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Decision-Making Analysis of Minimizing the Death Rate Due to COVID-19 by Using q-Rung Orthopair Fuzzy Soft Bonferroni Mean Operator

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Abstract

The q-Rung Orthopair Fuzzy Soft Set (q-ROFSS) theory is a significant extension of Pythagorean fuzzy soft set and intuitionistic fuzzy soft set theories for dealing with the imprecision and uncertainty in data. The purpose of this study is to improve and apply this theory in decision-making. To achieve this purpose, we firstly propose some Bonferroni Mean (BM) and Weighted Bonferroni Mean (WBM) aggregation operators for aggregating the data. Some desired properties are presented in detail and the existing aggregation operators are used as distinct cases of our proposed operators. Further, a decision-making analysis is presented based on our proposed operations and applied to decision-making in COVID-19 diagnosis. The preferred way is discussed to protect maximum human lives from COVID-19. A numerical example is given to support the claim. The experimental results demonstrate the proposed operators have an ability to make a precise decision with imprecision and uncertain information which will find a broad application in the decision-making area.

Keywords: Bonferroni operator, Aggregation, q-Rung orthopair fuzzy set, Decision-making analysis, COVID-19.

1 | Introduction

A study of rigorous scientific tools that help in making certain decisions in real-life problems has attracted the attention of artificial intelligent designers. Decision-making analysis is a significant need for a personal or collective. The analysis becomes increasingly complex with the increasing uncertainty and vagueness in input data coming either from individuals or from engineering or health industry. In such problems, the basic and the foremost problems are to design a rational mechanism to prefer one object over the other. In some sensitive real-world problems such as pandemic like the Spanish flu or COVID-19, the science of decision-making becomes even more relevant as it has a direct impact on human lives. An essential data required to decide the preference of one option over the other to save a maximum number of lives is itself ambiguous, imprecise and uncertain.

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For several decades, researchers have been working out different techniques and methods to deal with data uncertainties. The first notable step in this direction can be traced back form 1965 when Zadeh [1] initiated the concept of a fuzzy set and applied this concept successfully to overcome the flaws in a mathematical model of separating useable units from downstate units. After this successful idea, Atanassov [2] introduced the intuitionistic fuzzy set theory by extending the idea of Zadeh. For a systematic treatment of this subject, we refer the readers to [3] and [4]. Certain limitations are due to the definition of a membership function for each specific object, the lacking of appropriate parameterization under consideration which varies from situation to situation. The corresponding analysis based on the intuitionistic fuzzy set does not help the decision-makers to reach an appropriate decision. To overcome these difficulties, Molodtsov [5] presented the idea of a soft set to deal with the data where uncertainty is due to the inadequacy of involved parameters. The notions of Fuzzy Soft Sets (FSSs) and Intuitionistic Fuzzy Soft Sets (IFSSs) in [6] and [7] can be viewed as an extension of soft sets. A study of certain operations for intuitionistic FSSs was carried out in [8]. These investigations are being applied successfully in decision-making analysis especially in formulating the aggregation operators. In 2006, Xu and Yager [9] defined some important aggregation operators for aggregating the data in the decision-making process. Later on, several approaches have been used by researchers for aggregating the data given in different forms for dealing with uncertainty [10]-[20].

Aggregation operators have been extensively used in decision-making processes with the main focus on the attributes of the objects, not on the interrelationship of data items. There are several real-life problems where the interrelationship between the different objects becomes vital, for instance when a decision maker takes a decision based on thoughts of life risk and cost in a certain assignment, the importance is given to the risk than the cost. Yager [21] presented the idea of BM operator and its generalizations. BM operators have the ability to capture the features of the interrelationship between different objects. Afterward, Bonferroni mean operators have been extended by employing different approaches [22]-[27].

The concepts of intuitionistic fuzzy and intuitionistic FSSs theory do not constitute a suitable framework to deal with several problems in decision-making process. For instance, when the sum of the membership and non-memberships degree exceeds than 1, the existing theories mentioned afore are not applicable. Therefore, to overcome such problems Yager [28] introduced the notion of Pythagorean Fuzzy Set (PFS) and then Peng et al. [29] defined a Pythagorean Fuzzy Soft Set (PFSS). Yager [30] presented the notion of q-Rung Orthopair Fuzzy Sets (q-ROFSs). According to his idea, the sum of q-th power of both membership and non-membership is less than or equal to 1. So, the q-ROFSs are well suited to deal with imprecise data. Therefore, solving any decision-making problem using the tools of q-ROFSs are more efficient than those available in the literature. However, there are some complicated decision-making problems which cannot be handled using the techniques developed by q-ROFSs theory.

In Wuhan, China, an alarmingly contagious pandemic broke out in December 2019. It has been recognized as a zoonotic coronavirus. It has now become a great threat to the world. As discussed in [31], the generative number of COVID-19 is higher compared to the SARS coronavirus. Chen et al. [32] reviewed the clinical features and intrauterine vertical probable spread of COVID-19 infection in pregnant women. So, in COVID-19 scenario to save maximum human lives under some worst conditions is a big challenge for medical experts because they face the huge data in uncertain form. There is always a need to deal with vagueness of data in an efficient way according to the suggestions by different experts which is a complex decision-making problem. To deal with such complicated problems, the Bonferroni mean is very useful tool for group decision-making problems when arguments are interrelated to each other as Bonferroni mean can capture the interrelationship of the individual arguments. Recently, many authors proposed different decision-making techniques using Bonferroni mean operator. For more details we refer [33]-[36]. Motivated by this characteristic of Bonferroni mean, we define Bonferroni mean operator in q-rung orthopair fuzzy soft framework to deal such sensitive complicated problem.

The purpose of this paper is to present the concepts of aggregation operators called q-Rung Orthopair Fuzzy Soft Bonferroni-Mean (q-ROFSBM) operator and weighted q-rung orthopair fuzzy soft Bonferroni mean operator. Some basic properties of such operators are also presented. Finally, we employ our investigation for making appropriate decisions to secure maximum lives affected by coronavirus-oriented disease. We deliver a numerical example to illustrate the discussion made in this paper.

This paper is organized as follows: Section 2 contains basis concepts needed in the sequel. In Section 3, we define score function and hesitancy function for the ranking of q-Rung Orthopair Fuzzy Soft Numbers (q-ROFSNs). Moreover, we define q-rung orthopair fuzzy soft Bonferroni mean operator and discuss some of its important properties. Weighted q-rung orthopair fuzzy soft Bonferroni mean operator and its special cases are also discussed. In Section 4, we present a decision-making approach related to COVID-19 pandemic and show its capability in dealing with the vague data. In Section 5, a comparative study of the proposed approach is given with the existing methods in this direction. Finally, in Section 6, the conclusion and the scope of future research are outlined and discussed.

2 | Preliminaries

Definition 1 ([5]). Let E be the set of parameters, $A \subset E$ and U a universal set. A soft set can be identified by a pair(F, A), provided $F : A \longrightarrow P^{U}$, and P^{U} is the collection of all subsets of U.

Definition 2 ([6]). Let E be the set of parameters, $A \subset E$ and U a universal set. The pair (F, A) is called an intuitionistic fuzzy soft set, if $F\colon A \longrightarrow \mathrm{IF}^{U}$, where IF^{U} is the collection of all intuitionistic fuzzy subsets of U.

Definition 3 ([7]). Let E be the set of parameters, $A \subset E$ and U a universal set. The pair (F, A) is called a fuzzy soft set, given $F\colon A \longrightarrow F^{U}$, where F^{U} is a set of all fuzzy subsets of U.

Definition 4 ([28]). Let X be a universe, a PFS is defined on as follows:

$$\mathrm{F}(\mathrm{x_i}) = \left\{ \langle \mathrm{x_i}, \quad \mu_{\mathrm{F}}(\mathrm{x_i}), \quad \nu_{\mathrm{F}}(\mathrm{x_i}) \rangle \colon \mathrm{x_i} \in \mathrm{X} \right\}.$$

where $\mu_k : X \longrightarrow [0,1], and\ \nu_k : X \longrightarrow [0,1]$ are membership and non-memberships degrees of the elements of X with the condition that $0 \leq \mu_k(x_i)^2 + \nu_k(x_i)^2 \leq 1$. The degree of hesitancy is given by $I_k(x_i) = (1 - \mu_k(x_i)^2 + \nu_k(x_i)^2)^{\frac{1}{2}}$.

Definition 5 ([38]). Let X be a universe, Fermatean fuzzy set is defined as follows:

$$\mathrm{F}(\mathrm{x_i}) = \left\{ \langle \mathrm{x_i}, \quad \mu_{\mathrm{F}}(\mathrm{x_i}), \quad \nu_{\mathrm{F}}(\mathrm{x_i}) \rangle \colon \mathrm{x_i} \in \mathrm{X} \right\}.$$

where $\mu_k : X \longrightarrow [0,1], and\ \nu_k : X \longrightarrow [0,1]$ are membership and non-memberships degrees of the elements of X with the condition that $0 \leq \mu_k(x_i)^3 + \nu_k(x_i)^3 \leq 1$. The degree of hesitancy is given by $I_k(x_i) = (1 - \mu_k(x_i)^3 + \nu_k(x_i)^3)^{\frac{1}{3}}$.

Definition 6 ([30]). Let X be a universe, E a set of parameters. If PFS^{U} denotes the set of all Pythagorean fuzzy sets of X and $A \subset E$, then a pair (F, A) is called (PFSS) provided $F : A \longrightarrow \mathrm{PFS}^{U}$. This means that for any $e_k \in A$, the PFSS is defined as $F_{e_k}(x_i) = \{ \langle x_i,\ \mu_{e_k}(x_i),\ \nu_{e_k}(x_i) \rangle \colon x_i \in X\}$, where $\mu_k : X \longrightarrow [0,1]$, and $\nu_k : X \longrightarrow [0,1]$ are membership and non-memberships degrees of the elements of X with the condition that $0 \leq \mu_k(x_i)^2 + \nu_k(x_i)^2 \leq 1$. The degree of hesitancy is given as $I_k(x_i) = (1 - \mu_k(x_i)^2 + \nu_k(x_i)^2)^{\frac{1}{2}}$.

Definition 7 ([31]). Let X be a universe, the q-ROFS is defined as $F_k(x_i) = \{ \langle x_i,\ \mu_k(x_i),\ \nu_k(x_i) \rangle \colon x_i \in X\}$, where $\mu_k : X \longrightarrow [0,1]$, and $\nu_k : X \longrightarrow [0,1]$ are membership and non-memberships degrees of elements of

X with the given condition by $0 \leq \mu_k(x_i)^q + \nu_k(x_i)^q \leq 1, \quad (q \geq 1)$. The degree of the hesitancy is given as $I_k(x_i) = (\mu_k(x_i)^q + \nu_k(x_i)^q - \mu_k(x_i)^q \nu_k(x_i)^q)^{\frac{1}{q}}$.

Definition 8 ([22]). Let $X = \{x_1, x_2, \dots, x_n\}$ be a set of alternatives. For arbitrary real numbers $\lambda_1, \lambda_2 > 0$, a Bonferroni mean operator is defined as

$$\mathrm{BM}^{\lambda_1,\lambda_2}(x_1, x_2, \dots, x_n) = \left(\frac{1}{n(n-1)} \sum_{\substack{i,j=1 \\ i \neq j}}^{n} x_i^{\lambda_1} x_j^{\lambda_2}\right)^{\frac{1}{\lambda_1+\lambda_2}}.$$

Note that $BM^{\lambda_1,\lambda_2}(x_1, x_2, \dots, x_n)$ fulfils the followings:

I. $BM^{\lambda_1,\lambda_2}(0,0,0, \dots, 0) = 0$.
II. $BM^{\lambda_1,\lambda_2}(x, x, \dots, x) = x$, if $x_i = x \text{ for all } i$.
III. $BM^{\lambda_1,\lambda_2}(x_1, x_2, \dots, x_n) \leq BM^{\lambda_1,\lambda_2}(y_1, y_2, \dots, y_n)$ if $x_i \leq y_i$.
IV. $min\{x_i\} \leq BM^{\lambda_1,\lambda_2}(x_1, x_2, \dots, x_n) \leq max\{x_i\}$.

3 | q-Rung Orthopair Fuzzy Soft Bonferroni Mean Operators

This section is devoted to present the new generalized aggregation operators for solving the decision-making problems in the setting of (q-ROFSSs). More specifically, we define average operators namely, q-ROFSBM and Weighted q-Rung Orthopair Fuzzy Soft Bonferroni-Mean (Wq-ROFSBM) operators for collection Ω of q-ROFSNs $\alpha_{ik}(i = 1,2,3, \dots, n \ \& \ k = 1,2,3, \dots, m)$ and the weight vectors are denoted by η_i and ξ_k satisfy the condition that $\eta_i, \xi_k > 0 \ \sum_{i=1}^{n} \eta_i = 1$ and $\sum_{i=1}^{n} \xi_k = 1$.

Definition 9 ([39]). Let X be a universe, E set of parameters. If $\mathrm{q-ROFS}^U$ denotes the set of all q-ROFSs of X and $A \subset E$. Then the pair (F, A) is called q-Rung Orthopair Fuzzy Soft Sets (q-ROFSSs), where $F: A \longrightarrow \mathrm{q-ROFS}^U$. That is, for any $e_k \in A$ the q-ROFSS is identified by $F_{e_k}(x_i) = \{\langle x_i, \ \mu_k(x_i), \ \nu_k(x_i)\rangle : x_i \in X\}$, where $\mu_k: X \longrightarrow [0,1]$, and $\nu_k: X \longrightarrow [0,1]$ are membership and non-memberships degrees of the elements of X with the following condition $0 \leq \mu_k(x_i)^q + \nu_k(x_i)^q \leq 1, \quad (q \geq 1)$. The degree of hesitancy is given by $I_k(x_i) = (\mu_k(x_i)^q + \nu_k(x_i)^q - \mu_k(x_i)^q \nu_k(x_i)^q)^{\frac{1}{q}}$. For the sake of simplicity, we denote $F_{e_k}(x_i)$ by $\alpha_{ik} = \langle \mu_{ik}, \ \nu_{ik}\rangle$ and call it q-ROFSN.

Remark. Note that

– *If we take $q = 2$ in the above definition, it becomes a PFSS defined by Peng et al. [29].*
– *For $q = 1$ in the above definition, it becomes an IFSS defined by Maji et al. in [6]* and *[7].*

For applications of q-ROFSN in decision-making problems, we need to know how to rank the q-ROFSNs.

Definition 10. Define a score function for q-ROFSN $\alpha_{ik} = \langle \mu_{ik}, \ \nu_{ik}\rangle$ by

$$\mathrm{S}(\alpha_{ik}) = \frac{1}{2}(1 + \mu_{ik}{}^q - \nu_{ik}{}^q). \tag{1}$$

Note that $S(\alpha_{ik}) \in [0,1]$. Let $\alpha_{11} = \langle 0.8, \ 0.3\rangle$ and $\alpha_{12} = \langle 0.4, \ 0.5\rangle$ be two q- ROFSNs then by *Eq. (1)* for q=3 we have $S(\alpha_{11}) = 0.7425$ and $S(\alpha_{12}) = 0.6495$. So, $\alpha_{12} < \alpha_{11}$ as $S(\alpha_{12}) < S(\alpha_{11})$.

However, in some cases, the score function does not help rank the q- ROFSNs. For instance, if $\alpha_{11} = \langle 0.8, \ 0.6\rangle$ and $\alpha_{12} = \langle 0.5, \ 0.3\rangle$ are two q-ROFSNs then by *Eq. (1)* for q=1 we have $S(\alpha_{11}) = 0.6 = S(\alpha_{12})$. Here it is not possible to examine which one is the smaller. In such situations, another type of function called accuracy function ρ of α_{ik} can be used for ranking of q- ROFSNs.

Definition 11. ([15]). The accuracy function ρ of $\alpha_{ik} = \langle \mu_{ik},\ \nu_{ik} \rangle$ is defined as follows

$$\rho(\alpha_{ik}) = \mu_{ik}{}^{q} + \nu_{ik}{}^{q}. \tag{2}$$

Note that, $\rho(\alpha_{ik}) \in [0,1]$. For instance, let α_{11} and α_{12} be two q- ROFSNs. Suppose that by *Eq. (2)* for some value of q, we have $\rho\ (\alpha_{12}) < \rho(\alpha_{11})$ then $\alpha_{12} < \alpha_{11}$. For $\rho\ (\alpha_{11}) = \rho(\alpha_{12})$ we have $\alpha_{11} = \alpha_{12}$.

For ranking in the case $\rho\ (\alpha_{11}) = \rho(\alpha_{12})$, we suggest a definition based on the hesitancy degree. We define a hesitancy function as follows:

Definition 12. The hesitancy function φ of $\alpha_{ik} = \langle \mu_{ik},\ \nu_{ik} \rangle$ is defined as follows

$$\varphi(\alpha_{ik}) = (\mu_{ik}{}^{q} + \nu_{ik}{}^{q} - \mu_{ik}{}^{q}\nu_{ik}{}^{q})^{\frac{1}{q}}. \tag{3}$$

Note that $\varphi(\alpha_{ik}) \in [0,1]$. Unlike the above, the larger value of $\varphi(\alpha_{ik})$ makes the α_{ik} is smaller. For instance, let α_{11} and α_{12} be two q- ROFSNs. Then by *Eq. (3)* for some value of q, $\varphi(\alpha_{11}) < \varphi(\alpha_{12})$ imply that $\alpha_{12} < \alpha_{11}$.

3.1 | Basic Operations for q- ROFSNs

Definition 13. Let $\alpha_{11} = \langle \mu_{11},\ \nu_{11} \rangle$ and $\alpha_{12} = \langle \mu_{12},\ \nu_{12} \rangle$ be any two q- ROFSNs and λ be any real number which is positive, then the following is some basic operations on q- ROFSNs:

- $\alpha_{11} \vee \alpha_{11} = \langle\ max\{\mu_{11}, \mu_{12}\},\ min\{\ \nu_{11}, \nu_{12}\}\ \rangle.$
- $\alpha_{11} \wedge \alpha_{11} = \langle\ min\ \{\mu_{11}, \mu_{12}\},\ max\{\ \nu_{11}, \nu_{12}\}\ \rangle.$
- $\alpha_{11} \oplus \alpha_{12} = \langle\ (1-(1-\mu_{11}{}^{q})(1-\mu_{12}{}^{q}))^{\frac{1}{q}},\ \nu_{11}\nu_{12} \rangle = \langle (\mu_{11}{}^{q} + \mu_{12}{}^{q} - \mu_{11}{}^{q}\mu_{12}{}^{q})^{\frac{1}{q}},\ \nu_{11}\nu_{12} \rangle.$
- $\alpha_{11} \otimes \alpha_{12} = \langle \mu_{11}\mu_{12},\ (1-(1-\nu_{11}{}^{q})(1-\nu_{12}{}^{q}))^{\frac{1}{q}}\ \rangle = \langle \mu_{11}\mu_{12},\ (\nu_{11}{}^{q} + \nu_{12}{}^{q} - \nu_{11}{}^{q}\nu_{12}{}^{q})^{\frac{1}{q}}\ \rangle.$
- $\lambda\alpha_{11} = \langle\ (1-(1-\mu_{11}{}^{q})^{\lambda})^{\frac{1}{q}},\ \nu_{11}{}^{\lambda} \rangle.$
- $\alpha_{11}{}^{\lambda} = \langle\ \mu_{11}{}^{\lambda},\ (1-(1-\nu_{11}{}^{q})^{\lambda})^{\frac{1}{q}}\ \rangle.$
- $\alpha_{11}{}^{c} = \langle \nu_{11},\ \mu_{11} \rangle.$

Remarks.

- *All operations expressed in the definition above yield also q-ROFSNs.*
- *Commutative and associative laws hold in both operations under addition and multiplication expressed in definition 1.*

3.2 | Representation of q-ROFSS in Matrix Form

Let $X = \{x_1, x_2, \dots, x_m\}$ be the universal set and $A = \{\ e_1, e_2, \dots, e_n\}$ the set of parameters. If (F, A) is a q-ROFSS over X and $e_i \in A$, then there exist membership and non-membership degrees μ_{ik} and ν_{ik}, respectively with $0 \le \mu_k(x_i)^q + \nu_k(x_i)^q \le 1$, $(q \ge 1)$. Moreover, $F_{e_k}(x_i) = \alpha_{ik} = \langle \mu_{ik},\ \nu_{ik} \rangle$, we can write q-ROFS matrix $M_{m\times n}$ of (F, A) over X is given as $M_{m\times n} = [\ \langle \mu_{ik},\ \nu_{ik} \rangle\ ]_{m\times n}$ i.e.

$$M_{m\times n} = \begin{array}{c c} & \begin{array}{ccc} e_1 & \dots & e_m \end{array} \\ \begin{array}{c} x_1 \\ \vdots \\ x_m \end{array} & \begin{bmatrix} \alpha_{11} & \cdots & \alpha_{1n} \\ \vdots & \ddots & \vdots \\ \alpha_{m1} & \cdots & \alpha_{mn} \end{bmatrix} \end{array}.$$

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We represent the decision matrix by table as

Table 1. Representation of q-ROFSS.

(F, A)	e_1	e_2	………………	e_n
x_1	$\langle \mu_{11}, \nu_{11}\rangle$	$\langle \mu_{12}, \nu_{12}\rangle$	………………	$\langle \mu_{1n}, \nu_{1n}\rangle$
x_2	$\langle \mu_{21}, \nu_{21}\rangle$	$\langle \mu_{22}, \nu_{22}\rangle$	………………	$\langle \mu_{2n}, \nu_{2n}\rangle$
.	.	.		.
.	.	.		.
.	.	.		.
x_m	$\langle \mu_{m1}, \nu_{m1}\rangle$	$\langle \mu_{m21}, \nu_{m2}\rangle$	………………	$\langle \mu_{mn}, \nu_{mn}\rangle$

3.3 | q-Rung Orthopair Fuzzy Soft Bonferroni-Mean Operator

Definition 14. For any class of q-ROFSNs $\alpha_{ik} = \langle \mu_{ik}, \nu_{ik}\rangle$ for $(i = 1,2,3, \dots, n \ \& \ k = 1,2,3, \dots, m)$, the q-rung orthopair fuzzy soft Bonferroni mean (q-ROFSBM) is an operator

q-$ROFSBM\ :\ \Omega^n \longrightarrow \Omega$ is given by

$$\mathrm{q-ROFSBM}^{\lambda_1,\lambda_2}(\alpha_{11},\alpha_{12},\dots,\alpha_{mn}) = \left(\frac{1}{\mathrm{mn(n-1)(m-1)}}\sum_{\substack{\mathrm{k,l=1}\\ \mathrm{k\neq l}}}^{\mathrm{m}}\sum_{\substack{\mathrm{i,j=1}\\ \mathrm{i\neq j}}}^{\mathrm{n}}\left(\alpha_{\mathrm{ik}}^{\lambda_1}\otimes\alpha_{\mathrm{jl}}^{\lambda_2}\right)\right)^{\frac{1}{\lambda_1+\lambda_2}}, \tag{4}$$

where $\lambda_1, \lambda_2 > 0$ are real numbers and Ω is set of all q-ROFSNs.

Theorem 1. The aggregated values by applying q-ROFSBM operator on a collection of q-ROFSNs $\alpha_{ik} = \langle \mu_{ik}, \nu_{ik}\rangle$ for $(i = 1,2,3, \dots, n \ \& \ k = 1,2,3, \dots, m)$ is still q-ROFSN and given as follows:

$$\mathrm{q-ROFSBM}^{\lambda_1,\lambda_2}(\alpha_{11},\alpha_{12},\dots,\alpha_{mn}) = \left\langle \begin{array}{c} \left(\left(1-\prod_{\substack{\mathrm{l,k=1}\\ \mathrm{l\neq k}}}^{\mathrm{m}}\prod_{\substack{\mathrm{i,j=1}\\ \mathrm{i\neq j}}}^{\mathrm{n}}\left(1-\left(\mu_{\mathrm{ik}}^{\lambda_1}\mu_{\mathrm{jl}}^{\lambda_2}\right)^{\mathrm{q}}\right)^{\frac{1}{\mathrm{mn(n-1)(m-1)}}}\right)^{\frac{1}{\mathrm{q}}}\right)^{\frac{1}{\lambda_1+\lambda_2}}, \\ 1-\left(\left(1-\prod_{\substack{\mathrm{l,k=1}\\ \mathrm{l\neq k}}}^{\mathrm{m}}\prod_{\substack{\mathrm{i,j=1}\\ \mathrm{i\neq j}}}^{\mathrm{n}}\left(1-\left(1-\nu_{\mathrm{ik}}^{\mathrm{q}}\right)^{\lambda_1}\left(1-\nu_{\mathrm{jl}}^{\mathrm{q}}\right)^{\lambda_2}\right)^{\frac{1}{\mathrm{mn(n-1)(m-1)}}}\right)^{\frac{1}{\mathrm{q}}}\right)^{\frac{1}{\lambda_1+\lambda_2}} \end{array}\right\rangle. \tag{5}$$

Proof. The prove of this theorem is much lengthy but one can prove it by using the rule of mathematical induction on m and n. To illustrate the proof, we show its validity for $n = 2$. Indeed,

$$\mathrm{q-ROFSBM}^{\lambda_1,\lambda_2}(\alpha_{11},\alpha_{12},\dots,\alpha_{mn}) = \left(\frac{1}{\mathrm{mn(n-1)(m-1)}}\sum_{\substack{\mathrm{l,k=1}\\ \mathrm{l\neq k}}}^{\mathrm{m}}\sum_{\substack{\mathrm{i,j=1}\\ \mathrm{i\neq j}}}^{\mathrm{n}}\left(\alpha_{\mathrm{ik}}^{\lambda_1}\otimes\alpha_{\mathrm{jl}}^{\lambda_2}\right)\right)^{\frac{1}{\lambda_1+\lambda_2}},$$

where $\lambda_1, \lambda_2 > 0$. By using the value, we have

$$q-\mathrm{ROFSBM}^{\lambda_1,\lambda_2}(\alpha_{11},\alpha_{12},\dots,\alpha_{mn})=\left(\frac{1}{2m(m-1)}\sum_{\substack{l,k=1\\ l\neq k}}^{m}\sum_{\substack{i,j=1\\ i\neq j}}^{2}(\alpha_{ik}^{\lambda_1}\otimes\alpha_{jl}^{\lambda_2})\right)^{\frac{1}{\lambda_1+\lambda_2}}$$

$$=\left(\frac{1}{2m(m-1)}\sum_{\substack{l,k=1\\ l\neq k}}^{m}[(\alpha_{1k}^{\lambda_1}\otimes\alpha_{2l}^{\lambda_2})\oplus(\alpha_{2k}^{\lambda_1}\otimes\alpha_{1l}^{\lambda_2})]\right)^{\frac{1}{\lambda_1+\lambda_2}}.$$

Now

$$\left(\alpha_{1k}^{\lambda_1}\otimes\alpha_{2l}^{\lambda_2}\right)\oplus\left(\alpha_{2k}^{\lambda_1}\otimes\alpha_{1l}^{\lambda_2}\right)=\langle\,\mu_{1k}^{\lambda_1}\mu_{2l}^{\lambda_2},\left(\left(1-\left(1-\nu_{1k}^{q}\right)^{\lambda_1}\right)^{\frac{q}{q}}+\left(1-\left(1-\nu_{2l}^{q}\right)^{\lambda_2}\right)^{\frac{q}{q}}-\left(1-\left(1-\nu_{1k}^{q}\right)^{\lambda_1}\right)^{\frac{q}{q}}\left(1-\left(1-\nu_{2l}^{q}\right)^{\lambda_2}\right)^{\frac{q}{q}}\right)^{\frac{1}{q}}\rangle\oplus$$

$$\langle\mu_{2k}^{\lambda_1}\mu_{1l}^{\lambda_2},\left(\left(1-\left(1-\nu_{2k}^{q}\right)^{\lambda_1}\right)^{\frac{q}{q}}+\left(1-\left(1-\nu_{1l}^{q}\right)^{\lambda_2}\right)^{\frac{q}{q}}-\left(1-\left(1-\nu_{2k}^{q}\right)^{\lambda_1}\right)^{\frac{q}{q}}\left(1-\left(1-\nu_{1l}^{q}\right)^{\lambda_2}\right)^{\frac{q}{q}}\right)^{\frac{1}{q}}\rangle$$

$$=\langle\,\mu_{1k}^{\lambda_1}\mu_{2l}^{\lambda_2},\left(1-\left(1-\nu_{1k}^{q}\right)^{\lambda_1}\left(1-\nu_{2l}^{q}\right)^{\lambda_2}\right)^{\frac{1}{q}}\rangle\oplus\langle\,\mu_{2k}^{\lambda_1}\mu_{1l}^{\lambda_2},\left(1-\left(1-\nu_{2k}^{q}\right)^{\lambda_1}\left(1-\nu_{1l}^{q}\right)^{\lambda_2}\right)^{\frac{1}{q}}\rangle$$

$$=\langle(1-\left(1-(\mu_{1k}^{\lambda_1}\mu_{2l}^{\lambda_2})^{q}\right)(1-(\mu_{2k}^{\lambda_1}\mu_{1l}^{\lambda_2})^{q}))^{\frac{1}{q}},\left(1-\left(1-\nu_{1k}^{q}\right)^{\lambda_1}\left(1-\nu_{2l}^{q}\right)^{\lambda_2}\right)^{\frac{1}{q}}\left(1-\left(1-\nu_{2k}^{q}\right)^{\lambda_1}\left(1-\nu_{1l}^{q}\right)^{\lambda_2}\right)^{\frac{1}{q}}\rangle$$

$$=\langle\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{2}\left(1-\left(\mu_{ik}^{\lambda_1}\mu_{jl}^{\lambda_2}\right)^{q}\right)\right)^{\frac{1}{q}},1-\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{2}\left(1-\left(1-\nu_{ik}^{q}\right)^{\lambda_1}\left(1-\nu_{jl}^{q}\right)^{\lambda_2}\right)\right)^{\frac{1}{q}}\rangle.$$

Hence

$$\left(\frac{1}{2m(m-1)}\sum_{\substack{l,k=1\\ l\neq k}}^{m}\sum_{\substack{i,j=1\\ i\neq j}}^{2}\left(\alpha_{ik}^{\lambda_1}\otimes\alpha_{jl}^{\lambda_2}\right)\right)^{\frac{1}{\lambda_1+\lambda_2}}=\langle\left(\left(1-\prod_{\substack{l,k=1\\ l\neq k}}^{m}\prod_{\substack{i,j=1\\ i\neq j}}^{2}\left(1-\left(\mu_{ik}^{\lambda_1}\mu_{jl}^{\lambda_2}\right)^{q}\right)^{\frac{1}{2m(m-1)}}\right)^{\frac{1}{q}}\right)^{\frac{1}{\lambda_1+\lambda_2}},\ 1-\left(\left(1-\prod_{\substack{l,k=1\\ l\neq k}}^{m}\prod_{\substack{i,j=1\\ i\neq j}}^{2}\left(1-\left(1-\nu_{ik}^{q}\right)^{\lambda_1}\left(1-\nu_{jl}^{q}\right)^{\lambda_2}\right)^{\frac{1}{2m(m-1)}}\right)^{\frac{1}{q}}\right)^{\frac{1}{\lambda_1+\lambda_2}}\rangle.$$

Similarly, one can prove in general by using mathematical induction on n and m.

Example 1. Let $D = \{d_1, d_2, d_3\}$ denote the team of three doctors who give their preference to describe the "patient of coronavirus" by the symptoms over the set of parameters e_1 =cough, e_2 = temperature, e_3 = infection of the respiratory system. Correspondingly, the q-ROFSS in matrix form describes the "patient of coronavirus" to the medical officer, who decides about the admission of a patient to a hospital. The q-ROFSS is given as follows:

$$
\begin{array}{cc}
 & \begin{array}{ccc} \mathrm{e}_1 & \mathrm{e}_2 & \mathrm{e}_3 \end{array} \\
(\mathrm{F},\mathrm{A}) = \begin{array}{c} \mathrm{d}_1 \\ \mathrm{d}_2 \\ \mathrm{d}_3 \end{array} & \begin{bmatrix} \langle 0.7, 0.3\rangle & \langle 0.3, 0.5\rangle & \langle 0.8, 0.3\rangle \\ \langle 0.4, 0.3\rangle & \langle 0.4, 0.5\rangle & \langle 0.7, 0.4\rangle \\ \langle 0.5, 0.4\rangle & \langle 0.7, 0.6\rangle & \langle 0.4, 0.6\rangle \end{bmatrix}.
\end{array}
$$

For simplicity we have $\lambda_1, = \lambda_2 = 1, \quad q = 2.$

$$
\prod_{\substack{l,k=1 \\ l\neq k}}^{3} \prod_{\substack{i,j=1 \\ i\neq j}}^{3} \left(1 - \left(\mu_{ik}\mu_{jl}\right)^2\right)^{\frac{1}{3.3(3-1)(3-1)}}
$$

$$
= \left(1 - \left((\mu_{11})(\mu_{22})\right)^2\right)^{\frac{1}{36}} \times \left(1 - \left((\mu_{11})(\mu_{23})\right)^2\right)^{\frac{1}{36}} \times \ldots
$$

$$
\times \left(1 - \left((\mu_{33})(\mu_{22})\right)^2\right)^{\frac{1}{36}}
$$

$$
= \left(1 - \left((0.7)(0.4)\right)^2\right)^{\frac{1}{36}} \times \left(1 - \left((0.7)(0.7)\right)^2\right)^{\frac{1}{36}} \times \ldots \times \left(1 - \left((0.7)(0.4)\right)^2\right)^{\frac{1}{36}} = 0.8822.
$$

$$
.\left(1 - \prod_{\substack{l,k=1 \\ l\neq k}}^{3} \prod_{\substack{i,j=1 \\ i\neq j}}^{3} \left(1 - \left(\mu_{ik}\mu_{jl}\right)^2\right)^{\frac{1}{3.3(3-1)(3-1)}}\right) = 1 - 0.8822 = 0.1177
$$

$$
\left(1 - \prod_{\substack{l,k=1 \\ l\neq k}}^{3} \prod_{\substack{i,j=1 \\ i\neq j}}^{3} \left(1 - \left(\mu_{ik}\mu_{jl}\right)^2\right)^{\frac{1}{3.3(3-1)(3-1)}}\right)^{\frac{1}{4}} = (0.1177)^{\frac{1}{4}} = 0.5857.
$$

Similarly, we have

$$
1 - \left(\left(1 - \prod_{\substack{l,k=1 \\ l\neq k}}^{3} \prod_{\substack{i,j=1 \\ i\neq j}}^{3} \left(1 - \left(1 - v_{ik}^2\right)\left(1 - v_{jl}^2\right)\right)^{\frac{1}{3.3(3-1)(3-1)}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}} = 0.1876.
$$

Hence, by using *Eq. (5)*, we have 2-*ROFSBM*$^{1,1}(\alpha_{11}, \alpha_{12}, \ldots, \alpha_{33}) = \langle 0.5857,\ 0.1876\rangle$.

3.4 | Special Cases of Proposed q-ROFSBM Operator

Here some special cases are given of q-ROFSBM operator for different values of q which are followings:

- *For $q = 2$ in Eq. (5) then the projected operator condenses to PFS Bonferroni-mean operator.*
- *For $q = 1$ in Eq. (5) then the projected operator condenses to IFS Bonferroni-mean operator as discussed by Garg in [22].*

3.5 | Properties of Proposed q-ROFSBM Operator

From the *Theorem 3*, it is observed that our proposed q-ROFSBM satisfies the properties namely, idempotent property, boundedness property, monotonicity property which are design as fellows

Theorem 2 (Idempotent Property). If $\alpha_{ik} = \alpha$, $\forall\ i,k$. Let $\alpha = \langle \mu,\ \nu \rangle$ then $\mathrm{q-ROFSBM}^{\lambda_1,\lambda_2}(\alpha_{11}, \alpha_{12}, \dots, \alpha_{mn}) = \alpha$.

Proof.

$$\mathrm{q-ROFSBM}^{\lambda_1,\lambda_2}(\alpha_{11}, \alpha_{12}, \dots, \alpha_{mn}) = \mathrm{q-ROFSBM}^{\lambda_1,\lambda_2}(\alpha, \alpha, \dots, \alpha)$$

$$= \left\langle \begin{array}{c} \left(\left(1 - \prod_{\substack{l,k=1 \\ l\neq k}}^{m} \prod_{\substack{i,j=1 \\ i\neq j}}^{n} \left(1 - (\mu^{\lambda_1}\mu^{\lambda_2})^{q}\right)^{\frac{1}{mn(n-1)(m-1)}}\right)^{\frac{1}{q}}\right)^{\frac{1}{\lambda_1+\lambda_2}}, \\ 1 - \left(\left(1 - \prod_{\substack{l,k=1 \\ l\neq k}}^{m} \prod_{\substack{i,j=1 \\ i\neq j}}^{n} \left(1 - (1-\nu^{q})^{\lambda_1}(1-\nu^{q})^{\lambda_2}\right)^{\frac{1}{mn(n-1)(m-1)}}\right)^{\frac{1}{q}}\right)^{\frac{1}{\lambda_1+\lambda_2}} \end{array} \right\rangle$$

$$= \left\langle \begin{array}{c} \left(\left(1 - \prod_{\substack{l,k=1 \\ l\neq k}}^{m} \prod_{\substack{i,j=1 \\ i\neq j}}^{n} \left(1 - (\mu^{\lambda_1+\lambda_2})^{q}\right)^{\frac{1}{mn(n-1)(m-1)}}\right)^{\frac{1}{q}}\right)^{\frac{1}{\lambda_1+\lambda_2}}, \\ 1 - \left(\left(1 - \prod_{\substack{l,k=1 \\ l\neq k}}^{m} \prod_{\substack{i,j=1 \\ i\neq j}}^{n} \left(1 - (1-\nu^{q})^{\lambda_1+\lambda_2}\right)^{\frac{1}{mn(n-1)(m-1)}}\right)^{\frac{1}{q}}\right)^{\frac{1}{\lambda_1+\lambda_2}} \end{array} \right\rangle.$$

$$= \left\langle \left(\left(1 - \left(1 - \mu^{(\lambda_1+\lambda_2)q}\right)\right)^{\frac{1}{q}}\right)^{\frac{1}{\lambda_1+\lambda_2}}, 1 - \left(1 - \left(1 - (1-\nu^{q})^{\lambda_1+\lambda_2}\right)^{\frac{1}{q}}\right)^{\frac{1}{\lambda_1+\lambda_2}} \right\rangle$$

$$= \left\langle \left(\mu^{(\lambda_1+\lambda_2)q}\right)^{\frac{1}{q(\lambda_1+\lambda_2)}}, \left(1 - \left((1-\nu^{q})^{\lambda_1+\lambda_2}\right)^{\frac{1}{q}}\right)^{\frac{1}{\lambda_1+\lambda_2}} \right\rangle = \langle \mu, \nu \rangle = \alpha.$$

Hence proved.

Theorem 3 (Boundedness). Let $\beta = \left\langle \min_k \min_i \{\mu_{ik}\}, \max_k \max_i \{\nu_{ik}\} \right\rangle$ and $\gamma = \left\langle \max_k \max_i \{\mu_{ik}\}, \min_k \min_i \{\nu_{ik}\} \right\rangle$ then $\beta \le \mathrm{q-ROFSBM}^{\lambda_1,\lambda_2}(\alpha_{11}, \alpha_{12}, \dots, \alpha_{mn}) \le \gamma$.

Proof. For our easiness we write $\sigma = \frac{1}{mn(m-1)(n-1)}$. Here for all i, k we have

$$\min_k \min_i \{\mu_{ik}\} \le \mu_{ik} \le \max_k \max_i \{\mu_{ik}\}.$$

If and only if $\left(\min_k \min_i \{\mu_{ik}\}\right)^{(\lambda_1+\lambda_2)q} \le \left(\mu_{ik}^{\lambda_1} \mu_{jl}^{\lambda_2}\right)^q \le \left(\max_k \max_i \{\mu_{ik}\}\right)^{(\lambda_1+\lambda_2)q}$.

If and only if $\left(1 - \left(\max_k \max_i \{\mu_{ik}\}\right)^{(\lambda_1+\lambda_2)q}\right)^\sigma \le \left(1 - \left(\mu_{ik}^{\lambda_1} \mu_{jl}^{\lambda_2}\right)^q\right)^\sigma \le \left(1 - \left(\min_k \min_i \{\mu_{ik}\}\right)^{(\lambda_1+\lambda_2)q}\right)^\sigma$.

if and only if

$$\prod_{\substack{l,k=1 \\ l\neq k}}^{m} \prod_{\substack{i,j=1 \\ i\neq j}}^{n} \left(1 - \left(\max_k \max_i \{\mu_{ik}\}\right)^{(\lambda_1+\lambda_2)q}\right)^\sigma \le \prod_{\substack{l,k=1 \\ l\neq k}}^{m} \prod_{\substack{i,j=1 \\ i\neq j}}^{n} \left(1 - \left(\mu_{ik}^{\lambda_1} \mu_{jl}^{\lambda_2}\right)^q\right)^\sigma \le$$

$$\prod_{\substack{l,k=1 \\ l\neq k}}^{m} \prod_{\substack{i,j=1 \\ i\neq j}}^{n} \left(1 - \left(\min_k \min_i \{\mu_{ik}\}\right)^{(\lambda_1+\lambda_2)q}\right)^\sigma.$$

If and only if

$$\left(\min_k \min_i \{\mu_{ik}\}\right)^{(\lambda_1+\lambda_2)q} \le 1 - \prod_{\substack{l,k=1 \\ l\neq k}}^{m} \prod_{\substack{i,j=1 \\ i\neq j}}^{n} \left(1 - \left(\mu_{ik}^{\lambda_1} \mu_{jl}^{\lambda_2}\right)^q\right)^\sigma \le \left(\max_k \max_i \{\mu_{ik}\}\right)^{(\lambda_1+\lambda_2)q}.$$

This implies that

$$\min_k \min_i \{\mu_{ik}\} \le \left(\left(1 - \prod_{\substack{l,k=1 \\ l\neq k}}^{m} \prod_{\substack{i,j=1 \\ i\neq j}}^{n} \left(1 - \left(\mu_{ik}^{\lambda_1} \mu_{jl}^{\lambda_2}\right)^q\right)^{\frac{1}{mn(n-1)(m-1)}}\right)^{\frac{1}{q}}\right)^{\frac{1}{\lambda_1+\lambda_2}} \le \max_k \max_i \{\mu_{ik}\}. \tag{6}$$

Further,

$$\min_k \min_i \{\nu_{ik}\} \le \nu_{ik} \le \max_k \max_i \{\nu_{ik}\}.$$

If and only if

$$1 - \max_k \max_i \{\nu_{ik}\})^{(\lambda_1+\lambda_2)q} \le (1 - \nu_{ik}{}^q)^{\lambda_1} (1 - \nu_{ik}{}^q)^{\lambda_2} \le 1 - \min_k \min_i \{\nu_{ik}\})^{(\lambda_1+\lambda_2)q}.$$

In most of real-life problems, it is natural to study the weight of the experts and parameter.

i if and only if $\left(1 - (1 - \min_k \min_i \{\nu_{ik}\})^{(\lambda_1+\lambda_2)q}\right)^\sigma \le \left(1 - (1 - \nu_{ik}{}^q)^{\lambda_1} (1 - \nu_{ik}{}^q)^{\lambda_2}\right)^\sigma \le \left(1 - (1 - \max_k \max_i \{\nu_{ik}\})^{(\lambda_1+\lambda_2)q}\right)^\sigma$.

If and only if

$$\prod_{\substack{l,k=1\\ l\neq k}}^{m}\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-(1-\min_{k}\min_{i}\{\nu_{ik}\})^{(\lambda_1+\lambda_2)q}\right)^{\sigma} \leq \prod_{\substack{l,k=1\\ l\neq k}}^{m}\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-(1-\nu_{ik}{}^{q})^{\lambda_1}(1-\nu_{ik}{}^{q})^{\lambda_2}\right)^{\sigma} \leq \prod_{\substack{l,k=1\\ l\neq k}}^{m}\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-(1-\max_{k}\max_{i}\{\nu_{ik}\})^{(\lambda_1+\lambda_2)q}\right)^{\sigma}.$$

If and only if

$$1-(1-\min_{k}\min_{i}\{\nu_{ik}\})^{(\lambda_1+\lambda_2)q} \leq \prod_{\substack{l,k=1\\ l\neq k}}^{m}\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-(1-\nu_{ik}{}^{q})^{\lambda_1}(1-\nu_{ik}{}^{q})^{\lambda_2}\right)^{\sigma} \leq 1-(1-\max_{k}\max_{i}\{\nu_{ik}\})^{(\lambda_1+\lambda_2)q}.$$

If and only if

$$(1-\max_{k}\max_{i}\{\nu_{ik}\})^{(\lambda_1+\lambda_2)q} \leq 1-\prod_{\substack{l,k=1\\ l\neq k}}^{m}\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-(1-\nu_{ik}{}^{q})^{\lambda_1}(1-\nu_{ik}{}^{q})^{\lambda_2}\right)^{\sigma} \leq (1-\min_{k}\min_{i}\{\nu_{ik}\})^{(\lambda_1+\lambda_2)q}.$$

Therefore we have

$$\min_{k}\min_{i}\{\nu_{ik}\} \leq 1-\left(\left(1-\prod_{\substack{l,k=1\\ l\neq k}}^{m}\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-\left(1-\nu_{ik}^{q}\right)^{\lambda_1}\left(1-\nu_{jl}^{q}\right)^{\lambda_2}\right)^{\frac{1}{mn(n-1)(m-1)}}\right)^{\frac{1}{q}}\right)^{\frac{1}{\lambda_1+\lambda_2}} \leq \max_{k}\max_{i}\{\nu_{ik}\}. \quad (7)$$

If we take $\delta = q-ROFSBM^{\lambda_1,\lambda_2}(\alpha_{11},\alpha_{12},\dots,\alpha_{mn}) = \langle\mu_\delta,\nu_\delta\rangle$, and

$\beta = \left\langle \min_{k}\min_{i}\{\mu_{ik}\}, \max_{k}\max_{i}\{\nu_{ik}\}\right\rangle$ and $\gamma = \left\langle \max_{k}\max_{i}\{\mu_{ik}\}, \min_{k}\min_{i}\{\nu_{ik}\}\right\rangle$ then combining the *Eqs. (6)* and *(7)*, we get

$$\min_{k}\min_{i}\{\mu_{ik}\} \leq \mu_\delta \leq \max_{k}\max_{i}\{\mu_{ik}\}. \text{ and } \min_{k}\min_{i}\{\nu_{ik}\} \leq \nu_\delta \leq \max_{k}\max_{i}\{\nu_{ik}\}.$$

By definition of score function according to the *Eq. (1)*, we have

$$S(\delta) = \frac{1}{2}(1+\mu_\delta{}^{q}-\nu_\delta{}^{q}) \leq \frac{1}{2}\left(1+(\max_{k}\max_{i}\{\mu_{ik}\})^{q}-(\min_{k}\min_{i}\{\nu_{ik}\})^{q}\right) = S(\gamma). \quad (7.a)$$

$$S(\delta) = \frac{1}{2}(1+\mu_\delta{}^{q}-\nu_\delta{}^{q}) \geq \frac{1}{2}\left(1+(\min_{k}\min_{i}\{\mu_{ik}\})^{q}-(\max_{k}\max_{i}\{\nu_{ik}\})^{q}\right) = S(\beta). \quad (7.b)$$

By *Eqs. (7.a)* and *(7.b)* we have $S(\beta) \leq S(\delta) \leq S(\gamma)$, therefore, $\beta \leq \delta \leq \gamma$. Hence the result.

Theorem 4 (Monotonicity). Let β_{ik} be any collection of q-ROFSNs with $\alpha_{ik} \leq \beta_{ik}$ for all k, i. Then $\mathrm{q-ROFSBM}^{\lambda_1,\lambda_2}(\alpha_{11}, \alpha_{12}, \dots, \alpha_{mn}) \leq \mathrm{q-ROFSBM}^{\lambda_1,\lambda_2}(\beta_{11}, \beta_{12}, \dots, \beta_{mn})$.

Proof. The proof is similar to the proof of Theorem 3, so we overlook the proof.

3.6 | Weighted q-Rung Orthopair Fuzzy Soft Bonferroni Mean Operator

In most of real-life problems, it is natural to study the weight of the experts and parameters during the decision-making investigation. In this way, the governing person may get a extra precise consequences rendering to the current circumstances. Keeping in view this fact, based on the q-ROFSBM aggregation operator defined in the Section 3.3, we define a novel aggregation operator according to the weightage of the experts and the limitations called Wq-ROFSBM.

Definition 15. For any group of data of q-ROFSNs α_{ik} for $(i = 1, 2, 3, \dots, n \;\&\; k = 1, 2, 3, \dots, m)$, with weight vectors of the selected parameters and the experts $\eta = (\eta_1, \eta_2, \dots, \eta_n)^T$ and $\xi = (\xi_1, \xi_2, \dots, \xi_m)^T$ respectively such that $\eta_i > 0$ $\sum_{i=1}^{n} \eta_i = 1$ and $\xi_k > 0$, $\sum_{k=1}^{m} \xi_k = 1$, the Wq-ROFSBM is an operator $Wq-ROFSBM : \Omega^n \longrightarrow \Omega$ is given by

$$\mathrm{Wq-ROFSBM}^{\lambda_1,\lambda_2}(\alpha_{11}, \alpha_{12}, \dots, \alpha_{mn}) = \left(\frac{1}{mn(m-1)(n-1)} \sum_{\substack{l,k=1 \\ l \neq k}}^{m} \sum_{\substack{i,j=1 \\ i \neq j}}^{n} (\xi_k(\eta_i \alpha_{ik}))^{\lambda_1} \otimes (\xi_l(\eta_j \alpha_{jl}))^{\lambda_2} \right)^{\frac{1}{\lambda_1+\lambda_2}}, \tag{8}$$

where $\lambda_1, \lambda_2 > 0$ and Ω is the set of all q-ROFSNs.

Theorem 5. The aggregated value by applying Wq-ROFSBM operator on a collection of q-ROFSNs $\alpha_{ik} = \langle \mu_{ik}, \nu_{ik} \rangle$ for $(i = 1, 2, 3, \dots, n \;\&\; k = 1, 2, 3, \dots, m)$ with weight vectors of the selected parameters and the experts $\eta = (\eta_1, \eta_2, \dots, \eta_n)^T$ and $\xi = (\xi_1, \xi_2, \dots, \xi_m)^T$ respectively is still q-ROFSN and is given by

$$\mathrm{Wq-ROFSBM}^{\lambda_1,\lambda_2}(\alpha_{11}, \alpha_{12}, \dots, \alpha_{mn})$$

$$= \left\langle \begin{array}{c} \left(1 - \prod_{\substack{l,k=1 \\ l \neq k}}^{m} \prod_{\substack{i,j=1 \\ i \neq j}}^{n} \left(\left(1 - 1 - \left(1 - \mu_{ik}^{q}\right)^{\xi_k \eta_i} \right)^{\lambda_1} \right) 1 - \left(1 - \mu_{jl}^{q}\right)^{\xi_l \eta_j})^{\lambda_2} \right)^{\frac{1}{mn(n-1)(m-1)}} \right)^{\frac{1}{\lambda_1+\lambda_2}}, \\ 1 - \left(1 - \prod_{\substack{l,k=1 \\ l \neq k}}^{m} \prod_{\substack{i,j=1 \\ i \neq j}}^{n} \left(1 - \left(1 - (\nu_{ik}^{\xi_k \eta_i})^q\right)^{\lambda_1} \left(1 - (\nu_{jl}^{\xi_l \eta_j})^q\right)^{\lambda_2} \right)^{\frac{1}{mn(n-1)(m-1)}} \right)^{\frac{1}{\lambda_1+\lambda_2}} \end{array} \right\rangle. \tag{9}$$

Proof. The proof is alike to the proof of *Theorem 3*, so we omit the proof.

3.7 | Special Cases of the Proposed Wq-ROFSBM Operator

The aggregation operator defined by *Eq. (9)* is a more generalized operator for aggregation of the data in daily life problems involving vagueness for decision-making analysis. Special cases of our proposed operators are given below:

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I. For $q = 2$ and $\lambda_2 \longrightarrow 0$ in Eq. (9), the projected operator condenses in weighted PFS mean.
II. For $q = 2$, $\lambda_1 = 2$ and $\lambda_2 \longrightarrow 0$ in Eq. (9), the projected operator condenses in weighted PFS square-mean.
III. For $q = 2$, $\lambda_1 = 1$ and $\lambda_2 \longrightarrow 0$ in Eq. (9), the projected operator condenses in weighted PFS average operator.
IV. For $q = 1$ and $\lambda_2 \longrightarrow 0$ in Eq. (9), the projected operator condenses in weighted IFS mean.
V. For $q = 1$, $\lambda_1 = 2$ and $\lambda_2 \longrightarrow 0$ in Eq. (9), the projected operator condenses in weighted IFS square-mean.
VI. For $q = 1$, $\lambda_1 = 1$ and $\lambda_2 \longrightarrow 0$ in Eq. (9), the projected operator condenses in weighted IFS average operator.
VII. For $q = 1$ *and* $\lambda_1 = \lambda_2 = 1$ in Eq. (9), the projected operator condenses in weighted IFS interrelated square-mean.

Note that the cases (iv-vii) discussed by [22] are special cases of our projected operators.

4 | Application of a Proposed Approach to Save Maximum Lives by Covid-19

Coronaviruses are intimate of the viruses that reason diseases such as severe acute respiratory syndrome, common cold. At the end of 2019, a new virus was recognized as the source of a syndrome epidemic that invented in China. The caused disease is so-called coronavirus sickness (COVID-19). Many cases of COVID-19 have been testified all over the world, with the U.S. Public health-groups, such as WHO and U.S. The Centers for Disease-Control (CDC) and prevention, are nursing the condition and post informs on their sites. WHO stated it a worldwide disease in March 2020. The said groups have also delivered approvals for avoiding and considering the illness. Every welfare group and any NGO which is working for the prevention of COVID-19 want to save maximum lives. For this purpose, every group of experts has their suggestions. This problem depends on many factors such as age factor, the density of COVID-19 in each area, people's health, disease resistance in the human body and many more. In the case of a large number of COVID-19 patients appear for treatment; the first aim is to find ways to save maximum lives. For this, some are suggesting that doctors should adopt those patients who could be recovered more easily, like the young population and those people who have maximum resistance in their body against any disease. The children and old people should be their second choice and then the patients of other diseases like diabetes, cancer and harmful disease should be their last choice so that maximum lives could be saved in this difficult time. Given the fact that there is no authentic data available to deal with the virus, every group has its own set of strategies to handle the COVID-19 patients. This creates uncertainty for decision makers and this makes it too much difficult for those groups to make the decision those are working against COVID-19. This is a decision-making problem. Our proposed method has ability to solve this type of sensitive decision-making problem according to the weightage of experts and major parameters for COVID-19.

4.1 | Proposed Methodology

Let $U = \{u_1, u_2, \dots, u_t\}$ be set of alternatives and $D = \{d_1, d_2, \dots, d_n\}$ the set of specialists with their weight vectors $\eta_i > 0$ such that $\sum_{i=1}^{n} \eta_i = 1$. Let $E = \{e_1, e_2, \dots, e_m\}$ set of constraints with the weight vectors are $\xi_k > 0$, satisfying $\sum_{k=1}^{m} \xi_k = 1$. The experts give their partialities afterward assessing the substitutions over the considered constraints in terms of q-ROFSNs α_{ik} for $(i = 1, 2, 3, \dots, n \;\&\; k = 1, 2, 3, \dots, m)$. There are five steps which are summarizing for resolving the decision-analysis problem in the q-ROFSS environment by the defined operator.

Step 1. According to each alternative collect the information in term of q-ROFSNs which is given in matrix form by the experts as follows:

$$\cdots \quad e_m e_1$$

$$M = \begin{array}{c} d_1 \\ \vdots \\ d_n \end{array}\begin{bmatrix} \alpha_{11} & \cdots & \alpha_{1m} \\ \vdots & \ddots & \vdots \\ \alpha_{n1} & \cdots & \alpha_{nm} \end{bmatrix}.$$

Step 2. We normalize the given decision matrix by converting the value of cost type (C) into profit type (B) where necessary, by using this formula

$$z_{ik} = \begin{cases} \alpha_{ik} & e_k \in B \\ {\alpha_{ik}}^c & e_k \in C \end{cases},$$

where ${\alpha_{ik}}^c$ of $\alpha_{ik} = \langle \mu_{ik}, \ \nu_{ik} \rangle$ is defined as ${\alpha_{ik}}^c = \langle \nu_{ik}, \ \mu_{ik} \rangle$.

Step 3. According to a given decision matrix aggregate the values (b_{K_i}) equivalent to each alternate by using the projected aggregation operator Wq-ROFSBM, specified in *Eq. (9)*. Here b_{K_i}denote the aggregated value of $K_{i.}$

Step 4. Compute the ordering of aggregated q-ROFSNs by the *Eqs. (1)-(3)*.

Step 5. In view of order, rank the alternatives.

4.2 | Numerical Approach to Propose Problem

From the start of 2020, COVID-19 has become a serious problem everywhere in the world. Each government is trying to overcome this epidemic disease but many problems are faced by each organization that is working against it. The main reason to overcome this disease is that large numbers of patients cannot be admitted to hospitals immediately. The hospitals are overloaded and the number of doctors and surgical instruments required to meet the situation is not proportional. So, in very short time clinical inspection of a huge number of peoples are not possible. But the problem is still that Coronavirus spreads quickly and sometimes people carry the virus without showing any symptoms. The spread of a pandemic disease depends on two effects; how many people in each case are infected and how many times it requires for the contamination among people to spread. The first term is called the imitation number and the other is the time interval. The minimum time interval-propagation of COVID-19 means emergent epidemics will grow quickly and could be hard to break. So, to overcome the problem that which type of steps are required to save the maximum number of people is a complex decision-making problem.

We suggest that a method in q-rung orthopair fuzzy soft setting has the potential to handle the uncertainty about this epidemic and could be useful in making the decision to save maximum lives. The boards of medical specialists decide to admit a patient in a hospital or quarantine for treatment. But the symptoms of COVID-19 are in the majority of people. The head of the medical board is a decision maker and he will decide to admit a suspect in the hospital by the recommendations of the medical experts. The suffered population is in different stages of their ages $A = \{ K_1 = 1 - 15 \; years \; old, K_2 = 16 - 25 \; years \; old, K_3 = 25 - 40 \; years \; old, \; K_4 = above \; 40\}$ will be inspected by doctors $D = \{d_1, d_2, d_3\}$ having weights vector $\eta = (0.3, 0.5, 0.2)^T$. The doctors recommend the patient according to some parameters $\{ p_1 = \text{infection in the respiratory system}, \; p_2 = \text{fever}, \; p_3 = \text{cough}, p_4 = \text{pneumonia} \}$ having the weights vector $\xi = (0.3, 0.1, 0.1, 0.5)^T$. Supposed on their recommendation the decision maker allocates bed in the hospital according to the seriousness of this harmful disease. In this regard, the doctors inspect the patients and give their preference in terms of q-ROFSNs. The main procedure of the proposed approach is given below in the following steps.

Table 2. q-ROFS matrix for K_1 .

(K_1, A)	p_1	p_2	p_3	p_4
d_1	$\langle 0.4, 0.3\rangle$	$\langle 0.5, 0.4\rangle$	$\langle 0.3, 0.4\rangle$	$\langle 0.7, 0.2\rangle$
d_2	$\langle 0.6, 0.2\rangle$	$\langle 0.4, 0.3\rangle$	$\langle 0.9, 0.5\rangle$	$\langle 0.8, 0.3\rangle$
d_3	$\langle 0.7, 0.4\rangle$	$\langle 0.3, 0.2\rangle$	$\langle 0.5, 0.4\rangle$	$\langle 0.6, 0.2\rangle$

Table 3. q-ROFS matrix for K_2 .

(K_2, A)	p_1	p_2	p_3	p_4
d_1	$\langle 0.6, 0.2\rangle$	$\langle 0.4, 0.3\rangle$	$\langle 0.4, 0.2\rangle$	$\langle 0.7, 0.2\rangle$
d_2	$\langle 0.7, 0.3\rangle$	$\langle 0.5, 0.2\rangle$	$\langle 0.5, 0.2\rangle$	$\langle 0.8, 0.1\rangle$
d_3	$\langle 0.8, 0.2\rangle$	$\langle 0.6, 0.7\rangle$	$\langle 0.6, 0.3\rangle$	$\langle 0.6, 0.2\rangle$

Table 4. q-ROFS matrix for K_3 .

(K_3, A)	p_1	p_2	p_3	p_4
d_1	$\langle 0.4, 0.3\rangle$	$\langle 0.5, 0.4\rangle$	$\langle 0.4, 0.3\rangle$	$\langle 0.6, 0.1\rangle$
d_2	$\langle 0.5, 0.3\rangle$	$\langle 0.3, 0.2\rangle$	$\langle 0.5, 0.1\rangle$	$\langle 0.7, 0.2\rangle$
d_3	$\langle 0.7, 0.4\rangle$	$\langle 0.3, 0.2\rangle$	$\langle 0.5, 0.4\rangle$	$\langle 0.6, 0.2\rangle$

Table 5. q-ROFS matrix for K_4 .

(K_4, A)	p_1	p_2	p_3	p_4
d_1	$\langle 0.3, 0.2\rangle$	$\langle 0.6, 0.2\rangle$	$\langle 0.6, 0.2\rangle$	$\langle 0.4, 0.3\rangle$
d_2	$\langle 0.4, 0.1\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.4, 0.7\rangle$	$\langle 0.6, 0.2\rangle$
d_3	$\langle 0.5, 0.3\rangle$	$\langle 0.8, 0.3\rangle$	$\langle 0.9, 0.2\rangle$	$\langle 0.7, 0.4\rangle$

Step 1. The doctors give their evaluation values to each alternate $K_i = (i = 1, 2, 3, 4)$ under inspection along with the set of parameters $\{p_1, p_2, p_3, p_4\}$ in terms of q-ROFSNs and given in matrix *Tables 2* to *5*.

Step 2. All recommendations given by the doctors are of the same type, so here no need to normalize the data.

Step 3. According to given values, we aggregate the values consistent to each alternate by using the projected aggregation mean Wq-ROFSBM, given in *Eq. (9)*, for q=3 and $\lambda_1 = \lambda_2 = 2$. We get $b_{K_1} = \langle 0.4523,\ 0.4915\rangle, b_{K_2} = \langle 0.6415,\ 0.0271\rangle, b_{K_3} = \langle 0.4699,\ 0.0740\rangle$, and $b_{K_4} = \langle 0.2943,\ 0.7496\rangle$.

Step 4. Compute the score function of each aggregated number by the *Eq. (1)* as follows:

$$S(b_{K_1}) = 0.4868, S(b_{K_2}) = 0.6319, S(b_{K_3}) = 0.5516 \ \& \ S(b_{K_4}) = 0.3020.$$

Step 5. Given calculated score function, the ordering of the alternatives is as follows $, K_2 > K_3 > K_1 > K_4$where " $>$ " denoted the preference for alternatives. By this one can conclude that the best alternative is K_2, so the population lies in K_2 can get recover in a shorter time than other alternatives. Hence, by giving the preference to K_2 maximum lives could be saved.

Our proposed Wq-ROFSBM has a potential to capture the desired properties among arguments interrelationship in such problems which cannot be handled with weighted intuitionistic or weighted Pythagorean fuzzy soft Bonferroni mean operators. For an instant, in many daily life problems in which uncertainty occurs in such a way that the total of membership-degree and non-membership degrees are greater than 1. Also, as the values of $\lambda_1$and λ_2 increase, the scores of each alternate by Wq-ROFSBM operator will also rise. But the grade order of given alternatives remains the same in each case. Moreover, in special cases of Wq-ROFSBM in which one of λ_1 or λ_2 becomes zero, then Wq-ROFSBM operator cannot imprisonment the interrelation between the arguments. For expediency, we take the value of $\lambda_1$and λ_2 equal for the sake of simplicity in the calculation process. Thus, the governing person can choose the suitable alternate easily rendering to the situations in the decision-making process.

5 | Comparative Study

To show the significance and impact of the proposed approach in this paper, a comparison with comparable existing methods is presented. Note that different aggregation operators considered in [22] and [26] fail when the assigned value of membership of an element is 0.9 and non-membership is 0.7 because 0.9 + 0.7 > 1. So, in this case, Bonferroni means operators (aggregation operators) do not serve the purpose. The same is true for the operators given in [10] and [19]. Moreover, the operators defined in [10] and [19] become the special case of our proposed operator defined in *Eq. (9)* if we take $q = 1$, $\lambda_1 = 1$ and $\lambda_2 = 0$. Similarly, for $q = 2$, $\lambda_1 = 1$ and $\lambda_2 = 0$, the operator defined in [14] is a special case of our proposed operator. For $\lambda_1 = 1$ and $\lambda_2 = 0$, our operator reduced to an operator given in [16]. Moreover, operators defined in most of the literature mentioned above and in [24], are without prior knowledge of characterization of the individuals and hence are not appropriate to deal with imprecise data for a decision maker. The operator given in [38] is based on parameterization. However, it reduces to our proposed operator if we take $\lambda_1 = 1$ and $\lambda_2 = 0$ in the *Eq. (9)*. In a decision-making process when the arguments are interrelated to each other, the Bonferroni mean can capture the interrelationship of the individual arguments but the operators defined in [38] fail to deal with such situation. The advantage of operators presented in this paper lies in the fact that they have a potential to solve real-life problems by using their parameterizations properties when the arguments of the individuals are interrelated to each other. Hence, the developed concepts are more appropriate for solving the decision-making problems than existing operators in the setting of q-rung orthopair FSSs.

6 | Conclusions

The q-ROFSS theory is a very effective tool for considering the imprecision and uncertainty problems under consideration by the expert's recommendations and parameterization factors involved in any phenomena. We proposed aggregation operators namely Bonferroni mean and Weighted Bonferroni Mean (WBM) aggregation operator for aggregating the data in real-life problems in which sensitivity of decision is very important such as arising in the face of epidemic of COVID-19. In such sensitive problems, uncertainties occur at a very high level, so to deal with such decision-making process, the more generalized version of the existing operators are introduced. The study carried out in this direction is supported with the help of examples. Since the Bonferroni mean is a very useful tool for group decision-making problems when arguments are interrelated to each other as Bonferroni mean can capture the interrelationship of the individual arguments. For future research, one may use the defined q-rung orthopair Bonferroni fuzzy soft matrix and weighted q-rung orthopair fuzzy Bonferroni fuzzy soft matrices for data representation which could provide the optimum q-rung orthopair fuzzy soft constant. These constants then can be used to define q-rung orthopiar fuzzy soft differential equations which provide an effective way in decision making problems. Moreover, by utilizing the q-rung orthopair fuzzy soft constants, one can develop the system of q-rung orthopair fuzzy soft differential equations to study the dynamical process with nonlinear uncertain and vague data.

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