S_8, S_{11}, S_{13}, S_{20}\}$, $f(D) = \{S_4, S_7, S_{16}\}$, $f(F) = \{S_{10}, S_{19}\}$. Consequently, the soft set

$$(f, E) = \{(A, f(A)), (B, f(B)), (C, f(C)), (D, f(D)), (F, f(F))\}, \tag{5}$$

represents mathematically the student class overall performance in a parametric way.

Another advantage of using softs sets as tools for the assessment of student AR skills is the mathematical representation of each student's profile with respect to the steps of the AR process. In fact, consider a particular student of the class and let $V = \{R, S\text{-}R, M, A\}$ be the set of the steps of the AR process mentioned in the beginning of this section. Define a map $g: E \to P(V)$ assigning to each linguistic grade (parameter) of E the steps of the AR process in which the performance of the particular student was assessed by this grade. Then the soft set (g, E) represents mathematically the student's profile. For example, the soft set

$$(g, E) = \{(A, \{R, S-R\}), (B, \varnothing), (C, \{M, A\}), (D, \varnothing), (F, \varnothing)\}. \tag{6}$$

corresponds to a student who demonstrated excellent performance at the steps of representation and search-retrieval and good performance at the steps of mapping and adaptation.

Sometimes, however, it is necessary to assess the mean performance of a student class. This is needed, for example, when one wants to compare the performance of two different student classes. It becomes evident that, when using linguistic grades, the mean performance cannot be assessed by calculating the mean value of the student scores in the classical way. In order to overpass this difficulty we assigned a scale of numerical scores from 1 to 100 to the linguistic grades A, B, C, D and F as follows: A (85–100), B (75–84), C (60–74), D (50–59) and F (0–49)[1]. For simplifying our notation we used the same letters to correspond to each of those grades the GNs $A \in [85, 100]$, $B \in [75, 84]$, $C \in [60, 74]$, $D \in [50, 59]$ and $F \in [0, 49]$, respectively. This enabled us to assign one of the GNs A, B, C, D, F to each student of the class. Then, using equation (5), it is easy to check that the "mean value" M of all those GNs is equal to the GN $M = \frac{1}{20}(3A+6B+6C+3D+2F)$, or by (3) $M = \frac{1}{20}([255, 300]+[450, 504]+[360, 444]+[150, 177]+[0, 98])$, or by (2) $M = \frac{1}{20}[1215, 1523]$. Thus, by (3) $M \in [60.75, 76.15]$. Therefore, by (1) $W(M)=68.45$, which means that the student class demonstrated a good mean performance.

[1] The scores assigned to the linguistic grades are not standard and may differ from case to case. For instance, in a more rigorous assessment one could take A(90-100), B (80-89), C(70-79), D (60-69), F(<60), etc.

4 | Discussion and Conclusions

A hybrid method using soft sets and GNs as tools was developed in this work for the assessment of student AR skills. The use of soft sets enabled a mathematical representation of the overall student performance in terms of the linguistic grades (parameters) used for their assessment, whereas the use of GNs enabled to obtain a crisp representative evaluating their mean performance. The second is useful when one wants to compare the performance of two or more student groups.

The new hybrid assessment method developed here has a general character. This means that, apart from student assessment, it could be utilized for assessing a great variety of other human or machine (e.g. case–based reasoning or decision making systems) activities. This is an important direction for future research. The theory of fuzzy systems, however, and the alternative theories related to it which were mentioned in the beginning of Section 2 of the present work, give also many and good other opportunities of applied and theoretical research in almost all sectors of the human activity, e.g. see [25]-[28], etc.

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Type-2 Fuzzy Logic Controller Design Optimization Using the PSO Approach for ECG Prediction

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Abstract

In this study, a hybrid model for prediction issues based on IT2FLS and Particle Swarm Optimization (PSO) is proposed. The main contribution of this work is to discover the ideal strategy for creating an optimal value vector to optimize the membership function of the fuzzy controller. It should be emphasized that the optimized fuzzy controller is a type-2 interval fuzzy controller, which is better than a type-1 fuzzy controller in handling uncertainty. The limiting membership functions of the type-2 fuzzy set domain is type-1 fuzzy sets, which explains the trace of uncertainty in this situation. The proposed optimization strategy was tested using ECG signal data. The accuracy of the proposed IT2FLS_PSO estimation technique was evaluated using a number of performance metrics (MSE, RMSE, error mean, error STD). RMSE and MSE with IT2FI were calculated as 0.1183 and 0.0535, respectively. With IT2FISPSO, these values were calculated as 0.0140 and 0.0029, respectively. The proposed PSO-optimized IT2FIS controller significantly improved its performance under various operating conditions. The simulation results show that PSO is effective in designing optimal type-2 fuzzy controllers. The experimental results show that the proposed optimization strategy significantly improves the prediction accuracy.

Keywords: Interval type-2 fuzzy set, Particle swarm optimization, Optimization fuzzy controller, Fuzzy sets, Prediction problem.

1 | Introduction

Optimization is a branch of computer science that deals with finding the optimal solution to a problem. It is used to find the best solution or a suitable alternative from a set of proposed solutions. Bio-inspired optimization methods can be used to solve problems that involve optimizing a particular objective function that may be constrained by a set of constraints. Unlike traditional optimization techniques, this algorithm generates a set of solutions at each iteration. These strategies focus on generating, selecting, assembling, and modifying a collection of solutions. They require more computation time than other metaheuristics because they maintain and modify a collection of solutions rather than searching for a single answer [1]-[5].

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This paper presents an T2FIS-PSO hybrid model for developing IT2FIS fuzzy logic controllers and finding the best parameter values for membership functions using bio-inspired optimization techniques. The hybrid T2FIS-PSO strategy is expected to provide more accurate results than standard T2FIS approaches [6]. The type-2 fuzzy Logic System (FLS) proposed by Zadeh [7]-[9] is presented as an extension of the standard type-1 FLS, and then the associated ideas and computational techniques are developed [10]-[14]. The degrees of membership are similarly fuzzy in a type-2 fuzzy set. In this sense, a fuzzy set of type-1 is a subset of a fuzzy set of type-2 , since its secondary membership function is a one-element subset [13]. Mendel and other scientists [13] and [14] reduced the computational complexity of the type-2 FLS domain by basing it on the broad type-2 FLS. It is widely accepted by academics in a variety of fields because of its minimal computational complexity. However, the major obstacle is the difficulty in obtaining the control parameters. This increases the complexity of the FLS and complicates the computation. For this reason, several systems recommend the use of powerful fuzzy rules. There are several established methods for rule selection, including fuzzy C-means, fuzzy K-means, cluster subtraction, and singular value decomposition. Traditional parameter setting methods have a number of drawbacks, including non-spherical convergence, computational cost, and difficulty to obtain. With the advancement and refinement of intelligent algorithms, the application of intelligent algorithms to the development of FLS, such as Particle Swarm Optimization (PSO) and Genetic Algorithm (GA) based on FLS, has become a focus of research [6] and [15]. PSO has been applied to a variety of optimization problems, including mathematical functions, fuzzy controllers, control parameters, and planning problems [16]-[23]. Using the hybrid IT2FIS-PSO technique, we were able to design an optimal type-2 fuzzy logic controller that determines the optimal Membership Function Parameters (MF) for controlling unstable linear systems. The results of this study were validated by predicting arrhythmia symptoms on electrocardiograms using arrhythmia signals (ECG) [3] and [24]. The human body can recognize some forms of cardiac arrhythmia caused by the electrical activity of the heart. The word electrocardiogram (ECG) refers to biological signals. Continuous recording and interpretation of the ECG is critical for detecting irregularities and consequences that may develop during the monitoring phase of heart disease. Therefore, in today's clinical practice, the processing, storage, and transmission of ECG data over digital communication networks is critical [25]. ECGs are a type of chaotic time series that serve as a bridge between chaos theory and reality. They are the most common application of chaos and thus provide an entirely new domain for predicting complicated nonlinear signals. In the real world, numerous types of complex systems have been shown to exhibit chaotic behavior. With the rapid advances in chaos theory and its application approaches, time series data obtained from an observable chaotic system has become an extremely useful tool for understanding complicated systems. This is because the observed system has a wealth of dynamical information that allows the behavior of the complex system to be studied and analyzed by examining and evaluating the potential content of the chaotic time series generated. So far, numerous techniques have been offered for prediction based on chaos theory [26]-[28].

The basic objective of this work is to develop and validate an optimization procedure based on a meta-heuristic algorithm using a type-2 fuzzy controller for monitoring ECG signals. The objective is to determine the optimal settings of the type-2 fuzzy controller in conjunction with the provided optimization techniques. In order to ensure the smallest possible error margin in the tracking of the ECG signal, the control of the behavior of a system specified by a certain metric is studied. *Fig.1* graphically represents the proposed working method PSO-FLS. Each particle represents the fuzzy rules and associated MF for the FLC inputs and outputs in this process. Each particle represents a possible solution. These parameters are needed to define the particles of the PSO algorithm and calculate the global optimal fitness.

The structure of this paper is as follows: the second section is about materials and processes. The motivation and equations for the search and optimization of the PSO and IT2FIS algorithms are discussed in detail here, as is the proposed technique for the optimization of the type-2 fuzzy controller. Section 3 summarizes the results of the experiments conducted with the proposed technique. Finally, conclusions and future work are outlined in Section 4.

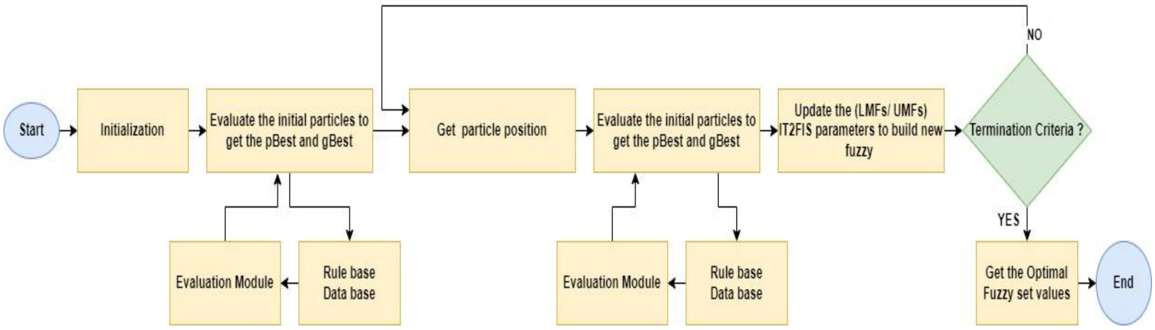

Fig. 1. Flowchart of the fuzzy membership function optimized with PSO.

2 | Material and Methods

Although previous research has focused on optimizing membership functions through PSO, none of these studies used the experimental settings used in this study to construct metrics. Since such a combination is rare in the literature, we will compare the performance of different computational techniques on these optimization problems. In this section, we provide an overview of the important ideas and concepts underlying our work.

2.1 | Particle Swarm Optimization

PSO is a kind of stochastic optimization technique invented by Kennedy [29]. It is a population-based search space determined by the social behavior of a flock of birds investigating a potential search area [30]-[32]. The PSO method is often used for multivariate optimization problems because it is a population-based probability optimization strategy. When an individual or particle performs an activity, it independently analyzes the fitness value of its position relative to the other swarm members. The computation of the individual's position, the rate of progress in each dimension of the solution set, and the optimal fitness value is critical [32]. The fitness function is used to calculate the best solution for the local or global swarm based on the optimization criteria of the PSO algorithm. *Eq. (1)* and *(2)* include the PSO velocity equation. The position and velocity of each swarm particle are fixed. Each particle gradually adjusts its position depending on two factors: the p_{best} of its nearest neighbor and the g_{best} of the swarm.

$$V_i(t+1) = w * V_i(t) + c_1 * r_1 * (p_{best} - x_i(t) + c_2 * r_2 * \left(g_{best} - x_i(t)\right). \quad (1)$$

$$x_i(t+1) = x_i(t) + (1-w) * V_i(t+1). \quad (2)$$

Here c_1 and c_2 are constant factors and w is the moment of inertia. r_1 and r_2 are two random integers from the range [0, 1]. The PSO approach is used to determine optimal parameters for intuitionistic fuzzy sets of type-2. Its performance is an evaluation of IT2FIS using the recommended upper and lower membership functions (UMFs and LMFs) for the IT2FIS membership functions. The algorithm presented in *Algorithm 1* shows how IT2FIS optimizes for particle swarms.

The PSO parameters and their meanings given in *Eq. (1)*, *Eq. (2)*, and Algorithm 1 are listed in *Table 1*.

Table 1. The PSO parameters and their meanings.

Population_Size	Initial Population Size
Pbest	Best movement.
Gbest	Best position movement.
C_1, C_2	Two acceleratin constants.
W^k, w_{max}, w_{min}	Inertia, initial and final weght.
$\overline{d}, \underline{d}$	Spread of MFs associate with upper and lower bound.
a	Tuned parameter.
v_i	Velocity.
x_i	Position of *i* th particle.

Algorithm 1. PSO in IT2FIS.

1. Intial papulation
2. Iteration=0
3. Setting papulation size, x= $\alpha,\bar{\mathrm{d}},\underline{\mathrm{d}}$
4. Setting C_1, C_2, W_{max}, W_{min}
5. While (iteration<MaxNumlt)
6. If $(x_i)<P_{best}$ then
7. $P_{best=}\ x_i$
8. end if
9. If $P_{best} < g_{best}$ then
10. $g_{best=}\ P_{best}$
11. end if
12. Calculation of Eq. (1) and (2)
13. Calculation of inertia weight (w)
14. Calculation of new position of particle ($x_i(t+1)$)
15. Iteration ++
16. End While
17. Return $\alpha,\bar{\mathrm{d}},\underline{\mathrm{d}}$.

2.2 | Interval Type-2 fuzzy Inference System (IT2FIS)

Interval type-2 is a specific uniform function of a T2F MF with a single value for the second stage. An IT2FIS, denoted by $\tilde{A}$, is expressed in *Eq. (3)* or *(4)*.

$$\widetilde{\mathrm{A}} = \left\{(\mathrm{x},\mathrm{y}),\mu_{\widetilde{\mathrm{A}}}(\mathrm{x},\mathrm{y})|\ \forall_{\mathrm{x}}\epsilon\ \mathrm{X}, \forall_{\mathrm{u}}\epsilon\ \mathrm{J}_{\mathrm{x}}\ \subseteq [0\ 1]\right\}. \tag{3}$$

$$\widetilde{\mathrm{A}} = \int_{x\epsilon X}\int_{u\epsilon J_x} 1/(x,u)\ J_x\ \subseteq [0\ 1]. \tag{4}$$

Where $\int\int$ denote the union of all acceptable x and u. An IT2FIS is defined in terms of an UMFs with $\bar{\mu}_{\tilde{A}}(x)$ and a LMFs with $\underline{\mu}_{\tilde{A}}(x)$. J_x is just the interval of $[\bar{\mu}_{\tilde{A}}(x), \underline{\mu}_{\tilde{A}}(x)]$. The UMF and LMF are determined using two T1F MFs representing the boundaries of the Footprint of Uncertainties (FOU). Consequently, $\mu_{\tilde{A}}(x,u) = 1$ and $\forall_u\epsilon\ J_x \subseteq [0\,1]$ are considered as IT2FIS membership functions. The form of MFs in IT2FIS is three-dimensional; the value of the third dimension is always 1. It has been shown to outperform T1FS in noisy and well-defined systems in real-time applications, corresponding to a case where $\mu_{\tilde{A}}(x,u) = 1$ and the integral form is shown in *Fig. 2*.

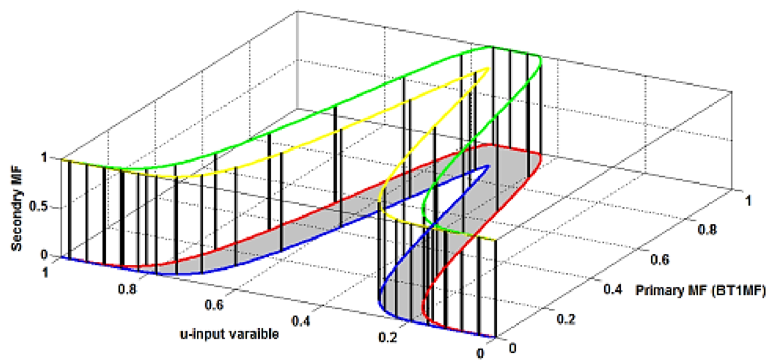

Fig. 2. 3D representation of the interval type-2 membership function [33].

A T2F is defined by IF-THEN rules, with T2F as antecedent and consecutive sets. To create a T2F controller, it is necessary to understand the block structure used in T1F, since the basic blocks are identical. A T2F, as shown in *Fig. 3*, consists of a fuzzifier, a rule base, a fuzzy inference engine, and an output processor. A Type Reducer (TR) and a defuzzifier are included in the output processor. TR is the main difference between T1F and T2F systems. The TR of T2F is used to generate a T1F output set of unique integers. Type reduction is included as it relates to the type of membership degrees of the elements [30].

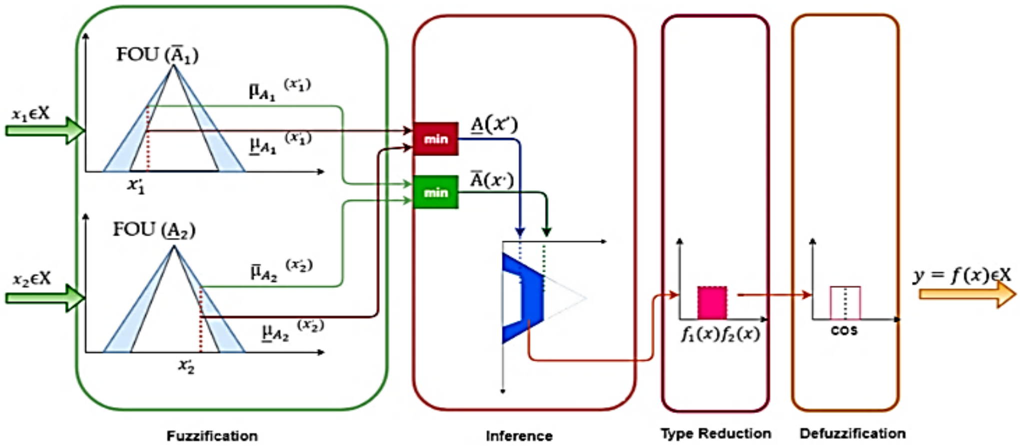

Fig. 3. Structure of a type-2 fuzzy logic system.

The stream RT of Karnik and Mendel [34] and [35] combines the output sets into a single output using one of their approaches, as shown in *Fig. 3*. The equation illustrates the mathematical expression for this *Procedure (5)*.

$$Y_{cos}(x) = [y_l, y_r] = \int_{y^1\in[y_l^1, y_r^1]} \cdots \int_{y^1\in[y_l^M, y_r^M]} \int_{f^1\in[\underline{f}^1, \bar{f}^1]} \cdots \int_{f^M\in[\underline{f}^M, \bar{f}^M]} / \frac{\sum_{i=1}^{M} f^i y^i}{\sum_{i=1}^{M} f^i}. \tag{5}$$

The left and right ends of this middle of the sets define it completely. The following set of the IT2FIS defines these two endpoints (y_l, y_r). Here, $\underline{f}^{\,i}$ and $\bar{f}^{\,i}$ are the lower and upper firing levels of the i-th rule, and M is the number of rules fired. These points are given in *Eq. (6)* and *(7)*. The outputs of the interval type-2 fuzzy system are represented by y_l and y_r.

$$y_l = \frac{\sum_{i=1}^{M} f_l^{\,i} y_l^{\,i}}{\sum_{i=1}^{M} f_l^{\,i}}. \tag{6}$$

$$y_r = \frac{\sum_{i=1}^{M} f_r^{\,i} y_r^{\,i}}{\sum_{i=1}^{M} f_r^{\,i}}. \tag{7}$$

3 | Simulation Results

The proposed optimization technique is used to determine the optimal settings for the type-2 fuzzy controller required to anticipate the behavior of the ECG signal in this study. The simulations were performed in MatLab®R2021 on an Intel®CoreTMi7-6700 CPU with 16GB RAM and a 3.40 GHz clock speed running Windows®10 (64 Bit). The goal of the fuzzy controller is to track a given signal with the lowest possible errors given by the controller's performance criteria. The main difference between dynamic and fixed parameter tuning in metaheuristic algorithms is that in dynamic tuning the selected parameters are changed during the iterations, leading to better solutions. The aim is to optimize the settings of the MFs of the type-2 fuzzy controller for signal tracking using the PSO algorithm to optimize the fuzzy controller using original techniques and their modifications. In this part, the results of fuzzy controller optimization for ECG signal behavior prediction are presented. The approach uses a metaheuristic algorithm to generate a vector containing the parameters required for the MFs of the optimized IT2FIS controller. Metaheuristic techniques in this case are versions of the PSO algorithm since they dynamically change the parameters using interval type-2 fuzzy systems. *Table 2* contains the parameters used in the proposed model.

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Table 2. Parameters of the PSO and IT2FIS.

Parameer	PSO	Parameter	IT2FIS
Function Tolerance	1.0000e-06	And Method	"prod"
Inertia Range	[0.1000, 1.1000]	Or Method	"probor"
Initial Swarm Span	2000	Implication Method	"prod"
Max Iteration	15	Aggregation Method	"sum"
Max Stal Iterations	20	Defuzzification Method	"wtaver"
Min Neighbors Fraction	0.2500	Inputs	1x4 fisvar
Objective Limit	0	Outputs	1x1fisvar
Seif-Adjustment Weight	1.4900	Rules	1x70 fisrule
Social Adjustment Weight	1.4900	Type Reduction Method	
Swarm Size	'min (100, 10*number of variables)'	"Karnikmendel"	
Distance Metric	"rmse"		

Mean Square Error (MSE), Root Mean Square Error (RMSE), root mean square error (EM), and standard deviation of error (Std) are all metrics used to measure algorithm performance during optimization.

$$\mathrm{MSE} = \frac{1}{N}\sum_{i=1}^{N}(x_i - y_i)^2. \quad [36] \text{ and } [37] \quad (8)$$

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - y_i)^2}. \quad [38] \quad (9)$$

$$\text{Error Mean} = \frac{1}{N}\sum_{i=1}^{N}(x_i - y_i). \quad [37] \quad (10)$$

$$\text{Error St. D} = \sqrt{\sum_{i=1}^{N}\frac{(x_i - \bar{x}_i)^2}{N-1}}. \quad [39] \text{ and } [40] \quad (11)$$

Where N is the number of data, $\bar{x}$ and $\bar{y}$ are the average of the predicted and actual, x_i and y_i are the predicted and actual values, respectively. Since the FIS has no predefined fuzzy rules, it is optimized using PSO, a global optimization technique for learning the rules. The maximum number of rules is limited by the number of possible MF combinations for the inputs. This limit can be exceeded because redundant rules are deleted during the adaptation process. The maximum iterations are set to 15. Training errors can be reduced by increasing the number of iterations. Increasing the number of iterations, on the other hand, prolongs the fitting process and may cause the rule parameters to be overfitted to the training data. The behavior of the FIS has changed with the training data and parameters shown in *Table 3*. Result of tuning the FIS using the given training data and options.

Table 3. Result of tuning the FIS using the given training data and options.

Interation	f-count	Best f (x)	Mean f (x)	Stall Iterations
0	100	0.1714	0.3262	0
1	200	0.1608	0.448	0
2	300	0.1397	0.3931	0
3	400	0.1345	0.3727	0
4	500	0.1345	0.3539	1
5	600	0.1345	0.3828	2
6	700	0.1345	.3788	3
7	800	0.1311	0.3629	0
8	900	0.1189	0.3407	0
9	1000	0.1189	0.3722	1
10	1100	0.1127	0.3787	0
11	1200	0.1127	0.3442	1
12	1300	0.1127	0.3702	2
13	1400	0.1127	0.3556	3
14	1500	0.1127	0.3716	4
15	1600	0.1127	0.3526	5

The graphical representation of the FIS created using the test data can be found in *Fig. 4*.

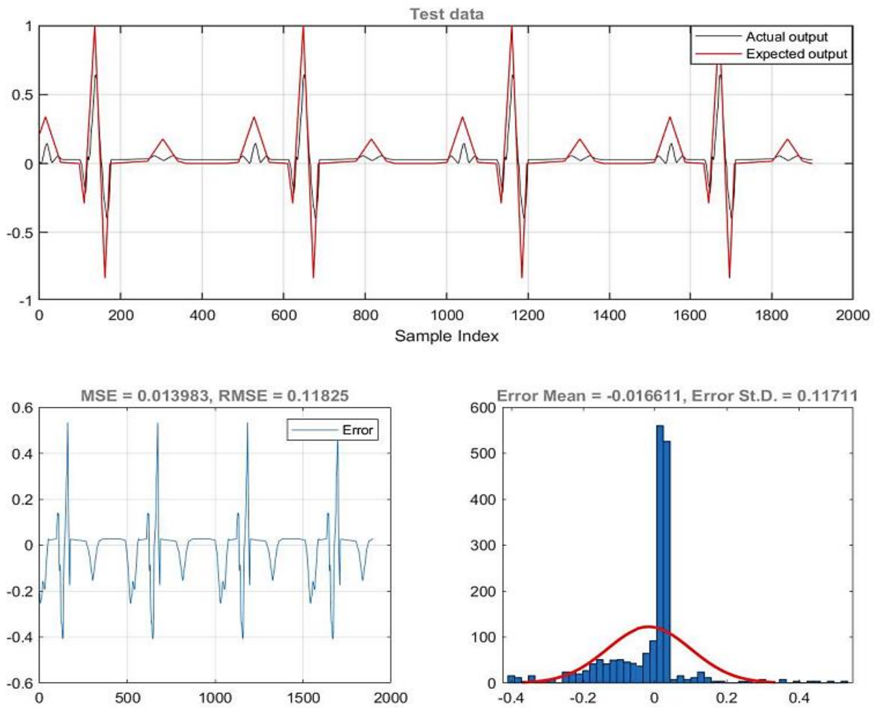

Fig. 4. The actual generated output with the expected validation output.

Fuzzy type-2 consists of both upper and lower membership functions. *Table 1* and *Fig. 5* show the results obtained when optimizing the upper membership functions.

Table 4. Results obtained using optimized upper MFs of FIS.

Interation	f-count	Best f (x)	Mean f (x)	Stall iterations
0	100	0.1127	0.3257	0
1	200	0.1127	0.3928	0
2	300	0.1103	0.2925	0
3	400	0.1014	0.2882	0
4	500	0.09827	0.2707	0
5	600	0.06759	0.2902	0
6	700	0.06759	0.2783	1
7	800	0.06759	0.277	2
8	900	0.06759	0.2746	3
9	1000	0.06285	0.2611	0
10	1100	0.06285	0.2656	1
11	1200	0.06285	0.2653	2
12	1300	0.06285	0.2454	3
13	1400	0.06285	0.2252	4
14	1500	0.06285	0.2135	5
15	1600	0.05985	0.1703	0

When comparing the predicted validation result of the modified FIS with the actual generated result using validation data and the actual produced result, it was found that changing the UMF parameters increased the performance of the FIS. The results obtained by adjusting the UMFs in the FIS structure are not reasonable. It is obvious that in this case, the functions of LMFs must be changed as well. The results of the functions and the graphical representations derived using the optimized FIS structure for both membership functions (LMFs and UMFs) can be found in *Table 5* and *Fig. 6*, respectively.

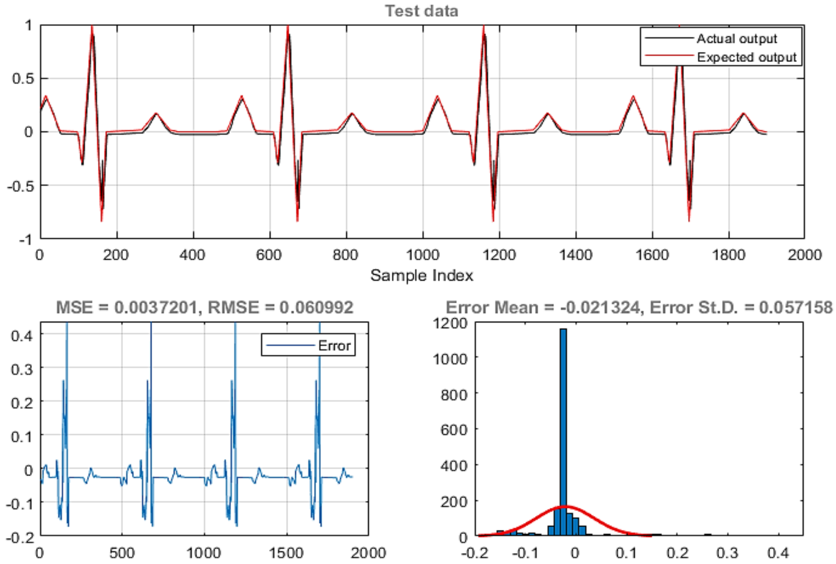

Fig. 5. The result of the actual, expected, error, and error histogram obtained with optimized upper MFs of FIS.

Table 5. Results obtained using optimized lower and upper MFs of FIS.

Iteration	f-count	Best f (x)	Mean f (x)	Stall Iterations
0	100	0.05985	0.1493	0
1	200	0.05985	0.1368	0
2	300	0.05491	0.1066	0
3	400	0.05421	0.09809	0
4	500	0.05421	0.1038	1
5	600	0.05421	0.1179	2
6	700	0.05421	0.09997	3
7	800	0.05421	0.07972	4
8	900	0.05421	0.07945	5
9	1000	0.05345	0.0911	0
10	1100	0.05345	0.08205	1
11	1200	0.05212	0.07336	0
12	1300	0.05212	0.0732	1
13	1400	0.05212	0.07687	2
14	1500	0.05209	0.0708	0
15	1600	0.05206	0.06181	0

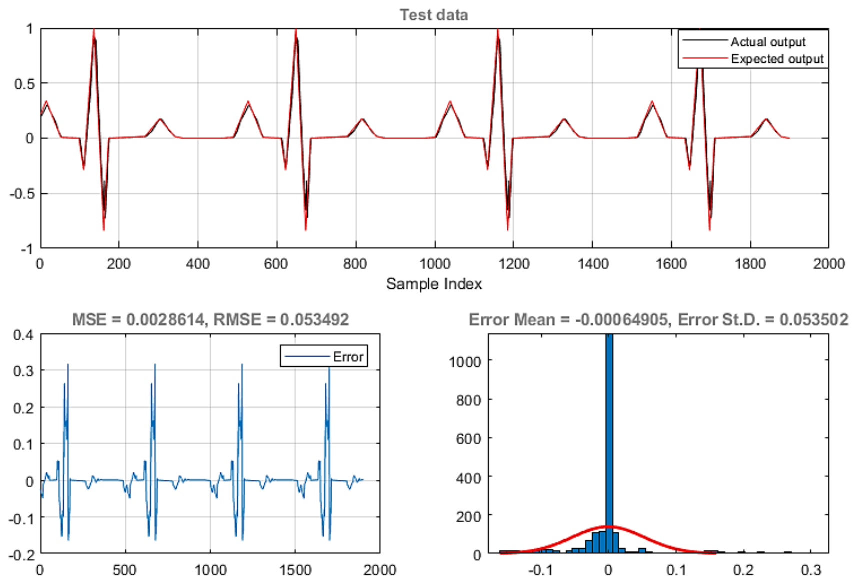

Fig. 6. The result of the actual, expected, error, and error histogram obtained with optimized upper and lower MFs of FIS.

When both modified upper and lower parameter values are included in the tuned and trained FIS with validation data, performance appears to increase. Compared to type-1 MFs, type-2 MFs have additional tunable properties. Thus, if the training data are suitable, a tuned type-2 FIS may fit better than a tuned Type-1 FIS. Changing any of the FIS attributes or tuning decisions, such as the number of inputs, the number of MFs, the type of MFs, the optimization technique, or the number of tuning iterations, can yield different tuning results. *Table 7* shows a statistical comparison of the performance of the metaheuristic algorithms used to test the proposed optimization method. The improvement in the values of MSE, RMSE, error-mean and error-Std shows the applicability of the proposed structure. After this modification, metaheuristic algorithms for type-2 fuzzy controllers were found to give satisfactory results on optimization problems. As can be seen in *Fig. 6*, we can say that there is enough statistical evidence that the PSO algorithm performs better in solving this problem, since it provides a small difference in the error in estimating the desired signal. Finally, in *Table 6*, we show a summary of all the above methods and can see that the best fuzzy controller was found by IT2FIS_PSO (UMFs and LMFs) with an average error of -6.4905e-04. This is important for the design of an optimal fuzzy controller because the goal is to find the best possible controller.

Table 6. An overview of the results of the proposed method.

	MSE	RMSE	ErrorMean	ErrorSTD
IT2FIS	0.0140	0.1183	-0.0166	0.1171
IT2FIS_ PSO (UMFs)	0.0037	0.0610	-0.0213	0.0572
IT2FIS_ PSO (UMFs and LMFs)	0.0029	0.0535	-6.4905e-04	0.0535

4 | Conclusions

Fuzzy controllers are widely used nowadays because they are able to solve problems that were once considered almost unsolvable. The results of parameter optimization of membership functions of type-2 fuzzy controllers for ECG signal estimation have been presented in this article as quite excellent. Moreover, it was found that the generated controller values were remarkably close to the desired signal trajectory. However, we used the PSO metaheuristic, which was found to be beneficial for optimizing the MF. The experiment used the original PSO metaheuristic in conjunction with dynamic parameter fitting by interval fuzzy systems of type-2. The collected results are presented in both tabular and graphical forms. From the results in *Table 6*, we can infer that parameter adjustment in the PSO algorithm using fuzzy logic is a viable choice for fuzzy control as these techniques provide competitive results. This is also evident in the simulation results, where we obtained fewer steady state errors and higher stability compared to the FLCs created using PSO. Therefore, we expect the hybrid method to perform better on increasingly difficult problems. The results show that the hybrid method performs better than the controls. Therefore, we can conclude that by integrating this bio-inspired method with others, we can improve the development of fuzzy logic controllers. In this regard, we intend to continue research in this area and investigate other types of problems that depend on the behavior of the algorithm and the use of type-1 and type-2 fuzzy logic. Various forms of fuzzy systems for dynamic parameter tuning in metaheuristic algorithms will be explored in future research and applied to a variety of control problems. In addition, it is intended to investigate multi-objective optimization techniques for dynamic parameter tuning using extended type-2 fuzzy systems to extend the controller to generalized type-2 fuzzy controllers and obtain better results for more complicated problems.

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A Novel Method for Solving Multi-Objective Linear Fractional Programming Problem under Uncertainty

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Abstract

This paper deals with a multi-objective linear fractional programming problem in fuzzy environment. The problem is considered by introducing all the parameters as piecewise quadratic fuzzy numbers. Through the use of the associated real number of the close interval approximation and the order relation of the piecewise quadratic fuzzy numbers, the problem is transformed into the corresponding crisp problem. A proposed method introduces to generate ideals and the set of all fuzzy efficient solutions. The advantage of it helps the decision maker to handle the real life problem. A numerical example is given illustrate the method.

Keywords: Linear fractional programming, Multi-objective decision making, Piecewise quadratic fuzzy number, Close interval approximation, Proposed method, Hungarian method, Fuzzy optimal solution.

1 | Introduction

Fractional Problem (FP) is a decision problem arises to optimize the ratio subject to constraints. In real world decision situations Decision Maker (DM) sometimes may face to evaluate ratio between inventory and sales, actual cost and standard cost, output and employee etc., with both denominator and numerator are linear. If only one ratio is considered as an objective function then under linear constraints the problem is said to be Linear Fractional Programming (LFP) problem. The fractional programming problem, i.e., the maximization of a fraction of two functions subject to given conditions, arises in various decision making situations; for instance , fractional programming is used in the fields of traffic planning [6], network flows [1], and game theory [8]. A review of various applications is given by Schaible [13] and [14]. Hassian et al. [21] introduced a parametric approach for solving multi-criteria LFP problem. Tantawy [17] introduced two approaches to solve the LFP problem namely; a feasible direction approach and a duality approach.

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Odior [10] introduced an algebraic approach based on the duality concept and the partial fractions to solve the LFP problem. Pandey and Punnen [11] introduced a procedure based on the Simplex method developed by Dantzig [6] to solve the LFP problem. Gupta and Chakraborty [7] solved the LFP problem depending on the sign of the numerator under the assumption that the denominator is non -vanishing in the feasible region using the fuzzy programming approach. Chakraborty [5] studied nonlinear fractional programming problem with multiple constraints under fuzzy environment. Stanojevic and Stancu- Minasian [16] proposed a method for solving fully fuzzified LFP problem. Buckley and Feuring [3] studied fully fuzzified linear programming involving coefficients and decision variables as fuzzy quantities. Li and Chen [9] introduced a fuzzy LFP problem with fuzzy coefficients and present the concept of fuzzy optimal solution. Sakawa et al. [20] introduced an interactive satisficing method for solving multi- objective fuzzy LFP problems with fuzzy parameters both in the objective functions and constraints. Pop and Stancu [12] studied LFP problem with all parameters and decision variables are triangular fuzzy numbers.

In this paper, fuzzy multi- objective LFP problem is introduced. The problem is converted into the crisp problem using the associated real number of the close interval approximation and the order relation of Piecewise Quadratic Fuzzy Numbers (PQFN), and hence the optimal transportation is obtained by applying optimal flowing method.

The following is a summary of the rest of the paper.

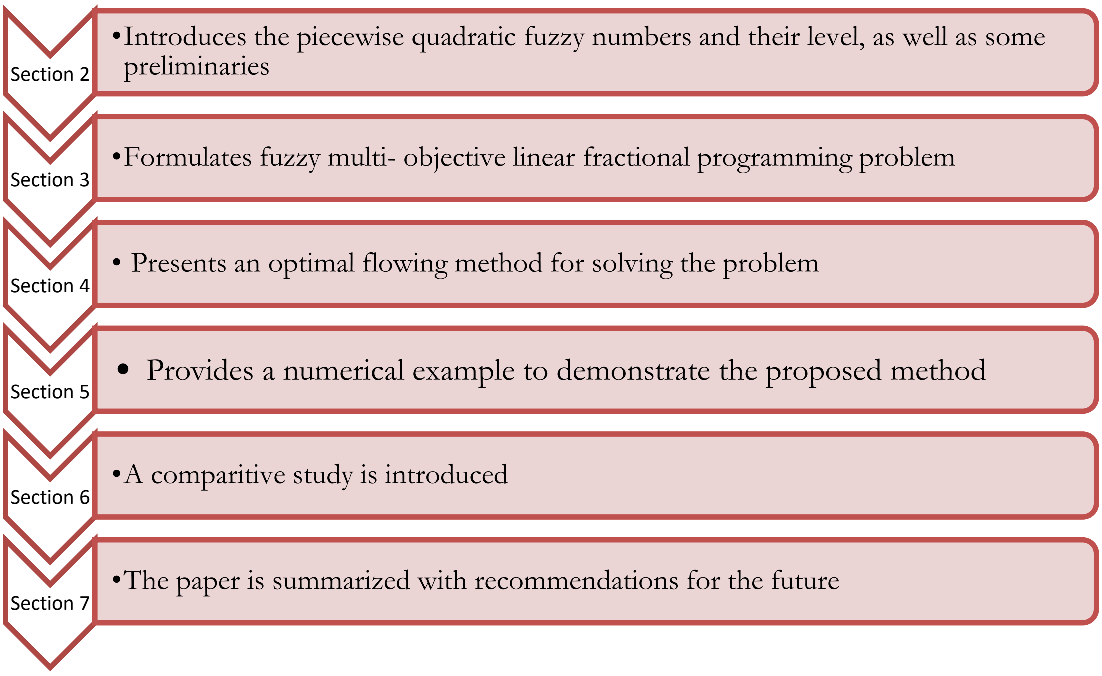

Fig. 1. Layout of remaining paper.

2 | Preliminaries

This section introduces some of basic concepts and results related to neutrosophic PQFN, close interval approximation, and their arithmetic operations are recalled.

Definition 1. [4]. A PQFN is denoted by $\tilde{A}_{PQ} = (a_1, a_2, a_3, a_4, a_5)$, where $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5$ are real numbers, and is defined by if its membership function $\mu_{\tilde{A}_{PQ}}$ is given by (see, *Fig. 1*).

$$\mu_{\tilde{A}_{PQ}} = \begin{cases} 0, & x < a_1; \\ \frac{1}{2}\frac{1}{(a_2-a_1)^2}(x-a_1)^2, & a_1 \le x \le a_2, \\ \frac{1}{2}\frac{1}{(a_3-a_2)^2}(x-a_3)^2+1, & a_2 \le x \le a_3, \\ \frac{1}{2}\frac{1}{(a_4-a_3)^2}(x-a_3)^2+1, & a_3 \le x \le a_4, \\ \frac{1}{2}\frac{1}{(a_5-a_4)^2}(x-a_5)^2, & a_4 \le x \le a_5, \\ 0, & x > a_5. \end{cases}$$

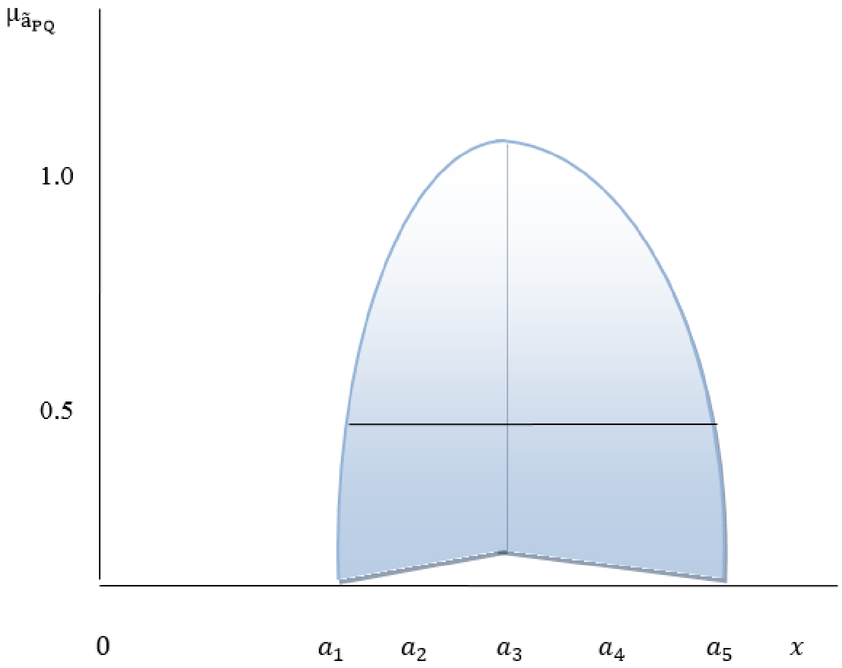

Fig. 1. Graphical representation of a Piecewise Quadratic Fuzzy Number (PQFN).

Definition 2. [4]. An interval approximation $[A] = \left[A_\alpha^L, A_\alpha^U\right]$ of a PQFN $\tilde{A}$ is called closed interval approximation if

$$A^L_\alpha = \inf\{x \in \mathbb{R}: \mu_{\tilde{A}} \ge 0.5\}, \text{ and } A^U_\alpha = \sup\{x \in \mathbb{R}: \mu_{\tilde{A}} \ge 0.5\}.$$

Definition 3. Associated ordinary number [4]. If $[A] = \left[A_\alpha^L, A_\alpha^U\right]$ is the close interval approximation of PQFN, the Associated ordinary number of $[A]$ is defined as $\hat{A} = \frac{A_\alpha^L + A_\alpha^U}{2}$.

Definition 4. [4]. Let $[A] = \left[A_\alpha^L, A_\alpha^U\right]$, and $[B] = \left[B_\alpha^L, B_\alpha^U\right]$ be two interval approximations of PQFN. Then the arithmetic operations are:

I. Addition: $[A] \oplus [B] = \left[A^L_\alpha + B^L_\alpha, A^U_\alpha + Bb^U_\alpha\right]$.

II. Subtraction: $[A] \ominus [B] = \left[A^L_\alpha - B^U_\alpha, A^U_\alpha - B^L_\alpha\right]$.

III. Scalar multiplication: $\alpha\,[A] = \begin{cases} \left[k\,A^L_\alpha,\ k\,A^U_\alpha\right], k > 0 \\ \left[\ k\,A^U_\alpha, k\,A^L_\alpha\right], k < 0. \end{cases}$

IV. Multiplication: $[A] \otimes [B]$

$$\left[\frac{A_\alpha^U B_\alpha^L + A_\alpha^L B_\alpha^U}{2}, \frac{A_\alpha^L B_\alpha^L + A_\alpha^U B_\alpha^U}{2}\right].$$

V. Division: $[A] \oslash [B]$

$$\begin{cases} \left[2\left(\frac{A_\alpha^L}{B_\alpha^L + B_\alpha^U}\right), 2\left(\frac{A_\alpha^U}{B_\alpha^L + B_\alpha^U}\right)\right], [B] > 0, B_\alpha^L + B_\alpha^U \neq 0 \\ \left[2\left(\frac{A_\alpha^U}{B_\alpha^L + B_\alpha^U}\right), 2\left(\frac{A_\alpha^L}{B_\alpha^L + B_\alpha^U}\right)\right], [B] < 0, B_\alpha^L + B_\alpha^U \neq 0. \end{cases}$$

VI. The order relation: $[A](\lesssim)[B]$

$$[A](\lesssim)[B] \text{ if } A_\alpha^L \leq B_\alpha^L \text{ and } A_\alpha^U \leq B_\alpha^U \text{ or } A_\alpha^L + A_\alpha^U \leq B_\alpha^L + B_\alpha^U.$$

It is noted that $P(\mathbb{R}) \subset F(\mathbb{R})$, where $F(\mathbb{R})$, and $P(\mathbb{R})$ are the sets of all PQFNs and close in interval approximation of PQFN, respectively.

3 | Problem Statement

Consider the following Fuzzy Multi-Objective Linear Fractional Programming (F-MOLFP) problem.

$$\max \tilde{Z}_{k_{PQ}} = \left(\tilde{Z}_{1_{PQ}}(x), \tilde{Z}_{2_{PQ}}(x), \dots, \tilde{Z}_{K_{PQ}}(x)\right), \quad K \geq 2$$

Subject to

$$\tilde{A}_{PQ} x \lesssim \tilde{b}_{PQ}.$$

$$x \geq 0.$$

Where, $\widetilde{M}_{PQ} = \left\{x \in \mathbb{R}^2 : \tilde{A}_{PQ} x \lesssim \tilde{b}_{PQ}, x \geq 0\right\}$ is the fuzzy feasible domain,

$\tilde{A}_{PQ} = \left(\tilde{a}_{ij_{PQ}}\right)_{m \times n}, x \in \mathbb{R}^n$ and $\tilde{b}_{PQ} = \left(\tilde{b}_{i_{PQ}}\right)_{1 \times m}$, $\tilde{Z}_{k_{PQ}}(x) = \frac{\tilde{f}_{k_{PQ}}(x)}{\tilde{g}_{k_{PQ}}(x)} = \frac{\tilde{c}_{k\,PQ}^T x + \tilde{\zeta}_{k_{PQ}}}{\tilde{d}_{k\,PQ}^T x + \tilde{\xi}_{k_{PQ}}}, \tilde{c}_k^T, \tilde{d}_k^T$ are $n-$dimensional PQF vectors; $\tilde{\zeta}_{k_{PQ}}, \tilde{\xi}_{k_{PQ}}$ are PQF fuzzy scalars and $\tilde{g}_{k_{PQ}}(x) > 0; \forall k = 1, K$ and for all $x \in \widetilde{M}_{PQ}$.

Definition 5. [19]. A point $\bar{x} = \left\{\bar{x}_j : j = \overline{1, n}\right\}$ is said to be fuzzy feasible solution to F-MOLFP if $\bar{x}$ satisfies the constraints in it.

Definition 6. A fuzzy feasible point $\bar{x} = \left\{\bar{x}_j : j = \overline{1, n}\right\}$ is called a fuzzy efficient solution to F-MOLFP if and only if there does not exists an $x \in \widetilde{M}_{PQ}$ and $\bigvee_{x \in \widetilde{H}_{PQ}} \tilde{Z}_{k_{PQ}}(x) \leq \tilde{Z}_{k_{PQ}}(\bar{x})$, where $\widetilde{H}_{PQ}$ is the set of all fuzzy efficient solutions and $\bigvee$ is the maximum.

Based on the close interval approximation of PQFN, the F-MOLFP is converted into the following CIA-MOLFP and MOLFP (i.e., (P_k)), respectively as

$$\max[Z_k(x)] = ([Z_1(x)], [Z_2(x)], \dots, [Z_K(x)]), K \geq 2$$

Subject to

$$[A]x \precsim [b],$$
$$x \geq 0.$$

Where, $[Z_k(x)] = \frac{[c_k]^T x + [\zeta_k]}{[d_k]^T x + [\xi_k]}, k = 1, 2, \dots, K,$ $[c_k] = \left(c^L_{k\alpha}, c^U_{k\alpha}\right), [d_k] = \left(d^L_{k\alpha}, d^U_{k\alpha}\right), [c\ \zeta_k], [\xi_k],$ $[b] = \left(b^L_\alpha, b^U_\alpha\right)$and $[A] = \left(A^L_\alpha, A^U_\alpha\right) \in P(\mathbb{R})$ ($P(\mathbb{R})$ is the set of PQFNs.

$$(P_k) \quad \max \widehat{Z}_k(x) = \left(\widehat{Z}_1(x), \widehat{Z}_2(x), \dots, \widehat{Z}_K(x)\right), \quad K \geq 2$$

Subject to

$$A^L_\alpha x \leq b^L_\alpha, A^U_\alpha x \leq b^U_\alpha,$$

$$x \geq 0.$$

4 | Solution Method

In this section, a method for finding the ideal and fuzzy efficient solution of the F-MOLFP is proposed as in the following steps:

Step1. Consider the F-MOLFP.

Step 2. Convert the F-MOLFP into the CIA-MOLFP and hence into (P_k).

Step 3. Construct " k" single objective from the (P_k).

Step 4. Solve each objective of the $(P_l), l = 1, 2, \dots, r, \dots, K$with respect to given constraints individually to obtain the optimal solution $X^*_l, l = 1, 2, \dots, K$ with the optimum value Z^*_l which is the ideal solution of the MOLFP.

Step 5. Use the optimal solution of problem (P_r), resulted from step4 in the problem $(P_l), l = 1, 2, \dots, r-1, r+1, \dots, K$.

Step 6. Repeat the setp5 for all $(P_l), l = 1, 2, \dots, r, \dots, K$ to obtain the efficient solution of the MOLFP.

5 | Numerical Example

Consider the following problem:

$$\max \tilde{Z}_{1PQ} = \frac{(1,4,7,10,12)x_1 \oplus (8,10,14,15,17)x_2 \oplus (1,2.5,4,7.5,11.5)x_3 \oplus (1,2,3,4,6)}{(10,14,20,22,24)x_1 \oplus (20,23.5,27.5,29,30)x_2 \oplus (16,18,20,25,28)x_3 \oplus (5,10,18,20,22)}.$$

$$\max \tilde{Z}_{2PQ} = \frac{(18,20,24,28,30)x_1 \oplus (14,16,18,25,30)x_2 \oplus (12,14,19,25,28)x_3 \oplus (0,1,6,10,12)}{(14,16,19,23,25)x_1 \oplus (16,18,21,25,27)x_2 \oplus (13,15,20,25,30)x_3 \oplus (8,10,15,20,25)}.$$

Subject to

$$(8,10,17,19,25)x_1 \oplus (12,14,16,22,24)x_2 \oplus (18,20,25,27,30)x_3$$
$$\leq (35,38,45,47,50),$$
$$(0.01,0.03,0,07,0.09,0.11)x_1 \oplus (0.03,0.05,0.08,0.1,0.15)\ x_2$$
$$\oplus (0.01,0.02,0.06,0.07,0.09)x_3 \leq (0.5,0.7,0.9163,.95,1.0),$$
$$(4,6,10,13,15)x_1 \oplus (0,5,10,15,18)x_2 \oplus (6,8,11,14,20)x_3 \leq (25,30,35,40,48),$$
$$x_1, x_2, x_3 \geq 0.$$

Using the close interval approximation of the PQFN, the problem (1) is converted into the following

$$\max Z_{CIA} = \frac{[4,10]x_1 \oplus [10,15]x_2 \oplus [2.5,7.5]x_3 \oplus [2,4]}{[14,22]x_1 \oplus [23.5,29.5]x_2 \oplus [18,25]x_3 \oplus [10,20]}.$$
$$\max Z_{2CIA} = \frac{[20,28]x_1 \oplus [16,25]x_2 \oplus [14,25]x_3 \oplus [1,10]}{[16,23]x_1 \oplus [18,25]x_2 \oplus [15,25]x_3 \oplus [10,20]}.$$

Subject to

$$[10,19]x_1 \oplus [14,22]x_2 \oplus [20,27]x_3 \leq [38,47],$$
$$[0.03,0.09]x_1 \oplus [0.05,0.1]\ x_2 \oplus [0.02,0.07]x_3 \leq [0.7,0.95],$$
$$[6,13]x_1 \oplus [5,15]x_2 \oplus [8,14]x_3 \leq [30,40],$$
$$x_1, x_2, x_3 \geq 0.$$

And hence

$$\max \widehat{Z}_1 = \frac{7x_1 + 12.5x_2 + 5x_3 + 3}{18x_1 + 32.5x_2 + 21.5x_3 + 15}.$$
$$\max \widehat{Z}_2 = \frac{24x_1 + 20.5x_2 + 19.5x_3 + 5.5}{19.5x_1 + 21.5x_2 + 20x_3 + 15}.$$

Subject to

$$10x_1 + 14x_2 + 20x_3 \leq 38,$$
$$19x_1 + 22x_2 + 27x_3 \leq 47,$$
$$0.03x_1 + 0.05\ x_2 + 0.07x_3 \leq 0.7,$$
$$0.09x_1 + 0.1\ x_2 + 0.07x_3 \leq 0.95,$$
$$6x_1 + 5x_2 + 8x_3 \leq 30,$$
$$13x_1 + 15x_2 + 14x_3 \leq 40,$$
$$x_1, x_2, x_3 \geq 0.$$

Step 4. $Z_1^* = 0.3518170$ at $x^* = (x_1^*, x_2^*, x_3^*) = (0, 2.136364, 0)$, and $Z_2^* = 1.02580$ at $x^* = (x_1^*, x_2^*, x_3^*) = (2.473684, 0, 0)$.

Step 5. Using the solution $x^* = (x_1^*, x_2^*, x_3^*) = (0, 2.136364, 0)$ into the second problem Z_2 with respect to the given constraints, we have $Z_2^* = 0.809903$. The efficient solution is $(\widehat{Z}_1, \widehat{Z}_2) = (0.3518170, 1.02580)$. similarly, use the solution $x^* = (x_1^*, x_2^*, x_3^*) = (2.473684, 0, 0)$ into Z_1 with respect to the given constraints, we obtain $Z_1^* = 0.341291$. The efficient solution is $(\widehat{Z}_1, \widehat{Z}_2) = (0.341291, 0.809903)$.

Therefore, the ideal solution is $(Z_1^*, Z_2^*) = (0.3518170, 1.02580)$, and the set of all efficient solution for $(\widehat{Z}_1, \widehat{Z}_2)$ are $(0.3518170, 1.02580)$ and $(0.341291, 0.809903)$.

In close interval approximation of piecewise quadratic fuzzy: the ideal solution is $(Z_1^*, Z_2^*) =$ ([0.281410,0.59872], [0.65640,1.59873]), and the set of all efficient solution for $(\widehat{Z}_1, \widehat{Z}_2)$ are ([0.281410,0.65640]: [0.59872,1.59873]) and ([0.15983,0.64387]: [0.465635,1.10225]).

6 | Comparative Study

In this section, the proposed approach is compared with some existing literature to illustrate the advantages of the proposed approach. *Table 1* investigates this comparison in the case of some parameters.

Table 1. Comparison of different researcher's contributions.

Author's Name	Multiobjective Programming	Hungarian Method	Optimal Solution	Environment
Simi and Talukder [15]	NO	NO	YES	Crisp set
Borza and Rambely [2]	NO	NO	YES	Crisp set
Xiao and Tian [18]	NO	NO	YES	Crisp set
Proposed approach	YES	YES	YES	Fuzzy Set

7 | Conclusions and Future Works

In this paper, a new algorithm for solving fuzzy multi- objective LFP has proposed which based purely on the Hungarian method. The idea of the algorithm is to find the ideal solution and the set of all efficient solutions. The advantage of it helps the DM to handle the real life problem. Therefore, future work may include the study of the problem in different uncertainty environment as neutrosophic, stochastic, etc.

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Machine Efficiency Measurement in Industry 4.0 Using Fuzzy Data Envelopment Analysis

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Abstract

Industry 4.0 implementations are competitive tools of recent production systems in which complex computerized systems are employed. Efficiency of these systems is generally measured by Data Envelopment Analysis (DEA) under certainty. However, the required data in modelling the system involve high degree of uncertainty, which necessitates the usage of fuzzy set theory. Fuzzy DEA models can successfully handle this problem and present efficient solutions for Industry 4.0 implementation. In this paper, efficiency of Industry 4.0 applications is measured by classical DEA and fuzzy DEA models, allowing the variables to have different units of measurement and to be independent from analytical production functions. Besides that, fuzzy algorithms for output-oriented DEA are proposed for BBC and CCR models. To the best of our knowledge, this article is the first quantitative academic study to measure the effects of Industry 4.0 applications on productivity. It also shows how fuzzy factors can affect decision-making by comparing fuzzy and classical DEA results. A real application of the models is realized in a company of home appliances manufacturing sector having Industry 4.0 applications. The effect of Industry 4.0 implementation on machine productivity, and superiority of fuzzy DEA over classical DEA are shown through the application.

Keywords: Industry 4.0, Fuzzy data envelopment analysis, Efficiency, Home appliance production.

1 | Introduction

Industry 4.0 concept, which is proposed firstly in 2011 by Kagermann et al. [8] and published as a manifesto in 2013 by ACATECH (German Academy of Science and Engineering), is a collective term that contains contemporary automation systems, data exchange, and production technologies. It describes production systems that accord the consumer requirements instantly and automation systems that stay in touch with each other continuously [17]. Industry 4.0 can be described as in which all units that are directly or indirectly associated with production, are worked each other having digital data, software, and information technologies integration [15]. According to Thames and Schaefer [16], smart systems that have autonomic self-properties will drive manufacturing ecosystems with the implementation of Industry 4.0, which causes an accelerated growth in productivity and unprecedented levels of operational efficiencies. Mrugalska and Wyrwicka [12] determined possible

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benefits of Industry 4.0 implementations as increasing the flexibility of business processes, elimination of failures in the demand chain, decision-making process optimization with instant end-to-end visibility, increased resource productivity, and efficiency, creating value opportunities, and reduction of energy and personal costs. Although there are many studies in the literature that mention the potential benefits of Industry 4.0 applications, no study has been found that examines existing applications and reveals these effects. To fill this gap in the literature, this study aims to perform an efficiency analysis to reveal the effects of current practices of Industry 4.0 applications.

Data Envelopment Analysis (DEA) is an adopted efficiency measurement system that has been researched and used for many years. DEA which can take place in different fields of application can also be used with fuzzy numbers in the application areas where it has unknown cases, obscurity, randomness. Measuring the efficiency of Industry 4.0 implementations involves high uncertainty with its unpredictable rapidly developing manner and its unmeasurable side effects. Industry 4.0 tools are frequently updated, and the gains it brings are constantly changing (increasing) and therefore it becomes difficult to measure these gains. The increase in the gains obtained by the combination of different applications of Industry 4.0 tools can also make measurement difficult and bring uncertainty in the evaluation process. The main aim of fuzzy logic and set theory is to model thinking and decision-making mechanisms in uncertain environments with inaccurate information [20]. Fuzzy sets enable analysts to define linguistic variables that have uncertain and ambiguous information to involve uncertainty in the analysis. Therefore, in this paper, it is decided to carry out a comprehensive study including DEA and fuzzy DEA to measure the efficiency of Industry 4.0 implementations for a real case. To be able to apply fuzzy algorithms for output-oriented DEA, fuzzy output-oriented CCR (Constant Return to Scale model) algorithm and fuzzy output-oriented BCC (Variable Return to Scale model) algorithm are proposed. In addition, the results of DEA and Fuzzy DEA methods are compared to determine the effect of uncertainty for Industry 4.0 implementations.

This paper will make important contributions to the literature by being the first quantitative academic study to measure the effects of Industry 4.0 applications on productivity. In addition, it shows how fuzzy factors can influence decision-making by comparing fuzzy and classical DEA results. The rest of the paper is organized as follows: In Section 2, the methods used in the paper are detailed. In Section 3, the information about the real case and the application details are given. Then the results of the application are discussed in Section 4 and the paper is concluded in Section 5.

2 | Literature Review

The subject of the efficiency of Industry 4.0 has been investigated by various authors. Most of the studies have been done in recent years and they are focused on the understanding and conceptualization of Industry 4.0. Costa et al. [6] simulated three different Industry 4.0 implementations in a flexible flow shop for production activity control. Liu et al. [10] proposed an industrial blockchain concept that integrates IoT, M2M, and efficient consensus algorithms and illustrated a blockchain-based application between the cooperating partners in four emerging product lifecycle stages. Cicconi and Raffaeli [5] determined Industry 3.0 technologies to support defect analysis for mechanical workpieces and proposed a knowledge-based tool to support the configurations of the quality control chain. Sorkun [15] investigated the Industry 4.0 enabling technologies in logistics operations using the fuzzy-total interpretative structure modeling. Dalmarco et al. [7] examined the challenges and opportunities of adopting Industry 4.0 from the perspective of technology provider companies using a research method based on interviews. Malik and Khan [11] presented an optimized IoT-based Job Shop Scheduler Monitoring System which will improve the efficiency of the whole shop floor. Kumar and Iyer [9] made exploratory research to explore the benefits of IIoT in engineering and manufacturing industries.

When efficiency analysis and Industry 4.0 are investigated together in the literature, it is seen that a limited number of studies have been carried out. Arora et al. [1] analyzed several factors hindering the growth of the agricultural supply chain and several industry 4.0 technologies. They presented a priority list that provides a ranking based on the relative efficiency of technologies' advances in addressing barriers. Woo et al. [19] investigated existing smart manufacturing and smart factories in the shipbuilding industry and developed a new framework for smart shipyard maturity level assessment using DEA. Pinheiro and Putnik [14] investigated the effects on the hierarchical structures of organizations to assess the possible benefits for the efficiency of the organizations resulting from the implementation of Industry 4.0.

In the literature review, it is seen that all papers focused on the effects of Industry 4.0 agreed that there will be an improvement in production systems with the utilization of Industry 4.0 tools, but no publication has been identified with efficiency impact analysis by empirical evaluations. Therefore, the main driver of this paper is to measure the effects of Industry 4.0 implementations on the manufacturing systems.

3 | Methodology

Efficiency measurement methods are mostly classified as ratio analysis, parametric techniques, and non-parametric techniques. In this study, DEA and FDEA which are non-parametric techniques, are used to determine the efficiency effect of Industry 4.0 applications.

3.1 | Data Envelopment Analysis

DEA which is one of the nonparametric methods is a structured method based on the principles of linear programming for measuring the relative effectiveness of units (DMUs) that are responsible for converting inputs into outputs. The efficiency of a Decision-Making Unit (DMU) in DEA is obtained by dividing the weighted sum of outputs by the weighted sum of inputs. The DMUs in which the objective function value is calculated to 1, are defined as active DMU, The DMUs in which the objective function value is not equal to 1, are defined as inactive DMU. The best DMUs form the efficiency frontier with the envelope algorithm and the effectiveness of other DMUs is measured according to their distance from this limit. In the envelope algorithm, DMU observations are drawn into an envelope by drawing an effective boundary with the use of algorithms and there is no observation beyond this limit [6] and[19]. Different DEA models are defined according to the way of enveloping and the distance from inactive units to the effective production limit [13].

According to the envelope type, there are two DEA models; Constant Return to Scale (CCR) model which was developed by Charnes et al. [4] in 1978, and Variable Return to Scale model which was developed by Banker et al. [2] (BCC) in 1984. According to the distance of inactive units to the effective production limit, there are two types of algorithms. The input-oriented model investigates how much input composition should be reduced to achieve the same output level most effectively without changing output level and the output-oriented model investigates how much output composition should be increased to achieve the same input level most effectively without changing input level.

3.2 | Fuzzy Data Envelopment Analysis

Data envelopment models are based on linear programming basics. The classical boundaries of DEA models that are created by classical mathematics are not able to overcome uncertain information, which could be clarified with fuzzy set theory, and in this way, the uncertainty could be included in the decision processes. When some of the observations are blurred, the objective function and constraints in the decision process become blurred. As data envelopment models are based on linear programming basics, fuzzy data envelopment problems are based on fuzzy linear programming techniques [21].

DEA methods that are using fuzzy set theory, are generally classified under five headlines in the literature, the tolerance approach, the fuzzy ranking approach, the possibility approach, the α-level-based approach, and Interval DEA (IDEA) approach. In this paper, the IDEA approach is determined as the efficiency measurement method to be used.

A pair of IDEA models are proposed by Wang et al. [18] for dealing with imprecise data such as interval data, ordinal preference information, fuzzy data, and their mixture. In these models, the efficiency scores are obtained as interval numbers and a minimax regret approach is used to rank the interval numbers.

To avoid the use of different efficient frontiers in measuring the effectiveness of different DMUs, the IDEA model is based on interval arithmetic, which is always using the same set of constraints (a unified and fixed efficient frontier), was developed.

As it is mentioned in Secion 2.1, four basic data envelopment model algorithms are input-oriented CCR, output-oriented CCR, input-oriented BCC, and output-oriented BCC. The classical and fuzzy models for these four DEA model algorithms are given below.

3.2.1 | Input oriented model – CCR input algorithm

In the crisp case, the model is given in *Eq. (1)* where E_0 represents the efficiency of o^{th} DMU, n represents the number of DMUs, m and s represent the number of inputs and outputs, respectively, u_r represents r^{th} output's weight value of o^{th} DMU, $v_i$represents i^{th} input's weight value of o^{th} DMU, x_{io} represents i^{th} input quantity of o^{th} DMU, y_{ro} represents r^{th} output quantity of o^{th} DMU:

$$
\begin{aligned}
& E_0 = \max \sum_{r=1}^{s} u_r y_{ro}, \\
& \text{s.t.} \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \qquad j = 1,2,\dots,n \\
& \sum_{i=1}^{m} v_i x_{io} = 1. \\
& v_i, u_r \ge 0 \qquad r = 1,2,\dots,s \qquad i = 1,2,\dots,m
\end{aligned}
\tag{1}
$$

In the fuzzy case of the model, the efficiency of o^{th} DMU is determined by an interval fuzzy number $\tilde{E}_o = [E_o^L, E_o^U]$ where E_o^L and E_o^U represent lower and upper levels of the efficiency of o^{th} DMU, respectively. The upper level of efficiency is calculated using *Eq. (2)* [18]:

$$
\begin{aligned}
& \max E_o^U = \frac{\sum_{r=1}^{s} u_r y_{ro}^U}{\sum_{i=1}^{m} v_i x_{io}^L}, \\
& \text{s.t.} \\
& E_j^U = \sum_{r=1}^{s} u_r y_{rj}^U - \sum_{i=1}^{m} v_i x_{ij}^L \le 0 . \quad j = 1, \dots, n \\
& u_r, v_i \ge \varepsilon \quad \forall\ r, i
\end{aligned}
\tag{2}
$$

where y_{ro}^U represents the upper bound of r^{th} output quantity of o^{th} DMU, x_{io}^L represents the lower bound of i^{th} input quantity of o^{th} DMU. The lower level of efficiency is calculated using *Eq. (3)* [18]:

$$
\begin{aligned}
& \max E_o^L = \frac{\sum_{r=1}^{s} u_r y_{ro}^L}{\sum_{i=1}^{m} v_i x_{io}^U}, \\
& \text{s.t.} \\
& E_j^U = \sum_{r=1}^{s} u_r y_{rj}^U - \sum_{i=1}^{m} v_i x_{ij}^L \le 0 . \quad j = 1, \dots, n \\
& u_r, v_i \ge \varepsilon \quad \forall\ r, I
\end{aligned}
\tag{3}
$$

where y_{ro}^L represents the lower bound of r^{th} output quantity of o^{th} DMU, x_{io}^U represents the upper bound of i^{th} input quantity of o^{th} DMU. As it is stated and proved in [19] using the same constraint for *Eq. (2)* and *Eq. (3)* determines a fixed production frontier for all the DMUs with fuzzy inputs and outputs.

3.2.2 | Output oriented model–CCR output algorithm

In the crisp case, the output-oriented CCR algorithm is given as in *Eq. (4)* as follows:

$$\begin{aligned}
&E_0 = \min \frac{\sum_{i=1}^{m} v_i x_{io}}{\sum_{r=1}^{s} u_r y_{ro}},\\
&\text{s.t.}\\
&\sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} \geq 0. \qquad j = 1,2, \dots, n\\
&v_i, u_r \geq \varepsilon. \qquad r = 1,2, \dots, s \qquad i = 1,2, \dots, m
\end{aligned} \tag{4}$$

In the literature review, it is seen that there is not any proposed algorithm for IDEA approach for output oriented CCR algorithm. Thus, the algorithms that are given in *Eq. (5)* and *Eq. (6)* are proposed for the output-oriented fuzzy CCR using the same assumptions of [18]. In the fuzzy case of the output-oriented CCR algorithm, the upper level of the efficiency of o^{th} DMU can be calculated using *Eq. (5)*:

$$\begin{aligned}
&\min E_o^U = \frac{\sum_{r=1}^{s} v_i x_{io}^U}{\sum_{i=1}^{m} u_r y_{ro}^L},\\
&\text{s.t.}\\
&E_j^U = \sum_{r=1}^{s} v_i x_{ij}^U - \sum_{i=1}^{m} u_r y_{rj}^L \leq 0. \qquad j = 1, \dots, n\\
&u_r, v_i \geq \varepsilon \qquad \forall\ r, i
\end{aligned} \tag{5}$$

The lower level of the efficiency of o^{th} DMU in fuzzy case of output-oriented CCR algorithm can be calculated using *Eq. (6)*:

$$\begin{aligned}
&\min E_o^L = \frac{\sum_{r=1}^{s} v_i x_{io}^L}{\sum_{i=1}^{m} u_r y_{ro}^U},\\
&\text{s.t.}\\
&E_j^U = \sum_{r=1}^{s} v_i x_{ij}^U - \sum_{i=1}^{m} u_r y_{rj}^L \leq 0\,. \qquad j = 1, \dots, n\\
&u_r, v_i \geq \varepsilon \qquad \forall\ r, i
\end{aligned} \tag{6}$$

In *Eq. (5)* and *Eq. (6)*, the same constraint is used to determine a fixed upper bound for the inputs.

3.2.3 | Input oriented model–BCC input algorithm

In the crisp case, input-oriented BCC algorithm is given in *Eq. (7)* where u_o represents unrestricted variable of o^{th} DMU.

$$\begin{aligned}
&E_0 = \max \frac{\sum_{r=1}^{s} u_r y_{ro} - u_o}{\sum_{i=1}^{m} v_i x_{io}},\\
&\text{s.t.}\\
&\sum_{r=1}^{s} u_r y_{rj} - u_o - \sum_{i=1}^{m} v_i x_{ij} \leq 0. \qquad j = 1,2, \dots, n\\
&v_i, u_r \geq \varepsilon \qquad r = 1,2, \dots, s \qquad i = 1,2, \dots, m
\end{aligned} \tag{7}$$

In the fuzzy case, same as previously determined algorithms first lower and upper levels of the efficiency of o^{th} DMU are calculated using *Eq. (8)* and *Eq. (9)*, respectively:

$$\max E_o^U = \frac{\sum_{r=1}^{s} u_r y_{ro}^U - u_o}{\sum_{i=1}^{m} v_i x_{io}^L},$$
$$\text{s.t.} \tag{8}$$
$$E_j^U = \sum_{r=1}^{s} u_r y_{rj}^U - u_o - \sum_{i=1}^{m} v_i x_{ij}^L \leq 0. \qquad j = 1, \dots, n$$
$$u_r, v_i \geq \varepsilon \quad \forall\ r, i$$

$$\max E_o^L = \frac{\sum_{r=1}^{s} u_r y_{ro}^L - u_o}{\sum_{i=1}^{m} v_i x_{io}^U},$$
$$\text{s.t.} \tag{9}$$
$$E_j^U = \sum_{r=1}^{s} u_r y_{rj}^U - u_o - \sum_{i=1}^{m} v_i x_{ij}^L \leq 0. \qquad j = 1, \dots, n$$
$$u_r, v_i \geq \varepsilon \quad \forall\ r, i$$

3.2.4 | Output oriented model – BCC output algorithm

In the crisp case, output-oriented BCC algorithm is given in *Eq. (10)* where v_o represents unrestricted variable of o^{th} DMU.

$$E_0 = \min \frac{\sum_{i=1}^{m} v_i x_{io} - v_o}{\sum_{r=1}^{s} u_r y_{ro}},$$
$$\text{s.t.} \tag{10}$$
$$\sum_{i=1}^{m} v_i x_{ij} - v_o - \sum_{r=1}^{s} u_r y_{rj} \geq 0. \qquad j = 1,2, \dots, n$$
$$v_i, u_r \geq \varepsilon \qquad r = 1,2, \dots, s \qquad i = 1,2, \dots, m$$

To the best of our knowledge, there is not any paper that proposed algorithms for output oriented fuzzy BCC algorithms in the literature. The upper level of the efficiency of o^{th} DMU in the fuzzy case of output-oriented BCC algorithm can be calculated using *Eq. (11)*.

$$\min E_o^U = \frac{\sum_{r=1}^{s} v_i x_{io}^U - v_o}{\sum_{i=1}^{m} u_r y_{ro}^L},$$
$$\text{s.t.} \tag{11}$$
$$E_j^U = \sum_{r=1}^{s} v_i x_{ij}^U - v_o - \sum_{i=1}^{m} u_r y_{rj}^L \leq 0\,. \qquad j = 1, \dots, n$$
$$u_r, v_i \geq \varepsilon \quad \forall\ r, i$$

Eq. (12) shows the calculation of the lower level of the efficiency of o^{th} DMU in the fuzzy case of output-oriented BCC algorithm.

$$\min E_o^L = \frac{\sum_{r=1}^{s} v_i x_{io}^L - v_o}{\sum_{i=1}^{m} u_r y_{ro}^U},$$
$$\text{s.t.} \tag{12}$$
$$E_j^U = \sum_{r=1}^{s} v_i x_{ij}^U - v_o - \sum_{i=1}^{m} u_r y_{rj}^L \leq 0\,. \qquad j = 1, \dots, n$$
$$u_r, v_i \geq \varepsilon\,. \quad \forall\ r, i$$

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In the evaluation of the efficiency of the interval data, in which the final efficiency scores for each DMU are defined by an interval, a minimax regret-based approach that is proposed by Wang et al. [18] is used to sort and compare the effectiveness of the different DMUs.

Input oriented. The maximum efficiency loss of all DMUs is calculated using *Eq. (13)*. Relative lowest efficient DMU with the highest efficiency loss is obtained with DMU removal iterations of lowest efficiency loss.

$$\mathrm{R(i.\,DMU)} = \max\left[\max\left(\text{Other DMUs' upper level efficiency}\left(\Theta_k^U\right)\right) - \text{i. DMU's lower level efficiency}(\Theta_k^L), 0\right]. \tag{13}$$

Output oriented. The maximum efficiency loss of all DMUs is calculated using *Eq. (14)*. Relative highest efficient DMU with the highest efficiency loss is obtained with DMU removal iterations of lowest efficiency loss.

$$\mathrm{(i.\,DMU)} = \max\left[\max\left(\text{Other DMUs' lower level efficiency}\left(\Theta_k^L\right)\right) - \text{i. DMU's upper level efficiency}(\Theta_k^U), 0\right]. \tag{14}$$

4 | Application in Home Appliances Sector

In this study, 44 machines of a company that is the leader of the sector according to many indicators including market share are examined. The factory to be examined within the scope of the research belongs to a group of companies, which supplies innovative white goods in 43 factories to consumers through 80 companies in 48 countries.

From the end of 2018, the company has started to implement the Industry 4.0 applications for the machines that were identified as bottlenecks (production capacity constraint). The company has decided to invest in the "Instantaneous System Condition Monitoring" and "Personnel Warning Systems" for the Preventive Maintenance Activity of the 3 of 44 machines that have the production capacity constraint. For a single machine 52 analog pressure sensors, 10 analog temperature sensors, 5 pieces of oil level sensors, 16 pcs laser distance sensors and 1 optical sensor are added. The working conditions of the machines are monitored instantaneously with the advanced precision measurement systems and the algorithms that are created specifically for the machine are provided to warn the machine without any downtimes (within the scope of the algorithm) by foreseeing the possible causes of stopping. In this study, machines (DMUs) that have Industry 4.0 applications are analyzed and denoted as M5, M12, and M26.

To avoid the effects of pandemics and to find out the real effect of Industry 4.0 implementations in the manufacturing process, data of 2018 and 2019 are used for the analysis.

4.1 | Definition of Variables

In the literature, a mathematical study on measuring the effects of Industry 4.0 applications could not be determined, however, Turanoglu Bekar and Kahraman [3] examined total productive maintenance by FDEA. The variables used in this literature (denoted with "*" in the tableau) is expanded and final input and output variables are determined in *Table 1* as follows:

Table 1. Input and output variables.

Inputs Variable No.	Variable Description	Unit
X1	Average Machine Operator Quantity	Unit
X2	Workpiece Technical Complexity Value	Between 1-10
X3	Workpiece Managerial Complexity Value	Between 1-10
X4	Machine Operator absenteeism Rate *	(%) Percent
X5	Machine Operator Turn-Over Rate *	(%) Percent
X6	1 / Machine Operator Working Year *	1/Year
X7	100 – Operator New Ideas Generated and Implemented *	Between 1-100
X8	100 – Level of 5S Point *	Between 1-100
X9	Availability of Maintenance Personnel *	Between 1-100
X10	Competence of Maintenance Personnel *	Between 1-100
Outputs Variable No.	**Variable Description**	**Unit**
Y1	Machine Available Working Time	Hour/Month
Y2	Machine OEE(Overall Equipment Effectiveness) Rate	(%) Percent
Y3	Workpiece Quantity Per Hour	Piece/Hour
Y4	Machine Autonomous Maintenance Level	Between 1-100
Y5	100 – Machine Work Accident Quantity *	100-Unit
Y6	100 / Machine Average Breakdown Quantity *	100/Unit
Y7	100 – Machine Mean Time to Repair (MTTR) *	100-Minute
Y8	Mean Time Between Failure (MTBF) *	Hour

In DEA, unit differences are eliminated by using weights according to the minimization of inputs and maximization of outputs. By this information, the variables to be used in the study were selected and their units were scaled.

Data control. In the study, Cronbach Alpha reliability analysis is performed in terms of internal consistency of statistical attitude scale. Reliability in terms of internal consistency is made to determine whether a single measurement tool measures the psychological conceptual structure in a consistent way by making a single application [22]. For the 2018 data set, Cronbach's alpha is found as 0.790, Cronbach's alpha based on standardized items is found as 0.895 for 18 items. In the 2019 data set, Cronbach's alpha is found as 0.744, Cronbach's alpha based on standardized items is found as 0.745 for 18 items.

4.2 | Classical Data Envelopment Analysis

In this subsection, four basic classical DEA algorithms are applied to data. The results of M5, M12, and M26 which are the Industry 4.0 machines are detailed to point out the effect of Industry 4.0 implementations.

4.2.1 | Classical data envelopment analysis: CCR–input

CCR-input-oriented classical DEA results and efficiency variations of the machines for 2018 and 2019 are given in *Table 2*.

Table 2. CCR – input-oriented classical DEA results and efficiency variations.

2018 - CCR-I Model Classical DEA Result		2019 - CCR-I Model Classical DEA Result		Classical DEA CCR-I Model 2018 - 2019 Efficiency Variation	
DMU	Efficiency rate	DMU	Efficiency rate	DMU	Efficiency rate
M5	62,9%	M5	100%	M5	37,1%
M12	66,4%	M12	92,4%	M12	26,0%
M26	77,1%	M26	100,0%	M26	22,9%
All machines average	92,4%	All machines average	97,8%	All machines average variation	5,4%

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According to the results, in 2019 the average relative efficiency of 44 machines increased by 5.4%. Machines M5, M12, and M26 are Industry 4.0 machines, their relative efficiency increased by 37.1%, 26%, and 22.9% respectively, and in 2019, M5 and M26 passed to the active border.

4.2.2 | Classical data envelopment analysis: CCR–output

CCR–output-oriented classical DEA results and efficiency variations of the machines for 2018 and 2019 are given in *Table 3*.

Table 3. CCR – output-oriented classical DEA results and efficiency variations.

2018 - CCR-O Model Classical DEA Result		2019 - CCR-O Model Classical DEA Result		Classical DEA CCR-O Model 2018 - 2019 Efficiency Variation	
DMU	Efficiency rate	DMU	Efficiency rate	DMU	Efficiency rate
M5	159,1%	M5	100,0%	M5	59,1%
M12	150,7%	M12	108,2%	M12	42,5%
M26	129,7%	M26	100,0%	M26	29,7%
All machines average	109,6%	All machines average	102,4%	All machines average variation	7,2%

According to these results in 2019, the average relative efficiency of 44 machines increased by 7.2%. Machines M5, M12, and M26 are Industry 4.0 applied machines and their relative efficiency increased by 59.1%, 42.5%, and 29.7% respectively. In 2019, M5 and M26 passed to the active border.

4.2.3 | Classical data envelopment analysis: BCC–input

BCC – input-oriented classical DEA results and efficiency variations of the machines for 2018 and 2019 are given in *Table 4*.

Table 4. CCR – input-oriented classical DEA results and efficiency variations.

2018 - BCC-I Model Classical DEA Result		2019 BCC-I Model Classical DEA Result		Classical DEA BCC-I Model 2018 - 2019 Efficiency Variation	
DMU	Efficiency rate	DMU	Efficiency rate	DMU	Efficiency rate
M5	64,8%	M5	100,0%	M5	35,2%
M12	66,4%	M12	92,4%	M12	26,0%
M26	78,1%	M26	100,0%	M26	21,9%
All machines average	94,1%	All machines average	98,3%	All machines average variation	4,2%

According to these results in 2019, the average relative efficiency of 44 machines increased by 4.2%. Machines M5, M12, and M26 are Industry 4.0 machines and their relative efficiency increased by 35.2%, 26%, and 21.9% respectively. In 2019, M5 and M26 passed to the active border.

4.2.4 | Classical data envelopment analysis: BCC–output

BCC–output-oriented classical DEA results and efficiency variations of the machines for 2018 and 2019 are given in *Table 5*.

According to these results in 2019, the average relative efficiency of 44 machines increased by 0.2%. Machines M5, M12, and M26 are machines with Industry 4.0 applications. According to the 2018 BCC-O model, the efficiency of the inactive M5 increased by 3.1% in 2019 and passed to the active border.

Table 5. BCC – output-oriented classical DEA results and efficiency variations.

2018 - BCC-O Model Classical DEA Result		2019 - BCC-O Model Classical DEA Result		Classical DEA BCC-O Model 2018 - 2019 Efficiency Variation	
DMU	Efficiency rate	DMU	Efficiency rate	DMU	Efficiency rate
M5	103,1%	M5	100,0%	M5	3,1%
M12	100,0%	M12	100,0%	M12	0,0%
M26	100,0%	M26	100,0%	M26	0,0%
All machines average	100,3%	All machines average	100,1%	All machines average variation	0,2%

4.3 | Fuzzy Data Envelopment Analysis

In this subsection, four basic fuzzy IDEA algorithms are applied to data. To fuzzify the crisp data, for quantitative variables (X1, X4, X5, X6, X7, Y1, Y2, Y3, Y5, Y6, Y7, and Y8) one standard deviation is subtracted and added to DMU value and for qualitative data (X2, X3, X8, X9, X10, and Y4) 10% of average is subtracted and added to DMU value to determine lower and upper boundary data, respectively. To measure the efficiency with IDEA type, upper and lower limit data must be adapted to the defined equations. For this purpose, the upper limit values of the output data and the lower limit values of the input data are used for the upper boundary activity equation. When measuring the lower limit activity values, the lower limit values of the output data and the upper limit values of the input data are used.

4.3.1 | Fuzzy data envelopment analysis: CCR–input oriented analysis results

CCR–input-oriented FDEA results and relative efficiency changes for 2018 and 2019 are given in *Table 6*.

Table 6. CCR – input-oriented FDEA results and efficiency changes.

	2018 - FDEA CCR-I Results		2019 - FDEA CCR-I Results		FDEA CCR-I 2018 - 2019 Efficiency Variation		
DMU	Lower bound technical efficiency rate	Upper bound technical efficiency rate	Lower bound technical efficiency rate	Upper bound technical efficiency rate	Lower bound technical efficiency variation	Upper bound technical efficiency variation	Min. variation
M5	66.3%	59.2%	99.8%	99.8%	33.6%	40.6%	33.6%
M12	69.8%	62.0%	93.0%	91.8%	23.2%	29.7%	23.2%
M26	79.4%	73.9%	100.0%	100.0%	20.6%	26.1%	20.6%
All machines average	93.9%	89.9%	98.3%	96.6%	4.4%	6.7%	4.4%

In 2018, the machines with the lowest relative activity of 44 machines are respectively; M5, M12, M39 and M26. As mentioned at the beginning of the study, M5, M12, and M26 are also the machines that are applied to Industry 4.0 applications as of 2019.

In 2019, the machines with the lowest relative efficiency of 44 machines are M3 and M39. M5, M12, and M26 were removed from the list of the lowest efficient machines. Even, M26 which was one of the lowest efficiency machines is at the effective boundary according to the FDEA CCR-I model as of 2019. As can be seen in M5 and M12 machines, efficiency improvement is 33.6% and 23.2% respectively.

4.3.2 | Fuzzy data envelopment analysis: CCR-output oriented analysis results

CCR–output-oriented FDEA results and relative efficiency changes for 2018 and 2019 are given in *Table 7*.

Table 7. CCR – input oriented classical DEA results and efficiency variations.

	2018 - FDEA CCR-I Results		2019 - FDEA CCR-I Results		FDEA CCR-I 2018 - 2019 Efficiency Variation		
DMU	Lower bound technical efficiency rate	Upper bound technical efficiency rate	Lower bound technical efficiency rate	Upper bound technical efficiency rate	Lower bound technical efficiency variation	Upper bound technical efficiency variation	Min. variation
M5	150.9%	168.9%	100.2%	100.2%	50.7%	68.7%	50.7%
M12	143.3%	161.2%	107.5%	109.0%	35.7%	52.2%	35.7%
M26	125.9%	135.2%	100.0%	100.0%	25.9%	35.2%	25.9%
All machines average	107.6%	113.4%	101.9%	103.8%	5.7%	9.6%	5.7%

In 2018, the machines with the lowest relative activity of 44 machines are respectively; M5, M12, M39, and M26.

In 2019, the machines with the lowest relative efficiency of 44 machines are M3 and M39. M5, M12, and M26 were removed from the list of the lowest efficient machines. Even, M26 which is one of the lowest efficiency machines in 2018, is at the effective boundary according to the FDEA CCR-O model as of 2019. As can be seen in M5 and M12 machines, efficiency improvement is 50.7% and 35.7%, respectively.

4.3.3 | Fuzzy data envelopment analysis: BCC-input oriented analysis results

BCC-input oriented FDEA results and relative efficiency changes for 2018 and 2019 are given in *Table 8*.

Table 8. BCC-input oriented FDEA results and efficiency changes.

	2018 FDEA BCC-I Results		2019 FDEA BCC-I Results		Relative Efficiency Variation		
DMU	Lower bound technical efficiency rate	Upper bound technical efficiency rate	Lower bound technical efficiency rate	Upper bound technical efficiency rate	Lower bound technical efficiency variation	Upper bound technical efficiency variation	Min. variation
M5	68.3%	60.4%	99.8%	99.8%	31.5%	39.4%	31.5%
M12	69.8%	62.0%	93.0%	91.8%	23.2%	29.7%	23.2%
M26	80.3%	75.7%	100.0%	100.0%	19.7%	24.3%	19.7%
All machines average	95.1%	91.5%	98.5%	97.9%	3.4%	6.4%	3.4%

In 2018, the machines with the lowest relative activity of 44 machines are respectively; M5, M12, M39, and M26.

In 2019, the machines with the lowest relative efficiency of 44 machines are M3 and M25. M5, M12, and M26 were removed from the list of the lowest efficient machines. Even, M26 which is one of the lowest efficiency machines in 2018, is at the effective boundary according to the FDEA BCC-I model as of 2019. As can be seen in M5 and M12 machines, efficiency improvement is 31.5% and 23.2% respectively.

4.3.4 | Fuzzy data envelopment analysis: BCC-output oriented analysis results

BCC – output-oriented FDEA results and relative efficiency changes for 2018 and 2019 are given in *Table 9*.

Table 9. BCC – output-oriented classical DEA results and efficiency variations.

	2018 FDEA BCC-O Results		2019 FDEA BCC-O Results		Relative Efficiency Variation		
DMU	Lower bound technical efficiency rate	Upper bound technical efficiency rate	Lower bound technical efficiency rate	Upper bound technical efficiency rate	Lower bound technical efficiency variation	Upper bound technical efficiency variation	Min. variation
M5	103.1%	102.0%	100.0%	100.0%	3.1%	2.02%	2.02%
M12	100.0%	100.0%	100.0%	100.0%	0.0%	0.00%	0.00%
M26	100.0%	100.0%	100.0%	100.0%	0.0%	0.00%	0.00%
All machines average	100.3%	100.1%	100.1%	100.0%	0.2%	0.04%	0.04%

In 2018, the machines with the lowest relative activity of 44 machines are respectively; M5, M34, and M39.

In 2019, the machines with the lowest relative efficiency of 44 machines are M3, M7, and M34. As of 2019, the M5 which has the lowest relative efficiency in 2018, is at the effective boundary according to the FDEA BCC-O model. M5 has a 2.02% efficiency improvement.

In addition, the sensitivity analysis is performed. It is seen that any of the input parameters and output parameters do not change the lowest efficient DMU for all of the algorithms used in the study when a single variable is removed.

5 | Discussions

In determining the efficiency measurement method of this study, there are three basic parting of the ways: efficiency analysis method, logic type selection (crisp, fuzzy), and working algorithm selection.

Method selection; has been carried out comprehensively because it contains unknowns and it has not been specialized in both theoretical and practical studies yet. As a result of the method search, it is decided to work with Interval type FDEA studies. To reveal the effect of the unknown, classical DEA studies are conducted and it is found appropriate to compare these two methods. Thus, it is aimed to reveal the exact efficiency effect.

Classical DEA results. According to the classical DEA results of 2018 and 2019, the efficiency of 44 machines shows an average improvement of at least 4.2% for all machines (The output-oriented BCC algorithm can be excluded because it cannot clearly show the relative separation of machine-specific relative to data). There are inherent differences in input-output-oriented or CRR-BCC enveloping algorithms. In the Classical DEA, the M5 and M26 machines have moved to the efficient frontier, regardless of the model.

Relative productivity of M12 is increased minimum of 26%. From 2018 to 2019, the efficiency increase for these three machines is between 21.9% and 59.1%.

FDEA results. According to the FDEA results of 2018 and 2019, the efficiency of 44 machines shows an average improvement of at least 3.4% for all machines (The output-oriented BCC algorithm can be excluded because it cannot clearly show the relative separation of machine-specific relative to data). There are inherent differences in input-output-oriented or CRR-BCC enveloping algorithms. In the FDEA, the M26 has moved to the efficient frontier, regardless of the model.

Relative productivity of M5 and M12 is increased minimum of 31.5% and 23.2% respectively. From 2018 to 2019, the efficiency increase for these three machines is between 19.7% and 50.7%.

Classical DEA-FDEA comparison-the uncertainty factor. According to the results of FDEA and Classic DEA, M5, M12, and M26 machines which have been applied to Industry 4.0 applications have a minimum 19.7% increase in 2019, regardless of the DEA model.

According to the data of 2018, the relative lowest efficiencies are M5, M12, M39, M26, respectively, regardless of the Classic, FDEA, or model.

According to 2019 data, the lowest relative efficiency is in the M3, M39, M25 machines, although they differ according to the Classic, FDEA models.

According to Classical DEA of 2019, the M5 and M26 which have Industry 4.0 applications, are among the most efficient machines with the highest efficiency in the efficient frontier with the highest efficiency. This situation applies only to M26 in FDEA. M5 and M12 are not within the active boundary (efficient frontier) even if their efficiency increased minimum of 23.2%. The fact that the full event status of any machine is different in FDEA and Classic DEA is expected in terms of application. Similar differences can be observed in M23, M24, M32, M33. These differences are occurred due to the uncertainty factor. Because classical DEA does not contain uncertainty, it has worked with crisp sets. However, FDEA (IDEA) works with interval data so it contains limited uncertainty that is converting qualitative comments to quantitative.

6 | Conclusion

Industry 4.0 studies require costly investments. This factor is also the basis of the lack of implementation. For this reason, companies want to make a detailed feasibility analysis and especially to see the effects of pilot applications before implementation. In this study, an application road map for efficiency impact analysis of Industry 4.0 applications is presented in the home appliance manufacturing sector, which can be used to find out the real effect of the applications.

As stated at the beginning of the study, Industry 4.0 applications contain high uncertainty. It is also understood that the uncertainty factor is critical to have successful implementations of new technologies. In the case of analysis involving uncertainties, the decision to conclude with the classical DEA will result in an effective interpretation of an inactive (M5) DMU as in practice. In this paper, it is shown that FDEA studies have more clear results in the areas of uncertainty and categorical evaluations.

The other objective of this study is to compare input and output-oriented DEA algorithms. The paper shows that these algorithms have similar results, but the order of the DMUs may differ. The limitation of the study is that the effects of constantly changing/developing industry 4.0 applications are measured with discrete data. Despite this, DEA can show the positive effect of these practices on productivity. For further research, it is suggested to analyze the results of Industry 4.0 implementations from a broader perspective including the effects on demand, cost of production, and customer satisfaction. It is clear how Industry 4.0 practices will affect our use of resources. The research can also be expanded by working with different sectors and machine groups.

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Conflicts of Interest

The authors state no conflict of interest.

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