E-ISSN: 2717-3453 | P-ISSN: 2783-1442

Journal of Fuzzy Extension and Applications

www.journal-fea.com

J. Fuzzy. Ext. Appl. Vol. 2, No. 1 (2021) 89–105.

Paper Type: Research Paper

A Simplified Method for Solving Transportation Problem with Triangular Fuzzy Numbers under Fuzzy Circumstances

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Citation:

Kané, L., Sidibé, H., Kané, S., Bado, H., Konaté, M., Diawara, D., & Diabat, L. (2021). A simplified method for solving transportation problem with triangular fuzzy numbers under fuzzy circumstances. *Journal of fuzzy extension and application, 2* (1), 89-105.

Received: 14/12/2020 Reviewed: 19/02/2021 Revised: 08/03/2021 Accept: 17/03/2021

Abstract

Transportation Problem (TP) is an important network structured linear programming problem that arises in several contexts and has deservedly received a great deal of attention in the literature. The central concept in this problem is to find the least total transportation cost of a commodity in order to satisfy demands at destinations using available supplies at origins in a crisp environment. In real life situations, the decision maker may not be sure about the precise values of the coefficients belonging to the transportation problem. The aim of this paper is to introduce a formulation of TP involving Triangular fuzzy numbers for the transportation costs and values of supplies and demands. We propose a two-step method for solving fuzzy transportation problem where all of the parameters are represented by non-negative triangular fuzzy numbers i.e., an Interval Transportation Problems (TPIn) and a Classical Transport Problem (TP). Since the proposed approach is based on classical approach it is very easy to understand and to apply on real life transportation problems for the decision makers. To illustrate the proposed approach two application examples are solved. The results show that the proposed method is simpler and computationally more efficient than existing methods in the literature.

Keywords: Fuzzy linear programming, Transportation problem, Triangular fuzzy numbers.

1 | Introduction

Transportation problem is an important network structured linear programming problem that arises in several contexts and has deservedly received a great deal of attention in the literature. The central concept in this problem is to find the least total transportation cost of a commodity in order to satisfy demands at destinations using available supplies at origins. Transportation problem can be used for a wide variety of situations such as scheduling, production, investment, plant location, inventory control, employment scheduling and many others.

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http://dx.doi.org/10.22105/jfea.2021.275280.1084a

In general, transportation problems are solved with the assumptions that the transportation costs and values of supplies and demands are specified in a precise way i.e., in crisp environment. However, in many cases the decision maker has no crisp information about the coefficients belonging to the transportation problem. If the nature of the information is vague, that is, if it has some lack of precision, the corresponding coefficients or elements defining the problem can be formulated by means of fuzzy sets, and thus fuzzy transportation problems arise. Several researchers have carried out investigations on fuzzy transportation problem. Zimmermann [4] developed Zimmermann's fuzzy linear programming into several fuzzy optimization methods for solving the transportation problems. ÓhÉigeartaigh [5] proposed an algorithm for solving transportation problems where the supplies and demands are fuzzy sets with linear or triangular membership functions. Chanas et al. [6] investigated the transportation problem with fuzzy supplies and demands and solved them via the parametric programming technique. Their method provided solution which simultaneously satisfies the constraints and the goal to a maximal degree.

In addition, Chanas et al. [7] formulated the classical, interval and fuzzy transportation problem and discussed the methods for solution for the fuzzy transportation problem. Chanas and Kuchta [8] discussed the type of transportation problems with fuzzy cost coefficients and converted the problem into a bicriterial transportation problem with crisp objective function. Their method only gives crisp solutions based on efficient solutions of the converted problems. Jimenez and Verdegay [9] and [10] investigated the fuzzy solid transportation problem in which supplies, demands and conveyance capacities are represented by trapezoidal fuzzy numbers and applied a parametric approach for finding the fuzzy solution. Liu and Kao [11] developed a procedure, based on extension principle to derive the fuzzy objective value of fuzzy transportation problem, in that the cost coefficients and the supply and demand quantities are fuzzy numbers.

Gani and Razak [12] presented a two-stage cost minimizing fuzzy transportation problem in which supplies and demands are as trapezoidal fuzzy numbers and used a parametric approach for finding a fuzzy solution with the aim of minimizing the sum of the transportation costs in the two stages. Li et al. [13] proposed a new method based on goal programming for solving fuzzy transportation problem with fuzzy costs. Lin [14] used genetic algorithm for solving transportation problems with fuzzy coefficients. Dinagar and Palanivel [15] investigated fuzzy transportation problem, with the help of trapezoidal fuzzy numbers and applied fuzzy modified distribution method to obtain the optimal solution in terms of fuzzy numbers. Pandian and Natarajan [16] introduced a new algorithm namely, fuzzy zero-point method for finding fuzzy optimal solution for such fuzzy transportation problem in which the transportation cost, supply and demand are represented by trapezoidal fuzzy numbers. Kumar and Kaur [17] proposed a new method based on fuzzy linear programming problem for finding the optimal solution of fuzzy transportation problem.

Gupta et al. [18] proposed a new method named as Mehar's method, to find the exact fuzzy optimal solution of fully fuzzy multi-objective transportation problems. Ebrahimnejad [19] applied a fuzzy bounded dual algorithm for solving bounded transportation problems with fuzzy supplies and demands. Shanmugasundari and Ganesan [20] developed the fuzzy version of Vogel's and MODI methods for obtaining the fuzzy initial basic feasible solution and fuzzy optimal feasible solution, respectively, without converting them into classical transportation problem. Also, Chandran and Kandaswamy [21] proposed an algorithm to find an optimal solution of a fuzzy transportation problem, where supply, demand and cost coefficients all are fuzzy numbers. Their algorithm provides decision maker with an effective solution in comparison to existing methods. Ebrahimnejad [22] using an example showed that their method will not always lead to a fuzzy optimal solution. Moreover, Kumar and Kaur [23] pointed out the limitations and shortcomings of the existing methods for solving fuzzy solid transportation problem and to overcome these limitations and shortcomings proposed a new method to find the fuzzy optimal solution of unbalanced fuzzy solid transportation problems.

In addition, Ebrahimnejad [24] proposed a two-step method for solving fuzzy transportation problem where all of the parameters are represented by non-negative triangular fuzzy numbers. Some researchers applied generalized fuzzy numbers for solving transportation problems. Kumar and Kaur [25] proposed a new method based on ranking function for solving fuzzy transportation problem by assuming that transportation cost, supply and demand of the commodity are represented by generalized trapezoidal fuzzy numbers. After that, Kaur and Kumar [26] introduced a similar algorithm for solving a special type of fuzzy transportation problem by assuming that a decision maker is uncertain about the precise values of transportation cost only but there is no uncertainty about the supply and demand of the product. Ebrahimnejad [27] demonstrated that once the ranking function is chosen, the fuzzy transportation problem introduced by Kaur and Kumar [26] is converted into crisp one, which is easily solved by the standard transportation algorithms.

The contributions of the present study are summarized as follows: (1) in the TPTri under consideration, all of the parameters, such as the transportation costs, supplies and demands are considered as fuzzy numbers, (2) according to the proposed approach, the TPTri is converted into an TPIn and a TP. The integration of the optimal solution of the four sub-problems provides the optimal solution of the TPTri, (3) in contrast to most existing approaches, which provide a precise solution, the proposed method provides a fuzzy optimal solution, (4) In contrast to existing methods that include negative parts in the obtained fuzzy optimal solution and fuzzy optimal cost, the proposed method provides a fuzzy optimal solution and optimal cost, (5) similarly, to the competing methods in the literature, the proposed method is applicable for all types of triangular fuzzy numbers. and (6) the complexity of computation is greatly reduced compared with commonly used existing methods in the literature.

The rest of this paper is organized as follows. In Section 2, we recall the definitions of interval number linear programming, interval numbers and the existing method for solving linear programming problem involving interval numbers. In Section 3, a new method is proposed for obtaining the fuzzy optimal solution of the TPTri. The advantages of the proposed method are discussed in Section 4. Two application examples are provided to illustrate the effectiveness of the proposed method in Section 5. Finally, concluding remarks are presented in Section 6.

2| Materials and Methods

In this section, some basic definitions, arithmetic operations for closed Intervals numbers and of linear programming problems involving interval numbers are presented [28].

2.1| A New Interval Arithmetic

In this section, some arithmetic operations for two intervals are presented [28].

Let $\Re = \{\bar{a} = [a^1, a^3]: a^1 \le a^3 \text{ } with \text{ } a^1, a^3 \in \mathbb{R}\}$ be the set of all proper intervals and $\bar{\Re} = \{\bar{a} = [a^1, a^3]: a^1 > a^3 \text{ with } a^1, a^3 \in \mathbb{R}\}$ be the set of all improper intervals on the real line $\mathbb{R}$. We shall use the terms "interval" and "interval number" interchangeably. The mid-point and width (or half-width) of an interval number are defined as the midpoint and width (or half-width) of an interval number $\bar{a} = [a^1, a^3]$ are defined as $m(\bar{a}) = \left(\frac{a^3+a^1}{2}\right)$ and $w(\bar{a}) = \left(\frac{a^3-a^1}{2}\right)$. The interval number $\bar{a}$ can also be expressed in terms of its midpoint and width as $\bar{a} = [a^1, a^3] = \langle m(\bar{a}), w(\bar{a})\rangle = \langle \frac{a^3+a^1}{2}, \frac{a^3-a^1}{2}\rangle$.

For any two intervals $\bar{a} = [a^1, a^3] = \langle m(\bar{a}), w(\bar{a})\rangle$ and $\bar{b} = [b^1, b^3] = \langle m(\bar{b}), w(\bar{b})\rangle$, the arithmetic operations on $\bar{a}$ and $\bar{b}$ are defined as:

Addition: $\bar{a} + \bar{b} = \langle m(\bar{a}) + m(\bar{b}), w(\bar{a}) + w(\bar{b})\rangle$;

Soustraction : $\bar{a} - \bar{b} = \langle m(\bar{a}) - m(\bar{b}), w(\bar{a}) + w(\bar{b})\rangle$, $\alpha\bar{a} = \begin{cases} \langle \alpha m(\bar{a}), \alpha w(\bar{a})\rangle \text{ if } \alpha \geq 0 \\ \langle \alpha m(\bar{a}), -\alpha w(\bar{a})\rangle \text{ if } \alpha < 0 \end{cases}$;

$$\text{Multiplication}: \ \bar{a} \times \bar{b} = \begin{cases} \langle m(\bar{a})m(\bar{b}) + w(\bar{a})w(\bar{b}), m(\bar{a})w(\bar{b}) + m(\bar{b})w(\bar{a}) \rangle \ if\ a^1 \geq 0, b^1 \geq 0 \\ \langle m(\bar{a})m(\bar{b}) + m(\bar{a})w(\bar{b}), m(\bar{b})w(\bar{a}) + w(\bar{b})w(\bar{a})\rangle \ if\ a^1 < 0, b^1 \geq 0 \\ \langle m(\bar{a})m(\bar{b}) - w(\bar{a})w(\bar{b}), m(\bar{b})w(\bar{a}) - m(\bar{a})w(\bar{b})\rangle \ if a^3 < 0, b^1 \geq 0 \end{cases}.$$

2.2| Formulation of a Linear Programming Problem Involving Interval Numbers (LPIn)

We consider the Linear Programming Problems involving Interval numbers (LPIn) as follows [28],

$$\begin{cases} \text{Max } \bar{Z}(\bar{x}) \approx \sum_{j=1}^{n} \bar{c}_j \bar{x}_j \\ \text{Subject to the constraints.} \\ \sum_{j=1}^{n} \bar{a}_{ij}\bar{x}_j \preccurlyeq \bar{b}_i \end{cases}$$

For all the rest of this paper, we will consider the following notations:

$$\bar{x} = [\bar{x}_j]_{n\times 1} = \left[[x_j^1, x_j^3]\right]_{n\times 1} = [\langle m(\bar{x}_j), w(\bar{x}_j)\rangle]_{n\times 1},$$

$$\bar{c} = [\bar{c}_j]_{1\times n} = \left[[c_j^1, c_j^3]\right]_{1\times n} = [\langle m(\bar{c}_j), w(\bar{c}_j)\rangle]_{1\times n},$$

$$\bar{b} = [\bar{b}_i]_{m\times 1} = \left[[b_i^1, b_i^3]\right]_{m\times 1} = [\langle m(\bar{b}_i), w(\bar{b}_i)\rangle]_{m\times 1} \text{ and}$$

$$\bar{A} = [\bar{a}_{ij}]_{m\times n} = \left[[a_{ij}^1, a_{ij}^3]\right]_{m\times n} = [\langle m(\bar{a}_{ij}), w(\bar{a}_{ij})\rangle]_{m\times n}.$$

For all the rest of this paper, we will consider the following LPIn [28],

$$\begin{cases} \text{Min/Max } \bar{Z}(\bar{x}_1, \dots, \bar{x}_n) \approx \sum_{j=1}^{n} [c_j^1, c_j^3][x_j^1, x_j^3] \\ \text{Subject to the constraints} \\ \sum_{j=1}^{n} [a_{ij}^1, a_{ij}^3]\ [x_j^1, x_j^3] \left(\substack{\preccurlyeq \\ = \\ \succcurlyeq}\right) [b_i^1, b_i^3] \\ 1 \leq j \leq n \text{ and } 1 \leq i \leq m \end{cases},$$

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LPIn is equivalent to

$$\begin{cases} \text{Min/Max } \bar{Z}(\bar{x}_1, \dots, \bar{x}_n) \approx \sum_{j=1}^{n} \langle m(\bar{c}_j), w(\bar{c}_j) \rangle \langle m(\bar{x}_j), w(\bar{x}_j) \rangle \\ \text{Subject to the constraints} \\ \sum_{j=1}^{n} \langle m(\bar{a}_{ij}), w(\bar{a}_{ij}) \rangle \ \langle m(\bar{x}_j), w(\bar{x}_j) \rangle \left(\begin{smallmatrix} \lesseqgtr \end{smallmatrix} \right) \langle m(\bar{b}_i), w(\bar{b}_i) \rangle \\ 1 \le j \le n \text{ and } 1 \le i \le m \end{cases}.$$

3| Main Results

In this section, we will describe our method of solving.

3.1| A New Interval Arithmetic for Triangular Fuzzy Numbers via Intervals Numbers

The aim of this section is to present some notations, notions and results which are of useful in our further consideration.

A number $\tilde{a} = (a^1, a^2, a^3)$ (where $a^1 \le a^2 \le a^3$) is said to be a triangular fuzzy number if its membership function is given by [1]-[3]:

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{x-a^1}{a^2-a^1}, & a^1 \le x \le a^2 \\ \frac{x-a^3}{a^2-a^3}, & a^2 \le x \le a^3 \end{cases}.$$

Assume that $\tilde{a} = (a^1, a^2, a^3) = (a^2|\bar{a}) = \left(a^2 \middle| \left[a^1, a^3\right] = \langle \frac{a^3+a^1}{2}, \frac{a^3-a^1}{2} \rangle\right)$ and $\tilde{b} = (b^1, b^2, b^3) = (b^2|\bar{b}) = \left(b^2 \middle| \left[b^1, b^3\right] = \langle \frac{b^3+b^1}{2}, \frac{b^3-b^1}{2} \rangle\right)$ are two triangular fuzzy numbers. For any two triangular fuzzy numbers $\tilde{a} = (a^2|\bar{a})$ and $\tilde{b} = (b^2|\bar{b})$, the arithmetic operations on $\tilde{a}$ and $\tilde{b}$ are defined as:

Addition: $\tilde{a} + \tilde{b} = \left(a^2 \middle| \left[a^1, a^3\right]\right) + \left(b^2 \middle| \left[b^1, b^3\right]\right) = \left(a^2 + b^2 \middle| \left[a^1, a^3\right] + \left[b^1, b^3\right]\right)$;

Soustraction: $\tilde{a} - \tilde{b} = \left(a^2 \middle| \left[a^1, a^3\right]\right) - \left(b^2 \middle| \left[b^1, b^3\right]\right) = \left(a^2 - b^2 \middle| \left[a^1, a^3\right] - \left[b^1, b^3\right]\right)$; .

Multiplication: $\tilde{a}\tilde{b} = \left(a^2 \middle| \left[a^1, a^3\right]\right)\left(b^2 \middle| \left[b^1, b^3\right]\right) = \left(a^2 b^2 \middle| \left[a^1, a^3\right]\left[b^1, b^3\right]\right)$.

For all the rest of this paper, we will consider the following notations:

Assume that $\tilde{c}_{ij} = (c_{ij}^1, c_{ij}^2, c_{ij}^3)$, $\tilde{x}_j = \left(x_j^2 \middle| \left[x_j^1, x_j^3\right]\right) = (x_{ij}^1, x_{ij}^2, x_{ij}^3)$, $\tilde{b}_j = \left(b_j^2 \middle| \left[b_j^1, b_j^3\right]\right) = (b_j^1, b_j^2, b_j^3)$ and $\tilde{a}_i = \left(a_i^2 \middle| \left[a_i^1, a_i^3\right]\right) = (a_i^1, a_i^2, a_i^3)$ are triangular fuzzy numbers with x_{ij}^1, x_{ij}^3, c_{ij}^1, c_{ij}^3, b_j^1, b_j^3, a_i^1 and a_i^3 are real numbers ($\mathbb{R}$).

3.2| Formulation of a Transportation Problems Involving Interval Numbers (TPIn)

We consider the TPIn as follows [28]:

$$\begin{cases} \text{Min } \bar{Z}(\bar{x}) \approx \sum_{j=1}^{n} \bar{c}_{ij}\bar{x}_{ij} \\ \text{Subject to the constraints} \\ \sum_{j=1}^{n} \bar{x}_{ij} \approx \bar{a}_i, 1 \le i \le m \\ \sum_{i=1}^{m} \bar{x}_{ij} \approx \bar{b}_j, 1 \le j \le n \end{cases}.$$

3.3| Formulation of a Transportation Problem with Triangular Fuzzy Numbers (TPTri)

A TPTri is a linear programming problem of a specific structure. If in transportation problem, all parameters and variables are fuzzy, we will have a fully fuzzy transportation problem as follows. Suppose that there are m warehouses and $\tilde{a}_i$ represents renders of warehouse i and n represents customer and $\tilde{b}_j$ is the demand of customer j. $\tilde{c}_{ij}$ is the cost of transporting one unit of product from warehouse i to the customer j and $\tilde{x}_{ij}$ is the value of transported product from warehouse i to the customer j. The objective is to minimize the cost of transporting a product from the warehouse to the customer.

We consider the TPTri as follows [1]-[3]:

$$\begin{cases} \text{Min } \tilde{Z}(\tilde{x}) \approx \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_{ij} \\ \text{Subject to the constraints} \\ \sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{a}_i, 1 \le i \le m \\ \sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{b}_j, 1 \le j \le n \end{cases}.$$

For all the rest of this paper, we will consider the following TPTri:

$$\begin{cases} \text{Min } \tilde{Z}(\tilde{x}) \approx \sum_{i=1}^{m} \sum_{j=1}^{n} \left(c_{ij}^{2} \middle| \bar{c}_{ij}^{13}\right)\left(x_{ij}^{2} \middle| \bar{x}_{ij}^{13}\right) \\ \qquad \text{Subject to the constraints} \\ \qquad \sum_{j=1}^{n} \left(x_{ij}^{2} \middle| \bar{x}_{ij}^{13}\right) \approx \left(a_{i}^{2} \middle| \bar{a}_{i}^{13}\right) \\ \qquad \sum_{i=1}^{m} \left(x_{ij}^{2} \middle| \bar{x}_{ij}^{13}\right) \approx \left(b_{j}^{2} \middle| \bar{b}_{j}^{13}\right) \\ \qquad 1 \le j \le n \text{ and } 1 \le i \le m. \end{cases}.$$

3.4| Our Method for Solving the Transportation Problem with Triangular Fuzzy Numbers (TPTri)

In this section, a method to find a fuzzy optimal solution of TPTri is presented.

For all the rest of this paper, we will consider the following primal TPIn13:

$$\begin{cases}\text{Min } \overline{Z}^{13}\left(\overline{x}^{13}\right) \approx \sum_{i=1}^{m}\sum_{j=1}^{n}\overline{c}_{ij}^{13}\overline{x}_{ij}^{13}\\ \quad \text{Subject to the constraints}\\ \quad \sum_{j=1}^{n}\overline{x}_{ij}^{13} \approx \overline{a}_{i}^{13}, 1 \le i \le m\\ \quad \sum_{i=1}^{m}\overline{x}_{ij}^{13} \approx \overline{b}_{j}^{13}, 1 \le j \le n\\ \quad \overline{x}_{ij}^{13} = \left[x_{j}^{1}, x_{j}^{3}\right] \ge 0\end{cases}.$$

3.4.1| Formulation of a transportation problem involving midpoint (TPMi13)

Thanks to the new interval arithmetic and TPIn13, we can write the following Transportation Problem involving Midpoint (TPMi13) [28]:

$$\begin{cases}\text{Min/Max } Z^{13}\left(x^{13}\right) = \sum_{i=1}^{m}\sum_{j=1}^{n} m\left(\overline{c}_{ij}^{13}\right)x_{ij}^{13}\\ \quad \text{Subject to the constraints}\\ \quad \sum_{j=1}^{n} x_{ij}^{13} = m\left(\overline{a}_{i}^{13}\right), 1 \le i \le m\\ \quad \sum_{i=1}^{n} x_{ij}^{13} = m\left(\overline{b}_{j}^{13}\right), 1 \le j \le n\\ \quad x_{ij}^{13} = m\left(\overline{x}_{ij}^{13}\right) = \frac{x_{ij}^{3}+x_{ij}^{1}}{2} \ge 0\end{cases}.$$

3.4.2| Formulation of a classical transportation problem (TP2)

The classical Transport Problem (PT2) is:

$$\begin{cases}\text{Min } Z^{2}\left(x^{2}\right) \approx \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{2}x_{ij}^{2}\\ \quad \text{Subject to the constraints}\\ \quad \sum_{j=1}^{n} x_{ij}^{2} \approx a_{i}^{2}, 1 \le i \le m\\ \quad \sum_{i=1}^{m} x_{ij}^{2} \approx b_{j}^{2}, 1 \le j \le n\\ \quad x_{ij}^{1} \le x_{ij}^{2} \le x_{ij}^{3}\end{cases}.$$

For all the rest of this paper, we will consider the following notations: $\overline{x}_{ij}^{13} = \left[x_{j}^{1}, x_{j}^{3}\right]$, $\overline{c}_{ij}^{13} = \left[c_{j}^{1}, c_{j}^{3}\right]$, $\overline{b}_{j}^{13} = \left[b_{j}^{1}, b_{j}^{3}\right]$ and $\overline{a}_{i}^{13} = \left[a_{i}^{1}, a_{i}^{3}\right]$.

Thanks to the new interval arithmetic, we can write the following Lemma [28]:

Lemma 1. $x^{13} = \left(x_{ij}^{13}\right)_{m\times n}$ is an optimal solution to the (TPMi13) if and only if $\overline{x}^{13} \approx \left(\overline{x}_{ij}^{13}\right)_{m\times n}$ is an optimal solution to the TPIn13.

Proof. [28]. Assuming that $\sum_{j=1}^{n} x_{ij}^{13} = \sum_{j=1}^{n}\frac{x_{ij}^{3}+x_{ij}^{1}}{2} = \frac{a_{i}^{3}+a_{i}^{1}}{2}$ and $\sum_{j=1}^{n} x_{ij}^{13} = \sum_{j=1}^{n}\frac{x_{ij}^{3}+x_{ij}^{1}}{2} = \frac{b_{j}^{3}+b_{j}^{1}}{2}$ with $w\left(\overline{x}_{ij}^{13}\right) = \frac{w\left(\overline{a}_{i}^{13}\right)}{N}$ where $N = \#\left\{x_{ij}^{13} \neq 0\right\}$ for $1 \le j \le n \text{ and } 1 \le i \le m,$ we can write that $\overline{x}_{ij}^{13} \approx \langle x_{ij}^{13}, w\left(\overline{x}_{ij}^{13}\right)\rangle = \left[x_{ij}^{13} - w\left(\overline{x}_{ij}^{13}\right), x_{ij}^{13} + w\left(\overline{x}_{ij}^{13}\right)\right]$ if and only if $x^{13} = \left(x_{ij}^{13}\right)_{m\times n}$ is an optimal solution to the TPMi13. Then $\overline{x}^{13} \approx \left(\overline{x}_{ij}^{13}\right)_{m\times n}$ is an optimal solution to the TPIn13.

Thanks, the Lemma above, we can write the following corollary [28]:

Corollary 1. If $\bar{x}_{ij}^{13} \approx \left[x_j^{*1}, x_j^{*3}\right]$ is an optimal solution to the TPIn13 and x_{ij}^2 is an optimal solution to the (TP2), then $\tilde{x}^* \approx \left(\tilde{x}_{ij}^*\right)_{m\times n}$ is an optimal solution to the TPTri with $\tilde{x}_{ij}^* \approx \left(x_{ij}^2 | \bar{x}_{ij}^{13}\right) = \left(x_{ij}^{*2} \left|\left[x_{ij}^{*1}, x_{ij}^{*3}\right]\right.\right) = \left(x_j^{*1}, x_{ij}^{*2}, x_{ij}^{*3}\right)$.

Notice that TPTri is equivalent to
$$\begin{cases} Min\ \tilde{Z}(\tilde{x}) \approx \sum_{i=1}^m \sum_{j=1}^n \left(c_{ij}^2 \left|\left[c_{ij}^1, c_{ij}^3\right]\right.\right)\left(x_{ij}^{*2} \left|\left[x_{ij}^{*1}, x_{ij}^{*3}\right]\right.\right) \\ \quad Subject\ to\ the\ constraints \\ \sum_{j=1}^n \left(x_{ij}^{*2} \left|\left[x_{ij}^{*1}, x_{ij}^{*3}\right]\right.\right) \approx \left(a_i^2 \left|\left[a_i^1, a_i^3\right]\right.\right) \\ \sum_{i=1}^m \left(x_{ij}^{*2} \left|\left[x_{ij}^{*1}, x_{ij}^{*3}\right]\right.\right) \approx \left(b_j^2 \left|\left[b_j^1, b_j^3\right]\right.\right) \\ 1 \le j \le n\ and\ 1 \le i \le m. \end{cases}.$$

$\sum_{j=1}^n \left(x_{ij}^2 \left|\left[x_{ij}^{*1}, x_{ij}^{*3}\right]\right.\right) \approx \left(a_i^2 \left|\left[a_i^1, a_i^3\right]\right.\right)$ is equivalent to $\sum_{j=1}^n x_{ij}^{*2} \approx a_i^2$ and $\sum_{j=1}^n \left[x_{ij}^{*1}, x_{ij}^{*3}\right] \approx \left[a_i^1, a_i^3\right]$.

$\sum_{j=1}^n \left(x_{ij}^2 \left|\left[x_{ij}^{*1}, x_{ij}^{*3}\right]\right.\right) \approx \left(b_j^2 \left|\left[b_j^1, b_j^3\right]\right.\right)$ is equivalent to $\sum_{j=1}^n x_{ij}^{*2} \approx b_j^2$ and $\sum_{j=1}^n \left[x_{ij}^{*1}, x_{ij}^{*3}\right] \approx \left[b_j^1, b_j^3\right]$.

Moreover $\sum_{j=1}^n \frac{x_{ij}^3 + x_{ij}^1}{2} = \frac{a_i^3 + a_i^1}{2}$ for $1 \le i \le m$ and $\sum_{j=1}^n \frac{x_{ij}^3 + x_{ij}^1}{2} = \frac{b_j^3 + b_j^1}{2}$ for $1 \le j \le n$ and $\sum_{j=1}^n \frac{x_{ij}^3 - x_{ij}^1}{2} = \frac{a_i^3 - a_i^1}{2}$ for $1 \le i \le m$ and $\sum_{j=1}^n \frac{x_{ij}^3 - x_{ij}^1}{2} = \frac{b_j^3 - b_j^1}{2}$ for $1 \le j \le n$.

3.4.3 | The steps of our computational method

The steps of our method for solving the TPTri as follows:

Step 1. Consider a TPTri.

Step 2. Identify TPIn13 and TP2.

Step 3. Ramesh and Ganesan's method [28]: solving the TPIn13 via TPMi13.

Applying the simplex method to the TPMi13 to determine the variables TPMi13:

$x^{13} = \left(x_{ij}^{13}\right)_{m\times n}$ and $\bar{x}_{ij}^{13} \approx \langle x_{ij}^{13}, w\left(\bar{x}_{ij}^{13}\right)\rangle = \left[x_{ij}^{13} - w\left(\bar{x}_{ij}^{13}\right), x_{ij}^{13} + w\left(\bar{x}_{ij}^{13}\right)\ \right]$ for $1 \le k \le m$ with $w\left(\bar{x}_{ij}^{13}\right) = \frac{w\left(\bar{a}_i^{13}\right)}{N_i}$ where $N_i = \# \left\{x_{ij}^{13} \neq 0\right\}$.

The associated value of the objective function: $Min\ \bar{Z}^{13}\left(\bar{x}^{13}\right) \approx \sum_{i=1}^m \sum_{j=1}^n \bar{c}_{ij}^{13} \bar{x}_{ij}^{13}$.

Step 4. Solving the TP2.

Applying the simplex method to the TP2 to determine the variables TP2:

$x^2 = \left(x_{ij}^2\right)_{m\times n}$ with the associated value of the objective function: $Min\ Z^2(x^2) \approx \sum_{i=1}^m \sum_{j=1}^n c_{ij}^2 x_{ij}^2$.

Step 5. Fuzzy optimal solution of TPTri: optimal solution: $\tilde{x}_{ij}^* \approx \left(x_{ij}^2 | \bar{x}_{ij}^{13}\right) = \left(x_{ij}^{*2} \left|\left[x_{ij}^{*1}, x_{ij}^{*3}\right]\right.\right) = \left(x_{ij}^{*1}, x_{ij}^{*2}, x_{ij}^{*3}\right)$ with the associated value of the objective function Min $\tilde{Z}^* = \left(Z^{*1}, Z^{*2}, Z^{*3}\right) = \left(Z^2 | \bar{Z}^{13}\right)$.

4| Advantages of the Proposed Method

Let us explore the main advantages of the proposed method:

The proposed technique does not use the goal and parametric approaches which are difficult to apply in real life situations.

By applying the proposed approach for finding the fuzzy optimal solution, there is no need of much knowledge of fuzzy linear programming technique, Zimmerman approach and crisp linear programming which are difficult to learn for a new decision maker.

The proposed method to solve TPTri is based on traditional transportation algorithms. Thus, the existing and easily available software can be used for the same. However, the existing method [1]-[3] to solve FTP should be implemented into a programming language.

To solve the TPTri by using the existing method [1]-[3], there is need to use arithmetic operations of generalized fuzzy numbers. While, if the proposed technique is used for the same then there is need to use arithmetic operations of real numbers. This proves that it is much easy to apply the proposed method as compared to the existing method [1]-[3].

Moreover, it is possible to assume a generic ranking index for comparing the fuzzy numbers involved in the TPTri , in such a way that each time in which the decision maker wants to solve the TPTri under consideration(s),he can choose (or propose) the ranking index that best suits the TPTri.

5| Numerical Illustration

This section covers the numerical problems to signify the methodology of the proposed algorithm.

Example 1.

Table 1. (TPTri) in triangular balanced form.

	R_1	R_2	Supply ($\tilde{a}_i$)
A	(22, 31, 34)	(15, 19, 29)	(150, 201, 246)
B	(30, 39, 54)	(8, 10, 12)	(50, 99, 154)
Demand ($\tilde{b}_j$)	(100, 150, 200)	(100, 150, 200)	$\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$

Step 1. Consider a TPTri.

$$\begin{cases} \text{Min } \tilde{Z}(\tilde{x}) \approx (22,31,34)\tilde{x}_{11} + (15,19,29)\tilde{x}_{12} + (30,39,54)\tilde{x}_{21} + (8,10,12)\tilde{x}_{22} \\ \text{Subject to the constraints} \\ \tilde{x}_{11} + \tilde{x}_{12} \approx (150,201,246) \\ \tilde{x}_{21} + \tilde{x}_{22} \approx (50,99,154) \\ \tilde{x}_{11} + \tilde{x}_{21} \approx (100,150,200) \\ \tilde{x}_{12} + \tilde{x}_{22} \approx (100,150,200) \\ \tilde{x}_{ij} \text{ are triangular fuzzy numbers} \end{cases}.$$

Step 2. Identify TPIn13 and TP2, respectivly.

$$\begin{cases} \text{Min } \overline{Z}^{13}(\overline{x}^{14}) \approx [22,34]\overline{x}_{11}^{13} + [15,29]\overline{x}_{12}^{13} + [30,54]\overline{x}_{21}^{13} + [8,12]\overline{x}_{22}^{13} \\ \quad \text{Subject to the constraints} \\ \overline{x}_{11}^{13} + \overline{x}_{12}^{13} \approx [150,246] = \langle 198, \quad 48 \rangle \\ \overline{x}_{21}^{13} + \overline{x}_{22}^{13} \approx [50,154] = \langle 102, \quad 52 \rangle \\ \overline{x}_{11}^{13} + \overline{x}_{21}^{13} \approx [100,200] = \langle 150,50 \rangle \\ \overline{x}_{12}^{13} + \overline{x}_{22}^{13} \approx [100,200] = \langle 150,50 \rangle \end{cases},$$

$$\begin{cases} \text{Min } Z^{2}(x^{2}) = 31x_{11}^{2} + 19x_{12}^{2} + 39x_{21}^{2} + 10x_{22}^{2} \\ \quad \text{Subject to the constraints} \\ x_{11}^{2} + x_{12}^{2} = 201 \\ x_{21}^{2} + x_{22}^{2} = 99 \\ x_{11}^{2} + x_{21}^{2} = 150 \\ x_{12}^{2} + x_{22}^{2} = 150 \\ x_{ij}^{1} \le x_{ij}^{2} \le x_{ij}^{3} \end{cases}.$$

Step 3. Ramesh and Ganesan's method [28]: solving the TPIn13 via TPMi13.

$$\begin{cases} \text{Min } Z^{13}(x^{13}) = 28x_{11}^{13} + 22x_{12}^{13} + 42x_{21}^{13} + 10x_{22}^{13} \\ \quad \text{Subject to the constraints} \\ x_{11}^{13} + x_{12}^{13} = 198 \\ x_{21}^{13} + x_{22}^{13} = 102 \\ x_{11}^{13} + x_{21}^{13} = 150 \\ x_{12}^{13} + x_{22}^{13} = 150 \end{cases}.$$

Applying the simplex method to the TPMi13 to determine the variables TPMi13: $x^{13} = \left(x_{ij}^{13}\right)_{m\times n}$. The optimal solution is: $x_{11}^{13} = 150,\ x_{12}^{13} = 48,\ x_{21}^{13} = 0$ and $x_{22}^{13} = 102$ with $Z^{13} = 6276$. We have $\overline{x}_{ij}^{13} \approx \langle x_{ij}^{13}, w(\overline{x}_{ij}^{13})\rangle \approx \left[x_{ij}^{13} - w(\overline{x}_{ij}^{13}), x_{ij}^{13} + w(\overline{x}_{ij}^{13})\ \right]$ with $w(\overline{a}_{1}^{13}) = 48$ with $N_1 = \# \left\{x_{11}^{13} \neq 0, x_{12}^{13} \neq 0\right\} = 2$ and $w(\overline{a}_{2}^{13}) = 52$ with $N_2 = \# \left\{x_{22}^{13} \neq 0\right\} = 1$.

We get $w(\overline{x}_{1j}^{13}) = \frac{w(\overline{a}_{1}^{13})}{N_1} = \frac{48}{2} = 24$ and $\overline{x}_{1j}^{13} \approx \left[x_{1j}^{13} - w(\overline{x}_{1j}^{13}), x_{1j}^{13} + w(\overline{x}_{1j}^{13})\ \right]$: $\overline{x}_{11}^{13} \approx \langle 150{,}24\rangle = [126{,}174]$ and $\overline{x}_{12}^{13} \approx \langle 48{,}24\rangle = [24{,}72]$. Furthermore $w(\overline{x}_{2j}^{13}) = \frac{w(\overline{a}_{2}^{13})}{N_2} = \frac{52}{1} = 52$ and $\overline{x}_{2j}^{13} \approx \left[x_{2j}^{13} - w(\overline{x}_{2j}^{13}), x_{2j}^{13} + w(\overline{x}_{2j}^{13})\ \right]$: $\overline{x}_{21}^{13} \approx \overline{0}$ and $\overline{x}_{22}^{13} \approx \langle 102{,}52\rangle = [50{,}154]$.

The associated value of the objective function: $Min\ \overline{Z}^{13}(\overline{x}^{13}) \approx \sum_{i=1}^{m}\sum_{j=1}^{n} \overline{c}_{ij}^{13}\overline{x}_{ij}^{13} = [3532,\ 9852]$.

Step 4. Solving the primal TP2.

$$\begin{cases} \text{Min } Z^2(x^2) = 31x_{11}^2 + 19x_{12}^2 + 39x_{21}^2 + 10x_{22}^2 \\ \quad\quad \text{Subject to the constraints} \\ \quad\quad x_{11}^2 + x_{12}^2 = 201 \\ \quad\quad x_{21}^2 + x_{22}^2 = 99 \\ \quad\quad x_{11}^2 + x_{21}^2 = 150 \\ \quad\quad x_{12}^2 + x_{22}^2 = 150 \\ \quad\quad 126 \le x_{11}^2 \le 174 \\ \quad\quad 24 \le x_{12}^2 \le 72 \\ \quad\quad 50 \le x_{22}^2 \le 154 \end{cases}.$$

Applying the simplex method to the TP2 to determine the primal variables TP2: $x^2 = \left(x_{ij}^2\right)_{m\times n}$. The optimal solution is: $x_{11}^2 = 150$, $x_{12}^2 = 51$, $x_{21}^2 = 0$ and $x_{22}^2 = 99$ with the associated value of the objective function: Min $Z^2 = 6609$.

Step 5. Fuzzy optimal solution of TPTri.

Optimal solution:

$\tilde{x}_{ij}^* \approx \left(x_{ij}^{*2}|\bar{x}_{ij}^{23}\right) = \left(x_{ij}^{*2}|\left[x_{ij}^{*1}, x_{ij}^{*3}\right]\right) = \left(x_{ij}^{*1}, x_{ij}^{*2}, x_{ij}^{*3}\right)$: $\tilde{x}_{11}^* \approx (150|[102, 198]) = (102, 150, 198)$, $\tilde{x}_{12}^* \approx (51|[0, 96]) = (0, 51, 96)$, $\tilde{x}_{21}^* \approx (0|\bar{0}) = \tilde{0}$, $\tilde{x}_{22}^* \approx (99|[50, 154]) = (50, 99, 154)$, with the associated value of the objective function Min $\tilde{Z}^* = \left(Z^{*1}, Z^{*2}, Z^{*3}\right) = \left(Z^2|\bar{Z}^{13}\right)$. We have Min $\tilde{Z}^* \approx (6609|[3532, 9852]) = (3532, 6609, 9852)$.

Interpretation of results. We will now interpret the minimum total fuzzy transportation cost obtained in *Example 1* by using the proposed methods presented in *Section 3* Similarly, the obtained fuzzy optimal solution will also be interpreted. By using the methods proposed the minimum total fuzzy transportation cost is (3532, 6609, 9852), which can be physically interpreted as follows:

- *The least amount of the minimum total transportation cost is 3532.*
- *The most possible amount of minimum total transportation cost is 6609.*
- *The greatest amount of the minimum total transportation cost is 9852 i.e., the minimum total transportation cost will always be greater than 3532 and less than 6609, and the highest chances are that the minimum total transportation cost will be 9852.*

Example 2. [1]-[3]. Dali Company is the leading producer of soft drinks and low-temperature foods in Taiwan. Currently, Dali plans to develop the South-East Asian market and broaden the visibility of Dali products in the Chinese market. Notably, following the entry of Taiwan to the World Trade Organization, Dali plans to seek strategic alliance with prominent international companies and introduced international bread to lighten the embedded future impact. In the domestic soft drinks market, Dali produces tea beverages to meet demand from four distribution centers in Taichung, Chiayi, Kaohsiung and Taipei, with production being based at three plants in Changhua, Touliu and Hsinchu. According to the preliminary environmental information, *Table 2* summarizes the potential supply available from these three plants, the forecast demand from the four distribution centers and the unit transportation costs for each route used by Dali for the upcoming season.

Table 2. Summarized data in the Dali case (in U.S. dollar).

Source	Destionation Taichung	Chiayi	Kaohsiung	Taipei	Supply ($\tilde{a}_i$) (000 dozen bottles)
Changhua	(\$8, \$10, \$10.8)	(\$20.4, \$22, \$24)	(\$8, \$10, \$10.6)	(\$18.8, \$20, \$22)	(7.2, 8, 8.8)
Touliu	(\$14, \$15, \$16)	(\$18.2, \$20, \$22)	(\$10, \$12, \$13)	(\$6, \$8, \$8.8)	(12, 14, 16)
Hsinchu	(\$18.4, \$20, \$21)	(\$9.6, \$12, \$13)	(\$7.8, \$10, \$10.8)	(\$14, \$15, \$16)	(10.2, 12, 13.8)
Demand ($\tilde{b}_j$) (000 dozen bottles)	(6.2, 7, 7.8)	(8.9, 10, 11.1)	(6.5, 8, 9.5)	(7.8, 9, 10.2)	$\sum_{i=1}^{m}\tilde{a}_i=\sum_{j=1}^{n}\tilde{b}_j$

Step 1. The TPTri is given by:

$$
\begin{cases}
\text{Min } \tilde{Z}(\tilde{x}) = (8,10,10.8)\left(x_{11}^1,x_{11}^2,x_{11}^3\right) + (20.4,22,24)\left(x_{12}^1,x_{12}^2,x_{12}^3\right) \\
+(8,10,10.6)\left(x_{13}^1,x_{13}^2,x_{13}^3\right) + (18.8,20,22)\left(x_{14}^1,x_{14}^2,x_{14}^3\right) \\
+(14,15,16)\left(x_{21}^1,x_{21}^2,x_{21}^3\right) + (18.2,20,22)\left(x_{22}^1,x_{22}^2,x_{22}^3\right) \\
+(10,12,13)\left(x_{23}^1,x_{23}^2,x_{23}^3\right) + (6,8,8.8)\left(x_{24}^1,x_{24}^2,x_{24}^3\right) \\
+(18.4,20,21)\left(x_{31}^1,x_{31}^2,x_{31}^3\right) + (9.6,12,13)\left(x_{32}^1,x_{32}^2,x_{32}^3\right) \\
+(7.8,10,10.8)\left(x_{33}^1,x_{33}^2,x_{33}^3\right) + (14,15,16)\left(x_{34}^1,x_{34}^2,x_{34}^3\right) \\
\text{Subject to the constraints} \\
\left(x_{11}^1,x_{11}^2,x_{11}^3\right) + \left(x_{12}^1,x_{12}^2,x_{12}^3\right) + \left(x_{13}^1,x_{13}^2,x_{13}^3\right) + \left(x_{14}^1,x_{14}^2,x_{14}^3\right) = (7.2,8,8.8) \\
\left(x_{21}^1,x_{21}^2,x_{21}^3\right) + \left(x_{22}^1,x_{22}^2,x_{22}^3\right) + \left(x_{23}^1,x_{23}^2,x_{23}^3\right) + \left(x_{24}^1,x_{24}^2,x_{24}^3\right) = (12,14,16) \\
\left(x_{31}^1,x_{31}^2,x_{31}^3\right) + \left(x_{32}^1,x_{32}^2,x_{32}^3\right) + \left(x_{33}^1,x_{33}^2,x_{33}^3\right) + \left(x_{34}^1,x_{34}^2,x_{34}^3\right) = (10.2,12,13.8) \\
\left(x_{11}^1,x_{11}^2,x_{11}^3\right) + \left(x_{21}^1,x_{21}^2,x_{21}^3\right) + \left(x_{31}^1,x_{31}^2,x_{31}^3\right) = (6.2,7,7.8) \\
\left(x_{12}^1,x_{12}^2,x_{12}^3\right) + \left(x_{22}^1,x_{22}^2,x_{22}^3\right) + \left(x_{32}^1,x_{32}^2,x_{32}^3\right) = (8.9,10,11.1) \\
\left(x_{13}^1,x_{13}^2,x_{13}^3\right) + \left(x_{23}^1,x_{23}^2,x_{23}^3\right) + \left(x_{33}^1,x_{33}^2,x_{33}^3\right) = (6.5,8,9.5) \\
\left(x_{14}^1,x_{14}^2,x_{14}^3\right) + \left(x_{24}^1,x_{24}^2,x_{24}^3\right) + \left(x_{34}^1,x_{34}^2,x_{34}^3\right) = (7.8,9,10.2)
\end{cases}
.
$$

Step 2. Identify TPIn13 and TP2, respectively.

$$
\begin{cases}
\text{Min } \bar{Z}^{13}\left(\bar{x}^{13}\right) = [8, \quad 10.8]\bar{x}_{11}^{13} + [20.4, \quad 24]\bar{x}_{12}^{13} \\
+[8, \quad 10.6]\bar{x}_{13}^{13} + [18.8, 22]\bar{x}_{14}^{13} \\
+[14,16]\bar{x}_{21}^{13} + [18.2, \quad 22]\bar{x}_{22}^{13} \\
+[10, \quad 13]\bar{x}_{23}^{13} + [6, \quad 8.8]\bar{x}_{24}^{13} \\
+[18.4, \quad 21]\bar{x}_{31}^{13} + [9.6, \quad 13]\bar{x}_{32}^{13} \\
+[7.8, \quad 10.8]\bar{x}_{33}^{13} + [14, \quad 16]\bar{x}_{34}^{13} \\
\text{Subject to the constraints} \\
\bar{x}_{11}^{13} + \bar{x}_{12}^{13} + \bar{x}_{13}^{13} + \bar{x}_{14}^{13} = [7.2, \quad 8.8] = \langle 8, \quad 0.8\rangle \\
\bar{x}_{21}^{13} + \bar{x}_{22}^{13} + \bar{x}_{23}^{13} + \bar{x}_{24}^{13} = [12, \quad 16] = \langle 14, 2\rangle \\
\bar{x}_{31}^{13} + \bar{x}_{32}^{13} + \bar{x}_{33}^{13} + \bar{x}_{34}^{13} = [10.2, \quad 13.8] = \langle 12, \quad 1.8\rangle \\
\bar{x}_{11}^{13} + \bar{x}_{21}^{13} + \bar{x}_{31}^{13} = [6.2, \quad 7.8] = \langle 7, \quad 0.8\rangle \\
\bar{x}_{12}^{13} + \bar{x}_{22}^{13} + \bar{x}_{32}^{13} = [8.9, \quad 11.1] = \langle 10, \quad 1.1\rangle \\
\bar{x}_{13}^{13} + \bar{x}_{23}^{13} + \bar{x}_{33}^{13} = [6.5, \quad 9.5] = \langle 8, \quad 1.5\rangle \\
\bar{x}_{14}^{13} + \bar{x}_{24}^{13} + \bar{x}_{34}^{13} = [7.8, \quad 10.2] = \langle 9, \quad 1.2\rangle
\end{cases}
,
$$

$$\begin{cases} \text{Min } Z^2(x^2) = 10x_{11}^2 + 22x_{12}^2 + 10x_{13}^2 + 20x_{14}^2 \\ 15x_{21}^2 + 20x_{22}^2 + 12x_{23}^2 + 8x_{24}^2 \\ 20x_{31}^2 + 12x_{32}^2 + 10x_{33}^2 + 15x_{34}^2 \\ \text{Subject to the constraints} \\ x_{11}^2 + x_{12}^2 + x_{13}^2 + x_{14}^2 = 8 \\ x_{21}^2 + x_{22}^2 + x_{23}^2 + x_{24}^2 = 14 \\ x_{31}^2 + x_{32}^2 + x_{33}^2 + x_{34}^2 = 12 \\ x_{11}^2 + x_{21}^2 + x_{31}^2 = 7 \\ x_{12}^2 + x_{22}^2 + x_{32}^2 = 10 \\ x_{13}^2 + x_{23}^2 + x_{33}^2 = 8 \\ x_{14}^2 + x_{24}^2 + x_{34}^2 = 9 \end{cases}.$$

Step 3. Ramesh and Ganesan's method [28]: solving the TPIn13 via TPMi13.

Applying the simplex method to the TPMi13 to determine the variables TPMi13:

$$\begin{cases} \text{Min } Z^{13}(x^{13}) = 9.4x_{11}^{13} + 22.2x_{12}^{13} \\ +9.3x_{13}^{13} + 20.4x_{14}^{13} \\ +15x_{21}^{13} + 20.1x_{22}^{13} \\ +11.5x_{23}^{13} + 7.4x_{24}^{13} \\ +19.7x_{31}^{13} + 11.3x_{32}^{13} \\ +9.3x_{33}^{13} + 15x_{34}^{13} \\ \text{Subject to the constraints} \\ x_{11}^{13} + x_{12}^{13} + x_{13}^{13} + x_{14}^{13} = 8 \\ x_{21}^{13} + x_{22}^{13} + x_{23}^{13} + x_{24}^{13} = 14 \\ x_{31}^{13} + x_{32}^{13} + x_{33}^{13} + x_{34}^{13} = 12 \\ x_{11}^{13} + x_{21}^{13} + x_{31}^{13} = 7 \\ x_{12}^{13} + x_{22}^{13} + x_{32}^{13} = 10 \\ x_{13}^{13} + x_{23}^{13} + x_{33}^{13} = 8 \\ x_{14}^{13} + x_{24}^{13} + x_{34}^{13} = 9 \end{cases}.$$

$x^{13} = \left(x_{ij}^{13}\right)_{m\times n}$. The optimal solution is: $x_{11}^{13} = 7, x_{12}^{13} = 0, x_{13}^{13} = 1$ $x_{14}^{13} = 0, x_{21}^{13} = 0, x_{22}^{13} = 0, x_{23}^{13} = 5,$ $x_{24}^{13} = 9$ and $x_{31}^{13} = 0, x_{32}^{13} = 10, x_{33}^{13} = 2$ $x_{34}^{13} = 0$.

$w\left(\bar{a}_1^{13}\right) = 0.8 = \frac{4}{5}$ with $N_1 = \# \left\{x_{11}^{13} \neq 0, x_{13}^{13} \neq 0 \right\} = 2$, $w\left(\bar{a}_2^{13}\right) = 2$ with $N_2 = \# \left\{x_{23}^{13} \neq 0, x_{24}^{13} \neq 0\right\} = 2$ and

$w\left(\bar{a}_3^{13}\right) = 1.8 = \frac{9}{5}$ with $N_3 = \# \left\{x_{32}^{13} \neq 0, x_{33}^{13} \neq 0\right\} = 2$.

We get $\bar{x}_{1j}^{13} \approx \left[x_{1j}^{13} - w\left(\bar{x}_{1j}^{13}\right), x_{1j}^{13} + w\left(\bar{x}_{1j}^{13}\right)\right]$ with $w\left(\bar{x}_{1j}^{13}\right) = \frac{4}{10} = \frac{2}{5}$:

$\bar{x}_{11}^{13} \approx \left[7 - \frac{2}{5}, 7 + \frac{2}{5}\right] = \left[\frac{33}{5}, \frac{37}{5}\right]$, $\bar{x}_{12}^{13} \approx \bar{0}$, $\bar{x}_{13}^{13} \approx \left[1 - \frac{2}{5}, 1 + \frac{2}{5}\right] = \left[\frac{3}{5}, \frac{7}{5}\right]$ and $\bar{x}_{14}^{13} \approx \bar{0}$.

$\bar{x}_{2j}^{13} \approx \left[x_{2j}^{13} - w\left(\bar{x}_{2j}^{13}\right), x_{2j}^{13} + w\left(\bar{x}_{2j}^{13}\right)\right]$ with $w\left(\bar{x}_{2j}^{13}\right) = \frac{2}{2} = 1$:

$\bar{x}^{13}_{21} \approx \bar{0}, \bar{x}^{13}_{22} \approx \bar{0}, \bar{x}^{13}_{23} \approx [5-1, 5+1] = [4,\ 6]$ and $\bar{x}^{13}_{24} \approx [9-1,\ 9+1] = [8,\ 10]$.

$\bar{x}^{13}_{3j} \approx \left[x^{13}_{3j} - w\left(\bar{x}^{13}_{3j}\right), x^{13}_{3j} + w\left(\bar{x}^{13}_{3j}\right)\ \right]$ with $w\left(\bar{x}^{13}_{3j}\right) = \frac{9}{10}$:

$\bar{x}^{13}_{31} \approx \bar{0}, \bar{x}^{13}_{32} \approx \left[10-\frac{9}{10}, 10+\frac{9}{10}\right] = \left[\frac{91}{10}, \frac{109}{10}\right], \bar{x}^{13}_{33} \approx \left[2-\frac{9}{10}, 2+\frac{9}{10}\right] = \left[\frac{11}{10}, \frac{29}{10}\right]$ and $\bar{x}^{13}_{34} \approx \bar{0}$.

The associated value of the objective function: $Min\ \bar{Z}^{13}\left(\bar{x}^{13}\right) \approx \sum_{i=1}^{m}\sum_{j=1}^{n} \bar{c}^{13}_{ij}\bar{x}^{13}_{ij} = \left[\frac{12081}{50}\$, \frac{21689}{50}\$\right]$.

Step 4. Solving the primal TP2.

$$\begin{cases} \text{Min } Z^2\left(x^2\right) = 10x^2_{11} + 22x^2_{12} + 10x^2_{13} + 20x^2_{14} \\ \quad 15x^2_{21} + 20x^2_{22} + 12x^2_{23} + 8x^2_{24} \\ \quad 20x^2_{31} + 12x^2_{32} + 10x^2_{33} + 15x^2_{34} \\ \quad \text{Subject to the constraints} \\ \quad x^2_{11} + x^2_{12} + x^2_{13} + x^2_{14} = 8 \\ \quad x^2_{21} + x^2_{22} + x^2_{23} + x^2_{24} = 14 \\ \quad x^2_{31} + x^2_{32} + x^2_{33} + x^2_{34} = 12 \\ \quad x^2_{11} + x^2_{21} + x^2_{31} = 7 \\ \quad x^2_{12} + x^2_{22} + x^2_{32} = 10 \\ \quad x^2_{13} + x^2_{23} + x^2_{33} = 8 \\ \quad x^2_{14} + x^2_{24} + x^2_{34} = 9 \\ \quad \frac{33}{5} \le x^2_{11} \le \frac{37}{5} \\ \quad \frac{3}{5} \le x^2_{13} \le \frac{7}{5} \\ \quad 4 \le x^2_{23} \le 6 \\ \quad 8 \le x^2_{24} \le 10 \\ \quad \frac{91}{10} \le x^2_{32} \le \frac{109}{10} \\ \quad \frac{11}{10} \le x^2_{33} \le \frac{29}{10} \end{cases}$$

The optimal solution is: $x^2_{11} = 7$, $x^2_{12} = 0$, $x^2_{13} = 1$ $x^2_{14} = 0$, $x^2_{21} = 0$, $x^2_{22} = 0$, $x^2_{23} = 5$, $x^2_{24} = 9$ and $x^2_{31} = 0$, $x^2_{32} = 10$, $x^2_{33} = 2$ $x^2_{34} = 0$ with Min $Z^2 = 352\$$.

Step 5. Fuzzy optimal solution of TPTri.

Optimal solution:

$\tilde{x}^*_{ij} \approx \left(x^{*2}_{ij}|\bar{x}^{23}_{ij}\right) = \left(\ x^{*2}_{ij}\middle|\left[x^{*1}_{ij}, x^{*3}_{ij}\right]\right) = \left(x^{*1}_{ij}, x^{*2}_{ij}, x^{*3}_{ij}\right)$:

$\tilde{x}^*_{11} \approx \left(\frac{33}{5},\ 7,\ \ \frac{37}{5}\right), \tilde{x}^*_{12} \approx \tilde{0},\ \tilde{x}^*_{13} \approx \left(\frac{3}{5},\ 1,\ \ \frac{7}{5}\right)$ and $\tilde{x}^*_{14} \approx \tilde{0}$;

$\tilde{x}^*_{21} \approx \tilde{0},\ \tilde{x}^*_{22} \approx \tilde{0},\ \ \tilde{x}^*_{23} \approx (4,\ 5,\ 6)$ and $\tilde{x}^*_{24} \approx (7,\ 9,\ 10)$;

$\tilde{x}^*_{31} \approx \tilde{0},\ \tilde{x}^*_{32} \approx \left(\frac{11}{10},\ 10,\ \ \frac{29}{10}\right),\ \tilde{x}^*_{33} \approx \left(\frac{91}{10},\ 2,\ \ \frac{109}{10}\right)$ and $\tilde{x}^*_{34} \approx \tilde{0}$.

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With the associated value of the objective function Min $\tilde{Z}^* = \left(Z^{*1}, Z^{*2}, Z^{*3}\right) = \left(Z^2 \middle| \overline{Z}^{13}\right)$.

We have Min $\tilde{Z}^* \approx \left(352\$ \middle| \left[\frac{12081}{50}\$, \frac{21689}{50}\$\right]\right) = \left(\frac{12081}{50}\$, 352\$, \frac{21689}{50}\$\right)$.

Interpretation of results:

We will now interpret the minimum total fuzzy transportation cost obtained in *Example 2* by using the proposed methods presented in Section 3 Similarly, the obtained fuzzy optimal solution will also be interpreted. By using the methods proposed the minimum total fuzzy transportation cost is $\left(\frac{12081}{50}\$, 352\$, \frac{21689}{50}\$\right)$, which can be physically interpreted as follows:

The least amount of the minimum total transportation cost is $\frac{12081}{50}\$$.

The most possible amount of minimum total transportation cost is 352$.

The greatest amount of the minimum total transportation cost is $\frac{21689}{50}\$$ i.e., the minimum total transportation cost will always be greater than $\frac{12081}{50}\$$ and less than 352$, and the highest chances are that the minimum total transportation cost will be $\frac{21689}{50}\$$.

5| Concluding Remarks and Future Research Directions

These days a number of researchers have shown interest in the area of fuzzy transportation problems and various attempts have been made to study the solution of these problems. In this paper, to overcome the shortcomings of the existing methods we introduced a new formulation of transportation problem involving Triangular fuzzy numbers for the transportation costs and values of supplies and demands. We propose a fuzzy linear programming approach for solving Triangular fuzzy numbers transportation problem based on the converting into an TPIn and a classical TP. To show the advantages of the proposed methods over existing methods, some fuzzy transportation problems, may or may not be solved by the existing methods, are solved by using the proposed methods and it is shown that it is better to use the proposed methods as compared to the existing methods for solving the transportation problems. Finally, we feel that, there are many other points of research and should be studied later on. Some of these points are discussed below.

The solid transportation problem considers the supply, the demand, and the conveyance to satisfy the transportation requirement in a cost-effective manner. Thus, research on the topic for developing the proposed method to derive the fuzzy objective value of the fuzzy solid transportation problem when the cost coefficients, the supply and demand quantities and conveyance capacities are interval-valued triangular fuzzy numbers, is left to the next research work.

Further research on introducing a new formulation of interval-valued triangular fuzzy numbers transportation problem that lead to a method for solving this problem based on the classical transportation algorithms is an interesting stream of future research.

From both theoretical and algorithmic considerations, and examples solved in this paper, it can be noticed that some shortcomings of the methods for solving the fuzzy transportation problems known from the literature can be resolved by using the new methods proposed in *Section 3*.

Acknowledgements

The authors are very grateful to the anonymous referees for their valuable comments and suggestions to improve the paper in the present form.

Conflicts of Interest

The authors declare no conflicts of interest.

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