E-ISSN: 2717-3453 | P-ISSN: 2783-1442

Journal of Fuzzy Extension and Applications

www.journal-fea.com

J. Fuzzy. Ext. Appl. Vol. 1, No. 4 (2020) 304–313.

J. Fuzzy. Ext. Appl

Paper Type: Research Paper

Some Similarity Measures of Rough Interval Pythagorean Fuzzy Sets

V. S. Subha[1],* , P. Dhanalakshmi[2]

[1] Department of Mathematics, Bharathidasan University, Tamilnadu, India; dharshinisuresh2002@gmail.com.

[2] Annamalai University, Chidambaram-608002, India; vpdhanam83@gmail.com.

Citation:

Subha, V. S., & Dhanalakshmi, P. (2020). Some similarity measures of rough interval pythagorean fuzzy sets. *Journal of fuzzy extension and application, 1* (4), 304-313.

Received: 07/08/2020 Reviewed: 13/09/2020 Revised: 21/10/2020 Accept: 09/11/2020

Abstract

The purpose of this study is to propose new similarity measures namely cosine, jaccard and dice similarity measures. The weighted cosine, weighted jaccard and weighted dice similarity measures has been also defined. Some of the important properties of the defined similarity measures and weighted similarity measures have been established. We develop a new multi attribute decision making problem based on the proposed similarity measures. To demonstrate the applicability, a numerical example is solved.

Keywords: Interval valued fuzzy set, Pythagorean fuzzy set, Rough set, Cosine similarity measure, Jaccard similarity measure, Dice similarity measure.

1 | Introduction

The concept of fuzzy set was introduced by Zadeh [15] in his classic paper in 1965 and has been applied to many branches in mathematics. Later Zadeh [14] also introduced the concept of interval valued fuzzy set by considering the values of membership functions as the intervals of numbers instead of the numbers alone. The notion of rough set theory was proposed by Pawlak [7]. The concept of rough set theory is an extension of crisp set theory for the study of intelligent systems characterized by inexact, uncertain or insufficient information. Dubois and Prade [3] were introduced the concept of rough fuzzy set. This theory was found to be more useful in decision making and medical diagnosis problems. A similarity measure is an important tool for determining the degree of similarity between two objects. Similarity measures between fuzzy sets is an important content in fuzzy mathematics. Yager [13] examined Pythagorean fuzzy set characterized by a membership degree and a non-membership degree that satisfies the case in which the square sum of its membership degree and non-membership degree is less than or equal to one.

Corresponding Author: dharshinisuresh2002@gmail.com

10.22105/JFEA.2021.281500.1061

Peng and Yang [9] introduced the concept of interval Pythagorean fuzzy sets which is a generalization of Pythagorean fuzzy sets and interval valued fuzzy sets. Hussain et al. [2] introduced the concept of rough Pythagorean fuzzy sets. The Pythagorean fuzzy set has been investigated from different perspectives, including decision-making technologies [8], medical diagnosis [10], and transportation problem [6]. In particular, an extension of Pythagorean fuzzy set, named Interval-Valued Pythagorean Fuzzy Sets in decision making [8], complex Pythagoren fuzzy set in pattern recognition [12].

To facilitate our discussion, the remainder of this paper is organized as follows. In Section 2 we review some fundamental conceptions rough sets, interval valued fuzzy sets, Pythgorean fuzzy sets. In Section 3 we propose cosine similarity measure of rough interval Pythagorean fuzzy sets and some properties of this similarity measure discussed. Sections 4 and 5 deals with jaccard, dice similarity measures. In Section 6 we present algorithm for proposed measures. Section 7 deals with numerical example of proposed measures.

2| Basic Concepts

In this section we list some basic concepts.

Definition 1. Let x be a nonempty set. A mapping $\tilde{\Omega}: x \to D[0,1]$ is called an interval valued fuzzy subset of x, where $\tilde{\Omega}(x) = [\Omega^-(x), \Omega^+(x)], x \in \mathrm{X}$, and Ω^- and Ω^+ are the fuzzy subets in X such that $\Omega^-(x) \leq \Omega^+(x)$ $x \in \mathrm{X}$. $D[0,1]$ denotes the set of closed subsets of [0,1].

Definition 2. [5]. Let ϑ be a congruence relation on X. Le Λ t be any nonempty subset of X. The sets $\underline{\vartheta}(\Lambda) = \{x \in X/[x]_\vartheta \subseteq \Lambda\}$ *and* $\overline{\vartheta}(\Lambda) = \{x \in X/[x]_\vartheta \cap \vartheta \neq \emptyset\}$ are called the lower and upper approximations of Λ. Then $\vartheta(\Lambda) = (\underline{\vartheta}(\Lambda), \overline{\vartheta}(\Lambda))$ is called rough set in $(\mathrm{X}, \vartheta) \Longleftrightarrow \underline{\vartheta}(\Lambda) \neq \overline{\vartheta}(\Lambda)$.

Definition 3. [3]. Let ϑ be an congruence relation on X. Let Λ fuzzy subset of X. The upper and lower approximations of Λ defined by $\overline{\vartheta}(\Lambda)(x) = \bigvee_{a \in [x]_\vartheta} \Lambda(a)$ and $\underline{\vartheta}(\Lambda)(x) = \bigwedge_{a \in [x]_\vartheta} \Lambda(a)$. $\vartheta(\Lambda) = (\underline{\vartheta}(\Lambda), \overline{\vartheta}(\Lambda))$ is called a rough fuzzy set of Λ with respect to ϑ if $\underline{\vartheta}(\Lambda) \neq \overline{\vartheta}(\Lambda)$.

Definition 4. [4]. Let $\tilde{\Omega}$ be an interval-valued fuzzy subset of X and let ϑ be the complete congruence relation on X. Let $\underline{\vartheta}(\tilde{\Omega})$ and $\overline{\vartheta}(\tilde{\Omega})$ be the interval-valued fuzzy subset of X defined by, $\underline{\vartheta}(\tilde{\Omega})(n) = \wedge_{n \in [y]_\vartheta} \tilde{\Omega}(n)$ and $\overline{\vartheta}(\tilde{\Omega})(n) = \vee_{n \in [y]_\vartheta} \tilde{\Omega}(n)$. Then $\vartheta(\tilde{\Omega}) = \left(\underline{\vartheta}(\tilde{\Omega}), \overline{\vartheta}(\tilde{\Omega})\right)$ is called an interval-valued rough fuzzy subset of X if $\underline{\vartheta}(\tilde{\Omega}) \neq \overline{\vartheta}(\tilde{\Omega})$.

Definition 5. [1]. Let X be a nonempty set then an Intutionistic fuzzy set can be defined as $\Lambda\Omega = \left\{\left(x, \mu_{\Omega}(x), \gamma_{\Omega_\Lambda}(x)\right)/x \in \mathrm{X}\right\}$ where $\mu_{\Omega_\Lambda}(x)$ *and* $\gamma_\Lambda(x)$ are mapping from X to [0,1] also $0 \leq \mu_{\Omega_\Lambda}(x) \leq 1, 0 \leq \gamma_{\Omega_{\Lambda_\Lambda}}(x) \leq 1, 0 \leq \mu_{\Omega_\Lambda}(x) + \gamma_{\Omega_\Lambda}(x) \leq 1$ for all $x \in \mathrm{X}$ and represent the degrees of membership and non-membership of element $x \in \mathrm{X}$ to set X.

Definition 6. [11]. Let X be a nonempty set then an Pythagorean fuzzy set can be defined as $\Omega = \left\{\left(x, \mu_\Omega(x), \gamma_{\Omega_\Lambda}(x)\right)/x \in \mathrm{X}\right\}$ where $\mu_\Omega(x)$ *and* $\gamma_{\Omega_\Lambda}(x)$ are mapping from X to [0,1] also $0 \leq \mu_{\Omega_\Lambda}(x) \leq 1, 0 \leq \gamma_\Omega(x) \leq 1, 0 \leq \mu_\Omega{}^2{}_\Lambda(x) + \gamma_\Omega{}^2{}_\Lambda(x) \leq 1$ for all $x \in \mathrm{X}$, and represent the degrees of membership and non membership of element $x \in \mathrm{X}$ to set X.

Definition 7. [7]. Let X be a non-empty set then an Interval Pythagorean fuzzy set can be defined as follows $\tilde{\Omega} = \left\{\left(x, \mu_{\tilde{\Omega}}(x), \gamma_{\tilde{\Omega}}(x)\right)/x \in \mathrm{X}\right\}$ where $\mu_{\tilde{\Omega}}(x) = [\mu_{\tilde{\Omega}}{}^-(x), \mu_{\tilde{\Omega}}{}^+(x)]$ and $\gamma_{\tilde{\Omega}}(x) = [\gamma_{\tilde{\Omega}}{}^-(x), \gamma_{\tilde{\Omega}}{}^+(x)]$ are the intervals in [0,1] also $0 \leq (\mu^+{}_{\tilde{\Omega}}(x))^2 + (\gamma^+{}_{\tilde{\Omega}}(x))^2 \leq 1$.

3| Cosine Similarity Measures (CSM) of Rough Interval Pythagorean Fuzzy (RIPF) Sets.

In this section we introduce the notion of CSM of *RIPF* sets also discuss some properties of *RIPF* sets. Also weighted CSM of *RIPF* sets are discussed.

Definition 8. Let X be a nonempty set. Let $\tilde{\Omega} = \{(n, \mu_{\tilde{\Omega}}(n), \gamma_{\tilde{\Omega}}(n))/n \in X\}$ be a pythagorean fuzzy set of X. Then rough interval Pythagorean fuzzy set is defined as $\vartheta(\tilde{\Omega}) = (\underline{\vartheta}(\tilde{\Omega}), \overline{\vartheta}(\tilde{\Omega}))$ where

$\underline{\vartheta}(\tilde{\Omega}) = \{\langle n, \underline{\vartheta}(\mu_{\tilde{\Omega}}), \underline{\vartheta}(\gamma_{\tilde{\Omega}})\rangle, n \in X\}$ and $\overline{\vartheta}(\tilde{\Omega}) = \{\langle n, \overline{\vartheta}(\mu_{\tilde{\Omega}}), \overline{\vartheta}(\gamma_{\tilde{\Omega}})\rangle, n \in X\}$,

with the condition that $0 \le (\underline{\vartheta}(\mu_{\tilde{\Omega}}))^2 + (\underline{\vartheta}(\gamma_{\tilde{\Omega}}))^2 \le 1$, $0 \le (\overline{\vartheta}(\mu_{\tilde{\Omega}}))^2 + (\overline{\vartheta}(\gamma_{\tilde{\Omega}}))^2 \le 1$.

Here, $\underline{\vartheta}(\mu_{\tilde{\Omega}})(n) = \wedge_{n\in[y]_\vartheta} \mu_{\tilde{\Omega}}(y)$ and $\underline{\vartheta}(\gamma_{\tilde{\Omega}})(n) = \vee_{n\in[y]_\vartheta} \gamma_{\tilde{\Omega}}(y)$ also,

$\overline{\vartheta}(\mu_{\tilde{\Omega}})(n) = \vee_{n\in[y]_\vartheta} \mu_{\tilde{\Omega}}(y)$ and $\overline{\vartheta}(\gamma_{\tilde{\Omega}})(n) = \wedge_{n\in[y]_\vartheta} \gamma_{\tilde{\Omega}}(y)$.

Definition 9. Let ϑ be an congruence relation on X. Consider two *RIPF* sets $\vartheta(\widetilde{\Omega_1})$, $\vartheta(\widetilde{\Omega_2})$ in $X = \{x_1, x_2 \dots \dots x_n\}$. A *CSM* between $\vartheta(\widetilde{\Omega_1})$ and $\vartheta(\widetilde{\Omega_2})$ is defined as follows:

$$C_{RIPF}(\vartheta(\widetilde{\Omega_1}), \vartheta(\widetilde{\Omega_2})) = \frac{1}{n}\sum_{i=1}^{n} \frac{\left(\delta\mu_{\vartheta(\widetilde{\Omega_1})}(x_i)\delta\mu_{\vartheta(\widetilde{\Omega_1})}(x_i) + \delta\gamma_{\vartheta(\widetilde{\Omega_1})}(x_i)\delta\gamma_{\vartheta(\widetilde{\Omega_2})}(x_i)\right)}{\sqrt{(\delta\mu_{\vartheta(\widetilde{\Omega_1})}(x_i))^2 + (\delta\gamma_{\vartheta(\widetilde{\Omega_1})}(x_i))^2}\sqrt{(\delta\mu_{\vartheta(\widetilde{\Omega_2})}(x_i))^2 + (\delta\gamma_{\vartheta(\widetilde{\Omega_2})}(x_i))^2}}. \quad (1)$$

Where

$$\delta\mu_{\vartheta(\widetilde{\Omega_1})}(x_i) = \frac{\left(\underline{\vartheta}(\mu^-(x_i)) + \underline{\vartheta}(\mu^+(x_i)) + \overline{\vartheta}(\mu^-(x_i)) + \overline{\vartheta}(\mu^+(x_i))\right)}{4};$$

$$\delta\gamma_{\vartheta(\widetilde{\Omega_1})}(x_i) = \frac{\left(\underline{\vartheta}(\gamma^-(x_i)) + \underline{\vartheta}(\gamma^+(x_i)) + \overline{\vartheta}(\gamma^-(x_i)) + \overline{\vartheta}(\gamma^+(x_i))\right)}{4};$$

$$\delta\mu_{\vartheta(\widetilde{\Omega_2})}(x_i) = \frac{\left(\underline{\vartheta}(\mu^-(x_i)) + \underline{\vartheta}(\mu^+(x_i)) + \overline{\vartheta}(\mu^-(x_i)) + \overline{\vartheta}(\mu^+(x_i))\right)}{4};$$

$$\delta\gamma_{\vartheta(\widetilde{\Omega_2})}(x_i) = \frac{\left(\underline{\vartheta}(\gamma^-(x_i)) + \underline{\vartheta}(\gamma^+(x_i)) + \overline{\vartheta}(\gamma^-(x_i)) + \overline{\vartheta}(\gamma^+(x_i))\right)}{4}.$$

Proposition 1. A *RIPCSM* between $\vartheta(\widetilde{\Omega_1})$ and $\vartheta(\widetilde{\Omega_2})$ satisfies the following properties:

$$0 \le C_{RIPF}\left(\vartheta(\widetilde{\Omega_1}), \vartheta(\widetilde{\Omega_2})\right) \le 1;$$

$$C_{RIPF}\left(\vartheta(\widetilde{\Omega_1}), \vartheta(\widetilde{\Omega_2})\right) = 1 \Longleftrightarrow \vartheta(\widetilde{\Omega_1}) = \vartheta(\widetilde{\Omega_2});$$

$$C_{RIPF}\left(\vartheta(\widetilde{\Omega_1}), \vartheta(\widetilde{\Omega_2})\right) = C_{RIPF}\left(\vartheta(\widetilde{\Omega_2}), \vartheta(\widetilde{\Omega_1})\right).$$

Subha and Dhanalakshmi | J. Fuzzy. Ext. Appl. 1(4) (2020) 304-313

Proof. It is obvious because all positive values of cosine function are within 0 and 1; it is obvious; for any two *RIPF* sets $\vartheta(\widetilde{\Omega_1})$ and $\vartheta(\widetilde{\Omega_2})$, if $\vartheta(\widetilde{\Omega_1}) = \vartheta(\widetilde{\Omega_2})$ then,

$\delta\mu_{\vartheta(\widetilde{\Omega_1})}(x_i) = \delta\mu_{\vartheta(\widetilde{\Omega_2})}(x_i)$ and $\delta\gamma_{\vartheta(\widetilde{\Omega_1})}(x_i) = \delta\gamma_{\vartheta(\widetilde{\Omega_2})}(x_i)$. Hence $cos(0) = 1$. Conversely, if $C_{RIPF}\left(\vartheta(\widetilde{\Omega_1}), \vartheta(\widetilde{\Omega_2})\right) = 1$, then $\delta\mu_{\vartheta(\widetilde{\Omega_1})}(x_i) = \delta\mu_{\vartheta(\widetilde{\Omega_2})}(x_i)$ and $\delta\gamma_{\vartheta(\widetilde{\Omega_1})}(x_i) = \delta\gamma_{\vartheta(\widetilde{\Omega_1})}(x_i)$. Hence $\vartheta(\widetilde{\Omega_1}) = \vartheta(\widetilde{\Omega_2})$.

If we consider weight ω_i of each elementx_i, a weighted *RICSM* between *RIPF* sets $\vartheta(\widetilde{\Omega_1})$ and $\vartheta(\widetilde{\Omega_2})$ is defined as follows:

$$C_{WRIPF}\left(\vartheta(\widetilde{\Omega_1}), \vartheta(\widetilde{\Omega_2})\right) = \frac{1}{n}\sum_{i=1}^{n} \frac{\left(\delta\mu_{\vartheta(\widetilde{\Omega_1})}(x_i)\delta\mu_{\vartheta(\widetilde{\Omega_1})}(x_i) + \delta\gamma_{\vartheta(\widetilde{\Omega_1})}(x_i)\delta\gamma_{\vartheta(\widetilde{\Omega_2})}(x_i)\right)}{\sqrt{(\delta\mu_{\vartheta(\widetilde{\Omega_1})}(x_i))^2 + (\delta\gamma_{\vartheta(\widetilde{\Omega_1})}(x_i))^2}\sqrt{(\delta\mu_{\vartheta(\widetilde{\Omega_2})}(x_i))^2 + (\delta\gamma_{\vartheta(\widetilde{\Omega_2})}(x_i))^2}}. \quad (2)$$

$\omega_i \in [0,1], i = 1,2,3 \dots n$ and $\sum_{i=1}^{n} \omega_i = 1$. If we take $\omega_i = \frac{1}{n}, i = 1,2, \dots . n$ then

$C_{WRIPF}\left(\vartheta(\widetilde{\Omega_1}), \vartheta(\widetilde{\Omega_2})\right) = C_{RIPF}\left(\vartheta(\widetilde{\Omega_2}), \vartheta(\widetilde{\Omega_1})\right)$.

The weighted *RICSM* between two *RIPF* sets $\vartheta(\widetilde{\Omega_1})$ and $\vartheta(\widetilde{\Omega_2})$ also satisfies the following properties.

Proposition 2.

$$0 \le C_{WRIPF}\left(\vartheta(\widetilde{\Omega_1}), \vartheta(\widetilde{\Omega_2})\right) \le 1;$$

$$C_{WRIPF}\left(\vartheta(\widetilde{\Omega_1}), \vartheta(\widetilde{\Omega_2})\right) = 1 \Longleftrightarrow \vartheta(\widetilde{\Omega_1}) = \vartheta(\widetilde{\Omega_2});$$

$$C_{WRIPF}\left(\vartheta(\widetilde{\Omega_1}), \vartheta(\widetilde{\Omega_2})\right) = C_{WRIPF}\left(\vartheta(\widetilde{\Omega_2}), \vartheta(\widetilde{\Omega_1})\right).$$

4| Jaccard Similarity Measure (JSM) of Rough Interval Pythagorean Fuzzy (RIPF) Set

In this section we introduce the concept of *JSM* of *RIPF* sets. Weighted *JSM* of *RIPF* also derived.

Definition 10. Let ϑ be an congruence relation on *X*. Consider two *RIPF* sets $\vartheta(\widetilde{\Omega_1})$, $\vartheta(\widetilde{\Omega_2})$ in $X = \{x_1, x_2 \dots \dots x_n\}$. A *JSM* between $\vartheta(\widetilde{\Omega_1})$and $\vartheta(\widetilde{\Omega_2})$ is defined as follows:

$$J_{IRPF}\left(\vartheta(\widetilde{\Omega_1}), \vartheta(\widetilde{\Omega_2})\right) = \frac{1}{n}\sum_{i=1}^{n} \frac{\left(\delta\mu_{\vartheta(\widetilde{\Omega_1})}(x_i)\delta\mu_{\vartheta(\widetilde{\Omega_2})}(x_i) + \delta\gamma_{\vartheta(\widetilde{\Omega_1})}(x_i)\delta\gamma_{\vartheta(\widetilde{\Omega_2})}(x_i)\right)}{\begin{array}{c}[(\delta\mu_{\vartheta(\widetilde{\Omega_1})}(x_i))^2 + \left(\delta\gamma_{\vartheta(\widetilde{\Omega_1})}(x_i)\right)^2 + \left(\delta\mu_{\vartheta(\widetilde{\Omega_2})}(x_i)\right)^2 + \left(\delta\gamma_{\vartheta(\widetilde{\Omega_2})}(x_i)\right)^2 + \\ \delta\mu_{\vartheta(\widetilde{\Omega_1})}(x_i)\delta\mu_{\vartheta(\widetilde{\Omega_2})}(x_i) + \delta\gamma_{\vartheta(\widetilde{\Omega_1})}(x_i)\delta\gamma_{\vartheta(\widetilde{\Omega_2})}(x_i)]\end{array}}, \quad (3)$$

where

$\delta\mu_{\vartheta(\widetilde{\Omega_1})}(x_i) = \frac{\left(\underline{\vartheta}(\mu^-(x_i))+\underline{\vartheta}(\mu^+(x_i))+\overline{\vartheta}(\mu^-(x_i))+\overline{\vartheta}(\mu^+(x_i))\right)}{4}$,

$\delta\gamma_{\vartheta(\widetilde{\Omega_1})}(x_i) = \frac{\left(\underline{\vartheta}(\gamma^-(x_i))+\underline{\vartheta}(\gamma^+(x_i))+\overline{\vartheta}(\gamma^-(x_i))+\overline{\vartheta}(\gamma^+(x_i))\right)}{4}$ and

$$\delta\mu_{\vartheta(\widetilde{\Omega_2})}(x_i) = \frac{\left(\underline{\vartheta}(\mu^-(x_i)) + \underline{\vartheta}(\mu^+(x_i)) + \overline{\vartheta}(\mu^-(x_i)) + \overline{\vartheta}(\mu^+(x_i))\right)}{4},$$

$$\delta\gamma_{\vartheta(\widetilde{\Omega_2})}(x_i) = \frac{\left(\underline{\vartheta}(\gamma^-(x_i)) + \underline{\vartheta}(\gamma^+(x_i)) + \overline{\vartheta}(\gamma^-(x_i)) + \overline{\vartheta}(\gamma^+(x_i))\right)}{4}.$$

Proposition 3. A RIPJSM between $\vartheta(\widetilde{\Omega_1})$ and $\vartheta(\widetilde{\Omega_2})$ satisfies the following properties:

$$0 \le J_{RIPF}\left(\vartheta(\widetilde{\Omega_1}), \vartheta(\widetilde{\Omega_2})\right) \le 1,$$

$$J_{RIPF}\left(\vartheta(\widetilde{\Omega_1}), \vartheta(\widetilde{\Omega_2})\right) = 1 \Longleftrightarrow \vartheta(\widetilde{\Omega_1}) = \vartheta(\widetilde{\Omega_2}),$$

$$J_{RIPF}\left(\vartheta(\widetilde{\Omega_1}), \vartheta(\widetilde{\Omega_2})\right) = J_{IRPF}\left(\vartheta(\widetilde{\Omega_2}), \vartheta(\widetilde{\Omega_1})\right).$$

Proof. It is obvious because all positive values of cosine function are within 0 and 1; it is obvious; for any two RIPF sets $\vartheta(\widetilde{\Omega_1})$ and $\vartheta(\widetilde{\Omega_2})$, if $\vartheta(\widetilde{\Omega_1}) = \vartheta(\widetilde{\Omega_2})$ then, $\delta\mu_{\vartheta(\widetilde{\Omega_1})}(x_i) = \delta\mu_{\vartheta(\widetilde{\Omega_2})}(x_i)$ and $\delta\gamma_{\vartheta(\widetilde{\Omega_1})}(x_i) = \delta\gamma_{\vartheta(\widetilde{\Omega_2})}(x_i)$. Hence $cos(0) = 1$. Conversely, if $J_{RIPF}\left(\vartheta(\widetilde{\Omega_1}), \vartheta(\widetilde{\Omega_2})\right) = 1$, then $\delta\mu_{\vartheta(\widetilde{\Omega_1})}(x_i) = \delta\mu_{\vartheta(\widetilde{\Omega_2})}(x_i)$ and $\delta\gamma_{\vartheta(\widetilde{\Omega_1})}(x_i) = \delta\gamma_{\vartheta(\widetilde{\Omega_2})}(x_i)$. Hence $\vartheta(\widetilde{\Omega_1}) = \vartheta(\widetilde{\Omega_2})$.

If we consider weight ω_i of each element x_i, a weighted RIPJSM between RIPF sets $\vartheta(\widetilde{\Omega_1})$ and $\vartheta(\widetilde{\Omega_2})$ is defined as follows:

$$J_{IRPF}\left(\vartheta(\widetilde{\Omega_1}), \vartheta(\widetilde{\Omega_2})\right) = \frac{1}{n}\sum_{i=1}^{n}\omega_i \frac{\left(\delta\mu_{\vartheta(\widetilde{\Omega_1})}(x_i)\delta\mu_{\vartheta(\widetilde{\Omega_2})}(x_i) + \delta\gamma_{\vartheta(\widetilde{\Omega_1})}(x_i)\delta\gamma_{\vartheta(\widetilde{\Omega_2})}(x_i)\right)}{\begin{gathered}[(\delta\mu_{\vartheta(\widetilde{\Omega_1})}(x_i))^2 + \left(\delta\gamma_{\vartheta(\widetilde{\Omega_1})}(x_i)\right)^2 + \left(\delta\mu_{\vartheta(\widetilde{\Omega_2})}(x_i)\right)^2 + \left(\delta\gamma_{\vartheta(\widetilde{\Omega_2})}(x_i)\right)^2 + \\ \delta\mu_{\vartheta(\widetilde{\Omega_1})}(x_i)\delta\mu_{\vartheta(\widetilde{\Omega_2})}(x_i) + \delta\gamma_{\vartheta(\widetilde{\Omega_1})}(x_i)\delta\gamma_{\vartheta(\widetilde{\Omega_2})}(x_i)]\end{gathered}}. \quad (4)$$

$\omega_i \in [0,1], i = 1,2,3 \dots n$ and $\sum_{i=1}^{n}\omega_i = 1$. If we take $\omega_i = \frac{1}{n}, i = 1,2, \dots . n$ then $J_{WRIPF}\left(\vartheta(\widetilde{\Omega_1}), \vartheta(\widetilde{\Omega_2})\right) = J_{RIPF}\left(\vartheta(\widetilde{\Omega_2}), \vartheta(\widetilde{\Omega_1})\right)$.

The weighted RIPJSM between two RIPF sets $\vartheta(\widetilde{\Omega_1})$ and $\vartheta(\widetilde{\Omega_2})$ also satisfies the following properties.

Proposition 4.

$$0 \le J_{WRIPF}\left(\vartheta(\widetilde{\Omega_1}), \vartheta(\widetilde{\Omega_2})\right) \le 1;$$

$$J_{WRIPF}\left(\vartheta(\widetilde{\Omega_1}), \vartheta(\widetilde{\Omega_2})\right) = 1 \Longleftrightarrow \vartheta(\widetilde{\Omega_1}) = \vartheta(\widetilde{\Omega_2});$$

Subha and Dhanalakshmi |J. Fuzzy. Ext. Appl. 1(4) (2020) 304-313

$$J_{WRIPF}\left(\vartheta\left(\widetilde{\Omega_1}\right),\vartheta\left(\widetilde{\Omega_2}\right)\right)=J_{WRIPF}\left(\vartheta\left(\widetilde{\Omega_2}\right),\vartheta\left(\widetilde{\Omega_1}\right)\right).$$

5| Dice Similarity Measure (DSM) of Rough Interval Pythagorean Fuzzy (RIPF) Set

This section deals with DSM of *RIPF* sets. Some properties of this similarity measure are discussed.

Definition 11. Let $\vartheta\left(\widetilde{\Omega_1}\right)$ and $\vartheta\left(\widetilde{\Omega_2}\right)$ be two *RIPF* set in$X=\{x_1,x_2\ldots\ldots x_n\}$. A *DSM* between $\vartheta\left(\widetilde{\Omega_1}\right)$and $\vartheta(\widetilde{\Omega_2})$ is defined as follows:

$$D_{RIPF}(\vartheta(\widetilde{\Omega_1}),\vartheta(\widetilde{\Omega_2}))=\frac{1}{n}\sum_{i=1}^{n}\frac{2\left(\delta\mu_{\vartheta(\widetilde{\Omega_1})}(x_i)\delta\mu_{\vartheta(\widetilde{\Omega_2})}(x_i)+\delta\gamma_{\vartheta(\widetilde{\Omega_1})}(x_i)\delta\gamma_{\vartheta(\widetilde{\Omega_2})}(x_i)\right)}{\sqrt{(\delta\mu_{\vartheta(\widetilde{\Omega_1})}(x_i))^2+(\delta\gamma_{\vartheta(\widetilde{\Omega_1})}(x_i))^2}\sqrt{(\delta\mu_{\vartheta(\widetilde{\Omega_2})}(x_i))^2+(\delta\gamma_{\vartheta(\widetilde{\Omega_2})}(x_i))^2}}. \tag{5}$$

Where

$$\delta\mu_{\ \vartheta(\widetilde{\Omega_1})}(x_i)=\frac{\left(\underline{\vartheta}(\mu^-(x_i))+\underline{\vartheta}(\mu^+(x_i))+\overline{\vartheta}(\mu^-(x_i))+\overline{\vartheta}(\mu^+(x_i))\right)}{4},$$

$$\delta\gamma_{\ \vartheta(\widetilde{\Omega_1})}(x_i)=\frac{\left(\underline{\vartheta}(\gamma^-(x_i))+\underline{\vartheta}(\gamma^+(x_i))+\overline{\vartheta}(\gamma^-(x_i))+\overline{\vartheta}(\gamma^+(x_i))\right)}{4} \text{ and}$$

$$\delta\mu_{\ \vartheta(\widetilde{\Omega_2})}(x_i)=\frac{\left(\underline{\vartheta}(\mu^-(x_i))+\underline{\vartheta}(\mu^+(x_i))+\overline{\vartheta}(\mu^-(x_i))+\overline{\vartheta}(\mu^+(x_i))\right)}{4},$$

$$\delta\gamma_{\ \vartheta(\widetilde{\Omega_2})}(x_i)=\frac{\left(\underline{\vartheta}(\gamma^-(x_i))+\underline{\vartheta}(\gamma^+(x_i))+\overline{\vartheta}(\gamma^-(x_i))+\overline{\vartheta}(\gamma^+(x_i))\right)}{4}.$$

Proposition 5. A *RIPJSM* between $\vartheta\left(\widetilde{\Omega_1}\right)$ and $\vartheta\left(\widetilde{\Omega_2}\right)$ satisfies the following properties:

$$0\le D_{RIPF}\left(\vartheta\left(\widetilde{\Omega_1}\right),\vartheta\left(\widetilde{\Omega_2}\right)\right)\le 1;$$

$$D_{RIPF}\left(\vartheta\left(\widetilde{\Omega_1}\right),\vartheta\left(\widetilde{\Omega_2}\right)\right)=1\Longleftrightarrow\vartheta\left(\widetilde{\Omega_1}\right)=\vartheta\left(\widetilde{\Omega_2}\right);$$

$$D_{RIPF}\left(\vartheta\left(\widetilde{\Omega_1}\right),\vartheta\left(\widetilde{\Omega_2}\right)\right)=D_{IRPF}\left(\vartheta\left(\widetilde{\Omega_2}\right),\vartheta\left(\widetilde{\Omega_1}\right)\right).$$

Proof. Proof is similar to *Proposition 3.*

If we consider weight ω_i of each elementx_i, a weighted *RIPDSM* between *RIPF* sets $\vartheta\left(\widetilde{\Omega_1}\right)$ and $\vartheta(\widetilde{\Omega_2})$ is defined $\sum_{i=1}^{n}\omega_i=1$.as follows:

$$D_{RIPF}(\vartheta(\widetilde{\Omega_1}),\vartheta(\widetilde{\Omega_2}))=\frac{1}{n}\sum_{i=1}^{n}\omega_i\frac{2\left(\delta\mu_{\vartheta(\widetilde{\Omega_1})}(x_i)\delta\mu_{\vartheta(\widetilde{\Omega_2})}(x_i)+\delta\gamma_{\vartheta(\widetilde{\Omega_1})}(x_i)\delta\gamma_{\vartheta(\widetilde{\Omega_2})}(x_i)\right)}{\sqrt{(\delta\mu_{\vartheta(\widetilde{\Omega_1})}(x_i))^2+(\delta\gamma_{\vartheta(\widetilde{\Omega_1})}(x_i))^2}\sqrt{(\delta\mu_{\vartheta(\widetilde{\Omega_2})}(x_i))^2+(\delta\gamma_{\vartheta(\widetilde{\Omega_2})}(x_i))^2}}. \tag{6}$$

$\omega_i\in[0,1], i=1,2,3\ldots n$ and If we take $\omega_i=\frac{1}{n}, i=1,2,\ldots.n$ then $D_{WRIPF}\left(\vartheta\left(\widetilde{\Omega_1}\right),\vartheta\left(\widetilde{\Omega_2}\right)\right)=D_{RIPF}\left(\vartheta\left(\widetilde{\Omega_2}\right),\vartheta\left(\widetilde{\Omega_1}\right)\right).$

The weighted *RIPDSM* between two *RIPF* sets $\vartheta(\widetilde{\Omega_1})$ and $\vartheta(\widetilde{\Omega_2})$ also satisfies the following properties.

Proposition 6.

$$0 \le D_{WRIPF}\left(\vartheta(\widetilde{\Omega_1}), \vartheta(\widetilde{\Omega_2})\right) \le 1;$$

$$D_{WRIPF}\left(\vartheta(\widetilde{\Omega_1}), \vartheta(\widetilde{\Omega_2})\right) = 1 \Longleftrightarrow \vartheta(\widetilde{\Omega_1}) = \vartheta(\widetilde{\Omega_2});$$

$$D_{WRIPF}\left(\vartheta(\widetilde{\Omega_1}), \vartheta(\widetilde{\Omega_2})\right) = D_{WRIPF}\left(\vartheta(\widetilde{\Omega_2}), \vartheta(\widetilde{\Omega_1})\right).$$

6| Decision Making Based on CSM, JSM and DSM under RIPF Environment

This section deals with *RIPSM* between *RIPF* sets to the multi-criteria decision making problem. Assume that $K = \{K_1, K_2, \dots . K_m\}$ be the set of attributes and $Q = \{Q_1, Q_2, \dots . Q_n\}$ be the set of alternatives. The proposed decision making approach is described by the following steps.

Algorithm 1. (See *Fig. 1*).

Step 1. Construct the Decision Matrix with RIPF Number. The decision maker forms a decision matrix with respect to n alternatives and m attributes in terms of *RIPF* numbers.

Step 2. Determine RIP Mean Operator.

$$\langle \delta\mu(x_i), \delta\gamma(x_i) \rangle = \left(\frac{\left(\underline{\vartheta}(\mu^-(x_i)) + \underline{\vartheta}(\mu^+(x_i)) + \overline{\vartheta}(\mu^-(x_i)) + \overline{\vartheta}(\mu^+(x_i))\right)}{4}, \frac{\left(\underline{\vartheta}(\gamma^-(x_i)) + \underline{\vartheta}(\gamma^+(x_i)) + \overline{\vartheta}(\gamma^-(x_i)) + \overline{\vartheta}(\gamma^+(x_i))\right)}{4} \right)$$

for $i = 1,2, \dots . n$.

Step 3. Determine the Weights of the Attributes. Assume that the weight of the attributes K_j *(j=1,2,...m)* considered by the decision maker is ω_j *(j=1,2,...m)* where all $\omega_j \in [0,1], j = 1,2,3 \dots m$ and $\sum_{j=1}^{m} \omega_j = 1$.

Step 4. Determine the Benefit Type Attributes and Cost Type Attributes. Generally, the evaluation attribute can be categorized into two types: benefit type attribute and cost type attribute.

For benefit type attribute: $Z^* = \{max(\mu_{Q_i}), min(\gamma_{Q_i})\}$.

For cost type attribute: $Z^* = \{min(\mu_{Q_i}), max(\gamma_{Q_i})\}$.

Step 5. Determine the Weighted RIPSM of the Alternatives.

$$C_{WRIPF}\left(\vartheta(\widetilde{\Omega_1}), \vartheta(\widetilde{\Omega_2})\right) = \sum_{i=1}^{n} \omega_i C_{RIPF}\left(\vartheta(\widetilde{\Omega_1}), \vartheta(\widetilde{\Omega_2})\right);$$

$$J_{WRIPF}\left(\vartheta(\widetilde{\Omega_1}), \vartheta(\widetilde{\Omega_2})\right) = \sum_{i=1}^{n} \omega_i J_{RIPF}\left(\vartheta(\widetilde{\Omega_1}), \vartheta(\widetilde{\Omega_2})\right);$$

Subha and Dhanalakshmi |J. Fuzzy. Ext. Appl. 1(4) (2020) 304-313

$$D_{WRIPF}\left(\vartheta\left(\widetilde{\Omega_1}\right),\vartheta\left(\widetilde{\Omega_2}\right)\right)=\sum_{i=1}^{n}\omega_i\, D_{RIPF}\left(\vartheta\left(\widetilde{\Omega_1}\right),\vartheta\left(\widetilde{\Omega_2}\right)\right).$$

Step 6. Ranking the Alternatives. The ranking order of all alternatives can be determined based on the descending order of similarity measures.

Step 7. End.

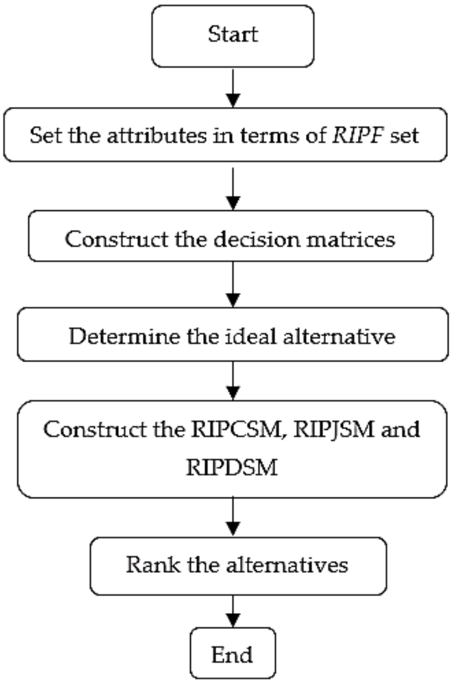

Fig 1. A flowchart of the proposed decision making.

7| Numerical Example for RIPCSM, RIPJSM and RIPDSM

Let us consider a decision maker wants to select the house from $Q = \{Q_1, Q_2, Q_3\}$ by considering four attributes, namely expensive (K_1), reasonable price (K_2), low price (K_3) and the risk factor (K_4). By proposed approach discussed above, the considered problem solved by the following steps:

Step 1. The decision maker forms a decision matrix with respect to the three alternatives and four attributes in terms of *RIP* number as follows.

Table 1. Decision matrix.

	K_1	K_2	K_3	K_4
Q_1	([.3,.4],[.5,.7]) , ([.3,.4],[.5,.7])	([.5,.6],[.8,.9]) , ([.5,.6],[.8,.9])	([.1,.2],[.7,.8]) , ([.5,.8],[.4,.6])	([.1,.2],[.7,.8]) , ([.5,.8],[.4,.6])
Q_2	([.7,.8],[.6,.7]) , ([.7,.8],[.6,.7])	([.7,.8],[.6,.7]) , ([.8,.9],[.4,.5])	([.5,.6],[.4,.5]) , ([.5,.6],[.4,.5])	([.7,.8],[.6,.7]) , ([.8,.9],[.4,.5])
Q_3	([.5,.7],[.3,.4]) , ([.8,.9],[.1,.2])	([.5,.7],[.3,.4]) , ([.8,.9],[.1,.2])	([.5,.7],[.3,.4]) , ([.8,.9],[.1,.2])	([.8,.9],[.1,.2]) , ([.8,.9],[.1,.2])

Step 2. Determine the RIP mean operator.

Table 2. Transformed decision matrix.

	K_1	K_2	K_3	K_4
Q_1	[.35,.6]	[.55,.85]	[.4,.625]	[.4,.625]
Q_2	[.75,.65]	[.8,.55]	[.55,.45]	[.8,.55]
Q_3	[.725,.25]	[.725,.25]	[.725,.25]	[.825,.15]

Step 3. The weight vectors considered by the decision maker are 0.35, 0.25, 0.25 and 0.15 respectively.

Step 4. Determine the benefit type attribute and cost type attribute. Here three benefit types attributes K_1,K_2,K_3 and one cost type attribute K_4.

$$Z^* = \{[0.75,0.25], [.8, .25], [.725, .25], [.825, .15]\}.$$

Step 5. Calculate the weighted *RIP* similarity measures of the alternatives. Calculated values of weighted *RIP* similarity values are

$C_{WIRPF}(Q_1, Z^*) = .7582;$

$C_{WIRPF}(Q_2, Z^*) = .9336;$

$C_{WIRPF}(Q_3, Z^*) = .9999;$

$J_{WIRPF}(Q_1, Z^*) = .6046;$

$J_{WIRPF}(Q_2, Z^*) = .8538;$

$J_{WIRPF}(Q_3, Z^*) = .9975;$

$D_{WIRPF}(Q_1, Z^*) = .7018;$

$D_{WIRPF}(Q_2, Z^*) = .9208;$

$D_{WIRPF}(Q_3, Z^*) = .9988.$

Step 6. Ranking the alternatives is prepared based on the descending order of similarity measures. Highest value reflects the best alternative. HencQ_3e is the best alternative.

8| Conclusion

In this paper, we have defined Cosine, Jaccard, Dice similarity measure, Weighted Cosine, Jaccard and Dice similarity measures. We have also proved their basic properties. We have developed MADM strategies based on the proposed measures respectively. We have presented an example for select a best house for live. The thrust of the concept presented in this article will be in pattern recognition, medical diagnosis etc. in rough interval Pythagorean fuzzy sets.

References

[1] Atanassov, K. (2016). Intuitionistic fuzzy sets. *International journal bioautomation*, *20*, 87-96.

[2] Hussain, A., Mahmood, T., & Ali, M. I. (2019). Rough Pythagorean fuzzy ideals in semigroups. *Computational and applied mathematics*, *38*(2), 67.

[3] Dubois, D., & Prade. H. (1990). Rough fuzzy sets and fuzzy rough sets. *Int.J.General Syst*, *17*, 191-209.

Subha and Dhanalakshmi |J. Fuzzy. Ext. Appl. 1(4) (2020) 304-313

[4] Ejegwa, P. A. (2020). New similarity measures for Pythagorean fuzzy sets with applications. *International journal of fuzzy computation and modelling*, *3*(1), 75-94.

[5] Kumar, T., Bajaj, R., & Ansari, M. D. (2020). On accuracy function and distance measures of interval-valued Pythagorean fuzzy sets with application to decision making. *Scientia iranica*, *27*(4), 2127-2139.

[6] Kumar, R., Edalatpanah, S. A., Jha, S., & Singh, R. (2019). A Pythagorean fuzzy approach to the transportation problem. *Complex & intelligent systems*, *5*(2), 255-263.

[7] Pawlak, Z. (1982). Rough sets. *International journal of computer & information sciences*, *11*(5), 341-356.

[8] Pawlak, Z. (2002). Rough sets and intelligent data analysis. *Information sciences*, *147*(1-4), 1-12.

[9] Peng, X., & Yang, Y. (2016). Fundamental properties of interval-valued Pythagorean fuzzy aggregation operators. *International journal of intelligent systems*, *31*(5), 444-487.

[10] Peng, X., & Li, W. (2019). Algorithms for interval-valued pythagorean fuzzy sets in emergency decision making based on multiparametric similarity measures and WDBA. *Ieee access*, *7*, 7419-7441.

[11] Peng, X., & Garg, H. (2019). Multiparametric similarity measures on Pythagorean fuzzy sets with applications to pattern recognition. *Applied intelligence*, *49*(12), 4058-4096.

[12] Ullah, K., Mahmood, T., Ali, Z., & Jan, N. (2020). On some distance measures of complex Pythagorean fuzzy sets and their applications in pattern recognition. *Complex & intelligent systems*, *6*(1), 15-27.

[13] Yager, R. R. (2013, June). Pythagorean fuzzy subsets. *2013 joint IFSA world congress and NAFIPS annual meeting (IFSA/NAFIPS)* (pp. 57-61). IEEE.

[14] Zadeh, L. A. (1975). The concept of a linguistic variable and it's application to approximation reasoning I. *Information sciences, 8*, 199-249.

[15] Zadeh, L. A. (1965). Fuzzy sets. *Information control*, *8* , 338-353.