E-ISSN: 2717-3453 | P-ISSN: 2783-1442

Journal of Fuzzy Extension and Applications

www.journal-fea.com

J. Fuzzy. Ext. Appl. Vol. 1, No. 4 (2020) 314–324.

Paper Type: Research Paper

Spherical Interval-Valued Fuzzy Bi-Ideals of Gamma Near-Rings

Chinnadurai Veerappan[1,*], Shakila Venkatesan[2]

[1] Department of Mathematics, Faculty of Mathematics, Annamalai University, Chidambaram, India; kv.chinnadurai@yahoo.com.
[2] Department of Mathematics, Annamalai University, Chidambaram, India; shaki04@gmail.com.

Citation:

Veerappan, C., & Venkatesan, S. (2020). Spherical interval-valued fuzzy bi-ideals of gamma near-rings. *Journal of fuzzy extension and application, 1* (4), 314-324.

Received: 08/07/2020 Reviewed: 30/07/2020 Revised: 12/09/2020 Accept: 24/10/2020

Abstract

In this paper we introduce the concept of the spherical interval-valued fuzzy bi-ideal of gamma near-ring $\mathcal{R}$ and its some results. The union and intersection of the spherical interval-valued fuzzy bi-ideal of gamma near-ring $\mathcal{R}$ is also a spherical interval-valued fuzzy bi-ideal of gamma near-ring $\mathcal{R}$. Further we discuss about the relationship between bi-ideal and spherical interval-valued fuzzy bi-ideal of gamma near-ring $\mathcal{R}$.

Keywords: Spherical fuzzy set, Interval-Valued fuzzy set, Γ -Near-Rings.

1 | Introduction

The Fuzzy Set (FS) was introduced by Zadeh [14] in 1965. It is identified as better tool for the scientific study of uncertainty, and came as a boost to the researchers working in the field of uncertainty. Many extensions and generalizations of FS was conceived by a number of researchers and a large number of real-life applications were developed in a variety of areas. In addition to this, parallel analysis of the classical results of many branches of Mathematics was also carried out in the fuzzy settings. Properties of fuzzy ideals in near-rings was studied by Hong et al. [3]. The monograph by Chinnadurai [1] gives a detailed discussion on fuzzy ideals in algebraic structures. Fuzzy ideals in Gamma near-ring $\mathscr{R}$ was discussed by Jun et al. [6] and [7] and Satyanarayana [8]. Thillaigovindan et al. [13] studied the interval valued fuzzy quasi-ideals of semigroups. Meenakumari and Tamizh chelvam [9] have defined fuzzy bi-ideal in $\mathscr{R}$ and established some properties of this structure. Srinivas and Nagaiah [11] have proved some results on T-fuzzy ideals of Γ-near-rings.

Corresponding Author: kv.chinnadurai@yahoo.com
10.22105/JFEA.2021.281500.1061

Thillaigovindan et al. [12] worked on interval valued fuzzy ideals of near-rings. Chinnadurai and Kadalarasi [2] have defined the direct product of fuzzy ideals in near-rings. Kutlu Gündoğdu and Kahraman [8] introduced spherical fuzzy sets as an extension of picture fuzzy sets. Chinnadurai and Shakila [3] and [4] discussed T-fuzzy bi-ideal of gamma near-ring and spherical fuzzy bi-ideals of gamma near-rings.

In this research work, we introduce the notion of Spherical Interval-Valued Fuzzy Bi-Ideal (SIVFBI) of gamma near-ring $\mathscr{R}$ as a generalization of spherical fuzzy bi-ideals of gamma near-rings $\mathscr{R}$. We will discuss some of the properties of spherical interval-valued fuzzy bi-ideal of gamma near-ring $\mathscr{R}$.

2| Preliminaries

In this section we present some definitions which are used for this research. Let $\mathcal{R}$ be a near-ring and Γ be a non-empty set such tha $\mathscr{R}$ t is a Gamma near-ring. A subgroup H of $(\mathscr{R},+)$ is a Bi-Ideal (BI) if and only if $H\Gamma\mathscr{R}\Gamma H \subseteq H$. A Spherical Fuzzy Set (SFS) $\tilde{A}_s$ of the universe of discourse U is given by, $\tilde{A}_s = \{u, (\tilde{\mu}(u), \tilde{\nu}(u), \tilde{\xi}(u)) | u \in U\}$ where $\tilde{\mu}(u): U \longrightarrow [0,1]$, $\tilde{\nu}(u): U \longrightarrow [0,1]$ and $\tilde{\xi}(u): U \longrightarrow [0,1]$ and $0 \leq \tilde{\mu}^2(u) + \tilde{\nu}^2(u) + \tilde{\xi}^2(u) \leq 1$, $u \in U$.

For each u, the numbers $\tilde{\mu}(u)$, $\tilde{\nu}(u)$ and $\tilde{\xi}(u)$ are the degree of membership, non-membership and hesitancy of u to $\tilde{A}_s$, respectively.

A SFS $A_s = (\mu, \nu, \xi)$, where $\mu: \mathscr{R} \longrightarrow [0,1]$, $\nu: \mathscr{R} \longrightarrow [0,1]$ and $\xi: \mathscr{R} \longrightarrow [0,1]$ of $\mathscr{R}$ is said to be a Spherical Fuzzy Bi-Ideal (SFBI) of $\mathscr{R}$ if the following conditions are satisfied

$$\mu(u - v) \geq \min\{\mu(u), \mu(v)\},$$

$$\nu(u - v) \geq \min\{\nu(u), \nu(v)\},$$

$$\xi(u - v) \leq \max\{\xi(u), \xi(v)\},$$

$$\mu(u\alpha v\beta w) \geq \min\{\mu(u), \mu(w)\},$$

$$\nu(u\alpha v\beta w) \geq \min\{\nu(u), \nu(w)\},$$

$$\xi(u\alpha v\beta w) \leq \max\{\xi(u), \xi(w)\},$$

for all $u, v, w \in \mathscr{R}$ and $\alpha, \beta \in \Gamma$.

3| Spherical Interval-Valued Fuzzy Bi-Ideals of Gamma Near-Rings

In this section we define SIVFBI of $\mathscr{R}$ and study some of it properties. We obtain the condition for an arbitrary fuzzy subset of $\mathscr{R}$ is said to be SIVFBI.

Definition 1. A spherical fuzzy *set* $\tilde{A}_s = (\tilde{\mu}, \tilde{\nu}, \tilde{\xi})$ *of* $\mathscr{R}$ is to be SIVFBI of $\mathscr{R}$ if the following conditions are satisfied

$$\tilde{\mu}(u - v) \geq \min^{i}\{\tilde{\mu}(u), \tilde{\mu}(v)\},$$

$$\tilde{\nu}(u - v) \geq \min^{i}\{\tilde{\nu}(u), \tilde{\nu}(v)\},$$

$$\tilde{\xi}(u - v) \leq \max^{i}\{\tilde{\xi}(u), \tilde{\xi}(v)\},$$

$$\tilde{\mu}(u\alpha v\beta w) \geq \min^{i}\{\tilde{\mu}(u), \tilde{\mu}(w)\},$$

$\tilde{v}(u\alpha v\beta w) \geq \min^i\{\tilde{v}(u), \tilde{v}(w)\}$,

$\tilde{\xi}(u\alpha v\beta w) \leq \max^i\{\tilde{\xi}(u), \tilde{\xi}(w)\}$,

for all $u, v, w \in \mathscr{R}$ and $\alpha, \beta \in \Gamma$, where $\tilde{\mu}: \mathscr{R} \longrightarrow D[0,1]$, $\tilde{v}: \mathscr{R} \longrightarrow D[0,1]$ and $\tilde{\xi}: \mathscr{R} \longrightarrow D[0,1]$. Here $D[0,1]$ denotes the family of closed subintervals of [0,1].

Example 1. Let $\mathscr{R} = \{0,1,2,3\}$ with binary operation $+$" on $\mathscr{R}, \Gamma = \{0,1\}$ and $\mathscr{R} \times \Gamma \times \mathscr{R} \longrightarrow \mathscr{R}$ be a mapping. From the cayley table,

+	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	1	0
3	3	2	0	1

0	0	1	2	3
0	0	0	0	0
1	0	1	1	1
2	0	2	2	2
3	0	3	3	3

1	0	1	2	3
0	0	0	0	0
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0

Define SFS $\tilde{\mu}: \mathscr{R} \longrightarrow D[0,1]$ by $\tilde{\mu}(0) = [0.2,0.3], \tilde{\mu}(1) = [0.3,0.6], \tilde{\mu}(2) = [0.7,0.9], \tilde{\mu}(3) = [0.5,0.9]$; $\tilde{v}: \mathscr{R} \longrightarrow D[0,1]$ by $\tilde{v}(0) = [0.2,0.4], \tilde{v}(1) = [0.5,0.6], \tilde{v}(2) = [0.6,0.7], \tilde{v}(3) = [0.7,0.9]$; $\tilde{\xi}: \mathscr{R} \longrightarrow D[0,1]$ by $\tilde{\xi}(0) = [0.1,0.3], \tilde{\xi}(1) = [0.4,0.6], \tilde{\xi}(2) = [0.8,0.9], \tilde{\xi}(3) = [0.5,0.7]$. Then $\tilde{A}_s$ is SIVFBI of $\mathscr{R}$.

Theorem 1. Let $\tilde{A}_s = [A_s^-; A_s^+]$ be a Spherical interval-valued fuzzy subset of a gamma near-ring $\mathscr{R}$, then $\tilde{A}_s$ is a SIVFBI of $\mathscr{R}$ if and only if A_s^-, A_s^+ are SFBI of $\mathscr{R}$.

Proof. If $\tilde{A}_s$ is a SIVFBI of $\mathscr{R}$. For any $u, v, w \in \mathscr{R}$. Now,

$[\mu^-(u - v), \mu^+(u - v)] = \tilde{\mu}(u - v)$

$\geq \min^i\{\tilde{\mu}(u), \tilde{\mu}(v)\}$

$= \min^i\{[\mu^-(u), \mu^+(u)], [\mu^-(v), \mu^+(v)]\}$

$= \min^i\{[\mu^-(u), \mu^-(v)]\}, \min^i\{[\mu^+(u), \mu^+(v)]\}$,

$[v^-(u - v), v^+(u - v)] = \tilde{v}(x - y)$

$\geq \min^i\{\tilde{v}(u), \tilde{v}(v)\}$

$= \min^i\{[v^-(u), v^+(u)], [v^-(v), v^+(v)]\}$

$= \min^i\{[v^-(u), v^-(v)]\}, \min^i\{[v^+(u), v^+(v)]\}$, and

$[\xi^-(u - v), \xi^+(u - v)] = \tilde{\xi}(u - v)$

J. Fuzzy. Ext. Appl

Veerappan and Venkatesan|J. Fuzzy. Ext. Appl. 1(4) (2020) 314-324

$$\leq \max^{i}\{\tilde{\xi}(u), \tilde{\xi}(v)\}$$

$$= \max^{i}\{[\xi^{-}(u), \xi^{+}(u)], [\xi^{-}(v), \xi^{+}(v)]\}$$

$$= \max^{i}\{[\xi^{-}(u), \xi^{-}(v)]\}, \max^{i}\{[\xi^{+}(u), \xi^{+}(v)]\};$$

$$[\mu^{-}(u\alpha v\beta w), \mu^{+}(u\alpha v\beta w)] = \tilde{\mu}(u\alpha v\beta w)$$

$$\geq \min^{i}\{\tilde{\mu}(u), \tilde{\mu}(w)\}$$

$$= \min^{i}\{[\mu^{-}(u), \mu^{+}(u)], [\mu^{-}(w), \mu^{+}(w)]\}$$

$$= \min^{i}\{[\mu^{-}(u), \mu^{-}(w)]\}, \min^{i}\{[\mu^{+}(u), \mu^{+}(w)]\};$$

$$[\nu^{-}(u\alpha v\beta w), \nu^{+}(u\alpha v\beta w)] = \tilde{\nu}(u\alpha v\beta w)$$

$$\geq \min^{i}\{\tilde{\nu}(u), \tilde{\nu}(w)\}$$

$$= \min^{i}\{[\nu^{-}(u), \nu^{+}(u)], [\nu^{-}(w), \nu^{+}(w)]\}$$

$$= \min^{i}\{[\nu^{-}(u), \nu^{-}(w)]\}, \min^{i}\{[\nu^{+}(u), \nu^{+}(w)]\}, \text{ and}$$

$$[\xi^{-}(u\alpha v\beta w), \xi^{+}(u\alpha v\beta w)] = \tilde{\xi}(u\alpha v\beta w)$$

$$\leq \max^{i}\{\tilde{\xi}(u), \tilde{\xi}(w)\}$$

$$= \max^{i}\{[\xi^{-}(u), \xi^{+}(u)], [\xi^{-}(w), \xi^{+}(w)]\}$$

$$= \max^{i}\{[\xi^{-}(u), \xi^{-}(w)]\}, \max^{i}\{[\xi^{+}(u), \xi^{+}(w)]\}.$$

Therefore A_s^-, A_s^+ are SFBI o $\mathscr{R}$ f.

Conversely let A_s^-, A_s^+ are SFBI of $\mathscr{R}$. Let $u, v, w \in \mathscr{R}$. Now,

$$\tilde{\mu}(u - v) = [\mu^{-}(u - v), \mu^{+}(u - v)]$$

$$\geq [\min^{i}\{\mu^{-}(u), \mu^{-}(v)\}, \min^{i}\{\mu^{+}(u), \mu^{+}(v)\}]$$

$$= \min^{i}\{\mu^{-}(u), \mu^{+}(u)\}, \min^{i}\{\mu^{-}(v), \mu^{+}(v)\}$$

$$= \min^{i}\{\tilde{\mu}(u), \tilde{\mu}(v)\};$$

$$\tilde{\nu}(u - v) = [\nu^{-}(u - v), \nu^{+}(u - v)]$$

$$\geq [\min^{i}\{\nu^{-}(u), \nu^{-}(v)\}, \min^{i}\{\nu^{+}(u), \nu^{+}(v)\}]$$

$$= \min^{i}\{\nu^{-}(u), \nu^{+}(u)\}, \min^{i}\{\nu^{-}(v), \nu^{+}(v)\}$$

$$= \min^{i}\{\tilde{\nu}(u), \tilde{\nu}(v)\}, \text{ and}$$

$$\tilde{\xi}(u - v) = [\xi^{-}(u - v), \xi^{+}(u - v)]$$

$\leq [\max^{i}\{\xi^{-}(u), \xi^{-}(v)\}, \max^{i}\{\xi^{+}(u), \xi^{+}(v)\}]$

$= \max^{i}\{\xi^{-}(u), \xi^{+}(u)\}, \max^{i}\{\xi^{-}(v), \xi^{+}(v)\}$

$= \max^{i}\{\tilde{\xi}(u), \tilde{\xi}(v)\};$

$\tilde{\mu}(u\alpha v\beta w) = [\mu^{-}(u\alpha v\beta w), \mu^{+}(u\alpha v\beta w)]$

$\geq [\min^{i}\{\mu^{-}(u), \mu^{-}(w)\}, \min^{i}\{\mu^{+}(u), \mu^{+}(w)\}]$

$= \min^{i}\{\mu^{-}(u), \mu^{+}(u)\}, \min^{i}\{\mu^{-}(w), \mu^{+}(w)\}$

$= \min^{i}\{\tilde{\mu}(u), \tilde{\mu}(w)\};$

$\tilde{\nu}(u\alpha v\beta w) = [\nu^{-}(u\alpha v\beta w), \nu^{+}(u\alpha v\beta w)]$

$\geq [\min^{i}\{\nu^{-}(u), \nu^{-}(w)\}, \min^{i}\{\nu^{+}(u), \nu^{+}(w)\}]$

$= \min^{i}\{\nu^{-}(u), \nu^{+}(u)\}, \min^{i}\{\nu^{-}(w), \nu^{+}(w)\}$

$= \min^{i}\{\tilde{\nu}(u), \tilde{\nu}(w)\}$, and

$\tilde{\xi}(u\alpha v\beta w) = [\xi^{-}(u\alpha v\beta w), \xi^{+}(u\alpha v\beta w)]$

$\leq [\max^{i}\{\xi^{-}(u), \xi^{-}(w)\}, \max^{i}\{\xi^{+}(u), \xi^{+}(w)\}]$

$= \max^{i}\{\xi^{-}(u), \xi^{+}(u)\}, \max^{i}\{\xi^{-}(w), \xi^{+}(w)\}$

$= \max^{i}\{\tilde{\xi}(u), \tilde{\xi}(w)\}.$

So $\tilde{A}_s$ is a SIVFBI of $\mathscr{R}$.

Hence the proof.

Theorem 2. If $\{\tilde{A}_{s_i}; i \in I\}$ be a family of SIVFBI of a gamma near-ring $\mathscr{R}$, then $\bigcap_{i \in I} \tilde{A}_{s_i}$ is also SIVFBI of $\mathscr{R}$, where I is an index set.

Proof. Let $\{\tilde{A}_{s_i}; i \in I\}$ be a family of SIVFBI of a gamma near-ring $\mathscr{R}$. For any $u, v, w \in \mathscr{R}$ and $\alpha, \beta \in \Gamma$.

$\bigcap_{i \in I} \tilde{\mu}_i(u - v) = \inf^{i}_{i \in I} \tilde{\mu}_i(u - v)$

$\geq \inf^{i}_{i \in I} \min^{i}\{\tilde{\mu}_i(u), \tilde{\mu}_i(v)\}$

$= \min^{i}\{\inf^{i}_{i \in I} \tilde{\mu}_i(u), \inf^{i}_{i \in I} \tilde{\mu}_i(v)\}$

$= \min^{i}\{\bigcap_{i \in I} \tilde{\mu}_i(u), \bigcap_{i \in I} \tilde{\mu}_i(v)\};$

$\bigcap_{i \in I} \tilde{\nu}_i(u - v) = \inf^{i}_{i \in I} \tilde{\nu}_i(u - v)$

Veerappan and Venkatesan | J. Fuzzy. Ext. Appl. 1(4) (2020) 314-324

$$\geq \inf^{i}_{i\in I}\min^{i}\{\tilde{\nu}_i(u),\tilde{\nu}_i(v)\}$$

$$= \min^{i}\{\inf^{i}_{i\in I}\tilde{\nu}_i(u),\inf^{i}_{i\in I}\tilde{\nu}_i(v)\}$$

$$= \min^{i}\{\bigcap_{i\in I}\tilde{\nu}_i(u),\bigcap_{i\in I}\tilde{\nu}_i(v)\};$$

$$\bigcap_{i\in I}\tilde{\xi}_i(u-v) = \inf^{i}_{i\in I}\tilde{\xi}_i(u-v)$$

$$\leq \inf^{i}_{i\in I}\max^{i}\{\tilde{\xi}_i(u),\tilde{\xi}_i(v)\}$$

$$= \max^{i}\{\inf^{i}_{i\in I}\tilde{\xi}_i(u),\inf^{i}_{i\in I}\tilde{\xi}_i(v)\}$$

$$= \max^{i}\{\bigcap_{i\in I}\tilde{\xi}_i(u),\bigcap_{i\in I}\tilde{\xi}_i(v)\};$$

$$\bigcap_{i\in I}\tilde{\mu}_i(u\alpha v\beta w) = \inf^{i}_{i\in I}\tilde{\mu}_i(u\alpha v\beta w)$$

$$\geq \inf^{i}_{i\in I}\min^{i}\{\tilde{\mu}_i(u),\tilde{\mu}_i(w)\}$$

$$= \min^{i}\{\inf^{i}_{i\in I}\tilde{\mu}_i(u),\inf^{i}_{i\in I}\tilde{\mu}_i(w)\}$$

$$= \min^{i}\{\bigcap_{i\in I}\tilde{\mu}_i(u),\bigcap_{i\in I}\tilde{\mu}_i(w)\};$$

$$\bigcap_{i\in I}\tilde{\nu}_i(u\alpha v\beta w) = \inf^{i}_{i\in I}\tilde{\nu}_i(u\alpha v\beta w)$$

$$\geq \inf^{i}_{i\in I}\min^{i}\{\tilde{\nu}_i(u),\tilde{\nu}_i(w)\}$$

$$= \min^{i}\{\inf^{i}_{i\in I}\tilde{\nu}_i(u),\inf^{i}_{i\in I}\tilde{\nu}_i(w)\}$$

$$= \min^{i}\{\bigcap_{i\in I}\tilde{\nu}_i(u),\bigcap_{i\in I}\tilde{\nu}_i(w)\}, \text{ and}$$

$$\bigcap_{i\in I}\tilde{\xi}_i(u\alpha v\beta w) = \inf^{i}_{i\in I}\tilde{\xi}_i(u\alpha v\beta w)$$

$$\leq \inf^{i}_{i\in I}\max^{i}\{\tilde{\xi}_i(u),\tilde{\xi}_i(w)\}$$

$$= \max^{i}\{\inf^{i}_{i\in I}\tilde{\xi}_i(u),\inf^{i}_{i\in I}\tilde{\xi}_i(w)\}$$

$$= \max^{i}\{\bigcap_{i\in I}\tilde{\xi}_i(u),\bigcap_{i\in I}\tilde{\xi}_i(w)\}.$$

Hence the proof.

Theorem 3. If $\{\tilde{A}_{s_i}; i\in I\}$ be a family of SIVFBI of a gamma near-ring $\mathscr{R}$, then $\bigcup_{i\in I}\tilde{A}_{s_i}$ is also SIVFBI of $\mathscr{R}$, where I is an index set.

Proof. Let $\{\tilde{A}_{s_i}; i\in I\}$ be a family of SIVFBI of a gamma near-ring $\mathscr{R}$. For any $u,v,w\in\mathscr{R}$ and $\alpha,\beta\in\Gamma$.

$$\bigcup_{i\in I} \tilde{\mu}_i(u-v) = \sup^i_{i\in I}\tilde{\mu}_i(u-v)$$

$$\geq \sup^i_{i\in I}\min{}^i\{\tilde{\mu}_i(u), \tilde{\mu}_i(v)\}$$

$$= \min{}^i\{\sup^i_{i\in I}\tilde{\mu}_i(u), \sup^i_{i\in I}\tilde{\mu}_i(v)\}$$

$$= \min{}^i\{\bigcup_{i\in I} \tilde{\mu}_i(u), \bigcup_{i\in I} \tilde{\mu}_i(v)\};$$

$$\bigcup_{i\in I} \tilde{\nu}_i(u-v) = \sup^i_{i\in I}\tilde{\nu}_i(u-v)$$

$$\geq \sup^i_{i\in I}\min{}^i\{\tilde{\nu}_i(u), \tilde{\nu}_i(v)\}$$

$$= \min{}^i\{\sup^i_{i\in I}\tilde{\nu}_i(u), \sup^i_{i\in I}\tilde{\nu}_i(v)\}$$

$$= \min{}^i\{\bigcup_{i\in I} \tilde{\nu}_i(u), \bigcup_{i\in I} \tilde{\nu}_i(v)\};$$

$$\bigcup_{i\in I} \tilde{\xi}_i(u-v) = \inf^i_{i\in I}\tilde{\xi}_i(u-v)$$

$$\leq \sup^i_{i\in I}\max{}^i\{\tilde{\xi}_i(u), \tilde{\xi}_i(v)\}$$

$$= \max{}^i\{\sup^i_{i\in I}\tilde{\xi}_i(u), \sup^i_{i\in I}\tilde{\xi}_i(v)\}$$

$$= \max{}^i\{\bigcup_{i\in I} \tilde{\xi}_i(u), \bigcup_{i\in I} \tilde{\xi}_i(v)\};$$

$$\bigcup_{i\in I} \tilde{\mu}_i(u\alpha v\beta w) = \sup^i_{i\in I}\tilde{\mu}_i(u\alpha v\beta w)$$

$$\geq \sup^i_{i\in I}\min{}^i\{\tilde{\mu}_i(u), \tilde{\mu}_i(w)\}$$

$$= \min{}^i\{\sup^i_{i\in I}\tilde{\mu}_i(u), \sup^i_{i\in I}\tilde{\mu}_i(w)\}$$

$$= \min{}^i\{\bigcap_{i\in I} \tilde{\mu}_i(u), \bigcup_{i\in I} \tilde{\mu}_i(w)\};$$

$$\bigcup_{i\in I} \tilde{\nu}_i(u\alpha v\beta w) = \sup^i_{i\in I}\tilde{\nu}_i(u\alpha v\beta w)$$

$$\geq \sup^i_{i\in I}\min{}^i\{\tilde{\nu}_i(u), \tilde{\nu}_i(w)\}$$

$$= \min{}^i\{\sup^i_{i\in I}\tilde{\nu}_i(u), \sup^i_{i\in I}\tilde{\nu}_i(w)\}$$

$$= \min{}^i\{\bigcup_{i\in I} \tilde{\nu}_i(u), \bigcup_{i\in I} \tilde{\nu}_i(w)\}, \text{ and}$$

$$\bigcup_{i\in I} \tilde{\xi}_i(u\alpha v\beta w) = \sup^i_{i\in I}\tilde{\xi}_i(u\alpha v\beta w)$$

$$\leq \inf^i_{i\in I}\max{}^i\{\tilde{\xi}_i(u), \tilde{\xi}_i(w)\}$$

$$= \max{}^i\{\sup^i_{i\in I}\tilde{\xi}_i(u), \sup^i_{i\in I}\tilde{\xi}_i(w)\}$$

$= \max^{i}\{\bigcup_{i\in I} \tilde{\xi}_i(u), \bigcup_{i\in I} \tilde{\xi}_i(w)\}.$

Hence the proof.

Theorem 4. If $\tilde{A}_s$ and $\tilde{\sigma}_s$ are SIVFBIs of $\mathscr{R}$, then $\tilde{A}_s \wedge \tilde{\sigma}_s$ is SIVFBI of $\mathscr{R}$.

Proof. Let and $\tilde{\sigma}_s$ are SFBIs of $\mathscr{R}$. Let $u, v, w \in \mathscr{R}$ and $\alpha, \beta \in \Gamma$. Then,

$(\tilde{\mu} \wedge \tilde{\sigma}_s)(u - v) = min^i\{\tilde{\mu}(u - v), \tilde{\sigma}_s(u - v)\}$, since by $(\tilde{\mu} \wedge \tilde{\sigma})(u) = min^i\{\tilde{\mu}(u), \tilde{\sigma}(u)\}$

$\geq \min^i\{\min^i\{\tilde{\mu}(u), \tilde{\mu}(v)\}, \min^i\{\tilde{\sigma}_s(u), \tilde{\sigma}_s(v)\}\}$

$= \min^i\{\min^i\{\min^i\{\tilde{\mu}(u), \tilde{\mu}(v), \tilde{\sigma}_s(u)\}, \tilde{\sigma}_s(v)\}\}$

$= \min^i\{\min^i\{\min^i\{\tilde{\mu}(u), \tilde{\sigma}_s(u)\}, \tilde{\mu}(v)\}, \tilde{\sigma}_s(v)\}$

$= \min^i\{\min^i\{\tilde{\mu}(u), \tilde{\sigma}_s(u)\}, \min^i\{\tilde{\mu}(v), \tilde{\sigma}_s(v)\}$

$= \min^i\{\tilde{\mu} \wedge \tilde{\sigma}(u)), (\tilde{\mu} \wedge \tilde{\sigma}(v))\}.$

Also $(\tilde{\nu} \wedge \tilde{\sigma}_s)(u - v) \geq \min^i\{\tilde{\nu} \wedge \tilde{\sigma}(u)), (\tilde{\nu} \wedge \tilde{\sigma}(v))\}$ and $(\tilde{\xi} \wedge \tilde{\sigma}_s)(u - v) \leq \max^i\{\tilde{\xi} \wedge \tilde{\sigma}(u)), (\tilde{\xi} \wedge \tilde{\sigma}(v))\}$.

Since $(\tilde{\mu}(u\alpha v\beta w) \geq min^i\{\tilde{\mu}(u), \tilde{\mu}(v))$.

$(\tilde{\mu} \wedge \tilde{\sigma}_s)(u\alpha v\beta w) = \min^i\{\tilde{\mu}(u\alpha v\beta w), \tilde{\sigma}_s(u\alpha v\beta w))$

$\geq \min^i\{\min^i\{\tilde{\mu}(u), \widetilde{A}_s(w)\}, \min^i\{\tilde{\sigma}_s(u), \tilde{\sigma}_s(w)\}\}$

$= \min^i\{\min^i\{\tilde{\mu}(u), \tilde{\sigma}_s(u)\}, \min^i\{\tilde{\mu}(w)\}, \tilde{\sigma}_s(w)\}\}$

$= \min^i\{(\tilde{\mu} \wedge \tilde{\sigma}_s)(u), (\tilde{\mu} \wedge \tilde{\sigma}_s)(w)\}.$

Also $(\tilde{\nu} \wedge \tilde{\sigma}_s)(u\alpha v\beta w) \geq min^i\{(\tilde{\nu} \wedge \tilde{\sigma}_s)(u), (\tilde{\nu} \wedge \tilde{\sigma}_s)(w)$ and $(\tilde{\xi} \wedge \tilde{\sigma}_s)(u\alpha v\beta w) \leq max^i\{(\tilde{\xi} \wedge \tilde{\sigma}_s)(u), (\tilde{\xi} \wedge \tilde{\sigma}_s)(w))$.

Hence $(\tilde{A}_s \wedge \tilde{\sigma}_s)$ is a SIVFBI of $\mathscr{R}$.

Lemma 1. Let A be BI of $\mathscr{R}$. For any $0 < m < 1$, there exists a SIVFBI A_s of $\mathscr{R}$ such that $\tilde{A}_{s_m} = A$.

Proof. Let A be BI of $\mathscr{R}$. Define $\tilde{A}_s: \mathscr{R} \longrightarrow [0,1]$ by

$$\widetilde{A}_s(u) = \begin{cases} m, & \text{if } u \in A \\ 0, & \text{if } u \notin A. \end{cases}$$

where m be a constant in (0,1). Clearly $\widetilde{A}_{s_m} = A$. Let $u, v \in \mathscr{R}$. If $u, v \in A$, then $\tilde{\mu}(u - v) = m \geq min^i\{\tilde{\mu}(u), \tilde{\mu}(v)\}, \widetilde{\nu}(u - v) = m \geq min^i\{\tilde{\nu}(u), \tilde{\nu}(v)\}$ and $\tilde{\xi}(u - v) = m \leq max^i\{\tilde{\xi}(u), \tilde{\xi}(v)\}$.

If at least one of u and v is not in A, then $u - v \notin A$ and so $\tilde{\mu}(u - v) = 0 = min^i\{\tilde{\mu}(u), \tilde{\mu}(v)\}$, $\tilde{\nu}(u - v) = 0 = min^i\{\tilde{\nu}(u), \tilde{\nu}(v)\}$ and $\tilde{\xi}(u - v) = 0 = max^i\{\tilde{\xi}(u), \tilde{\xi}(v)\}$.

Let $u, v, w \in \mathscr{R}$ and $\alpha, \beta \in \Gamma$. If $u, w \in A$, then $\tilde{\mu}(u), \tilde{\nu}(u), \tilde{\xi}(u) = m; \tilde{\mu}(w), \tilde{\nu}(w), \tilde{\xi}(w) = m$. Also $\tilde{\mu}(u\alpha v\beta w) = m \geq min^i\{\tilde{\mu}(u), \tilde{\mu}(w)\}$, $\tilde{\nu}(u\alpha v\beta w) = m \geq min^i\{\tilde{\nu}(u), \tilde{\nu}(w)\}$ and $\tilde{\xi}(u\alpha v\beta w) = m \leq max^i\{\tilde{\xi}(u), \tilde{\xi}(w)\}$.

If at least one of u and w is not in A, then $\tilde{\mu}(u\alpha v\beta w) \geq 0 = min^i\{\tilde{\mu}(u), \tilde{\mu}(w)\}$, $\tilde{\nu}(u\alpha v\beta w) \geq 0 = min^i\{\tilde{\nu}(u), \tilde{\nu}(w)\}$ and $\tilde{\xi}(u\alpha v\beta w) \leq 0 = max^i\{\tilde{\xi}(u), \tilde{\xi}(w)\}$.

Thus $\widetilde{A}_s$ is SIVFBI of $\mathscr{R}$.

Theorem 5. If $\tilde{A}_s$ be SIVFBI of $\mathscr{R}$, then the complement $\tilde{A}_s$ is also SIVFBI o $\mathscr{R}$ f.

Proof. For $u, v, w \in \mathscr{R}$ and $\alpha, \beta \in \Gamma$, we have

$\tilde{\mu}(u-v) = 1 - \tilde{\mu}(u-v) \geq 1 - min^i\{\tilde{\mu}(u), \tilde{\mu}(v)\}$=$min^i\{1-\tilde{\mu}(u), 1-\tilde{\mu}(v)\}$= $min^i\{\tilde{\mu}(u), \tilde{\mu}(v)\}$, and also $\tilde{\nu}(u-v) \geq min^i\{\tilde{\nu}(u), \tilde{\nu}(v)\}$, $\tilde{\xi}(u-v) \leq max^i\{\tilde{\xi}(u), \tilde{\xi}(v)\}$.

$\tilde{\mu}(u\alpha v\beta w) = 1 - \tilde{\mu}(u\alpha v\beta w) \geq 1 - min^i\{\tilde{\mu}(u), \tilde{\mu}(w)\}$=$min^i\{1-\tilde{\mu}(u), 1-\tilde{\mu}(w)\}$= $min^i\{\tilde{\mu}(u), \tilde{\mu}(w))$, and also $\tilde{\nu}(u\alpha v\beta w) \geq min^i\{\tilde{\nu}(u), \tilde{\nu}(w)\}$, $\tilde{\xi}(u\alpha v\beta w) \leq max^i\{\tilde{\xi}(u), \tilde{\xi}(w)\}$.

Hence $\tilde{A}_s$ is also SIVFBI of $\mathscr{R}$.

Lemma 6. Let U is fuzzy subset of $\mathscr{R}$. Then U is BI of $\mathscr{R}$ if and only if $\tilde{A}_{s_U}$ is SIVFBI of $\mathscr{R}$.

Proof. Let U be BI of $\mathscr{R}$. For $u, v \in U$, $u - v \in U$.

Let $u, v \in \mathscr{R}$.

case(a): If $u, v \in U$, then $\tilde{\mu}_U(u) = 1$ and $\tilde{\mu}_U(v) = 1$. Thus $\tilde{\mu}_U(u-v) = 1 \geq min^i\left\{\tilde{\mu}(u), \tilde{\mu}(v)\right\}$.

case(b): If $u \in U$ and $v \notin U$, then $\tilde{\mu}_U(u) = 1$ and $\tilde{\mu}_U(v) = 0$. Thus $\tilde{\mu}_U(u-v) = 0 \geq min^i\{\tilde{\mu}(u), \tilde{\mu}(v))$.

case(c): If $u \notin U$ and $v \in U$, then $\tilde{\mu}_U(u) = 0$ and $\tilde{\mu}_U(v) = 1$. Thus $\tilde{\mu}_U(u-v) = 0 \geq min^i\{\tilde{\mu}(u), \tilde{\mu}(v)\}$.

case(d): If $u \notin U$ and $v \notin U$, then $\tilde{\mu}_U(u) = 0$ and $\tilde{\mu}_U(v\tilde{\xi}_U(u-v) \leq max^i\{\tilde{\xi}(u), \tilde{\xi}(v)\}) = 0$. Thu$\widetilde{\mu}_U(u-v) = 0s \geq min^i\{\tilde{\mu}(u), \tilde{\mu}(v)\}$.

In the above four cases $\tilde{\nu}_U(u-v) \geq min^i\{\tilde{\nu}(u), \tilde{\nu}(v)\}$.

Let $u, v, w \in \mathscr{R}$.

case(a): If $u \in U$ and $w \in U$, then $\tilde{\mu}_U(u) = 1$ and $\tilde{\mu}_U(w) = 1$. Thus $\tilde{\mu}_U(u\alpha v\beta w) = 1 \geq min^i\{\tilde{A}_{s_U}\tilde{\mu}(u), \tilde{\mu}(w)\}$.

case(b): If $u \in U$ an $w \notin U$ d, then$\widetilde{\mu}_U(u) = 1$ and $\tilde{\mu}_U(w) = 0$. Thus $\tilde{\mu}_U(u\alpha v\beta w) = 0 \geq min^i\{\tilde{\mu}(u), \tilde{\mu}(w)\}$.

case(c): If $u \notin U$ and $w \in U$, then $\tilde{\mu}_U(u) = 0$ and $\tilde{\mu}_U(w) = 1$. Thus $\mu_U(u\alpha v\beta w) = 0 \geq min^i\{\tilde{\mu}(u), \tilde{\mu}(w)\}$.

case(d): If $u \notin U$ and $w \notin U$, then $\tilde{\mu}_U(u) = 0$ and $\tilde{\mu}_U(w) = 0$. Thus $\tilde{\mu}_U(u\alpha v\beta w) = 0 \geq min^i\{\tilde{\mu}(u), \tilde{\mu}(w)\}$.

Also $\tilde{\nu}_U(u\alpha v\beta w) \geq min^i\{\tilde{\nu}(u), \tilde{\nu}(w)\}$ and$\tilde{\xi}_U(u\alpha v\beta w) \leq max^i\{\tilde{\xi}(u), \tilde{\xi}(w)\}$ Thus $\tilde{A}_{s_U}$ is a SIVFBI of $\mathscr{R}$.

Conversely, suppose is a SIVFBI of $\mathscr{R}$. Then by *Lemma 4* $\tilde{A}_{s_U}$ has only two elements.

Veerappan and Venkatesan|J. Fuzzy. Ext. Appl. 1(4) (2020) 314-324

Hence U is BI of $\mathscr{R}$.

Theorem 7. If $\mathscr{R}$ be a gamma near-ring and $\tilde{A}_s$ be SIVFBI of $\mathscr{R}$, then the set $\mathscr{R}_{\tilde{A}_s} = \{u \in \mathscr{R} | \tilde{A}_s(u) = \tilde{A}_s(0)\}$ is BI of $\mathscr{R}$.

Proof. Let $\tilde{A}_s$ be SIVFBI of and le $u, v, w \in \mathscr{R}$ t. Then

$\tilde{\mu}(u - v) \geq min^i\{\tilde{\mu}(u), \tilde{\mu}(v)\} = min^i\{\tilde{\mu}(0), \tilde{\mu}(0)\} = \tilde{\mu}(0)$. So $\tilde{\mu}(u - v) = \tilde{\mu}(0)$, *then* $u - v \in \mathscr{R}_{\tilde{A}_s}$.

$\tilde{\nu}(u - v) \geq min^i\{\tilde{\nu}(u), \tilde{\nu}(v)\} = min^i\{\tilde{\nu}(0), \tilde{\nu}(0)\} = \tilde{\nu}(0)$. So $\tilde{\nu}(u - v) = \tilde{\nu}(0)$, *then* $u - v \in \mathscr{R}_{\tilde{A}_s}$.

$\tilde{\xi}(u - v) \leq max^i\{\tilde{\xi}(u), \tilde{\xi}(v)\} = max^i\{\tilde{\xi}(0), \tilde{\xi}(0)\} = \tilde{\xi}(0)$. So $\tilde{\xi}(u - v) = \tilde{\xi}(0)$, *then* $u - v \in \mathscr{R}_{\tilde{A}_s}$.

$\tilde{\mu}(u\alpha v\beta w) \geq min^i\{\tilde{\mu}(u), \tilde{\mu}(w)\} = min^i\{\tilde{\mu}(0), \tilde{\mu}(0)\} = \tilde{\mu}(0)$. So $\tilde{\mu}(u\alpha v\beta w) = \tilde{\mu}(0)$, *then* $u\alpha v\beta w \in \mathscr{R}_{\tilde{A}_s}$.

$\tilde{\nu}(u\alpha v\beta w) \geq min^i\{\tilde{\nu}(u), \tilde{\nu}(w)\} = min^i\{\tilde{\nu}(0), \tilde{\nu}(0)\} = \tilde{\nu}(0)$. So $\tilde{\nu}(u\alpha v\beta w) = \tilde{\nu}(0)$, *then* $u\alpha v\beta w \in \mathscr{R}_{\tilde{A}_s}$.

$\tilde{\xi}(u\alpha v\beta w) \leq max^i\{\tilde{\xi}(u), \tilde{\xi}(w)\} = max^i\{\tilde{\xi}(0), \tilde{\xi}(0)\} = \tilde{\xi}(0)$. So $\tilde{\xi}(u\alpha v\beta w) = \tilde{\xi}(0)$, *then* $u\alpha v\beta w \in \mathscr{R}_{\tilde{A}_s}$.

Then $\mathscr{R}_{\tilde{A}_s}$ is BI of $\mathscr{R}$.

Theorem 8. If B be a non-empty subset of $\mathscr{R}$ and $\widetilde{A}_{s_B}$ be a Spherical Interval-Valued Fuzzy Set (SIVFS) $\mathscr{R}$ defined by

$$\widetilde{A}_{s_B}(u) = \begin{cases} \tilde{p}, & \text{if } u \in B \\ \tilde{q}, & \text{otherw}\tilde{p} \geq \tilde{q}\text{ise.} \end{cases}$$

for $u \in \mathscr{R}$, $\tilde{p}, \tilde{q} \in D[0,1]$ and. Then $\tilde{A}_{s_B}(u)$ is a SIVFBI of $\mathscr{R}$ if and only if B is a BI of $\mathscr{R}$. Als$\mathscr{R}_{\tilde{A}_{s_B}} = B$ *o.*

Proof. Let $\widetilde{A}_{s_B}$t be a SIVFS $\mathcal{R}$ and le $u, v, w \in B$ t. Then $\tilde{A}_{s_B}(u) = \tilde{p} = \tilde{A}_{s_B}(v) = \tilde{A}_{s_B}(w)$. Now,

$$\widetilde{A}_{s_B}(u - v) \geq \min^i\{\widetilde{A}_{s_B}(u), \widetilde{A}_{s_B}(v)\}$$

$$= \min^i\{\tilde{p}, \tilde{p}\} = \tilde{p}.$$

$$\widetilde{A}_{s_B}(u - v) = \tilde{p}, \text{so } u - v \in B.$$

$$\widetilde{A}_{s_B}(u\alpha v\beta w) \geq \min^i\{\widetilde{A}_{s_B}(u), \widetilde{A}_{s_B}(w)\}$$

$$= \min^i\{\tilde{p}, \tilde{p}\} = \tilde{p}.$$

$$\widetilde{A}_{s_B} B(u\alpha v\beta w) = \tilde{p}, \text{so } u\alpha v\beta w \in B.$$

Then is a BI of $\mathscr{R}$.

Conversely let B be a BI of $\mathscr{R}$ and le $u, v, w \in \mathscr{R}$ t.

If at $\tilde{A}_{s_B}(u)$ least one u, v is not in B, then $u - v \notin B$ and so$\tilde{A}_{s_B}(u - v) \geq min^i\{\tilde{A}_{s_B}(u), \tilde{A}_{s_B}(v)\} = \tilde{q}$.

If at least one u, w is not in B, then $u\alpha v\beta w \notin B$ and so $\tilde{A}_{s_B}(u\alpha v\beta w) \geq min^i\{\tilde{A}_{s_B}(u), \tilde{A}_{s_B}(w)\} = \tilde{q}$.

Thus is a SIVFBI of $\mathscr{R}$.

4| Conclusion

We obtained the union and intersection of the spherical interval-valued fuzzy bi-ideal of gamma near-ring $\mathscr{R}$ is also a spherical interval-valued fuzzy bi-ideal of gamma near-ring. And for that condition, $\tilde{A}_{s_m} = A$, for any $0 < m < 1$, bi-ideal of gamma near-ring $\mathscr{R}$ becomes spherical interval-valued fuzzy bi-ideal of gamma near-ring $\mathscr{R}$. In future we will discuss the spherical fuzzy sets in some other algebraic structures.

References

[1] Chinnadurai, V. (2013). *Fuzzy ideals in algebraic structures*. LAP LAMBERT Academic Publishing.

[2] Chinnadurai, V., & Kadalarasi, S. (2016). Direct product of fuzzy ideals of near-rings. *Annals of fuzzy mathematics and informatics*, *12* (2), 193-204.

[3] Chinnadurai, V., & Shakila, V. (2020, November). T-fuzzy bi-ideal of Gamma near-ring. *AIP conference proceedings* (Vol. 2277, No. 1, p. 090012). AIP Publishing LLC.

[4] Chinnadurai, V., & Shakila, V. (2020). Spherical fuzzy bi-ideals of gamma near-rings. *Advances in mathematics: scientific journal*, *9*(10), 7793-7802.

[5] Hong, S. M., Jun, Y. B., & Kim, H. S. (1998). Fuzzy ideals in near-rings. *Bulletin of the korean mathematical society*, *35*(3), 455-464.

[6] Jun, Y. B., Sapanci, M., & Öztürk, M. A. (1999). Fuzzy ideals in gamma near-rings. *Turkish journal of mathematics*, *22*(4), 449-460.

[7] Jun, Y. B., Kim, K. H., & Ozturk, M. A. (2001). On fuzzy ideals of gamma near-rings. *Journal of fuzzy mathematics*, *9*(1), 51-58.

[8] Kutlu Gündoğdu, F., & Kahraman, C. (2019). Spherical fuzzy sets and spherical fuzzy TOPSIS method. *Journal of intelligent & fuzzy systems*, *36*(1), 337-352.

[9] Meenakumari, N., & Chelvam, T. T. (2011). Fuzzy bi-ideals in gamma near-rings. *Journal of algebra discrete structures*, *9*(1-2), 43-52.

[10] Satyanarayana, B. (1999). A note on Gamma near-rings. *Indian J.Mathematics*, *41*, 427-433.

[11] Srinivas, T., & Nagaiah, T. (2012). Some results on T-fuzzy ideals of Γ-near-rings. *Annals of fuzzy mathematics and informatics*, *4*(2), 305-319.

[12] Thillaigovindan, N., & Chinnadurai, V. (2009). Interval valued fuzzy quasi-ideals of semigroups. *East asian mathematics journal*, *25*(4), 449-467.

[13] Thillaigovindan, N., Chinnadurai, V., & Kadalarasi, S. (2015). Interval valued fuzzy ideals of nera-rings. *The journal of fuzzy mathematics*, *23*(2), 471-484.

[14] Zadeh, L. A. (1996). Fuzzy sets. In *Fuzzy sets, fuzzy logic, and fuzzy systems: selected papers by Lotfi A Zadeh* (pp. 394-432). https://doi.org/10.1142/9789814261302_0021

Veerappan and Venkatesan|J. Fuzzy. Ext. Appl. 1(4) (2020) 314-324