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Some Results on Certain Properties of Intuitionistic Fuzzy Sets

Jaydip Bhattacharya*

Department of Mathematics, Bir Bikram Memorial College, Agartala, West Tripura, India; jay73bhatta@gmail.com.

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Abstract

An operator is a special symbol for performing a specific function. Several operators like modal operators, topological operators, level operators, etc. have been defined over intuitionistic fuzzy sets. At the same time, so many operations were introduced and studied. The key objective of this paper is to study those operations over intuitionistic fuzzy sets and to investigate their properties. Some new results are obtained and proved.

Keywords: Fuzzy sets, Intuitionistic fuzzy sets, Modal operators, Operations.

1 | Introduction

The notion of fuzzy set was introduced and developed by Zadeh [15]. In fuzzy set concept, a membership function is defined to assign each element of the reference system, a real value in the interval [0,1]. The membership value of an element is 0 indicates that the element does not belong to the class whereas the membership value of an element is 1 indicates that the element belong to that class. Other values between 0 and 1 indicate the degree of membership to a class. The main limitation of fuzzy set theory is that the concept of nonmembership function and the hesitation margin are ignored. After eighteen years that is in 1983, Atanassov [1] rectified this concept and introduced intuitionistic fuzzy sets as an extension of fuzzy sets accommodating both membership and nonmembership function along with hesitation margin. It is to be noted that in intuitionistic fuzzy set theory, the sum of the membership function and nonmembership function is a value between 0 and 1.

Many mathematicians and researchers [4], [5], [7], [8], [9], [12] worked hard to develop and enrich this subject. The notion of modal operator was first introduced by Atanassov [2]. Modal operators ($\Box$, $\Diamond$) defined over the set of all intuitionistic fuzzy sets that transform every intuitionistic fuzzy set into a fuzzy set. Atanassov [4] also defined the operators ($\boxplus$, $\boxtimes$) on intuitionistic fuzzy sets and generalized these operators into ($\boxplus_\alpha$, $\boxtimes_\alpha$) where $\alpha \in [0,1]$. The behavior of these operators over the basic operations and algebraic laws on intuitionistic fuzzy sets were rigorously studied by many researchers [3], [9], [10], [11] [13], [14].

Corresponding Author: jay73bhatta@gmail.com
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In this paper, we concentrate deeply on the operations $*$, $\odot$, $\bowtie$, ∞, $\lhd$ and $\rhd$ which are playing vital role on intuitionistic fuzzy sets. In the main section of this paper, we try to investigate various properties of these operations using modal operators and other generalized operators. Finally we construct two tables to investigate the behavioral change of intuitionistic fuzzy sets under different operations and indicate the observations.

2 | Preliminaries

Throughout this paper, intuitionistic fuzzy set and fuzzy set are denoted by IFS and FS respectively.

Definition 1. **[12].** Let X be a nonempty set. A fuzzy set A drawn from X is defined as $A = \{<x, \mu_A(x)> : x \in X\}$, where $\mu_A : X \to [0,1]$ is the membership function of the fuzzy set A. Fuzzy set is a collection of objects with graded membership,i.e. having degrees of membership.

Definition 2. **[2].** Let X be a nonempty set. An intuitionistic fuzzy set A in X is an object having the form $A= \{<x, \mu_A(x),\nu_A(x)>:x\in X\}$, where the functions μ_A, $\nu_A : X\to[0,1]$ define respectively, the degree of membership and degree of non-membership of the element $x\in X$ to the set A, which is a subset of X, and for every element $x\in X$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Furthermore, we have $\pi_A(x)= 1- \mu_A(x) - \nu_A(x)$ called the intuitionistic fuzzy set index or hesitation margin of x in A. $\pi_A(x)$ is the degree of indeterminacy of $x\in X$ to the IFS A and $\pi_A(x) \in[0,1]$,i.e, $\pi_A(x): X \to[0,1]$ and $0 \leq \pi_A(x) \leq 1$ for every $x\in X$.

$\pi_A(x)$ expresses the lack of knowledge of whether x belongs to IFS A or not.

Definition 3. **[2].** Let A and B be two IFSs in X. The basic operations are defined as follows:

I. [inclution] $A\subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)\ \forall\ x\in X$.
II. [equality] $A= B \Leftrightarrow \mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)\ \forall\ x\in X$.
III. [complement] $A^c =\{<x, \nu_A(x), \mu_A(x) >: x\in X\}$,
IV. [union] $A\cup B =\{<x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x) , \nu_B(x))>: x\in X\}$.
V. [intersection] $A\cap B =\{<x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x) , \nu_B(x))>: x\in X\}$.
VI. [addition] $A\oplus B =\{<x, \mu_A(x) + \mu_B(x) - \mu_A(x)\, \mu_B(x), \nu_A(x)\, \nu_B(x)>: x\in X\}$.
VII. [multiplication] $A\otimes B =\{<x, \mu_A(x)\, \mu_B(x), \nu_A(x) + \nu_B(x) -\nu_A(x)\, \nu_B(x)>: x\in X\}$.
VIII. [difference] $A- B =\{<x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x) , \mu_B(x))>: x\in X\}$.
IX. [symmetric difference] $A\Delta B =\{<x, \max[\min(\mu_A(x), \nu_B(x)), \min(\mu_B(x),\nu_A(x))], \min[\max(\nu_A(x), \mu_B(x)), \max(\nu_B(x), \mu_A(x))]>: x\in X\}$.
X. [cartesian product] $A\times B =\{<x, (\mu_A(x)\, \mu_B(x)), (\nu_A(x)\, \nu_B(x))>: x\in X\}$.

Definition 4. Let A and B be two IFSs in a nonempty set X. Then

I. [7]. $A \ominus B = \{< x,½[\mu_A(x) + \mu_B(x)], ½[\nu_A(x) + \nu_B(x)] > : x\in X\}$.
II. [4]. $A \$ B = \{< x, (\mu_A(x).\mu_B(x))^{1/2}, (\nu_A(x). \nu_B(x))^{1/2} > : x\in X\}$.

Definition 5. **[2].** A is said to be a proper subset of B, i.e. $A\subset B$ if $A\subseteq B$ and $A \neq B$. It means $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ but $\mu_A(x) \neq \mu_B(x)$ and $\nu_A(x) \neq \nu_B(x)$ for $x\in X$.

Definition 6. **[4].** [modal operators] Let X be a nonempty set. If A is an IFS drawn from X, then

I. $\Box A = \{< x, \mu_A(x) , 1- \mu_A(x) > : x \in X \}$,
II. $\Diamond A = \{< x, 1- \nu_A(x) , \nu_A(x) > : x \in X\}$.

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For a proper IFS, $\Box A \subset A \subset \Diamond A$ and $\Box A \neq A \neq \Diamond A$.

Definition 7. [4]. Let X be a nonempty set. If A is an IFS drawn from X, then

$$\boxplus A = \{< x, \frac{\mu_A(x)}{2}, \frac{\nu_A(x)+1}{2} >\} \text{ and}$$

$$\boxtimes A = \{< x, \frac{\mu_A(x)+1}{2}, \frac{\nu_A(x)}{2} >\}.$$

Clearly $\boxplus A \subset A \subset \boxtimes A$.

Definition 8. [4]. Let $\alpha \in [0,1]$ and A be an IFS drawn from X, then

I. $\boxplus_\alpha A = \{ < x, \alpha.\mu_A(x), \alpha.\nu_A(x) + 1 - \alpha > : x \in X\}$,
II. $\boxtimes_\alpha A = \{ < x, \alpha.\mu_A(x) + 1 - \alpha, \alpha.\nu_A(x) > : x \in X\}$.

Definition 9. [4], [6]. Let X be a nonempty set. If A and B be two IFSs drawn from X, then

I. $A * B = \{< x, \frac{\mu_A(x) + \mu_B(x)}{2(\mu_A(x) + \mu_B(x)+1)}, \frac{\nu_A(x) + \nu_B(x)}{2(\nu_A(x) + \nu_B(x)+1)} > : x \in X \}$,
II. $A \odot B = \{< x, \frac{\mu_A(x)\ \mu_B(x)}{2(\mu_A(x)\ \mu_B(x)+1)}, \frac{\nu_A(x)\ \nu_B(x)}{2(\nu_A(x)\ \nu_B(x)+1)} > : x \in X \}$,
III. $A \bowtie B = \{< x, \frac{\mu_A(x) + \mu_B(x)}{2(\mu_A(x) + \mu_B(x))+1}, \frac{\nu_A(x) + \nu_B(x)}{2(\nu_A(x) + \nu_B(x))+1} > : x \in X \}$,
IV. $A \infty B = \{< x, \frac{\mu_A(x)\ \mu_B(x)}{2(\mu_A(x)\ \mu_B(x))+1}, \frac{\nu_A(x)\ \nu_B(x)}{2(\nu_A(x)\ \nu_B(x))+1} > : x \in X \}$,
V. $A \lhd B = \{< x, \frac{\mu_A(x) + \mu_B(x)}{\mu_A(x) + \mu_B(x)+1}, \frac{\nu_A(x) + \nu_B(x)}{\nu_A(x) + \nu_B(x)+1} > : x \in X \}$,
VI. $A \rhd B = \{< x, \frac{\mu_A(x)\ \mu_B(x)}{\mu_A(x)\ \mu_B(x)+1}, \frac{\nu_A(x)\ \nu_B(x)}{\nu_A(x)\ \nu_B(x)+1} > : x \in X \}$.

Some characteristics of the operations $*$, $\odot$, $\bowtie$, ∞, $\lhd$ and $\rhd$ for two IFSs A and B are as follows:

I. $A * B$, $A \odot B$, $A \bowtie B$, $A \infty B$, $A \lhd B$, and $A \rhd B$ are all intuitionistic fuzzy sets.
II. Clearly,

i. $\Box (A * B) \subseteq (A * B) \subseteq \Diamond (A * B)$,
ii. $\Box (A \odot B) \subseteq (A \odot B) \subseteq \Diamond (A \odot B)$,
iii. $\Box (A \bowtie B) \subseteq (A \bowtie B) \subseteq \Diamond (A \bowtie B)$,
iv. $\Box (A \infty B) \subseteq (A \infty B) \subseteq \Diamond (A \infty B)$,
v. $\Box (A \lhd B) \subseteq (A \lhd B) \subseteq \Diamond (A \lhd B)$,
vi. $\Box (A \rhd B) \subseteq (A \rhd B) \subseteq \Diamond (A \rhd B)$.

III. Furthermore,

i. $\boxplus (A * B) \subseteq (A * B) \subseteq \boxtimes (A * B)$,
ii. $\boxplus (A \odot B) \subseteq (A \odot B) \subseteq \boxtimes (A \odot B)$,
iii. $\boxplus (A \bowtie B) \subseteq (A \bowtie B) \subseteq \boxtimes (A \bowtie B)$,
iv. $\boxplus (A \infty B) \subseteq (A \infty B) \subseteq \boxtimes (A \infty B)$,
v. $\boxplus (A \lhd B) \subseteq (A \lhd B) \subseteq \boxtimes (A \lhd B)$
vi. $\boxplus (A \rhd B) \subseteq (A \rhd B) \subseteq \boxtimes (A \rhd B)$.

IV. $\boxplus (A * B) = \boxplus_{.5} (A * B)$ and $\boxtimes (A * B) = \boxtimes_{.5} (A * B)$. These equations are also true for the operations $\odot$, $\bowtie$, ∞, $\lhd$ and $\rhd$.

V. For $\alpha \in [0,1]$,

i. $⊞_\alpha (A * B) \subset ⊠_\alpha (A * B$.
ii. $⊞_\alpha (A \odot B) \subset ⊠_\alpha (A \odot B)$,
iii. $⊞_\alpha (A \bowtie B) \subset ⊠_\alpha (A \bowtie B)$,
iv. $⊞_\alpha (A \infty B) \subset ⊠_\alpha (A \infty B)$,
v. $⊞_\alpha (A \lhd B) \subset ⊠_\alpha (A \lhd B)$,
vi. $⊞_\alpha (A \rhd B) \subset ⊠_\alpha (A \rhd B)$.

VI. $⊞\alpha (A * B) \subset ⊞\beta (A * B)$ for $\alpha < \beta$ and $⊠\gamma (A * B) \subset ⊠\delta (A * B)$ for $\gamma > \delta$; $\alpha, \beta, \gamma, \delta \in [0,1]$.

3 | Main Results

Theorem 1. Let X be a nonempty set. If A and B be two IFSs drawn from X, then

I. $[\square \lozenge (A \cup B)]^C = \lozenge \square (A \cup B)^C$,
II. $[\lozenge \square (A \cup B)]^C = \square \lozenge (A \cup B)^C$,
III. $[\square \lozenge (A \cap B)]^C = \lozenge \square (A \cap B)^C$,
IV. $[\lozenge \square (A \cap B)]^C = \square \lozenge (A \cap B)^C$,
V. $[\square \lozenge (A \oplus B)]^C = \lozenge \square (A \oplus B)^C$,
VI. $[\lozenge \square (A \oplus B)]^C = \square \lozenge (A \oplus B)^C$,
VII. $[\square \lozenge (A \otimes B)]^C = \lozenge \square (A \otimes B)^C$,
VIII. $[\lozenge \square (A \otimes B)]^C = \square \lozenge (A \otimes B)^C$,
IX. $[\square \lozenge (A - B)]^C = \lozenge \square (A - B)^C$,
X. $[\lozenge \square (A - B)]^C = \square \lozenge (A - B)^C$,
XI. $[\square \lozenge (A \Delta B)]^C = \lozenge \square (A \Delta B)^C$,
XII. $[\lozenge \square (A \Delta B)]^C = \square \lozenge (A \Delta B)^C$,
XIII. $[\square \lozenge (A \times B)]^C = \lozenge \square (A \times B)^C$,
XIV. $[\lozenge \square (A \times B)]^C = \square \lozenge (A \times B)^C$,
XV. $[\square \lozenge (A ⊜ B)]^C = \lozenge \square (A ⊜ B)^C$,
XVI. $[\lozenge \square (A ⊜ B)]^C = \square \lozenge (A ⊜ B)^C$,
XVII. $[\square \lozenge (A \$ B)]^C = \lozenge \square (A \$ B)^C$,
XVIII. $[\lozenge \square (A \$ B)]^C = \square \lozenge (A \$ B)^C$.

Proof. Now $(A \cup B) = \{< \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x)) >\}$

$\lozenge (A \cup B) = \{< 1 - \min(\nu_A(x), \nu_B(x)), \min(\nu_A(x), \nu_B(x)) >\}$

$$[\square \lozenge (A \cup B)] = \{< 1 - \min(\nu_A(x), \nu_B(x)), \min(\nu_A(x), \nu_B(x)) >\}$$

$$[\square \lozenge (A \cup B)]^C = \{< \min(\nu_A(x), \nu_B(x)), 1 - \min(\nu_A(x), \nu_B(x)) >\}.$$

Again $(A \cup B)^C = \{< \min(\nu_A(x), \nu_B(x)), \max(\mu_A(x), \mu_B(x)) >\}$

$\square (A \cup B)^C = \{< \min(\nu_A(x), \nu_B(x)), 1 - \min(\nu_A(x), \nu_B(x)) >\}$

$$\lozenge \square (A \cup B)^C = \{< \min(\nu_A(x), \nu_B(x)), 1 - \min(\nu_A(x), \nu_B(x)) >\}.$$

Hence $[\square \lozenge (A \cup B)]^C = \lozenge \square (A \cup B)^C$

Similarly (II) to (XVIII) can be proved.

Theorem 2. Let X be a nonempty set. If A and B be two IFSs drawn from X, then

I. $[(\Box A) \cup (\Diamond B)]^{C} = (\Diamond A^{C}) \cap (\Box B^{C})$,
II. $[(\Diamond A) \cup (\Box B)]^{C} = (\Box A^{C}) \cap (\Diamond B^{C})$,
III. $[(\Box A) \cap (\Diamond B)]^{C} = (\Diamond A^{C}) \cup (\Box B^{C})$,
IV. $[(\Diamond A) \cap (\Box B)]^{C} = (\Box A^{C}) \cup (\Diamond B^{C})$,
V. $[(\Box A) \oplus (\Diamond B)]^{C} = (\Diamond A^{C}) \otimes (\Box B^{C})$,
VI. $[(\Diamond A) \oplus (\Box B)]^{C} = (\Box A^{C}) \otimes (\Diamond B^{C})$,
VII. $[(\Box A) \otimes (\Diamond B)]^{C} = (\Diamond A^{C}) \oplus (\Box B^{C})$,
VIII. $[(\Diamond A) \otimes (\Box B)]^{C} = (\Box A^{C}) \oplus (\Diamond B^{C})$.

Proof. Now $[(\Box A) \cup (\Diamond B)] = \{< \max(\mu_A(x), 1- \nu_B(x)), \ \min(1- \mu_A(x), \nu_B(x)) >\}$

$[(\Box A) \cup (\Diamond B)]^{C} = \{< \min(1- \mu_A(x), \nu_B(x)), \max(\mu_A(x), 1- \nu_B(x)) >\}$.

Again $(\Diamond A^{C}) \cap (\Box B^{C}) = \ < 1- \mu_A(x), \mu_A(x) > \ \cap \ < \nu_B(x), 1- \nu_B(x) >\}$

$= \{< \min(1- \mu_A(x), \nu_B(x)), \max(\mu_A(x), 1- \nu_B(x)) >\}$.

Thus $[(\Box A) \cup (\Diamond B)]^{C} = (\Diamond A^{C}) \cap (\Box B^{C})$

Hence the proof.

Similarly (II) to (VIII) can be proved.

Theorem 3. Let X be a nonempty set. If A and B be any two IFSs drawn from X, then

I. $[\Box \Diamond(A * B)] C = \Diamond \Box (A * B)C$,
II. $[\Diamond \Box (A * B)] C = \Box \Diamond (A * B)C$,
III. $[\Box \Diamond(A \odot B)] C = \Diamond \Box (A \odot B)C$,
IV. $[\Diamond \Box (A \odot B)] C = \Box \Diamond (A \odot B)C$,
V. $[\Box \Diamond(A \bowtie B)] C = \Diamond \Box (A \bowtie B)C$,
VI. $[\Diamond \Box (A \bowtie B)] C = \Box \Diamond (A \bowtie B)C$,
VII. $[\Box \Diamond(A \infty B)] C = \Diamond \Box (A \infty B)C$,
VIII. $[\Diamond \Box (A \infty B)] C = \Box \Diamond (A \infty B)C$,
IX. $[\Box \Diamond(A \lhd B)] C = \Diamond \Box (A \lhd B)C$,
X. $[\Diamond \Box (A \lhd B)] C = \Box \Diamond (A \lhd B)C$,
XI. $[\Box \Diamond(A \rhd B)] C = \Diamond \Box (A \rhd B)C$,
XII. $[\Diamond \Box (A \rhd B)] C = \Box \Diamond (A \rhd B)C$,
XIII. $[\Box \Diamond(A \ominus B)] C = \Diamond \Box (A \ominus B)C$,
XIV. $[\Diamond \Box (A \ominus B)] C = \Box \Diamond (A \ominus B)C$,
XV. $[\Box \Diamond(A \$ B)] C = \Diamond \Box (A \$ B)C$,
XVI. $[\Diamond \Box (A \$ B)] C = \Box \Diamond (A \$ B) C$.

Proof. Now $\Diamond(A * B) = \{< \ 1 - \frac{\nu_A(x) + \nu_B(x)}{2(\nu_A(x)+ \nu_B(x)+1)}, \frac{\nu_A(x) + \nu_B(x)}{2(\nu_A(x)+ \nu_B(x)+1)} >\}$

$\Box \Diamond(A * B) = \{< \ 1 - \frac{\nu_A(x) + \nu_B(x)}{2(\nu_A(x)+ \nu_B(x)+1)}, \frac{\nu_A(x) + \nu_B(x)}{2(\nu_A(x)+ \nu_B(x)+1)} >\}$

$[\Box \Diamond(A * B)]^{C} = \{< \frac{\nu_A(x) + \nu_B(x)}{2(\nu_A(x)+ \nu_B(x)+1)}, 1 - \frac{\nu_A(x) + \nu_B(x)}{2(\nu_A(x)+ \nu_B(x)+1)} >\}$.

Again $(A * B)^C$ $\{< \frac{\nu_A(x) + \nu_B(x)}{2(\nu_A(x)+ \nu_B(x)+1)}, \frac{\mu_A(x) + \mu_B(x)}{2(\mu_A(x)+ \mu_B(x)+1)} >\}$

$$\square (A * B)^C = \{< \frac{\nu_A(x) + \nu_B(x)}{2(\nu_A(x)+ \nu_B(x)+1)}, 1 - \frac{\nu_A(x) + \nu_B(x)}{2(\nu_A(x)+ \nu_B(x)+1)} >\}$$

$$\lozenge \square (A * B)^C = \{< \frac{\nu_A(x) + \nu_B(x)}{2(\nu_A(x)+ \nu_B(x)+1)}, 1 - \frac{\nu_A(x) + \nu_B(x)}{2(\nu_A(x)+ \nu_B(x)+1)} >\}.$$

Hence $[\square \lozenge(A * B)]^C = \lozenge \square (A * B)^C$

Similarly (II) to (XVI) can be proved.

Theorem 4. Let X be a nonempty set. If A and B be any two IFSs drawn from X, then

I. $[(\square A) * (\lozenge B)]^C = (\lozenge A^C) * (\square B^C)$,
II. $[(\lozenge A) * (\square B)]^C = (\square A^C) * (\lozenge B^C)$,
III. $[(\square A) \odot (\lozenge B)]^C = (\lozenge A^C) \odot (\square B^C)$,
IV. $[(\lozenge A) \odot (\square B)]^C = (\square A^C) \odot (\lozenge B^C)$,
V. $[(\square A) \bowtie (\lozenge B)]^C = (\lozenge A^C) \bowtie (\square B^C)$,
VI. $[(\lozenge A) \bowtie (\square B)]^C = (\square A^C) \bowtie (\lozenge B^C)$,
VII. $[(\square A) \infty (\lozenge B)]^C = (\lozenge A^C) \infty (\square B^C)$,
VIII. $[(\lozenge A) \infty (\square B)]^C = (\square A^C) \infty (\lozenge B^C)$,
IX. $[(\square A) \lhd (\lozenge B)]^C = (\lozenge A^C) \lhd (\square B^C)$,
X. $[(\lozenge A) \lhd (\square B)]^C = (\square A^C) \lhd (\lozenge B^C)$,
XI. $[(\square A) \rhd (\lozenge B)]^C = (\lozenge A^C) \rhd (\square B^C)$,
XII. $[(\lozenge A) \rhd (\square B)]^C = (\square A^C) \rhd (\lozenge B^C)$,
XIII. (m $[(\square A) \ominus (\lozenge B)]^C = (\lozenge A^C) \ominus (\square B^C)$,
XIV. $[(\lozenge A) \ominus (\square B)]^C = (\square A^C) \ominus (\lozenge B^C)$,
XV. $[(\square A) \$ (\lozenge B)]^C = (\lozenge A^C) \$ (\square B^C)$,
XVI. $[(\lozenge A) \$ (\square B)]^C = (\square A^C) \$ (\lozenge B^C)$.

Proof. These results can also be proved by similar manner as done in *Theorem 3*.

Theorem 5. Let X be a nonempty set. If A and B be any two IFSs drawn from X, then

I. $\square \square (A * B) = \square [(\square A) * (\square B)]$,
II. $\lozenge \lozenge (A * B) = \lozenge [(\lozenge A) * (\lozenge B)]$,
III. $\square \square (A \odot B) = \square [(\square A) \odot (\square B)]$,
IV. $\lozenge \lozenge (A \odot B) = \lozenge [(\lozenge A) \odot (\lozenge B)]$,
V. $\square \square (A \bowtie B) = \square [(\square A) \bowtie (\square B)]$,
VI. $\lozenge \lozenge (A \bowtie B) = \lozenge [(\lozenge A) \bowtie (\lozenge B)]$,
VII. $\square \square (A \infty B) = \square [(\square A) \infty (\square B)]$,
VIII. $\lozenge \lozenge (A \infty B) = \lozenge [(\lozenge A) \infty (\lozenge B)]$,
IX. $\square \square (A \lhd B) = \square [(\square A) \lhd (\square B)]$,
X. $\lozenge \lozenge (A \lhd B) = \lozenge [(\lozenge A) \lhd (\lozenge B)]$,
XI. $\square \square (A \rhd B) = \square [(\square A) \rhd (\square B)]$,
XII. $\lozenge \lozenge (A \rhd B) = \lozenge [(\lozenge A) \rhd (\lozenge B)]$,
XIII. $\square \square (A \ominus B) = \square [(\square A) \ominus (\square B)]$,
XIV. $\lozenge \lozenge (A \ominus B) = \lozenge [(\lozenge A) \ominus (\lozenge B)]$,
XV. $\square \square (A \$ B) = \square [(\square A) \$ (\square B)]$,
XVI. $\lozenge \lozenge (A \$ B) = \lozenge [(\lozenge A) \$ (\lozenge B)]$.

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Proof. Here $\Box\,(A * B) = \{< \frac{\mu_A(x)+\mu_B(x)}{2(\mu_A(x)+\mu_B(x)+1)},\ 1-\frac{\mu_A(x)+\mu_B(x)}{2(\mu_A(x)+\mu_B(x)+1)} >\}$

or $\Box\,\Box\,(A * B) = \{< \frac{\mu_A(x)+\mu_B(x)}{2(\mu_A(x)+\mu_B(x)+1)},\ 1-\frac{\mu_A(x)+\mu_B(x)}{2(\mu_A(x)+\mu_B(x)+1)} >\}$.

Again $(\Box A * \Box B) = \{< \frac{\mu_A(x)+\mu_B(x)}{2(\mu_A(x)+\mu_B(x)+1)},\ \frac{1-\mu_A(x)+1-\mu_B(x)}{2(\mu_A(x)+\mu_B(x)+1)} >\}$

or $\Box\,(\Box A * \Box B) = \{< \frac{\mu_A(x)+\mu_B(x)}{2(\mu_A(x)+\mu_B(x)+1)},\ 1-\frac{\mu_A(x)+\mu_B(x)}{2(\mu_A(x)+\mu_B(x)+1)} >\}$.

Hence $\Box\,\Box\,(A * B) = \Box\,(\Box A * \Box B)$.

Similarly (II) to (XVI) can be proved.

Theorem 6. Let X be a nonempty set. If A and B be any two IFSs drawn from X, then

I. $[\boxplus\boxtimes\,(A * B)]^C = \boxtimes\,\boxplus\,(A * B)^C$,
II. $[\boxtimes\boxplus\,(A * B)]^C = \boxplus\boxtimes\,(A * B)^C$,
III. $[\boxplus\boxtimes\,(A \odot B)]^C = \boxtimes\,\boxplus\,(A \odot B)^C$,
IV. $[\boxtimes\boxplus\,(A \odot B)]^C = \boxplus\boxtimes\,(A \odot B)^C$,
V. $[\boxplus\boxtimes\,(A \bowtie B)]^C = \boxtimes\,\boxplus\,(A \bowtie B)^C$,
VI. $[\boxtimes\boxplus\,(A \bowtie B)]^C = \boxplus\boxtimes\,(A \bowtie B)^C$,
VII. $[\boxplus\boxtimes\,(A \infty B)]^C = \boxtimes\,\boxplus\,(A \infty B)^C$,
VIII. $[\boxtimes\boxplus\,(A \infty B)]^C = \boxplus\boxtimes\,(A \infty B)^C$,
IX. $[\boxplus\boxtimes\,(A \lhd B)]^C = \boxtimes\,\boxplus\,(A \lhd B)^C$,
X. $[\boxtimes\boxplus\,(A \lhd B)]^C = \boxplus\boxtimes\,(A \lhd B)^C$,
XI. $[\boxplus\boxtimes\,(A \rhd B)]^C = \boxtimes\,\boxplus\,(A \rhd B)^C$,
XII. $[\boxtimes\boxplus\,(A \rhd B)]^C = \boxplus\boxtimes\,(A \rhd B)^C$,
XIII. $[\boxplus\boxtimes\,(A \ominus B)]^C = \boxtimes\,\boxplus\,(A \ominus B)^C$,
XIV. $[\boxtimes\boxplus\,(A \ominus B)]^C = \boxplus\boxtimes\,(A \ominus B)^C$,
XV. $[\boxplus\boxtimes\,(A \$ B)]^C = \boxtimes\,\boxplus\,(A \$ B)^C$,
XVI. $[\boxtimes\boxplus\,(A \$ B)]^C = \boxplus\boxtimes\,(A \$ B)^C$.

Proof. Here $\boxtimes\,(A * B) = \boxtimes\{< \frac{\mu_A(x)+\mu_B(x)}{2(\mu_A(x)+\mu_B(x)+1)},\ \frac{\nu_A(x)+\nu_B(x)}{2(\nu_A(x)+\nu_B(x)+1)} >\}$

$$= \{< \frac{1}{2}\left(\frac{\mu_A(x)+\mu_B(x)}{2(\mu_A(x)+\mu_B(x)+1)}+1\right),\ \frac{\nu_A(x)+\nu_B(x)}{4(\nu_A(x)+\nu_B(x)+1)} >\}$$

or $\boxplus\boxtimes\,(A * B) = \{< \frac{1}{4}\left(\frac{\mu_A(x)+\mu_B(x)}{2(\mu_A(x)+\mu_B(x)+1)}+1\right),\ \frac{1}{2}\left(\frac{\nu_A(x)+\nu_B(x)}{4(\nu_A(x)+\nu_B(x)+1)}+1\right) >\}$

or $[\boxplus\boxtimes\,(A * B)]^C = \{< \frac{1}{2}\left(\frac{\nu_A(x)+\nu_B(x)}{4(\nu_A(x)+\nu_B(x)+1)}+1\right),\ \frac{1}{4}\left(\frac{\mu_A(x)+\mu_B(x)}{2(\mu_A(x)+\mu_B(x)+1)}+1\right) >\}$.

Again $\boxplus\,(A * B)^C = \boxplus\,\{< \frac{\nu_A(x)+\nu_B(x)}{2(\nu_A(x)+\nu_B(x)+1)},\ \frac{\mu_A(x)+\mu_B(x)}{2(\mu_A(x)+\mu_B(x)+1)} >\}$

$$= \{< \frac{\nu_A(x)+\nu_B(x)}{4(\nu_A(x)+\nu_B(x)+1)},\ \frac{1}{2}\left(\frac{\mu_A(x)+\mu_B(x)}{2(\mu_A(x)+\mu_B(x)+1)}+1\right) >\}$$

or $\boxtimes\,\boxplus\,(A * B)^C = \{< \frac{1}{2}\left(\frac{\nu_A(x)+\nu_B(x)}{4(\nu_A(x)+\nu_B(x)+1)}+1\right),\ \frac{1}{4}\left(\frac{\mu_A(x)+\mu_B(x)}{2(\mu_A(x)+\mu_B(x)+1)}+1\right) >\}$.

Thus [⊞⊠ (A ∗ B)] ^{C} = ⊠ ⊞ (A ∗ B) ^{C}

Hence the proof.

Similarly (II) to (XVI) can be proved.

Theorem 7. Let X be a nonempty set. If A and B be two IFSs drawn from X, then

I. [⊞⊞ (A ∗ B)] C = ⊠ [⊞ (A ∗ B)] C,
II. [⊠⊠ (A ∗ B)] C = ⊞ [⊠ (A ∗ B)] C,
III. [⊞⊞ (A ⊙ B)] C = ⊠ [⊞ (A ⊙ B)] C,
IV. [⊠⊠ (A ⊙ B)] C = ⊞ [⊠ (A ⊙ B)] C,
V. [⊞⊞ (A ⋈ B)] C = ⊠ [⊞ (A ⋈ B)] C,
VI. [⊠⊠ (A ⋈ B)] C = ⊞ [⊠ (A ⋈ B)] C,
VII. [⊞⊞ (A ∞ B)] C = ⊠ [⊞ (A ∞ B)] C,
VIII. [⊠⊠ (A ∞ B)] C = ⊞ [⊠ (A ∞ B)] C,
IX. [⊞⊞ (A ⊲ B)] C = ⊠ [⊞ (A ⊲ B)] C,
X. [⊠⊠ (A ⊲ B)] C = ⊞ [⊠ (A ⊲ B)] C,
XI. [⊞⊞ (A ⊳ B)] C = ⊠ [⊞ (A ⊳ B)] C,
XII. [⊠⊠ (A ⊳ B)] C = ⊞ [⊠ (A ⊳ B)] C,
XIII. [⊞⊞ (A ⊜ B)] C = ⊠ [⊞ (A ⊜ B)] C,
XIV. [⊠⊠ (A ⊜ B)] C = ⊞ [⊠ (A ⊜ B)] C,
XV. [⊞⊞ (A $ B)] C = ⊠ [⊞ (A $ B)] C,
XVI. [⊠⊠ (A $ B)] C = ⊞ [⊠ (A $ B)] C.

Proof. Obvious.

Theorem 8. Let X be a nonempty set. If A and B be two IFSs drawn from X for α ∈ [0,1], then

I. [$\boxplus_\alpha \boxtimes_\alpha$ (A ∗ B)] ^{C} = $\boxtimes_\alpha \boxplus_\alpha$ (A ∗ B)^C,
II. [$\boxtimes_\alpha \boxplus_\alpha$ (A ∗ B)] ^{C} = $\boxplus_\alpha \boxtimes_\alpha$ (A ∗ B)^C,
III. [$\boxplus_\alpha \boxtimes_\alpha$ (A ⊙ B)] ^{C} = $\boxtimes_\alpha \boxplus_\alpha$ (A ⊙ B)^C,
IV. [$\boxtimes_\alpha \boxplus_\alpha$ (A ⊙ B)] ^{C} = $\boxplus_\alpha \boxtimes_\alpha$ (A ⊙ B)^C,
V. [$\boxplus_\alpha \boxtimes_\alpha$ (A ⋈ B)] ^{C} = $\boxtimes_\alpha \boxplus_\alpha$ (A ⋈ B)^C,
VI. [$\boxtimes_\alpha \boxplus_\alpha$ (A ⋈ B)] ^{C} = $\boxplus_\alpha \boxtimes_\alpha$ (A ⋈ B)^C,
VII. [$\boxplus_\alpha \boxtimes_\alpha$ (A ∞ B)] ^{C} = $\boxtimes_\alpha \boxplus_\alpha$ (A ∞ B)^C,
VIII. [$\boxtimes_\alpha \boxplus_\alpha$ (A ∞ B)] ^{C} = $\boxplus_\alpha \boxtimes_\alpha$ (A ∞ B)^C,
IX. [$\boxplus_\alpha \boxtimes_\alpha$ (A ⊲ B)] ^{C} = $\boxtimes_\alpha \boxplus_\alpha$ (A ⊲ B)^C,
X. [$\boxtimes_\alpha \boxplus_\alpha$ (A ⊲ B)] ^{C} = $\boxplus_\alpha \boxtimes_\alpha$ (A ⊲ B)^C,
XI. [$\boxplus_\alpha \boxtimes_\alpha$ (A ⊳ B)] ^{C} = $\boxtimes_\alpha \boxplus_\alpha$ (A ⊳ B)^C,
XII. [$\boxtimes_\alpha \boxplus_\alpha$ (A ⊳ B)] ^{C} = $\boxplus_\alpha \boxtimes_\alpha$ (A ⊳ B)^C,
XIII. [$\boxplus_\alpha \boxtimes_\alpha$ (A ⊜ B)] ^{C} = $\boxtimes_\alpha \boxplus_\alpha$ (A ⊜ B)^C,
XIV. [$\boxtimes_\alpha \boxplus_\alpha$ (A ⊜ B)] ^{C} = $\boxplus_\alpha \boxtimes_\alpha$ (A ⊜ B)^C,
XV. [$\boxplus_\alpha \boxtimes_\alpha$ (A $ B)] ^{C} = $\boxtimes_\alpha \boxplus_\alpha$ (A $ B)^C,
XVI. [$\boxtimes_\alpha \boxplus_\alpha$ (A $ B)] ^{C} = $\boxplus_\alpha \boxtimes_\alpha$ (A $ B)^C.

Proof. Now $\boxtimes_\alpha (A * B) = \{< \alpha\frac{\mu_A(x)+\mu_B(x)}{2(\mu_A(x)+\mu_B(x)+1)} + 1 - \alpha\,,\ \alpha\frac{\nu_A(x)+\nu_B(x)}{2(\nu_A(x)+\nu_B(x)+1)} >\}$

or $\boxplus_\alpha \boxtimes_\alpha (A * B) = \{< \alpha(\alpha\frac{\mu_A(x)+\mu_B(x)}{2(\mu_A(x)+\mu_B(x)+1)} + 1 - \alpha)\,,\ \alpha.\alpha\frac{\nu_A(x)+\nu_B(x)}{2(\nu_A(x)+\nu_B(x)+1)} + 1 - \alpha >\}$

or $[\boxplus_\alpha \boxtimes_\alpha (A * B)]^C = \{< \alpha.\alpha\frac{\nu_A(x)+\nu_B(x)}{2(\nu_A(x)+\nu_B(x)+1)} + 1-\alpha,\ \alpha(\alpha\frac{\mu_A(x)+\mu_B(x)}{2(\mu_A(x)+\mu_B(x)+1)}+1-\alpha) >\}$

Again $\boxplus_\alpha (A * B)^C = \{< \alpha\frac{\nu_A(x)+\nu_B(x)}{2(\nu_A(x)+\nu_B(x)+1)},\ \alpha\frac{\mu_A(x)+\mu_B(x)}{2(\mu_A(x)+\mu_B(x)+1)} + 1-\alpha >\}$

or $\boxtimes_\alpha \boxplus_\alpha (A * B)^C = \{< \alpha.\alpha\frac{\nu_A(x)+\nu_B(x)}{2(\nu_A(x)+\nu_B(x)+1)} + 1-\alpha,\ \alpha(\alpha\frac{\mu_A(x)+\mu_B(x)}{2(\mu_A(x)+\mu_B(x)+1)}+1-\alpha) >\}$.

Thus $[\boxplus_\alpha \boxtimes_\alpha (A * B)]^C = \boxtimes_\alpha \boxplus_\alpha (A * B)^C$

Hence the proof.

Similarly (II) to (XVI) can be proved.

Theorem 9. Let X be a nonempty set. If A and B be two IFSs drawn from X for $\alpha \in [0,1]$, then

I. $[\boxplus_\alpha\boxplus_\alpha (A * B)]^C = \boxtimes_\alpha [\boxplus_\alpha (A * B)]^C$,
II. $[\boxtimes_\alpha\boxtimes_\alpha (A * B)]^C = \boxplus_\alpha [\boxtimes_\alpha (A * B)]^C$,
III. $[\boxplus_\alpha \boxplus_\alpha (A \odot B)]^C = \boxtimes_\alpha[\boxplus_\alpha (A \odot B)]^C$,
IV. $[\boxtimes_\alpha\boxtimes_\alpha (A \odot B)]^C = \boxplus_\alpha [\boxtimes_\alpha (A \odot B)]^C$,
V. $[\boxplus_\alpha \boxplus_\alpha (A \bowtie B)]^C = \boxtimes_\alpha[\boxplus_\alpha (A \bowtie B)]^C$,
VI. $[\boxtimes_\alpha\boxtimes_\alpha (A \bowtie B)]^C = \boxplus_\alpha [\boxtimes_\alpha (A \bowtie B)]^C$,
VII. $[\boxplus_\alpha \boxplus_\alpha (A \infty B)]^C = \boxtimes_\alpha[\boxplus_\alpha (A \infty B)]^C$,
VIII. $[\boxtimes_\alpha\boxtimes_\alpha (A \infty B)]^C = \boxplus_\alpha [\boxtimes_\alpha (A \infty B)]^C$,
IX. $[\boxplus_\alpha \boxplus_\alpha (A \lhd B)]^C = \boxtimes_\alpha[\boxplus_\alpha (A \lhd B)]^C$,
X. $[\boxtimes_\alpha\boxtimes_\alpha (A \lhd B)]^C = \boxplus_\alpha [\boxtimes_\alpha (A \lhd B)]^C$,
XI. $[\boxplus_\alpha \boxplus_\alpha (A \rhd B)]^C = \boxtimes_\alpha[\boxplus_\alpha (A \rhd B)]^C$,
XII. $[\boxtimes_\alpha\boxtimes_\alpha (A \rhd B)]^C = \boxplus_\alpha [\boxtimes_\alpha (A \rhd B)]^C$,
XIII. $[\boxplus_\alpha \boxplus_\alpha (A ⊜ B)]^C = \boxtimes_\alpha[\boxplus_\alpha (A ⊜ B)]^C$,
XIV. $[\boxtimes_\alpha\boxtimes_\alpha (A ⊜ B)]^C = \boxplus_\alpha [\boxtimes_\alpha (A ⊜ B)]^C$,
XV. $[\boxplus_\alpha \boxplus_\alpha (A \$ B)]^C = \boxtimes_\alpha[\boxplus_\alpha (A \$ B)]^C$,
XVI. $[\boxtimes_\alpha\boxtimes_\alpha (A \$ B)]^C = \boxplus_\alpha [\boxtimes_\alpha (A \$ B)]^C$.

Proof. $[\boxplus_\alpha\boxplus_\alpha (A * B)] = \boxplus_\alpha\{< \alpha(\frac{\mu_A(x)+\mu_B(x)}{2(\mu_A(x)+\mu_B(x)+1)}),\ \alpha\frac{\nu_A(x)+\nu_B(x)}{2(\nu_A(x)+\nu_B(x)+1)}+1-\alpha >\}$

$$= \{< \alpha(\alpha\frac{\mu_A(x)+\mu_B(x)}{2(\mu_A(x)+\mu_B(x)+1)}),\ \alpha\left(\alpha\frac{\nu_A(x)+\nu_B(x)}{2(\nu_A(x)+\nu_B(x)+1)}+1-\alpha\right)+1-\alpha >\}$$

or $[\boxplus_\alpha\boxplus_\alpha (A * B)]^C = \{< \alpha\left(\alpha\frac{\nu_A(x)+\nu_B(x)}{2(\nu_A(x)+\nu_B(x)+1)}+1-\alpha\right)+1-\alpha,\ \alpha(\alpha\frac{\mu_A(x)+\mu_B(x)}{2(\mu_A(x)+\mu_B(x)+1)}) >\}$.

Again $[\boxplus_\alpha (A * B)]^C = \{< \alpha\frac{\nu_A(x)+\nu_B(x)}{2(\nu_A(x)+\nu_B(x)+1)}+1-\alpha,\ \alpha(\frac{\mu_A(x)+\mu_B(x)}{2(\mu_A(x)+\mu_B(x)+1)}) >\}$

or $\boxtimes_\alpha [\boxplus_\alpha (A * B)]^C = \{< \alpha\left(\alpha\frac{\nu_A(x)+\nu_B(x)}{2(\nu_A(x)+\nu_B(x)+1)}+1-\alpha\right)+1-\alpha,\ \alpha(\alpha\frac{\mu_A(x)+\mu_B(x)}{2(\mu_A(x)+\mu_B(x)+1)}) >\}$.

Thus $[\boxplus_\alpha\boxplus_\alpha (A * B)]^C = \boxtimes_\alpha [\boxplus_\alpha (A * B)]^C$.

Hence the proof.

Similarly (II) to (XVI) can be proved.

Example 1. Let A= <.7, .2 > and B= <.6, .1 > be two intuitionistic fuzzy sets and # be any operation defined in *Definitions 9* and *4*.

Operator	A # B	$\boxplus$(A#B)	$\boxplus_{.2}$ (A#B)	$\boxplus_{.5}$ (A#B)	$\boxplus_{.8}$ (A#B)
$*$	<.2826, .1154>	<.1413, .5577>	<.0565, .8231>	<.1413, .5577>	<.2261, .2923>
$\odot$	<.1479, .0098>	<.0740, .5049>	<.0296, .8020>	<.0740, .5049>	<.1183, .2078>
$\bowtie$	<.3611, .1875>	<.1806, .5938>	<.0722, .8375>	<.1806, .5938>	<.2889, .35>
∞	<.2283, .0192>	<.1142, .5096>	<.0457, .8038>	<.1142,.5096>	<.1826, .2154>
$\lhd$	<.5652, .2308>	<.2826, .6154>	<.1130, .8462>	<.2826, .6154>	<.4522, .3846>
$\rhd$	<.2958, .0196>	<.1479, .5098>	<.0592, .8039>	<.1479, .5098>	<.2366, .5157>
$\ominus$	<.65, .15>	<.325, .575>	<.13, .83>	<.325, .575>	<.52, .32>
$	<.6481, .1414>	<.3241, .5707>	<.1296, .8283>	<.3241, .5707>	<.5185, .3131>

Operator	A # B	$\boxtimes$(A#B)	$\boxtimes_{.2}$ (A#B)	$\boxtimes_{.5}$ (A#B)	$\boxtimes_{.8}$ (A#B)
$*$	<.2826, .1154>	<.6413, .0577>	<.8565, .0231>	<.6413, .0577>	<.4261, .0923>
$\odot$	<.1479, .0098>	<.5740, .0049>	<.8296, .0020>	<.5740, .0049>	<.3183, .0078>
$\bowtie$	<.3611, .1875>	<.6806, .0938>	<.8722, .0375>	<.6806, .0938>	<.4889, .15>
∞	<.2283, .0192>	<.6142, .0096>	<.8457, .0038>	<.6142,.0096>	<.3826, .0154>
$\lhd$	<.5652, .2308>	<.7826, .1154>	<.9130, .0462>	<.7826, .1154>	<.6522, .1846>
$\rhd$	<.2958, .0196>	<.6479, .0098>	<.8592, .0039>	<.6479, .0098>	<.4366, .0157>
$\ominus$	<.65, .15>	<.825, .075>	<.93, .03>	<.825, .075>	<.72, .12>
$	<.6481, .1414>	<.8241, .0707>	<.9296, .0283>	<.8241, .0707>	<.7185, .1131>

Taking two IFSs A= <.7, .2 > and B= <.6, .1 > we have constructed two above tables and observe the various properties of IFSs on the basis of different operations. The observations are as follows:

I. $\boxtimes_{.2}(A\#B) \not\subset \boxtimes_{.5}(A\#B) \not\subset \boxtimes_{.8}(A\#B)$ while $\boxplus_{.2}(A\#B) \subset \boxplus_{.5}(A\#B) \subset \boxplus_{.8}(A\#B)$.

II. $\boxplus_{.8}(A\#B) \not\subset \boxplus_{.5}(A\#B) \not\subset \boxplus_{.2}(A\#B)$ while $\boxtimes_{.8}(A\#B) \subset \boxtimes_{.5}(A\#B) \subset \boxtimes_{.2}(A\#B)$.

III. $\boxplus_{.2}(A\#B) \subset \boxplus_{.5}(A\#B) \subset \boxplus_{.8}(A\#B) \subset \boxtimes_{.8}(A\#B) \subset \boxtimes_{.5}(A\#B) \subset \boxtimes_{.2}(A\#B)$.

4| Conclusion

Some new properties are established in intuitionistic fuzzy sets with the help of certain operations together with the modal operators. These will certainly give a new dimension for developing the literature. As the new results are interesting and meaningful, these may be used in the practical field also.

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Conflicts of Interest

The author declares no conflict of interest.

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