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The New Algorithm for Fully Fuzzy Transportation Problem by Trapezoidal Fuzzy Number (A Generalization of Triangular Fuzzy Number)

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Abstract

Transportation problem is an important network structured linear programming problem that arises in several contexts and has deservedly received a great deal of attention in the literature. The central concept in this problem is to find the least total transportation cost of a commodity in order to satisfy demands at destinations using available supplies at origins in a crisp environment. In real life situations, the decision maker may not be sure about the precise values of the coefficients belonging to the transportation problem. The aim of this paper is to introduce a formulation of fully fuzzy transportation problem involving trapezoidal fuzzy numbers for the transportation costs and values of supplies and demands. We propose a two-step method for solving fuzzy transportation problem where all of the parameters are represented by triangular fuzzy numbers i.e. two interval transportation problems. Since the proposed approach is based on classical approach it is very easy to understand and to apply on real life transportation problems for the decision makers. To illustrate the proposed approach four application examples are solved. The results show that the proposed method is simpler and computationally more efficient than existing methods in the literature.

Keywords: Fully fuzzy linear programming, Transportation problem, Trapezoidal fuzzy numbers, Triangular fuzzy numbers.

1 | Introduction

Transportation problem is an important network structured linear programming problem that arises in several contexts and has deservedly received a great deal of attention in the literature. The central concept in this problem is to find the least total transportation cost of a commodity in order to satisfy demands at destinations using available supplies at origins. Transportation problem can be used for a wide variety of situations such as scheduling, production, investment, plant location, inventory control, employment scheduling and many others.

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In general, transportation problems are solved with the assumptions that the transportation costs and values of supplies and demands are specified in a precise way i.e., in crisp environment. However, in many cases the decision maker has no crisp information about the coefficients belonging to the transportation problem. If the nature of the information is vague, that is, if it has some lack of precision, the corresponding coefficients or elements defining the problem can be formulated by means of fuzzy sets, and thus fuzzy transportation problems arise. Several researchers have carried out investigations on fuzzy transportation problem. Zimmermann [4] developed Zimmermann's fuzzy linear programming into several fuzzy optimization methods for solving the transportation problems. ÓhÉigeartaigh [5] proposed an algorithm for solving transportation problems where the supplies and demands are fuzzy sets with linear or triangular membership functions. Chanas et al. [6] investigated the transportation problem with fuzzy supplies and demands and solved them via the parametric programming technique. Their method provided solution which simultaneously satisfies the constraints and the goal to a maximal degree.

In addition, Chanas et al. [7] formulated the classical, interval and fuzzy transportation problem and discussed the methods for solution for the fuzzy transportation problem. Kuchta [8] discussed the type of transportation problems with fuzzy cost coefficients and converted the problem into a bicriterial transportation problem with crisp objective function. Their method only gives crisp solutions based on efficient solutions of the converted problems. Jiménez and Verdegay [9], [10] investigated the fuzzy solid transportation problem in which supplies, demands and conveyance capacities are represented by trapezoidal fuzzy numbers and applied a parametric approach for finding the fuzzy solution. Liu and Kao [11] developed a procedure, based on extension principle to derive the fuzzy objective value of fuzzy transportation problem, in that the cost coefficients and the supply and demand quantities are fuzzy numbers. Gani and Razak [12] presented a two-stage cost minimizing fuzzy transportation problem in which supplies and demands are as trapezoidal fuzzy numbers and used a parametric approach for finding a fuzzy solution with the aim of minimizing the sum of the transportation costs in the two stages. Li et al. [13] proposed a new method based on goal programming for solving fuzzy transportation problem with fuzzy costs. Lin [14] used genetic algorithm for solving transportation problems with fuzzy coefficients. Dinagar and Palanivel [15] investigated fuzzy transportation problem, with the help of trapezoidal fuzzy numbers and applied fuzzy modified distribution method to obtain the optimal solution in terms of fuzzy numbers. Pandian and Natarajan [16] introduced a new algorithm namely, fuzzy zero-point method for finding fuzzy optimal solution for such fuzzy transportation problem in which the transportation cost, supply and demand are represented by trapezoidal fuzzy numbers. Kumar and Kaur [17] proposed a new method based on fuzzy linear programming problem for finding the optimal solution of fuzzy transportation problem. Gupta et al. [18] proposed a new method named as Mehar's method, to find the exact fuzzy optimal solution of fully fuzzy multi-objective transportation problems. Ebrahimnejad [19] applied a fuzzy bounded dual algorithm for solving bounded transportation problems with fuzzy supplies and demands. Shanmugasundari and Ganesan [20] developed the fuzzy version of Vogel's and MODI methods for obtaining the fuzzy initial basic feasible solution and fuzzy optimal feasible solution, respectively, without converting them into classical transportation problem. Also, [21] Chandran and Kandaswamy [21] proposed an algorithm to find an optimal solution of a fuzzy transportation problem, where supply, demand and cost coefficients all are fuzzy numbers. Their algorithm provides decision maker with an effective solution in comparison to existing methods. Ebrahimnejad [22] using an example showed that their method will not always lead to a fuzzy optimal solution.

Moreover, Kumar and Kaur [23] pointed out the limitations and shortcomings of the existing methods for solving fuzzy solid transportation problem and to overcome these limitations and shortcomings proposed a new method to find the fuzzy optimal solution of unbalanced fuzzy solid transportation problems. In addition, Ebrahimnejad [24] proposed a two-step method for solving fuzzy transportation problem where all of the parameters are represented by non-negative triangular fuzzy numbers. Some researchers applied generalized fuzzy numbers for solving transportation problems. Kumar and Kaur [25] proposed a new method based on ranking function for solving fuzzy transportation problem by assuming that transportation cost, supply and demand of the commodity are represented by generalized trapezoidal fuzzy numbers. After that, Kaur and Kumar [26] introduced a similar algorithm for solving a special type of fuzzy

transportation problem by assuming that a decision maker is uncertain about the precise values of transportation cost only but there is no uncertainty about the supply and demand of the product. Ebrahimnejad [27] demonstrated that once the ranking function is chosen, the fuzzy transportation problem introduced by Kaur and Kumar [26] is converted into crisp one, which is easily solved by the standard transportation algorithms.

The contributions of the present study are summarized as follows: (a) in the *Eq. (15)* under consideration, all of the parameters, such as the transportation costs, supplies and demands are considered as fuzzy numbers. (b) According to the proposed approach, the *Eq. (15)* is converted into two interval transportation problems *Eq. (16)* and *Eq. (17)*. The integration of the optimal solution of the two sub-problems provides the optimal solution of the *Eq. (15)*. (c) In contrast to most existing approaches, which provide a precise solution, the proposed method provides a fuzzy optimal solution. (d) In contrast to existing methods that include negative parts in the obtained fuzzy optimal solution and fuzzy optimal cost, the proposed method provides a fuzzy optimal solution and optimal cost. (e) Similarly, to the competing methods in the literature, the proposed method is applicable for all types of trapezoidal fuzzy numbers. (f) The complexity of computation is greatly reduced compared with commonly used existing methods in the literature.

The rest of this paper is organized as follows: In Section 2, we recall the definitions of interval number linear programming, interval numbers and fully fuzzy transportation problem. In Section 3, a new method is proposed for obtaining the fuzzy optimal solution of the *Eq. (15)*. The advantages of the proposed method are discussed in Section 4. Two application examples are provided to illustrate the effectiveness of the proposed method in Section 5, and a comparative study in Section 6. Finally, concluding remarks are presented in Section 7.

2| Materials and Methods

In this section, some basic definitions, arithmetic operations for closed Intervals numbers and of linear programming problems involving interval numbers are presented [28].

2.1| A New Interval Arithmetic

In this section, some arithmetic operations for two intervals are presented [28].

Let $\Re = \left\{ \bar{a} = \left[a^1, a^4 \right] : a^1 \le a^4,\ \ a^1, a^4 \in \mathbb{R} \right\}$ be the set of all proper intervals. We shall use the terms "interval" and "interval number" interchangeably. The mid-point and width (or half-width) of an interval number $\bar{a} = \left[a^1, a^4 \right]$ are defined as $m(\bar{a}) = \dfrac{a^4 + a^1}{2}$ and $w(\bar{a}) = \dfrac{a^4 - a^1}{2}$. The interval number $\bar{a}$ can also be expressed in terms of its midpoint and width as

$$\bar{a} = \left[a^1, a^4 \right] = \langle m(\bar{a}), w(\bar{a}) \rangle = \langle \frac{a^4 + a^1}{2}, \frac{a^4 - a^1}{2} \rangle . \tag{1}$$

For any two intervals $\bar{a} = \left[a^1, a^4 \right] = \langle m(\bar{a}), w(\bar{a}) \rangle$ and $\bar{b} = \left[b^1, b^4 \right] = \langle m(\bar{b}), w(\bar{b}) \rangle$, the arithmetic operations on $\bar{a}$ and $\bar{b}$ are defined as:

Addition: $$\bar{a} + \bar{b} = \langle m(\bar{a}) + m(\bar{b}), w(\bar{a}) + w(\bar{b}) \rangle . \tag{2}$$

Subtraction: $$\bar{a} - \bar{b} = \langle m(\bar{a}) - m(\bar{b}), w(\bar{a}) + w(\bar{b}) \rangle . \tag{3}$$

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Multiplication: $$\alpha\bar{a}=\begin{cases}\langle \alpha m(\bar{a}),\alpha w(\bar{a})\rangle \text{ if } \alpha\geq 0,\\ \langle \alpha m(\bar{a}),-\alpha w(\bar{a})\rangle \text{ if } \alpha<0.\end{cases} \tag{4}$$

$$\bar{a}\times\bar{b}=\begin{cases}\langle m(\bar{a})m(\bar{b})+w(\bar{a})w(\bar{b}),m(\bar{a})w(\bar{b})+m(\bar{b})w(\bar{a})\rangle \text{ if } a^1\geq 0,b^1\geq 0,\\ \langle m(\bar{a})m(\bar{b})+m(\bar{a})w(\bar{b}),m(\bar{b})w(\bar{a})+w(\bar{b})w(\bar{a})\rangle \text{ if } a^1<0,b^1\geq 0,\\ \langle m(\bar{a})m(\bar{b})-w(\bar{a})w(\bar{b}),m(\bar{b})w(\bar{a})-m(\bar{a})w(\bar{b})\rangle \text{ if } a^4<0,b^1\geq 0.\end{cases} \tag{5}$$

2.2| A New Interval Arithmetic for Trapezoidal Fuzzy Numbers via Intervals Numbers

The aim of this section is to present some notations, notions and results which are of useful in our further consideration.

A number $\tilde{a}=(a^1,a^2,a^3,a^4)$ (where $a^1\leq a^2\leq a^3\leq a^4$) is said to be a trapezoidal fuzzy number if its membership function is given by [1]-[3]:

$$\mu_{\tilde{a}}(x)=\begin{cases}\dfrac{x-a^1}{a^2-a^1},\ a^1\leq x\leq a^2\ ,\\[2ex] \dfrac{x-a^4}{a^3-a^4},\ a^3\leq x\leq a^4\ .\end{cases} \tag{6}$$

Assume that $\tilde{a}=(a^1,a^2,a^3,a^4)=(\bar{a}^{14}/\bar{a}^{23})=([a^1,a^4]/[a^2,a^3])$,

and $\tilde{b}=(b^1,b^2,b^3,b^4)=(\bar{b}^{14}/\bar{b}^{23})=([b^1,b^4]/[b^2,b^3])$ are two trapezoidal fuzzy numbers. For any two trapezoidal fuzzy numbers $\tilde{a}=(\bar{a}^{14}/\bar{a}^{23})$ and $\tilde{b}=(\bar{b}^{14}/\bar{b}^{23})$, the arithmetic operations on $\tilde{a}$ and $\tilde{b}$ are defined as:

Addition: $$\tilde{a}+\tilde{b}=(\bar{a}^{14}|\bar{a}^{23})+(\bar{b}^{14}|\bar{b}^{23})=([a^1,a^4]+[b^1,b^4]|[a^2,a^3]+[b^2,b^3]). \tag{7}$$

Multiplication: $$\tilde{a}\tilde{b}=(\bar{a}^{14}|\bar{a}^{23})\times(\bar{b}^{14}|\bar{b}^{23})=([a^1,a^4]\times[b^1,b^4]|[a^2,a^3]\times[b^2,b^3]). \tag{8}$$

2.3| Formulation of a Transportation Problems Involving Interval Numbers

We consider the Transportation Problem involving Interval numbers as follows [28]:

$$\begin{cases} \text{Min}\,\bar{Z}(\bar{x}) \approx \sum_{j=1}^{n}\sum_{i=1}^{m} \bar{c}_{ij}\bar{x}_{ij} \\ \text{Subject to the constraints} \\ \sum_{j=1}^{n} \bar{x}_{ij} \approx \bar{a}_i, \text{for } i = 1,2,..,m\ , \\ \sum_{i=1}^{m} \bar{x}_{ij} \approx \bar{b}_j,\ \text{for } j = 1,2,..,n. \end{cases} \tag{9}$$

where $\bar{c}_{ij} = \left[\, c_{ij}^1, c_{ij}^4 \right]$, $\bar{a}_i = \left[\, a_i^1, a_i^4 \right]$, $\bar{b}_j = \left[\, b_j^1, b_j^4 \right]$ are non-negative interval numbers and

$\bar{x}_{ij} = \left[\, x_{ij}^1, x_{ij}^4 \right]$ are unrestricted interval numbers.

Objective function transformation.

$$\bar{Z}(\bar{x}) \approx \sum_{j=1}^{n}\sum_{i=1}^{m} \bar{c}_{ij}\bar{x}_{ij} = \sum_{j=1}^{n}\sum_{i=1}^{m} \left[\, c_{ij}^1, c_{ij}^4 \right]\left[\, x_{ij}^1, x_{ij}^4 \right] = \sum_{j=1}^{n}\sum_{i=1}^{m} \langle\, m\left(\bar{c}_{ij}\bar{x}_{ij}\right), w\left(\bar{c}_{ij}\bar{x}_{ij}\right) \rangle\ ,$$

Where

$$m\left(\bar{c}_{ij}\bar{x}_{ij}\right) = \begin{cases} m\left(\bar{c}_{ij}\right)m\left(\bar{x}_{ij}\right) + w\left(\bar{c}_{ij}\right)w\left(\bar{x}_{ij}\right) \text{if } x_{ij}^1 \geq 0, \\ m\left(\bar{c}_{ij}\right)m\left(\bar{x}_{ij}\right) + w\left(\bar{c}_{ij}\right)m\left(\bar{x}_{ij}\right) \text{if } x_{ij}^1 \langle 0 \text{ and } x_{ij}^4 \rangle 0, \\ m\left(\bar{c}_{ij}\right)m\left(\bar{x}_{ij}\right) - w\left(\bar{c}_{ij}\right)w\left(\bar{x}_{ij}\right) \text{if } x_{ij}^4 < 0. \end{cases} \tag{10}$$

And

$$w\left(\bar{c}_{ij}\bar{x}_{ij}\right) = \begin{cases} m\left(\bar{c}_{ij}\right)w\left(\bar{x}_{ij}\right) + w\left(\bar{c}_{ij}\right)m\left(\bar{x}_{ij}\right) \text{if } x_{ij}^1 \geq 0, \\ m\left(\bar{c}_{ij}\right)w\left(\bar{x}_{ij}\right) + w\left(\bar{c}_{ij}\right)w\left(\bar{x}_{ij}\right) \text{if } x_{ij}^1 \langle 0 \text{ and } x_{ij}^4 \rangle 0, \\ m\left(\bar{c}_{ij}\right)w\left(\bar{x}_{ij}\right) - w\left(\bar{c}_{ij}\right)m\left(\bar{x}_{ij}\right) \text{if } x_{ij}^4 < 0. \end{cases} \tag{11}$$

Transformation of constraints.

$$\sum_{j=1}^{n} \bar{x}_{ij} \approx \bar{a}_i \Leftrightarrow \sum_{j=1}^{n} \langle\, m\left(\bar{x}_{ij}\right), w\left(\bar{x}_{ij}\right) \rangle \approx \langle\, m\left(\bar{a}_i\right), w\left(\bar{a}_i\right) \rangle \text{ for } i = 1,2,..,m \text{ and}$$

$$\sum_{i=1}^{m} \bar{x}_{ij} \approx \bar{b}_j \Leftrightarrow \sum_{i=1}^{m} \langle\, m\left(\bar{x}_{ij}\right), w\left(\bar{x}_{ij}\right) \rangle \approx \langle\, m\left(\bar{b}_j\right), w\left(\bar{b}_j\right) \rangle \text{ for } j = 1,2,..,n.$$

Now we can say that

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$$\begin{cases} \text{Min}\,\bar{Z}(\bar{x}) \approx \sum_{j=1}^{n}\sum_{i=1}^{m} \langle m(\bar{c}_{ij}\bar{x}_{ij}), w(\bar{c}_{ij}\bar{x}_{ij})\rangle \\ \text{Subject to the constraints} \\ \sum_{j=1}^{n} \langle m(\bar{x}_{ij}), w(\bar{x}_{ij})\rangle \approx \langle m(\bar{a}_i), w(\bar{a}_i)\rangle, i=1,2,..,m, \\ \sum_{i=1}^{m} \langle m(\bar{x}_{ij}), w(\bar{x}_{ij})\rangle \approx \langle m(\bar{b}_j), w(\bar{b}_j)\rangle, j=1,2,..,n. \end{cases} \tag{12}$$

is equivalent to

$$\begin{cases} m(\text{Min}\,\bar{Z}(\bar{x})) = \sum_{j=1}^{n}\sum_{i=1}^{m} m(\bar{c}_{ij}\bar{x}_{ij}) \\ \text{Subject to the constraints} \\ \sum_{j=1}^{n} m(\bar{x}_{ij}) = m(\bar{a}_i), \text{for } i=1,2,..,m, \\ \sum_{i=1}^{m} m(\bar{x}_{ij}) = m(\bar{b}_j), \text{ for } j=1,2,..,n. \end{cases} \tag{13}$$

Optimal Solution (9) according to the choice of the decision maker:

$$\bar{x}_{ij}^{*} = \left[x_{ij}^{1}, x_{ij}^{4}\right] = \left[m(\bar{x}_{ij}) - w(\bar{x}_{ij}), m(\bar{x}_{ij}) + w(\bar{x}_{ij})\right] \text{ where } \sum_{j=1}^{n} w(\bar{x}_{ij}) \geq w(\bar{a}_i),\ i = 1,2,..,m.$$

Remark.

I. $\sum_{j=1}^{n} \bar{x}_{ij} = \bar{a}_i$ if and only if $\sum_{j=1}^{n} m(\bar{x}_{ij}) = m(\bar{a}_i)$ and $\sum_{j=1}^{n} w(\bar{x}_{ij}) = w(\bar{a}_i)$ for $i = 1,2,..,m$.

II. $\sum_{j=1}^{n} \bar{x}_{ij} \neq \bar{a}_i$ if and only if $\sum_{j=1}^{n} m(\bar{x}_{ij}) = m(\bar{a}_i)$ and $\sum_{j=1}^{n} w(\bar{x}_{ij}) > w(\bar{a}_i)$ for $i = 1,2,..,m$.

III. $\sum_{i=1}^{m} \bar{x}_{ij} = \bar{b}_j$ if and only if $\sum_{i=1}^{m} m(\bar{x}_{ij}) = m(\bar{b}_j)$ and $\sum_{i=1}^{m} w(\bar{x}_{ij}) = w(\bar{b}_j)$ for $j = 1,2,..,n$.

IV. $\sum_{i=1}^{m} \bar{x}_{ij} \neq \bar{b}_j$ if and only if $\sum_{i=1}^{m} m(\bar{x}_{ij}) = m(\bar{b}_j)$ and $\sum_{i=1}^{m} w(\bar{x}_{ij}) > w(\bar{b}_j)$ for $j = 1,2,..,n$.

2.4 | Formulation of a Fully Fuzzy Transportation Problem

The fuzzy linear programming formulation of a fully fuzzy transportation problem can be written as follows as follows [2], [3]:

$$\begin{cases} \text{Min}\,\tilde{Z}(\tilde{x}) \approx \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}\tilde{x}_{ij} \\ \text{Subject to the constraints} \\ \sum_{j=1}^{n}\tilde{x}_{ij} \approx \tilde{a}_i,\ \text{for } i=1,2,..,m \\ \sum_{i=1}^{m}\tilde{x}_{ij} \approx \tilde{b}_j,\ \text{for } j=1,2,..,n \end{cases} \tag{14}$$

with $\sum_{i=1}^{m}\tilde{a}_i = \sum_{j=1}^{n}\tilde{b}_j$ where $\tilde{x}_{ij}$ are unrestricted fuzzy numbers and $\tilde{c}_{ij}$, $\tilde{a}_i$ and $\tilde{b}_j$ are non-negatives fuzzy numbers.

3| Main Results

In this section, we will describe our method of solving.

3.1| Transportation Problem with Trapezoidal Fuzzy Numbers

For all the rest of this paper, we will consider the following transportation problem with trapezoidal fuzzy numbers as follows:

$$\begin{cases} \text{Min}\,\tilde{Z}(\tilde{x}) \approx \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}\tilde{x}_{ij} \\ \text{Subject to the constraints} \\ \sum_{j=1}^{n}\tilde{x}_{ij} \approx \tilde{a}_i,\text{for } i=1,2,..,m, \\ \sum_{i=1}^{m}\tilde{x}_{ij} \approx \tilde{b}_j,\ \text{for } j=1,2,..,n. \end{cases} \tag{15}$$

where $\tilde{c}_{ij} = \left(c_{ij}^1, c_{ij}^2, c_{ij}^3, c_{ij}^4\right)$, $\tilde{x}_j = \left(x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4\right)$, $\tilde{b}_j = \left(b_j^1, b_j^2, b_j^3, b_j^4\right)$ and $\tilde{a}_i = \left(a_i^1, a_i^2, a_i^3, a_i^4\right)$ are trapezoidal fuzzy numbers with

$\tilde{x}_{ij} = \left(\bar{x}_{ij}^{14} / \bar{x}_{ij}^{23}\right) = \left(\left[x_{ij}^1, x_{ij}^4\right] / \left[x_{ij}^2, x_{ij}^3\right]\right)$ where $\bar{x}_{ij}^{14} = \left[x_{ij}^1, x_{ij}^4\right]$ and $\bar{x}_{ij}^{23} = \left[x_{ij}^2, x_{ij}^3\right]$,

$\tilde{c}_{ij} = \left(\bar{c}_{ij}^{14} / \bar{c}_{ij}^{23}\right) = \left(\left[c_{ij}^1, c_{ij}^4\right] / \left[c_{ij}^2, c_{ij}^3\right]\right)$ where $\bar{c}_{ij}^{14} = \left[c_{ij}^1, c_{ij}^4\right]$ and $\bar{c}_{ij}^{23} = \left[c_{ij}^2, c_{ij}^3\right]$,

$\tilde{b}_j = \left(\bar{b}_j^{14} / \bar{b}_j^{23}\right) = \left(\left[b_j^1, b_j^4\right] / \left[b_j^2, b_j^3\right]\right)$ where $\bar{b}_j^{14} = \left[b_j^1, b_j^4\right]$ and $\bar{b}_j^{23} = \left[b_j^2, b_j^3\right]$ and

$\tilde{a}_i = \left(\bar{a}_i^{14} / \bar{a}_i^{23}\right) = \left(\left[a_i^1, a_i^4\right] / \left[a_i^2, a_i^3\right]\right)$ where $\bar{a}_i^{14} = \left[a_i^1, a_i^4\right]$ and $\bar{a}_i^{23} = \left[a_i^2, a_i^3\right]$.

For all the rest of this paper, we will consider the following transportation problems involving interval numbers $\bar{x}_{ij}^{14}$, $\bar{c}_{ij}^{14}$, $\bar{a}_i^{14}$ and $\bar{b}_j^{14}$ as follows:

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$$\begin{cases} \text{Min}\,\bar{Z}^{14}\left(\bar{x}^{14}\right) \approx \sum_{i=1}^{m}\sum_{j=1}^{n} \bar{c}_{ij}^{14}\bar{x}_{ij}^{14} \\ \text{Subject to the constraints} \\ \sum_{j=1}^{n} \bar{x}_{ij}^{14} \approx \bar{a}_i^{14}, \text{for } i=1,2,..,m, \\ \sum_{i=1}^{m} \bar{x}_{ij}^{14} \approx \bar{b}_j^{14}, \text{for } j=1,2,..,n. \end{cases} \tag{16}$$

And the transportation problems involving interval numbers $\bar{x}_{ij}^{23}$, $\bar{c}_{ij}^{23}$, $\bar{a}_i^{23}$ and $\bar{b}_j^{23}$ as follows:

$$\begin{cases} \text{Min}\,\bar{Z}^{23}\left(\bar{x}^{23}\right) \approx \sum_{i=1}^{m}\sum_{j=1}^{n} \bar{c}_{ij}^{23}\bar{x}_{ij}^{23} \\ \text{Subject to the constraints} \\ \sum_{j=1}^{n} \bar{x}_{ij}^{23} \approx \bar{a}_i^{23}, \text{for } i=1,2,..,m, \\ \sum_{i=1}^{m} \bar{x}_{ij}^{23} \approx \bar{b}_j^{23}, \text{for } j=1,2,..,n. \end{cases} \tag{17}$$

3.2| Formulation of a Transportation Problem involving Midpoint

Thanks to the new interval arithmetic and *Eq. (16)*, we can write the following transportation problem involving midpoint ($\bar{x}_{ij}^{14}$, $\bar{c}_{ij}^{14}$, $\bar{a}_i^{14}$ and $\bar{b}_j^{14}$) [28] as follows:

$$\begin{cases} \text{Min}\,Z^{14}\left(x^{14}\right) = \sum_{i=1}^{m}\sum_{j=1}^{n} m\left(\bar{c}_{ij}^{14}\right)x_{ij}^{14} \\ \text{Subject to the constraints} \\ \sum_{j=1}^{n} x_{ij}^{14} = m\left(\bar{a}_i^{14}\right), \text{for } i=1,2,..,m, \\ \sum_{i=1}^{m} x_{ij}^{14} = m\left(\bar{b}_j^{14}\right), \text{for } j=1,2,..,n. \end{cases} \tag{18}$$

where $m\left(\bar{c}_{ij}^{14}\right) = \dfrac{c_{ij}^4 + c_{ij}^1}{2}$, $m\left(\bar{a}_i^{14}\right) = \dfrac{a_i^4 + a_i^1}{2}$, $m\left(\bar{b}_j^{14}\right) = \dfrac{b_j^4 + b_j^1}{2}$ and $w\left(\bar{a}_i^{14}\right) = \dfrac{a_i^4 - a_i^1}{2}$.

Thanks to the new interval arithmetic and *Eq. (17)*, we can write the following transportation problem involving midpoint ($\bar{x}_{ij}^{23}$, $\bar{c}_{ij}^{23}$, $\bar{a}_i^{23}$ and $\bar{b}_j^{23}$) [28] as follows:

$$\begin{cases} \text{Min}Z^{23}\left(x^{23}\right)=\sum_{i=1}^{m}\sum_{j=1}^{n} m\left(\bar{c}_{ij}^{23}\right)x_{ij}^{23} \\ \quad \text{Subject to the constraints} \\ \sum_{j=1}^{n} x_{ij}^{23}=m\left(\bar{a}_{i}^{23}\right), \text{for } i=1,2,..,m, \\ \sum_{i=1}^{m} x_{ij}^{23}=m\left(\bar{b}_{j}^{23}\right), \text{for } j=1,2,..,n. \end{cases} \tag{19}$$

where $m\left(\bar{c}_{ij}^{23}\right)=\frac{c_{ij}^{3}+c_{ij}^{2}}{2}$, $m\left(\bar{a}_{i}^{23}\right)=\frac{a_{i}^{3}+a_{i}^{2}}{2}$, $m\left(\bar{b}_{j}^{23}\right)=\frac{b_{j}^{3}+b_{j}^{2}}{2}$ and $w\left(\bar{a}_{i}^{23}\right)=\frac{a_{i}^{3}-a_{i}^{2}}{2}$.

Thanks to the new interval arithmetic, we can write the following proposition [28]:

Proposition 1. If $\bar{x}_{ij}^{14}=\left\lfloor x_{ij}^{*1}, x_{ij}^{*4}\right\rfloor$ is an optimal solution to the *Eq. (16)* and $\bar{x}_{ij}^{23}=\left\lfloor x_{ij}^{*2}, x_{ij}^{*3}\right\rfloor$ is an optimal solution to the *Eq. (17)*, then $\tilde{x}^{*}=\left(\tilde{x}_{ij}^{*}\right)_{m\times n}$ is an optimal solution to the *Eq. (15)* with

$$\tilde{x}_{ij}^{*}=\left(\bar{x}_{ij}^{14}/\bar{x}_{ij}^{23}\right)=\left(\left[x_{ij}^{*1}, x_{ij}^{*4}\right]/\left[x_{ij}^{*2}, x_{ij}^{*3}\right]\right)=\left(x_{ij}^{*1}, x_{ij}^{*2}, x_{ij}^{*3}, x_{ij}^{*4}\right).$$

3.3| The Steps of Our Computational Method

The steps of our method for solving the fully fuzzy transportation problem involving trapezoidal fuzzy numbers as follows.

3.3.1| Solution procedure for transportation problem with trapezoidal fuzzy numbers

Step 1. Construct the fuzzy transportation *Problem (15)*, and then convert it into a balanced one if it is not. Else, go to Step 2.

Step 2. Solving *Eq. (17)* via *Eq. (19)*. Determine $\bar{x}_{ij}^{23}=\left[x_{ij}^{*2}, x_{ij}^{*3}\right]=\left\lfloor x_{ij}^{23}-w\left(\bar{x}_{ij}^{23}\right), x_{ij}^{23}+w\left(\bar{x}_{ij}^{23}\right)\right\rfloor$ by applying the following conditions:

I. $w\left(\bar{x}_{ij}^{23}\right)=0$ if and only if $x_{ij}^{23}=0$ and

II. $\sum_{x_{ij}^{23}\neq 0} w\left(\bar{x}_{ij}^{23}\right)=w\left(\bar{a}_{i}^{23}\right)$ with $w\left(\bar{x}_{ip}^{23}\right)\geq w\left(\bar{x}_{ik}^{23}\right)$ if $c_{ip}^{4}\leq c_{ik}^{4}$ for $i=1,..,m$.

III. Go to Step 3.

Step 3. Solving *Eq. (16)* via *Eq. (18)*. Determine $\bar{x}_{ij}^{14}=\left[x_{ij}^{*1}, x_{ij}^{*4}\right]=\left\lfloor x_{ij}^{14}-w\left(\bar{x}_{ij}^{14}\right), x_{ij}^{14}+w\left(\bar{x}_{ij}^{14}\right)\right\rfloor$ for $i=1,..,m$. Considering the following cases:

Case 1. If $E_{i}=\sum_{j=1}^{n}\left|x_{ij}^{14}-x_{ij}^{23}\right|+w\left(\bar{a}_{i}^{23}\right)\leq w\left(\bar{a}_{i}^{14}\right)$, then

$\bar{x}_{ij}^{14}=\left[x_{ij}^{*1}, x_{ij}^{*4}\right]=\left[x_{ij}^{14}-w\left(\bar{x}_{ij}^{14}\right), x_{ij}^{14}+w\left(\bar{x}_{ij}^{14}\right)\right]$ with $\sum_{x_{ij}^{14}\neq 0} w\left(\bar{x}_{ij}^{14}\right)=w\left(\bar{a}_{i}^{14}\right)$. Else, go to Case 2.

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Case 2. If $E_i = \sum_{j=1}^{n} \left| x_{ij}^{14} - x_{ij}^{23} \right| + w\left(\bar{a}_i^{23}\right) > w\left(\bar{a}_i^{14}\right)$, then

$\bar{x}_{ij}^{14} = \left[x_{ij}^{*1}, x_{ij}^{*4} \right] = \left\lfloor x_{ij}^{14} - w\left(\bar{x}_{ij}^{14}\right), x_{ij}^{14} + w\left(\bar{x}_{ij}^{14}\right) \right\rfloor$ with $w\left(\bar{x}_{ij}^{14}\right) = \left| x_{ij}^{14} - x_{ij}^{23} \right| + w\left(\bar{x}_{ij}^{23}\right)$ and go to Step 4.

Step 4. The *Optimal Solution (15)* according to the choice of the decision maker is:

$$Min\ \tilde{Z}(\tilde{x}) \approx \sum_{i=1}^{m}\sum_{j=1}^{n} \left(c_{ij}^{1}, c_{ij}^{2}, c_{ij}^{3}, c_{ij}^{4}\right)\left(x_{ij}^{*1}, x_{ij}^{*2}, x_{ij}^{*3}, x_{ij}^{*4}\right) = \left(Z^{*1}, Z^{*2}, Z^{*3}, Z^{*4}\right) \text{ where}$$

$$\tilde{x}_{ij}^{*} = \left(\bar{x}_{ij}^{14} / \bar{x}_{ij}^{23}\right) = \left(\left[x_{ij}^{*1}, x_{ij}^{*4} \right] / \left[x_{ij}^{*2}, x_{ij}^{*3} \right]\right) = \left(x_{ij}^{*1}, x_{ij}^{*2}, x_{ij}^{*3}, x_{ij}^{*4} \right).$$

3.3.2 | Solution procedure for transportation problem with triangular fuzzy numbers

The steps of our method for solving the fully fuzzy transportation problem involving triangular fuzzy numbers as follows:

The fuzzy number $\tilde{x}_{ij}^{*} = \left(x_{ij}^{*1}, x_{ij}^{*2}, x_{ij}^{*3}, x_{ij}^{*4} \right)$ is said to be triangular if and only if $x_{ij}^{*2} = x_{ij}^{*3}$. Then $w\left(\bar{x}_{ij}^{23}\right) = \dfrac{x_{ij}^{*3} - x_{ij}^{*2}}{2} = 0$. Therefore $w\left(\bar{x}_{ij}^{14}\right) = \left| x_{ij}^{14} - x_{ij}^{23} \right|$ and

$$\tilde{x}_{ij}^{*} = \left(x_{ij}^{*1}, x_{ij}^{*2}, x_{ij}^{*3}, x_{ij}^{*4} \right) = \left(x_{ij}^{*1}, x_{ij}^{*2}, x_{ij}^{*2}, x_{ij}^{*4} \right) = \left(x_{ij}^{*1}, x_{ij}^{*2}, x_{ij}^{*4} \right).$$

In this article, consider fuzzy triangular numbers by: $\tilde{x}_{ij}^{*} = \left(x_{ij}^{2} / \bar{x}_{ij}^{13} \right) = \left(x_{ij}^{2} / \left[x_{ij}^{*1}, x_{ij}^{*3} \right] \right) = \left(x_{ij}^{*1}, x_{ij}^{*2}, x_{ij}^{*3} \right)$, where $\left[x_{ij}^{*1}, x_{ij}^{*3} \right] = \left\lfloor x_{ij}^{13} - w\left(\bar{x}_{ij}^{13}\right), x_{ij}^{13} + w\left(\bar{x}_{ij}^{13}\right) \right\rfloor$.

Step 1. Construct the fuzzy transportation *Problem (15)*, and then convert it into a balanced one if it is not. Else, go to Step 2.

Step 2. Solving *Eq. (17)* via *Eq. (19)*. Determine $\bar{x}_{ij}^{23} = \left\lfloor x_{ij}^{*2}, x_{ij}^{*3} \right\rfloor = \left\lfloor x_{ij}^{*2}, x_{ij}^{*2} \right\rfloor = x_{ij}^{*2}$, for $i = 1, \ldots, m$ and go to Step 3.

Step 3. Solving *Eq. (16)* via *Eq. (18)*. Determine $\bar{x}_{ij}^{13} = \left[x_{ij}^{*1}, x_{ij}^{*3} \right] = \left[x_{ij}^{13} - w\left(\bar{x}_{ij}^{13}\right), x_{ij}^{13} + w\left(\bar{x}_{ij}^{13}\right) \right]$ for $i = 1, \ldots, m$. Considering the following cases:

Case 1. If $E_i = \sum_{j=1}^{n} \left| x_{ij}^{13} - x_{ij}^{2} \right| \le w\left(\bar{a}_i^{13}\right)$, then $\bar{x}_{ij}^{13} = \left[x_{ij}^{*1}, x_{ij}^{*3} \right] = \left\lfloor x_{ij}^{13} - w\left(\bar{x}_{ij}^{13}\right), x_{ij}^{13} + w\left(\bar{x}_{ij}^{13}\right) \right\rfloor$ with $\sum_{x_{ij}^{13} \neq 0} w\left(\bar{x}_{ij}^{13}\right) = w\left(\bar{a}_i^{13}\right)$. Else, go to Case 2.

Case 2. If $E_i = \sum_{j=1}^{n} \left| x_{ij}^{13} - x_{ij}^{2} \right| > w\left(\bar{a}_i^{13}\right)$, then $\bar{x}_{ij}^{13} = \left[x_{ij}^{*1}, x_{ij}^{*3} \right] = \left\lfloor x_{ij}^{13} - w\left(\bar{x}_{ij}^{13}\right), x_{ij}^{13} + w\left(\bar{x}_{ij}^{13}\right) \right\rfloor$ with $w\left(\bar{x}_{ij}^{13}\right) = \left| x_{ij}^{13} - x_{ij}^{2} \right|$ and go to Step 4.

Step 4. The *Optimal Solution (15)* according to the choice of the decision maker is:

$$Min\ \tilde{Z}(\tilde{x}) \approx \sum_{i=1}^{m}\sum_{j=1}^{n} \left(c_{ij}^{1}, c_{ij}^{2}, c_{ij}^{3}\right)\left(x_{ij}^{*1}, x_{ij}^{*2}, x_{ij}^{*3}\right) = \left(Z^{*1}, Z^{*2}, Z^{*3}\right) \text{with}$$

$$\tilde{x}_{ij}^{*} = \left(x_{ij}^{2} / \tilde{x}_{ij}^{13}\right) = \left(x_{ij}^{2} / \left[\ x_{ij}^{*1}\ , x_{ij}^{*3}\right]\right) = \left(x_{ij}^{*1}, x_{ij}^{*2}, x_{ij}^{*3}\right).$$

4 | Advantages of the Proposed Method

Let us explore the main advantages of the proposed method:

- *The new proposed method is applicable to all Eq. (15) where $\tilde{x}_{ij}$ are unrestricted triangular or trapezoidal fuzzy numbers and $\tilde{c}_{ij}$, $\tilde{a}_i$ and $\tilde{b}_j$ are non-negatives triangular or trapezoidal fuzzy numbers.*
- *The proposed technique does not use the goal and parametric approaches which are difficult to apply in real life situations.*
- *By applying the proposed approach for finding the fuzzy optimal solution, there is no need of much knowledge of fuzzy linear programming technique, Zimmerman approach and crisp linear programming which are difficult to learn for a new decision maker.*
- *The proposed method to solve Eq. (15) is based on traditional transportation algorithms. Thus, the existing and easily available software can be used for the same. However, the existing method [1], [2], [3], [11], [29] to solve Eq. (15) should be implemented into a programming language.*
- *To solve the Eq. (15) by using the existing method [1], [2], [3], [11], [29], there is need to use arithmetic operations of generalized fuzzy numbers. While, if the proposed technique is used for the same then there is need to use arithmetic operations of real numbers. This proves that it is much easy to apply the proposed method as compared to the existing method [1], [2], [3], [11], [29].*
- *Moreover, it is possible to assume a generic ranking index for comparing the fuzzy numbers involved in the Eq. (15), in such a way that each time in which the decision maker wants to solve the FFTP under consideration (s)he can choose (or propose) the ranking index that best suits the Eq. (15).*

5 | Numerical Illustration

In this section, to illustrate the new method proposed, and the existing fully fuzzy transportation problem due to [1], [2], [3], [11], [29], [30], presented in *Examples 1-4*, are solved by the proposed method.

Example 1. [3].

Step 1. Construct the fuzzy transportation *Problem (15)*, and then convert it into a balanced one if it is not.

Table 1. Eq. (15) in trapezoidal unbalanced form.

	R_1	R_2	R_3	R_3	Supply ($\tilde{a}_i$)
A	$(10,10,10,10)$	$(50,50,50,50)$	$(80,80,80,80)$	$(0,0,0,0)$	$(70,90,90,100)$
B	$(60,70,80,90)$	$(60,60,60,60)$	$(20,20,20,20)$	$(0,0,0,0)$	$(40,60,70,80)$
C	$(0,0,0,0)$	$(0,0,0,0)$	$(0,0,0,0)$	$(0,0,0,0)$	$(0,0,10,50)$
Demand ($\tilde{b}_j$)	$(30,40,50,70)$	$(20,30,40,50)$	$(40,50,50,80)$	$(20,30,30,30)$	$\sum_{i=1}^{m}\tilde{a}_i = \sum_{j=1}^{n}\tilde{b}_j$

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Step 2. Solving *Eq. (17)* via *Eq. (19)*. $Min Z^{23}\left(x^{23}\right)=25x_{11}^{23}+65x_{12}^{23}+100x_{13}^{23}+75x_{21}^{23}+90x_{22}^{23}+40x_{23}^{23}$ subject to the constraints $x_{11}^{23}+x_{12}^{23}+x_{13}^{23}+x_{14}^{23}=90$, $x_{21}^{23}+x_{22}^{23}+x_{23}^{23}+x_{24}^{23}=65$, $x_{31}^{23}+x_{32}^{23}+x_{33}^{23}+x_{34}^{23}=5$, $x_{11}^{23}+x_{21}^{23}+x_{31}^{23}=45$, $x_{12}^{23}+x_{22}^{23}+x_{32}^{23}=35$, $x_{13}^{23}+x_{23}^{23}+x_{33}^{23}=50$ and $x_{14}^{23}+x_{24}^{23}+x_{34}^{23}=30$.

Optimal solution is: $x_{11}^{23}=45$, $x_{12}^{23}=30$, $x_{13}^{23}=0$, $x_{14}^{23}=15$, $x_{21}^{23}=0$, $x_{22}^{23}=0$, $x_{23}^{23}=50$, $x_{24}^{23}=15$, $x_{31}^{23}=0$, $x_{32}^{23}=5$, $x_{33}^{23}=0$ and $x_{34}^{23}=0$. Furthermore $w\left(\bar{a}_1^{23}\right)=0$, $w\left(\bar{a}_2^{23}\right)=5$ and $w\left(\bar{a}_3^{23}\right)=5$.

For $i=1$, then $\sum_{x_{1j}^{23}\neq 0} w\left(\bar{x}_{1j}^{23}\right)=w\left(\bar{a}_1^{23}\right)=0$ with $w\left(\bar{x}_{11}^{23}\right)=w\left(\bar{x}_{12}^{23}\right)=w\left(\bar{x}_{13}^{23}\right)=w\left(\bar{x}_{14}^{23}\right)=0$. We get $\bar{x}_{11}^{23}=\lfloor 45,45\rfloor$, $\bar{x}_{12}^{23}=\lfloor 30,30\rfloor$, $\bar{x}_{13}^{23}=\bar{0}$ and $\bar{x}_{14}^{23}=\lfloor 15,15\rfloor$.

For $i=2$, then $\sum_{x_{2j}^{23}\neq 0} w\left(\bar{x}_{2j}^{23}\right)=w\left(\bar{a}_2^{23}\right)=5$ with $w\left(\bar{x}_{21}^{23}\right)=w\left(\bar{x}_{22}^{23}\right)=0$, $w\left(\bar{x}_{23}^{23}\right)=1$ and

$w\left(\bar{x}_{24}^{23}\right)=4$. We get $\bar{x}_{21}^{23}=\lfloor 0,0\rfloor$, $\bar{x}_{22}^{23}=\lfloor 0,0\rfloor$, $\bar{x}_{23}^{23}=\lfloor 49,51\rfloor$ and $\bar{x}_{24}^{23}=\lfloor 11,19\rfloor$.

For $i=3$, then $\sum_{x_{3j}^{23}\neq 0} w\left(\bar{x}_{3j}^{23}\right)=w\left(\bar{a}_3^{23}\right)=5$ with $w\left(\bar{x}_{31}^{23}\right)=0$, $w\left(\bar{x}_{32}^{23}\right)=5$, $w\left(\bar{x}_{33}^{23}\right)=0$ and

$w\left(\bar{x}_{34}^{23}\right)=0$. We get $\bar{x}_{31}^{23}=\bar{0}$, $\bar{x}_{32}^{23}=\lfloor 0,10\rfloor$, $\bar{x}_{33}^{23}=\bar{0}$ and $\bar{x}_{34}^{23}=\bar{0}$.

Step 3. Solving *Eq. (16)* via *Eq. (18)*. We get
$Min Z^{14}\left(x^{14}\right)=25x_{11}^{14}+70x_{12}^{14}+100x_{13}^{14}+75x_{21}^{14}+95x_{22}^{14}+40x_{23}^{14}$

subject to the constraints $x_{11}^{14}+x_{12}^{14}+x_{13}^{14}+x_{14}^{14}=85$, $x_{21}^{14}+x_{22}^{14}+x_{23}^{14}+x_{24}^{14}=60$,

$x_{31}^{14}+x_{32}^{14}+x_{33}^{14}+x_{34}^{14}=25$, $x_{11}^{14}+x_{21}^{14}+x_{31}^{14}=50$, $x_{12}^{14}+x_{22}^{14}+x_{32}^{14}=35$, $x_{13}^{14}+x_{23}^{14}+x_{33}^{14}=60$ and

$x_{14}^{14}+x_{24}^{14}+x_{34}^{14}=25$.

Optimal solution is: $x_{11}^{14}=50$, $x_{12}^{14}=10$, $x_{13}^{14}=0$, $x_{14}^{14}=25$, $x_{21}^{14}=0$, $x_{22}^{14}=0$, $x_{23}^{14}=60$, $x_{24}^{14}=0$,

$x_{31}^{14}=0$, $x_{32}^{14}=25$, $x_{33}^{14}=0$ and $x_{34}^{14}=0$. Furthermore $w\left(\bar{a}_1^{14}\right)=15$, $w\left(\bar{a}_2^{14}\right)=20$ and $w\left(\bar{a}_3^{14}\right)=25$

For $i=1$, we have

$$E_1=\sum_{j=1}^{4}\left|x_{1j}^{14}-x_{1j}^{23}\right|+w\left(\bar{a}_1^{23}\right)=|50-45|+|10-30|+|0-0|+|25-15|+0=35>15=w\left(\bar{a}_1^{14}\right),$$

then $w\left(\bar{x}_{1j}^{14}\right)=\left|x_{1j}^{32}-x_{1j}^{14}\right|+w\left(\bar{x}_{1j}^{23}\right)$. We have $w\left(\bar{x}_{11}^{14}\right)=5$, $w\left(\bar{x}_{12}^{14}\right)=20$, $w\left(\bar{x}_{13}^{14}\right)=0$ and

$w\left(\bar{x}_{14}^{14}\right)=10$. We get $\bar{x}_{11}^{14}=\lfloor 45,55\rfloor$, $\bar{x}_{12}^{14}=\lfloor -10,30\rfloor$, $\bar{x}_{13}^{14}=\bar{0}$ and $\bar{x}_{14}^{14}=\lfloor 15,35\rfloor$.

For $i=2$, we have

$$E_2=\sum_{j=1}^{4}\left|x_{2j}^{14}-x_{2j}^{23}\right|+w\left(\bar{a}_2^{23}\right)=|0-0|+|0-0|+|60-50|+|0-15|+5=30>20=w\left(\bar{a}_2^{14}\right),$$

then $w\left(\bar{x}_{2j}^{14}\right)=\left|x_{2j}^{14}-x_{2j}^{23}\right|+w\left(\bar{x}_{2j}^{23}\right)$. We have $w\left(\bar{x}_{21}^{14}\right)=w\left(\bar{x}_{22}^{14}\right)=0$,

$w\left(\bar{x}_{23}^{23}\right)=11$ and $w\left(\bar{x}_{24}^{14}\right)=19$. We get $\bar{x}_{21}^{14}=\lfloor 0,0\rfloor$, $\bar{x}_{22}^{14}=\lfloor 0,0\rfloor$, $\bar{x}_{23}^{14}=\lfloor 49,71\rfloor$ and

$\bar{x}_{24}^{14}=\lfloor -19,19\rfloor$.

For $i=3$, we have

$$E_3=\sum_{j=1}^{4}\left|x_{3j}^{14}-x_{3j}^{23}\right|+w\left(\bar{a}_3^{23}\right)=|0-0|+|25-5|+|0-0|+|0-0|+5=25\le 25=w\left(\bar{a}_3^{14}\right),$$

then $\sum_{x_{3j}^{14}\neq 0} w\left(\bar{x}_{3j}^{14}\right)=w\left(\bar{a}_3^{14}\right)=25$. We have $w\left(\bar{x}_{31}^{14}\right)=0$, $w\left(\bar{x}_{32}^{14}\right)=25$, $w\left(\bar{x}_{33}^{14}\right)=0$ and $w\left(\bar{x}_{34}^{14}\right)=0$

We get $\bar{x}_{31}^{14}=\bar{0}$, $\bar{x}_{32}^{14}=\lfloor 0,50\rfloor$, $\bar{x}_{33}^{14}=\bar{0}$ and $\bar{x}_{34}^{14}=\bar{0}$.

Step 4. The optimal solution according to the choice of the decision maker is:

$Min\ \tilde{Z}(\tilde{x})\approx\sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij}^1,c_{ij}^2,c_{ij}^3,c_{ij}^4\right)\left(x_{ij}^{*1},x_{ij}^{*2},x_{ij}^{*3},x_{ij}^{*4}\right)=\left(Z^{*1},Z^{*2},Z^{*3},Z^{*4}\right)=(930,2930,2970,3470)$ and

$\tilde{x}_{ij}^{*}=\left(\bar{x}_{ij}^{14}/\bar{x}_{ij}^{23}\right)=\left(\left[x_{ij}^{*1},x_{ij}^{*4}\right]/\left[x_{ij}^{*2},x_{ij}^{*3}\right]\right)=\left(x_{ij}^{*1},x_{ij}^{*2},x_{ij}^{*3},x_{ij}^{*4}\right)$ with

$\tilde{x}_{11}^{*}=(45,\ 45,\ 45,55)$, $\tilde{x}_{12}^{*}=(-10,\ 30,\ 30,\ 30)$, $\tilde{x}_{13}^{*}=\tilde{0}$, $\tilde{x}_{14}^{*}=(15,\ 15,\ 15,\ 35)$,

$\tilde{x}_{21}^{*}=\tilde{0}$, $\tilde{x}_{22}^{*}=\tilde{0}$, $\tilde{x}_{23}^{*}=(49,\ 49,\ 51,\ 71)$, $\tilde{x}_{24}^{*}=(-19,\ 11,\ 19,\ 19)$,

$\tilde{x}_{31}^{*}=\tilde{0}$, $\tilde{x}_{32}^{*}=(0,0,10,50)$, $\tilde{x}_{33}^{*}=\tilde{0}$ and $\tilde{x}_{34}^{*}=\tilde{0}$.

Example 2. [29].

Step 1. Construct the fuzzy transportation *Problem (15)*, and then convert it into a balanced one if it is not.

Table 2. Eq. (15) in trapezoidal balanced form.

	R_1	R_2	R_3	Supply $\tilde{a}_i$
A	$(1,4,9,19)$	$(1,2,5,9)$	$(2,5,8,18)$	$(1,5,7,9)$
B	$(8,9,12,26)$	$(3,5,8,12)$	$(7,9,13,28)$	$(4,7,8,10)$
C	$(11,12,20,27)$	$(0,5,10,15)$	$(4,5,8,11)$	$(4,5,8,11)$
Demand $\tilde{b}_j$	$(3,5,8,12)$	$(4,8,9,10)$	$(2,4,6,8)$	$\sum_{i=1}^{m}\tilde{a}_i=\sum_{j=1}^{n}\tilde{b}_j$

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Step 2. Solving *Eq. (17)* via *Eq. (19)*. Optimal solution is: $x_{11}^{23}=6$, $x_{12}^{23}=0$, $x_{13}^{23}=0$, $x_{21}^{23}=\frac{1}{2}$, $x_{22}^{23}=7$, $x_{23}^{23}=0$, $x_{31}^{23}=0$, $x_{32}^{23}=\frac{3}{2}$ and $x_{33}^{14}=5$. Furthermore $w\left(\bar{a}_1^{23}\right)=\frac{2}{2}=1$, $w\left(\bar{a}_2^{23}\right)=\frac{1}{2}$, $w\left(\bar{a}_3^{23}\right)=\frac{3}{2}$.

For $i=1$, then $\sum_{x_{1j}^{23}\neq 0} w\left(\bar{x}_{1j}^{23}\right)=w\left(\bar{a}_1^{23}\right)=1$ with $w\left(\bar{x}_{11}^{23}\right)=1$. We get $\bar{x}_{11}^{23}=\lfloor 5,7\rfloor$, $\bar{x}_{12}^{23}=\lfloor 0,0\rfloor$ and $\bar{x}_{13}^{23}=\lfloor 0,0\rfloor$.

For $i=2$, then $\sum_{x_{2j}^{23}\neq 0} w\left(\bar{x}_{2j}^{23}\right)=w\left(\bar{a}_2^{23}\right)=\frac{1}{2}$ with $w\left(\bar{x}_{21}^{23}\right)=\frac{1}{4}$ and $w\left(\bar{x}_{22}^{23}\right)=\frac{1}{4}$. We get $\bar{x}_{21}^{23}=\left[\frac{1}{4},\frac{3}{4}\right]$, $\bar{x}_{22}^{23}=\left[\frac{27}{4},\frac{29}{4}\right]$ and $\bar{x}_{23}^{23}=\lfloor 0,0\rfloor$.

For $i=3$, then $\sum_{x_{3j}^{23}\neq 0} w\left(\bar{x}_{3j}^{23}\right)=w\left(\bar{a}_3^{23}\right)=\frac{3}{2}$ with $w\left(\bar{x}_{32}^{23}\right)=\frac{1}{2}$ and $w\left(\bar{x}_{33}^{23}\right)=1$. We get $\bar{x}_{31}^{23}=\lfloor 0,0\rfloor$, $\bar{x}_{32}^{23}=\lfloor 1,2\rfloor$ and $\bar{x}_{33}^{23}=\lfloor 4,6\rfloor$.

Step 3. Solving *Eq. (16)* via *Eq. (18)*. Optimal solution is: $x_{11}^{14}=5$, $x_{12}^{14}=0$ and $x_{13}^{14}=0$, $x_{21}^{14}=\frac{5}{2}$, $x_{22}^{14}=\frac{9}{2}$, $x_{23}^{14}=0$, $x_{31}^{14}=0$, $x_{32}^{14}=\frac{5}{2}$ and $x_{33}^{14}=5$. Furthermore $w\left(\bar{a}_1^{14}\right)=4$, $w\left(\bar{a}_2^{14}\right)=3$, $w\left(\bar{a}_3^{14}\right)=\frac{7}{2}$.

For $i=1$, we have $E_1=\sum_{j=1}^{n}\left|x_{1j}^{14}-x_{1j}^{23}\right|+w\left(\bar{a}_1^{23}\right)=|5-6|+|0-0|+|0-0|+1=2\leq 4=w\left(\bar{a}_1^{14}\right)$, then $\sum_{x_{1j}^{14}\neq 0} w\left(\bar{x}_{1j}^{14}\right)=w\left(\bar{a}_1^{14}\right)=4$ with $w\left(\bar{x}_{11}^{14}\right)=4$. We get $\bar{x}_{11}^{14}=\lfloor 1,9\rfloor$, $\bar{x}_{12}^{14}=\lfloor 0,0\rfloor$ and $\bar{x}_{13}^{14}=\lfloor 0,0\rfloor$.

For $i=2$, we have $E_2=\sum_{j=1}^{n}\left|x_{2j}^{14}-x_{2j}^{23}\right|+w\left(\bar{a}_2^{23}\right)=\left|\frac{5}{2}-\frac{1}{2}\right|+\left|\frac{9}{2}-7\right|+|0-0|+\frac{1}{2}=5>w\left(\bar{a}_2^{14}\right)=3$, then $w\left(\bar{x}_{2j}^{14}\right)=\left|x_{2j}^{23}-x_{2j}^{14}\right|+w\left(\bar{x}_{2j}^{23}\right)$ with $w\left(\bar{x}_{21}^{14}\right)=\frac{9}{4}$ and $w\left(\bar{x}_{22}^{14}\right)=\frac{11}{4}$.

We get $\bar{x}_{21}^{14}=\left[\frac{1}{4},\frac{19}{4}\right]$, $\bar{x}_{22}^{14}=\left[\frac{7}{4},\frac{29}{4}\right]$ and $\bar{x}_{23}^{14}=\lfloor 0,0\rfloor$.

For $i=3$, we have $E_3=\sum_{j=1}^{n}\left|x_{3j}^{14}-x_{3j}^{23}\right|+w\left(\bar{a}_3^{23}\right)=\left|\frac{5}{2}-\frac{3}{2}\right|+|5-5|+|0-0|+\frac{3}{2}=\frac{5}{2}\leq w\left(\bar{a}_3^{14}\right)=\frac{7}{2}$, then $\sum_{x_{3j}^{14}\neq 0} w\left(\bar{x}_{3j}^{14}\right)=w\left(\bar{a}_3^{14}\right)=\frac{7}{2}$ with $w\left(\bar{x}_{32}^{14}\right)=\frac{3}{2}$ and $w\left(\bar{x}_{33}^{14}\right)=2$.

We get $\bar{x}_{31}^{14}=\lfloor 0,0\rfloor$, $\bar{x}_{32}^{14}=\lfloor 1,4\rfloor$ and $\bar{x}_{33}^{14}=\lfloor 3,7\rfloor$.

Step 4. The optimal solution according to the choice of the decision maker is:

$\tilde{x}^*_{ij} = \left(\bar{x}^{14}_{ij} / \bar{x}^{23}_{ij}\right) = \left(\left[\, x^{*1}_{ij}, x^{*4}_{ij} \right] / \left[\, x^{*2}_{ij}, x^{*3}_{ij} \right]\right) = \left(x^{*1}_{ij}, x^{*2}_{ij}, x^{*3}_{ij}, x^{*4}_{ij}\right)$. $Min\ \tilde{Z}(\tilde{x}) \approx \left(\frac{81}{4}, 81, 173, \frac{1037}{2}\right)$ with $\tilde{x}^*_{11} = (1, 5, 7, 9)$, $\tilde{x}^*_{12} = \tilde{0}$, $\tilde{x}^*_{13} = \tilde{0}$,

$\tilde{x}^*_{21} = \left(\frac{1}{4}, \frac{1}{4}, \frac{3}{4}, \frac{19}{4}\right)$, $\tilde{x}^*_{22} = \left(\frac{7}{4}, \frac{27}{4}, \frac{29}{4}, \frac{29}{4}\right)$, $\tilde{x}^*_{23} = \tilde{0}$, $\tilde{x}^*_{31} = \tilde{0}$, $\tilde{x}^*_{32} = (1,1,2,4)$ and $\tilde{x}^*_{33} = (3,4,6,7)$.

Example 3. [29].

Step 1. Construct the fuzzy transportation *Problem (15)*, and then convert it into a balanced one if it is not.

Table 3. Eq. (15) in triangular balanced form.

	R_1	R_2	Supply ($\tilde{a}_i$)
A	$(22,31,34)$	$(15,19,29)$	$(150,201,246)$
B	$(30,39,54)$	$(8,10,12)$	$(50,99,154)$
Demand ($\tilde{b}_j$)	$(100,150,200)$	$(100,150,200)$	$\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$

Step 2. Solving *Eq. (17)* via *Eq. (19)*. We get $Min Z^2\left(x^2\right) = 31x^2_{11} + 19x^2_{12} + 39x^2_{21} + 10x^2_{22}$ subject to the constraints $x^2_{11} + x^2_{12} = 201$, $x^2_{21} + x^2_{22} = 99$, $x^2_{11} + x^2_{21} = 150$ and $x^2_{12} + x^2_{22} = 150$.

Optimal solution is: $x^2_{11} = 150$, $x^2_{12} = 51$, $x^2_{21} = 0$ and $x^2_{22} = 99$.

Step 3. Solving *Eq. (16)* via *Eq. (18)*. We have $Min Z^{13}\left(x^{13}\right) = 28x^{13}_{11} + 22x^{13}_{12} + 42x^{13}_{21} + 10x^{13}_{22}$ subject to the

constraints $x^{13}_{11} + x^{13}_{12} = 198$, $x^{13}_{21} + x^{13}_{22} = 102$, $x^{13}_{11} + x^{13}_{21} = 150$ and $x^{13}_{12} + x^{13}_{22} = 150$.

Optimal solution is: $x^{13}_{11} = 150$, $x^{13}_{12} = 48$, $x^{13}_{21} = 0$ and $x^{13}_{22} = 102$. Furthermore $w\left(\bar{a}^{13}_1\right) = 48$,

$w\left(\bar{a}^{13}_2\right) = 52$.

For $i = 1$, we have $E_1 = \sum_{j=1}^{2} \left| x^{13}_{1j} - x^2_{1j} \right| = |150 - 150| + |48 - 51| = 3 \le 48 = w\left(\bar{a}^{13}_1\right)$,

then $\sum_{x^{13}_{1j} \neq 0} w\left(\bar{x}^{13}_{1j}\right) = w\left(\bar{a}^{13}_1\right) = 48$ with $w\left(\bar{x}^{13}_{11}\right) = 10$ and $w\left(\bar{x}^{13}_{12}\right) = 38$.

We get $\bar{x}^{13}_{11} = \lfloor 140, 160 \rfloor$ and $\bar{x}^{13}_{12} = \lfloor 10, 86 \rfloor$.

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For $i=2$, we have $E_2=\sum_{j=1}^{2}\left|x_{2j}^{13}-x_{2j}^{2}\right|=\left|0-0\right|+\left|102-99\right|=3\le 52=w\left(\bar{a}_2^{13}\right)$,

then $\sum_{x_{2j}^{13}\neq 0} w\left(\bar{x}_{2j}^{13}\right)=w\left(\bar{a}_2^{13}\right)=52$ with $w\left(\bar{x}_{21}^{13}\right)=0$ and $w\left(\bar{x}_{22}^{13}\right)=52$.

We get $\bar{x}_{21}^{13}=\left\lfloor 0,0\right\rfloor$ and $\bar{x}_{22}^{13}=\left\lfloor 50,154\right\rfloor$.

Step 4. The optimal solution according to the choice of the decision maker is

$Min\tilde{Z}\left(\tilde{x}\right)\approx\sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij}^{1},c_{ij}^{2},c_{ij}^{3}\right)\tilde{x}_{ij}$ with $\tilde{x}_{ij}^{*}=\left(x_{ij}^{2}/\bar{x}_{ij}^{13}\right)=\left(x_{ij}^{2}/\left[x_{ij}^{*1},x_{ij}^{*3}\right]\right)=\left(x_{ij}^{*1},x_{ij}^{*2},x_{ij}^{*3}\right)$. We have

Min $\tilde{Z}^{*}\approx\left(6609/\left[3630,\ 9782\right]\right)=\left(3630,6609,\ 9782\right)$ where

$\tilde{x}_{11}^{*}=\left(140,150,\ 160\right)$, $\tilde{x}_{12}^{*}=\left(10,51,86\right)$,

$\tilde{x}_{21}^{*}=\tilde{0}$ and $\tilde{x}_{22}^{*}=\left(50,99,154\right)$.

5.1| Interpretation of Results

We will now interpret the minimum total fuzzy transportation cost obtained in *Example 3*. by using the proposed methods presented in Section 3. Similarly, the obtained fuzzy optimal solution will also be interpreted. By using the methods proposed the minimum total fuzzy transportation cost is $\left(3630,6609,\ 9782\right)$, which can be physically interpreted as follows:

- *The least amount of the minimum total transportation cost is* 3630.
- *The most possible amount of minimum total transportation cost is* 6609.
- *The greatest amount of the minimum total transportation cost is* 9782 *i.e., the minimum total transportation cost will always be greater than* 3630 *and less than* 6609*, and the highest chances are that the minimum total transportation cost will be* 9782.

Example 4. [1]-[3].

Step 1. Construct the fuzzy transportation *Problem (15)*, and then convert it into a balanced one if it is not.

Step 2. Solving *Eq. (17)* via *Eq. (19)*. We get

$$Min Z^{2}\left(x^{2}\right)=10x_{11}^{2}+22x_{12}^{2}+10x_{13}^{2}+20x_{14}^{2}15x_{21}^{2}+20x_{22}^{2}+12x_{23}^{2}+8x_{24}^{2}+20x_{31}^{2}+12x_{32}^{2}+10x_{33}^{2}+15x_{34}^{2}$$

subject to the constraints $x_{11}^{2}+x_{12}^{2}+x_{13}^{2}+x_{14}^{2}=8$,

$x_{21}^{2}+x_{22}^{2}+x_{23}^{2}+x_{24}^{2}=14$, $x_{31}^{2}+x_{32}^{2}+x_{33}^{2}+x_{34}^{2}=12$, $x_{11}^{2}+x_{21}^{2}+x_{31}^{2}=7$, $x_{12}^{2}+x_{22}^{2}+x_{32}^{2}=10$,

$x_{13}^{2}+x_{23}^{2}+x_{33}^{2}=8$ and $x_{14}^{2}+x_{24}^{2}+x_{34}^{2}=9$.

Optimal solution is: $x_{11}^2 = 7$, $x_{12}^2 = 0$, $x_{13}^2 = 1$, $x_{14}^2 = 0$, $x_{21}^2 = 0$, $x_{22}^2 = 0$, $x_{23}^2 = 5$, $x_{24}^2 = 9$,

$x_{31}^2 = 0$, $x_{32}^2 = 10$, $x_{33}^2 = 2$ and $x_{34}^2 = 0$.

Table 4. Eq. (15) in triangular balanced form (in U.S. dollar).

Source	Destination Taichung	Chiayi	Kaohsiung	Taipei	Supply ($\tilde{a}_i$) (000 dozen bottles)
Changhua	(\$8, \$10, \$10.8)	(\$20.4, \$22, \$24)	(\$8,\$10,\$10.6)	(\$18.8, \$20, \$22)	(7.2,8,8.8)
Touliu	(\$14, \$15, \$16)	(\$18.2, \$20, \$22)	(\$10, \$12, \$13)	(\$6, \$8, \$8.8)	(12,14,16)
Hsinchu	(\$18.4, \$20, \$21)	(\$9.6, \$12, \$13)	(\$7.8, \$10, \$10.8)	(\$14, \$15, \$16)	(10.2,12,13.8)
Demand ($\tilde{b}_j$) (000 dozen bottles)	(6.2,7,7.8)	(8.9,10,11.1)	(6.5,8,9.5)	(7.8,9,10.2)	$\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$

Step 3. Solving *Eq. (16)* via *Eq. (18)*. We get

$$Min Z^{13}\left(x^{13}\right) = 9.4x_{11}^{13} + 22.2x_{12}^{13} + 9.3x_{13}^{13} + 20.4x_{14}^{13} + 15x_{21}^{13} + 20.1x_{22}^{13}$$

$+11.5x_{23}^{13} + 7.4x_{24}^{13} + 19.7x_{31}^{13} + 11.3x_{32}^{13} + 9.3x_{33}^{13} + 15x_{34}^{13}$ subject to the constraints

$x_{11}^{13} + x_{12}^{13} + x_{13}^{13} + x_{14}^{13} = 8$, $x_{21}^{13} + x_{22}^{13} + x_{23}^{13} + x_{24}^{13} = 14$, $x_{31}^{13} + x_{32}^{13} + x_{33}^{13} + x_{34}^{13} = 12$,

$x_{11}^{13} + x_{21}^{13} + x_{31}^{13} = 7$, $x_{12}^{13} + x_{22}^{13} + x_{32}^{13} = 10$, $x_{13}^{13} + x_{23}^{13} + x_{33}^{13} = 8$ and $x_{14}^{13} + x_{24}^{13} + x_{34}^{13} = 9$.

Optimal solution is: $x_{11}^{13} = 7$, $x_{12}^{13} = 0$, $x_{13}^{13} = 1$, $x_{14}^{13} = 0$, $x_{21}^{13} = 0$, $x_{22}^{13} = 0$, $x_{23}^{13} = 5$, $x_{24}^{13} = 9$,

$x_{31}^{13} = 0$, $x_{32}^{13} = 10$, $x_{33}^{13} = 2$ and $x_{34}^{13} = 0$. We have $w\left(\bar{a}_1^{13}\right) = \frac{4}{5}$, $w\left(\bar{a}_2^{13}\right) = 2$ and $w\left(\bar{a}_3^{13}\right) = \frac{9}{5}$.

Furthermore:

- $\bar{x}_{11}^{13} = \left[\frac{34}{5}, \frac{36}{5}\right]$, $\bar{x}_{12}^{13} = \lfloor 0,0 \rfloor$, $\bar{x}_{13}^{13} = \left[\frac{2}{5}, \frac{8}{5}\right]$ *and* $\bar{x}_{14}^{13} = \lfloor 0,0 \rfloor$ *where* $w\left(\bar{x}_{11}^{13}\right) = \frac{1}{5}$, $w\left(\bar{x}_{12}^{13}\right) = 0$, $w\left(\bar{x}_{13}^{13}\right) = \frac{3}{5}$ *and* $w\left(\bar{x}_{14}^{13}\right) = 0$.
- $\bar{x}_{21}^{13} = \lfloor 0,0 \rfloor$, $\bar{x}_{22}^{13} = \lfloor 0,0 \rfloor$, $\bar{x}_{23}^{13} = \left[\frac{9}{2}, \frac{11}{2}\right]$ *and* $\bar{x}_{24}^{13} = \left[\frac{15}{2}, \frac{21}{2}\right]$ *where* $w\left(\bar{x}_{21}^{13}\right) = 0$, $w\left(\bar{x}_{22}^{13}\right) = 0$, $w\left(\bar{x}_{23}^{13}\right) = \frac{1}{2}$ *and* $w\left(\bar{x}_{24}^{13}\right) = \frac{3}{2}$.

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– $\bar{x}_{31}^{13} = \lfloor 0,0 \rfloor$, $\bar{x}_{32}^{13} = \left[\frac{47}{5}, \frac{53}{5}\right]$, $\bar{x}_{33}^{13} = \left[\frac{4}{5}, \frac{16}{5}\right]$ *and* $\bar{x}_{34}^{13} = \lfloor 0,0 \rfloor$ *with* $w\left(\bar{x}_{31}^{13}\right) = 0$, $w\left(\bar{x}_{32}^{13}\right) = \frac{3}{5}$, $w\left(\bar{x}_{33}^{13}\right) = \frac{6}{5}$ *and* $w\left(\bar{x}_{34}^{13}\right) = 0$.

Step 4. The optimal solution according to the choice of the decision maker is

The value of the objective function: $Min\ \tilde{Z}(\tilde{x}) \approx \sum_{i=1}^{m}\sum_{j=1}^{n} \left(c_{ij}^{1}, c_{ij}^{2}, c_{ij}^{3}\right)\left(x_{ij}^{*1}, x_{ij}^{*2}, x_{ij}^{*3}\right) = \left(Z^{*1}, Z^{*2}, Z^{*3}\right)$

with $\tilde{x}_{ij}^{*} = \left(x_{ij}^{2} / \bar{x}_{ij}^{13}\right) = \left(x_{ij}^{2} / \left[x_{ij}^{*1}, x_{ij}^{*3}\right]\right) = \left(x_{ij}^{*1}, x_{ij}^{*2}, x_{ij}^{*3}\right)$.

We have Min $\tilde{Z}^{*} \approx \left(352\$ / \left[\frac{12204}{50}\$, \frac{21549}{50}\$\right]\right) = \left(\frac{12204}{50}\$, 352\$, \frac{21549}{50}\$\right)$ where

$\tilde{x}_{11}^{*} = \left(\frac{34}{5}, 7, \frac{36}{5}\right)$, $\tilde{x}_{12}^{*} = \tilde{0}$, $\tilde{x}_{13}^{*} = \left(\frac{2}{5}, 1, \frac{8}{5}\right)$, $\tilde{x}_{14}^{*} = \tilde{0}$,

$\tilde{x}_{21}^{*} = \tilde{0}$, $\tilde{x}_{22}^{*} = \tilde{0}$, $\tilde{x}_{23}^{*} = \left(\frac{9}{2}, 5, \frac{11}{2}\right)$, $\tilde{x}_{24}^{*} = \left(\frac{15}{2}, 9, \frac{21}{2}\right)$,

$\tilde{x}_{31}^{*} = \tilde{0}$, $\tilde{x}_{32}^{*} = \left(\frac{47}{5}, 10, \frac{53}{5}\right)$, $\tilde{x}_{33}^{*} = \left(\frac{4}{5}, 2, \frac{16}{5}\right)$ and $\tilde{x}_{34}^{*} = \tilde{0}$.

5.2| Interpretation of Results

We will now interpret the minimum total fuzzy transportation cost obtained in *Example 4* by using the proposed methods presented in *Section 3*. Similarly, the obtained fuzzy optimal solution will also be interpreted. By using the methods proposed the minimum total fuzzy transportation cost is $\left(\frac{12204}{50}\$, 352\$, \frac{21549}{50}\$\right)$, which can be physically interpreted as follows:

– *The least amount of the minimum total transportation cost is* $\frac{12204}{50}\$$.

– *The most possible amount of minimum total transportation cost is* $352\$$.

– *The greatest amount of the minimum total transportation cost is* $\frac{21549}{50}\$$ *i.e., the minimum total transportation cost will always be greater than* $\frac{12204}{50}\$$ *and less than* $352\$$, *and the highest chances are that the minimum total transportation cost will be* $\frac{21549}{50}\$$.

6| A Comparative Study

Author's of [3], [11], [29] have proposed a method to find the crisp optimal solution of such fuzzy transportation problems in which all the parameters are represented by triangular or trapezoidal fuzzy numbers. Then, they have used their new method proposed to find the crisp optimal solution of a real-life fuzzy transportation problem.

However, it is often better to find a fuzzy optimal solution than a crisp optimal solution. In this section we will therefore show how in the problem considered by [3], [11], [29], we can obtain a fuzzy optimal solution of the same real-life problem using the new method proposed.

Example	Minimum Total Fuzzy Transportation Cost		
	[3]	[29]	Method Proposed in This Paper
1	$(2100,2900,3500,4200)$		$(930,2930,2970,3470)$
2		$(31,80,199,460)$	$\left(\frac{81}{4},81,173,\frac{1037}{2}\right)$
3			$(3630,6609,9782)$
4	$(238.44,347.8,428.9)$		$\left(\frac{12204}{50},352,\frac{21549}{50}\right)$

7| Concluding Remarks and Future Research Directions

7.1| Concluding Remarks

These days a number of researchers have shown interest in the area of fuzzy transportation problems and various attempts have been made to study the solution of these problems. In this paper, to overcome the shortcomings of the existing methods we introduced a new formulation of transportation problem involving trapezoidal (or triangular) fuzzy numbers for the transportation costs and values of supplies and demands. We propose a fuzzy linear programming approach for solving trapezoidal (or triangular) fuzzy numbers transportation problem based on the converting into two interval transportation problems *Eq. (16)* and *Eq. (17)*. To show the advantages of the proposed methods over existing methods, some fuzzy transportation problems, may or may not be solved by the existing methods, are solved by using the proposed methods and it is shown that it is better to use the proposed methods as compared to the existing methods for solving the transportation problems. From both theoretical and algorithmic considerations, and examples solved in this paper, it can be noticed that some shortcomings of the methods for solving the fuzzy transportation problems known from the literature can be resolved by using the new methods proposed in Section 5.

7.2| Future Research Directions

Finally, we feel that, there are many other points of research and should be studied later on interval numbers or fuzzy numbers. Some of these points are below:

We will consider the following transportation *Problems (14)* with fuzzy numbers as follows:

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$Min\tilde{Z}(\tilde{x}) \approx \sum_{j=1}^{n}\sum_{i=1}^{m}\tilde{c}_{ij}\tilde{x}_{ij}$ subject to the constraints $\sum_{j=1}^{n}\tilde{x}_{ij} \approx \tilde{a}_i$ and $\sum_{i=1}^{m}\tilde{x}_{ij} \approx \tilde{b}_j$ where

$\tilde{c}_{ij} = \left(c_{ij}^1, c_{ij}^2, \ldots, c_{ij}^t\right)$, $\tilde{x}_{ij} = \left(x_{ij}^1, x_{ij}^2, \ldots, x_{ij}^t\right)$, $\tilde{a}_i = \left(a_i^1, a_i^2, \ldots, a_i^t\right)$ and $\tilde{b}_j = \left(b_j^1, b_j^2, \ldots, b_j^t\right)$ with

$t \in \mathbb{N}_{\geq 1}$. Let $\bar{x}_{ij}^{pq} = \left\lfloor x_{ij}^p, x_{ij}^q \right\rfloor$ where $p \leq q$ and $p, q \in \mathbb{N}_{\geq 1}$. The same applies to $\bar{c}_{ij}^{pq}$, $\bar{a}_i^{pq}$ and $\bar{b}_j^{pq}$.

I. Solution procedure for classical Transportation Problem $(t=1)$: $x_{ij} = \left(x_{ij}^1\right)$.

II. Solution procedure for Transportation Problem with Interval numbers $(t=2)$: $\bar{x}_{ij}^{12} = \left\lfloor x_{ij}^1, x_{ij}^2 \right\rfloor$.

III. Solution procedure for Transportation Problem with Triangular fuzzy numbers $(t=3)$:

$\tilde{x}_{ij} = \left(x_{ij}^1, x_{ij}^2, x_{ij}^3\right) = \left(x_{ij}^2, \bar{x}_{ij}^{13}\right)$.

IV. Solution procedure for Transportation Problem with Trapezoidal fuzzy numbers $(t=4)$:

$\tilde{x}_{ij} = \left(x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4\right) = \left(\bar{x}_{ij}^{23}, \bar{x}_{ij}^{14}\right)$.

V. Solution procedure for Transportation Problem with Pentagonal fuzzy numbers $(t=5)$:

$\tilde{x}_{ij} = \left(x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4, x_{ij}^5\right) = \left(x_{ij}^3, \bar{x}_{ij}^{24}, \bar{x}_{ij}^{15}\right)$

VI. Solution procedure for Transportation Problem with Hexagonal fuzzy numbers $(t=6)$:

$\tilde{x}_{ij} = \left(x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4, x_{ij}^5, x_{ij}^6\right) = \left(\bar{x}_{ij}^{34}, \bar{x}_{ij}^{25}, \bar{x}_{ij}^{16}\right)$.

VII. Solution procedure for Transportation Problem with Heptagonal fuzzy numbers $(t=7)$

: $\tilde{x}_{ij} = \left(x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4, x_{ij}^5, x_{ij}^6, x_{ij}^7\right) = \left(x_{ij}^4, \bar{x}_{ij}^{35}, \bar{x}_{ij}^{26}, \bar{x}_{ij}^{17}\right)$.

VIII. Solution procedure for Transportation Problem with Octagonal fuzzy numbers $(t=8)$

: $\tilde{x}_{ij} = \left(x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4, x_{ij}^5, x_{ij}^6, x_{ij}^7, x_{ij}^8\right) = \left(\bar{x}_{ij}^{45}, \bar{x}_{ij}^{36}, \bar{x}_{ij}^{27}, \bar{x}_{ij}^{18}\right)$.

IX. Solution procedure for Transportation Problem with Nonagonal fuzzy numbers $(t=9)$:

$\tilde{x}_{ij} = \left(x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4, x_{ij}^5, x_{ij}^6, x_{ij}^7, x_{ij}^8, x_{ij}^9\right) = \left(x_{ij}^5, \bar{x}_{ij}^{46}, \bar{x}_{ij}^{37}, \bar{x}_{ij}^{28}, \bar{x}_{ij}^{19}\right)$.

X. Solution procedure for Transportation Problem with Decagonal fuzzy numbers $(t=10)$:

$\tilde{x}_{ij} = \left(x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4, x_{ij}^5, x_{ij}^6, x_{ij}^7, x_{ij}^8, x_{ij}^9, x_{ij}^{10}\right) = \left(\bar{x}_{ij}^{56}, \bar{x}_{ij}^{47}, \bar{x}_{ij}^{38}, \bar{x}_{ij}^{29}, \bar{x}_{ij}^{110}\right)$.

XI. Solution procedure for Transportation Problem with Hendecagonal fuzzy numbers $(t=11)$:

$\tilde{x}_{ij} = \left(x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4, x_{ij}^5, x_{ij}^6, x_{ij}^7, x_{ij}^8, x_{ij}^9, x_{ij}^{10}, x_{ij}^{11}\right) = \left(x_{ij}^6, \bar{x}_{ij}^{57}, \bar{x}_{ij}^{48}, \bar{x}_{ij}^{39}, \bar{x}_{ij}^{210}, \bar{x}_{ij}^{111}\right)$.

XII. Solution procedure for Transportation Problem with Dodecagonal fuzzy numbers $(t=12)$

: $\tilde{x}_{ij} = \left(x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4, x_{ij}^5, x_{ij}^6, x_{ij}^7, x_{ij}^8, x_{ij}^9, x_{ij}^{10}, x_{ij}^{11}, x_{ij}^{12}\right) = \left(\bar{x}_{ij}^{67}, \bar{x}_{ij}^{58}, \bar{x}_{ij}^{49}, \bar{x}_{ij}^{310}, \bar{x}_{ij}^{211}, \bar{x}_{ij}^{112}\right)$.

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Conflicts of interest

The authors declare no conflicts of interest.

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