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Basic Fuzzy Arithmetic Operations Using α–Cut for the Gaussian Membership Function

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Abstract

Currently fuzzy set theory has a wide range to model real life problems with incomplete or vague information which perfectly suits the reality and its application is theatrically increasing. This work explored the basic fuzzy operations with the Gaussian Membership using the α-cut method. As it is known that, the Gaussian membership function has a great role in modelling the fuzzy problems this is what impelled to explore its operation which can further be used in analysis of fuzzy problems. Primarily the basic operations which has been discussed here are addition, subtraction, multiplication, division, reciprocal, exponential, logarithmic and nth power.

Keywords: Fuzzy arithmetic, α–cut and Gaussian membership function.

1 | Introduction

Over the recent years, since its foundation by Zadeh [13] fuzzy sets and logic has been usefully in solving the real life problems which has partial or vague information which has wider scope of application and has dramatically increased in 1990's in area of decision making problem till this time for instance in the field like of pattern classification and information processing [7]. In the future by studying effectually and reconnoitering fuzzy implications can be the step towards the simulation of human thinking [3].

Furthermore, as fuzzy can be applied in different other areas like that of optimization which were specifically done by Shirin [9] in their work they proposed optimization solution to the problem by computing using three methods, which were the Bellman-Zadeh's method, Zimmerman's method, and fuzzy version of Simplex method, are compared to each other. Others are Tang et al. [12] and Sahayasudha and Vijayalakshmi [11] who did in fuzzy optimization and transportation problem, respectively.

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Largely, fuzzy arithmetic operation has a great role in the analysis of fuzzy problems a number of works has been done involving fuzzy arithmetic operations like that of Bobillo and Straccia [1], Dutta et al. [4], Mazeika et al. [7], Stefanini et al. [10], Sahayasudha and Vijayalakshmi [11], Raju and Jayagopal [8] and Iliadis et al. [6] but almost all of them did not use the approach of operations using α–cut for the Gaussian membership function in their works.

Other work is that of Hassanzadeh et al. [5] in their paper titled "an α-cut approach for fuzzy product and its use in computing solutions of fully fuzzy linear systems" whereby they used a regression model to obtain the membership function of the product.

The gaussian membership function has now been used in solving the fuzzy problems, for instance Mazeika et al. [7] in their work they used trapezoidal and Gaussian membership function and introduced a new general set approach to compute fuzzy sets based on interval analysis. They protracted their work techniques to handle multidimensional continuous membership functions, also Bundy and Wallen [2], Iliadis et al. [6] used gaussian distribution in their papers, whereby in this work we showed the Gaussian membership function using α–cut which can operated to both the continuous and discrete fuzzy set.

The core principle of the fuzzy set is the construction of the membership function and its operations in the fuzzy environment. Therefore, for this work focused on the basic arithmetic operation using alpha cut for the Gaussian function as this approach will be usefully for other scientists in handling and analyzing the real-life problem in the fuzzy environment.

2 | Basic Concepts and Notations

2.1 | Fuzzy Set and Membership Function

Let X be a non-empty crisp set, a fuzzy set A is define as set of pairs $A = \{(x, \mu_A(x))\}$, where $x \in X$ and $\mu_A(x)$ is membership value for the corresponding crisp value $x \in X$ which is defined by a membership function as $\mu_A\colon X \to [0,1]$ [13].

There are many membership functions so far which has been commonly used are triangular function, trapezoidal function, singleton function, L–function, gamma function, S–function, Gaussian function, Sigmoidal and Pseudo–Exponential function [14].

2.2 | Normal Fuzzy Set

A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exists at least one $x \in X$ such that $\mu_A(x) = 1$.

2.3 | Support

The support of a fuzzy set A defined on X is a crisp set defined as

$$\text{Support(A)} = x\epsilon X\colon \mu_A(x) > 0.$$

2.4 | Fuzzy Number

A fuzzy set A defined on the set of real numbers $\mathbb{R}$ is said to be a fuzzy number if its membership function: $\mu_A\colon \mathbb{R} \to [0,1]$ has the following properties

I. A must be a normal fuzzy set.
II. A_α must be a closed interval for every a $\alpha \in (0,1]$.
III. The support of A, must be bounded.

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2.5 | Gaussian Function with an Example

The Gaussian membership function of a crisp set A of a non-empty universal set X is defined by

$$\mu_A(x) = \exp\left[\frac{-(x-m)^2}{2\sigma^2}\right]. \tag{1}$$

For all $x \in X$ or generally $x \in \mathbb{R}$ as the crisp set, m can be taken as the mid value or mean and $\sigma > 0$ can be taken as the standard deviation of the crisp set. This Gaussian function will take a bell-shaped curve and the smaller σ the narrower the bell [14].

For example, to express a specific Gaussian membership function for the following discrete crisp set B which is chosen arbitrary as, $B = 2,3,5,9,8,14$, using the data for set B here we can find $m = 6.8$ as mean and $\sigma = 4.1$ as standard deviation therefore the membership function will be

$$\mu_B(x) = \exp\left[\frac{-(x-6.8)^2}{2 \times 4.1^2}\right]. \tag{2}$$

Solving using the given Gaussian membership function then we can have the fuzzy set which will be as $\beta = \{(2, 0.50), (3, 0.65), (5, 0.91), (8, 0.96), (9, 0.87), (14, 0.21)\}$.

Generally, the above Gaussian membership geometrically as continuous can be seen as shown:

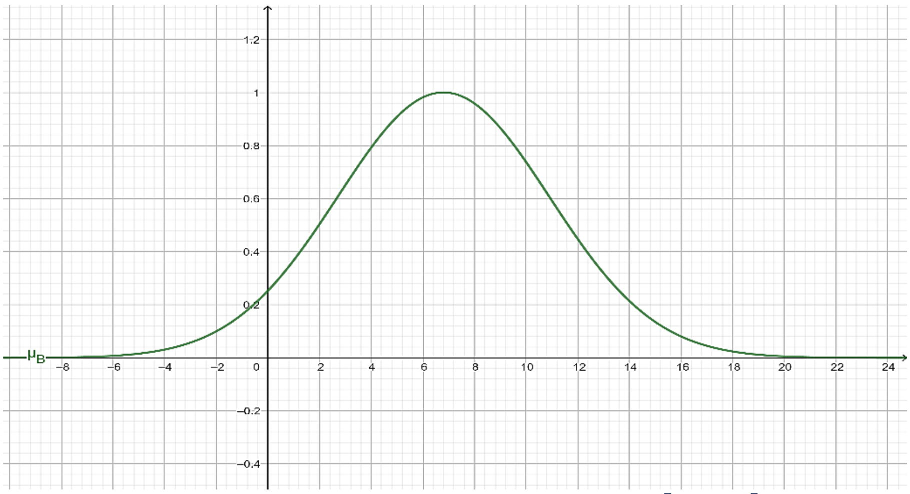

Fig. 1. Gaussian membership for $\mu_B(x) = \exp\left[\frac{-(x-6.8)^2}{2\times 4.1^2}\right]$.

2.6 | Alpha (α)-Cut

Let X be a non- empty crisp set, an α–cut for a given fuzzy set A denoted by A_α is defined as the crisp set of all elements of A whose membership grades are greater than α,

$$\forall\ \alpha \in (0,1], \quad \text{that is } A_\alpha = \{x \in X | \mu_A(x) \geq \alpha\}. \tag{3}$$

Now from the Gaussian membership function given above we can find the alpha-cut as follows:

$$\mu_A(x) = \exp\left[\frac{-(x-m)^2}{2\sigma^2}\right] \geq \alpha. \tag{4}$$

To simplify the calculations, we have taken the alpha – cut for equality, that is

$$\exp\left[\frac{-(x-m)^2}{2\sigma^2}\right] = \alpha. \tag{5}$$

Then we solve for x to obtain the α–cut for the corresponding fuzzy set, apply logarithm throughout we have

$$\left[\frac{-(x-m)^2}{2\sigma^2}\right] = \ln\alpha. \tag{6}$$

$$x = m + \sigma\sqrt{-2\ln\alpha}.$$

Simplifying and taking the term with plus we have,

Therefore, the α – cut will be given by

$$A_\alpha = m + \sigma\sqrt{-2\ln\alpha}. \tag{7}$$

3 | Results and Discussions

Here we have explored all the basic operations and showing in detail how to find the membership of all the operations with α–cut using the Gaussian Membership function, the operations discussed here are addition, subtraction, multiplication, division, reciprocal, exponential, logarithmic and nth power. The graphs to help the analysis were drawn using GeoGebra Calculator suite for graphing software.

3.1 | Basic Operations

3.1.1 | Addition

Let the fuzzy sets A and B with their corresponding membership as $\mu_A(x) = \exp\left[\frac{-(x-m_A)^2}{2\sigma_A{}^2}\right]$, $\mu_B(x) = exp\left[\frac{-(x-m_B)^2}{2\sigma_B{}^2}\right]$ respectively Solving the α–cut for the two fuzzy set we have, $A_\alpha = m_A + \sigma_A\sqrt{-2\,ln\,\alpha}$ and $B_\alpha = m_B + \sigma_B\sqrt{-2\,ln\,\alpha}$ respectively for fuzzy set A and B.

By adding we have, $A_\alpha + B_\alpha = m_A + \sigma_A\sqrt{-2\,ln\,\alpha} + m_B + \sigma_B\sqrt{-2\,ln\,\alpha}$.

Upon simplifying we have, $A_\alpha + B_\alpha = (m_A + m_B) + (\sigma_A + \sigma_B)\sqrt{-2\ln\alpha}$.

Now to get the membership for addition we have to let $x = A_\alpha + B_\alpha$ and solve for α, that is $x = (m_A + m_B) + (\sigma_A + \sigma_B)\sqrt{-2\ln\alpha}$, then $\ln\alpha = \left(\frac{-(x-(m_A+m_B))^2}{2(\sigma_A+\sigma_B)^2}\right)$ henceforth, we have $\alpha = exp\left(\frac{-(x-(m_A+m_B))^2}{2(\sigma_A+\sigma_B)^2}\right)$, lastly the membership will be

$$\mu_{(A+B)}(x) = \exp\left(\frac{-(x-(m_A+m_B))^2}{2(\sigma_A+\sigma_B)^2}\right)\ \forall x \in \mathbb{R}. \tag{8}$$

For example, if we choose arbitrary the specific values for $\sigma_A = 1$, $\sigma_B = 2$, $m_A = 10$ and $m_B = 20$, therefore we will have the membership as, $\mu_A(x) = exp\left(\frac{-(x-10)^2}{2(1)^2}\right)$, $\mu_B(x) = exp\left(\frac{-(x-20)^2}{2(2)^2}\right)$ and $\mu_{(A+B)}(x) = exp\left(\frac{-\left(x-(10+20)\right)^2}{2(1+2)^2}\right)$. On the same axes their graphs will be see as in the *Fig. 2* below ($\mu_A(x)$ in green, $\mu_B(x)$ in Blue and $\mu_{(A+B)}(x)$ in Red).

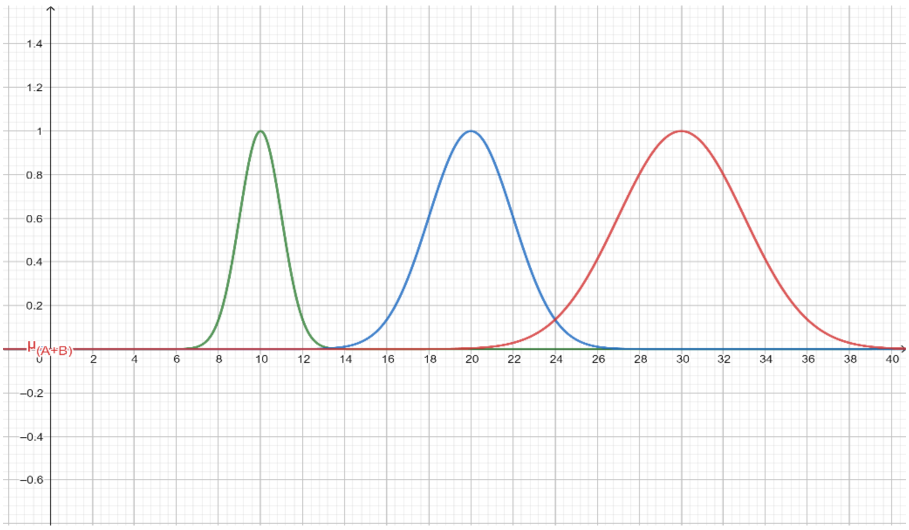

Fig. 2. Graphs of $\mu_A(x) = \exp\left(\frac{-(x-10)^2}{2(1)^2}\right)$, $\mu_B(x) = \exp\left(\frac{-(x-20)^2}{2(2)^2}\right)$ and $\mu_{(A+B)}(x) = \exp\left(\frac{-\left(x-(10+20)\right)^2}{2(1+2)^2}\right)$.

3.2 | Subtraction

Let the fuzzy sets A and B with their corresponding membership as $\mu_A(x) = exp\left[\frac{-(x-m_A)^2}{2\sigma_A{}^2}\right]$, $\mu_B(x) = exp\left[\frac{-(x-m_B)^2}{2\sigma_B{}^2}\right]$, respectively.

Solving the α–cut for the two fuzzy set we have, $A_\alpha = m_A + \sigma_A\sqrt{-2\,ln\,\alpha}$ and $B_\alpha = m_B + \sigma_B\sqrt{-2\,ln\,\alpha}$ respectively for fuzzy set A and B.

By subtracting we have, $A_\alpha - B_\alpha = m_A + \sigma_A\sqrt{-2\,ln\,\alpha} - (m_B + \sigma_B\sqrt{-2\,ln\,\alpha})$.

Upon simplifying we have, $A_\alpha - B_\alpha = (m_A - m_B) + (\sigma_A - \sigma_B)\sqrt{-2\,ln\,\alpha}$.

Now to get the membership for subtraction we have to let $x = A_\alpha - B_\alpha$ and solve for α, that is $x = (m_A - m_B) + (\sigma_A - \sigma_B)\sqrt{-2\,ln\,\alpha}$, then $ln\,\alpha = \left(\frac{-(x-(m_A-m_B))^2}{2(\sigma_A-\sigma_B)^2}\right)$ hence, we have $\alpha = exp\left(\frac{-(x-(m_A-m_B))^2}{2(\sigma_A-\sigma_B)^2}\right)$, lastly the membership will be

$$\mu_{(A+B)}(x) = \exp\left(\frac{-(x-(m_A-m_B))^2}{2(\sigma_A-\sigma_B)^2}\right) \forall x \in \mathbb{R}. \qquad (9)$$

For example, if we choose arbitrary the specific values for $\sigma_A = 1$, $\sigma_B = 2$, $m_A = 10$ and $m_B = 20$, therefore we will have the membership as, $\mu_A(x) = exp\left(\frac{-(x-10)^2}{2(1)^2}\right)$, $\mu_B(x) = exp\left(\frac{-(x-20)^2}{2(2)^2}\right)$ and $\mu_{(A-B)}(x) = exp\left(\frac{-\left(x-(10-20)\right)^2}{2(1-2)^2}\right)$.

On the same axes their graphs will be see as in the *Fig. 3* below $\mu_A(x)$ (in green, $\mu_B(x)$ in Blue and $\mu_{(A-B)}(x)$ in Red).

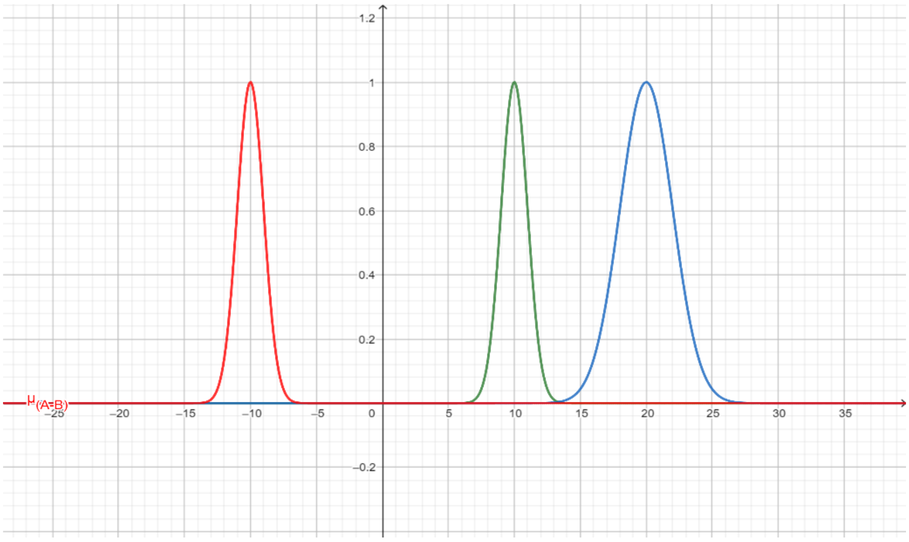

Fig. 3. Graphs of $\mu_A(x) = \exp\left(\frac{-(x-10)^2}{2(1)^2}\right)$, $\mu_B(x) = \exp\left(\frac{-(x-20)^2}{2(2)^2}\right)$ and $\mu_{(A-B)}(x) = \exp\left(\frac{-\left(x-(10-20)\right)^2}{2(1-2)^2}\right)$.

3.3 | Multiplication

For the fuzzy sets A and B with their corresponding membership as $\mu_A(x) = exp\left[\frac{-(x-m_A)^2}{2\sigma_A{}^2}\right]$, $\mu_B(x) = exp\left[\frac{-(x-m_B)^2}{2\sigma_B{}^2}\right]$, respectively.

Solving the $\alpha-$ cut for the two fuzzy set we have $A_\alpha = m_A + \sigma_A\sqrt{-2\ln\alpha}$ and $B_\alpha = m_B + \sigma_B\sqrt{-2\ln\alpha}$ respectively for fuzzy set A and B.

Now, multiplication $A_\alpha * B_\alpha = (m_A + \sigma_A\sqrt{-2\ln\alpha}) * (m_B + \sigma_B\sqrt{-2\ln\alpha})$

Let $x = (m_A + \sigma_A\sqrt{-2\ln\alpha}) * \left(m_B + \sigma_B\sqrt{-2\ln\alpha}\right)$.

$$x = (m_A m_B) + (m_B\sigma_A + m_A\sigma_B) * \left(\sqrt{-2\ln\alpha}\right) + (\sigma_A\sigma_B) * \left(\sqrt{-2\ln\alpha}\right)^2. \tag{10}$$

Upon simplifying and making α the subject we have

$$\alpha = \exp\left(-\frac{1}{2}\left(\frac{(m_B\sigma_A + m_A\sigma_B) \pm \sqrt{(m_B\sigma_A + m_A\sigma_B)^2 + 4(\sigma_A\sigma_B)*(x-(m_A m_B))}}{(2\sigma_A\sigma_B)}\right)^2\right). \tag{11}$$

Therefore,

$$\mu_{(A\times B)}(x) = \exp\left(-\frac{1}{2}\left(\frac{(m_B\sigma_A + m_A\sigma_B) + \sqrt{(m_B\sigma_A + m_A\sigma_B)^2 + 4(\sigma_A\sigma_B)*(x-(m_A m_B))}}{(2\sigma_A\sigma_B)}\right)^2\right). \tag{12}$$

or

$$\mu_{(A\times B)}(x) = \exp\left(-\frac{1}{2}\left(\frac{(m_B\sigma_A + m_A\sigma_B) - \sqrt{(m_B\sigma_A + m_A\sigma_B)^2 + 4(\sigma_A\sigma_B)*(x-(m_A m_B))}}{(2\sigma_A\sigma_B)}\right)^2\right). \tag{13}$$

For example, if we choose arbitrary the specific values for $\sigma_A = 1$, $\sigma_B = 2$, $m_A = 10$ and $m_B = 20$, therefore we will have the membership as, $\mu_A(x) = exp\left(\frac{-(x-10)^2}{2(1)^2}\right)$ and

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$\mu_{(A\times B)}(x) = exp\left(-\frac{1}{2}\left(\frac{(20\times1+10\times2)+\sqrt{(20\times1+10\times2)^2+4(2\times1)(x-(10\times20))}}{(2\times1\times2)}\right)^2\right)$ Simplifying, $\mu_{(A\times B)}(x) = exp\left(-\frac{1}{2}\left(\frac{40+\sqrt{8x}}{4}\right)^2\right)$.

On the same axes their graphs will be see as in the *Fig. 4* below $\mu_A(x)$ in green $\mu_{(A\times B)}(x)$ in Blue.

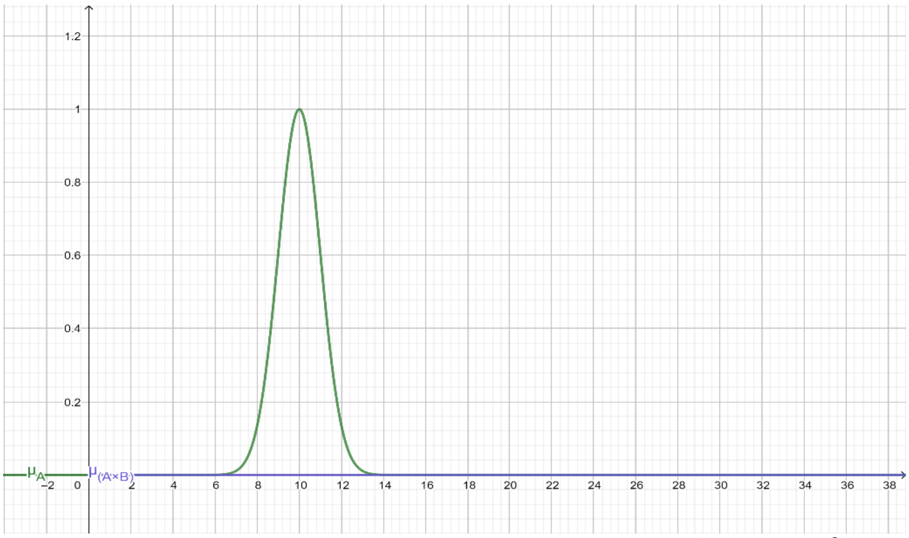

Fig. 4. Graphs $\mu_A(x) = \exp\left(\frac{-(x-10)^2}{2(1)^2}\right)$ and $\mu_{(A\times B)}(x) = \exp\left(-\frac{1}{2}\left(\frac{40+\sqrt{8x}}{4}\right)^2\right)$.

3.4 | Division

For the fuzzy sets A and B with their corresponding membership as $\mu_A(x) = exp\left[\frac{-(x-m_A)^2}{2\sigma_A{}^2}\right]$, $\mu_B(x) = exp\left[\frac{-(x-m_B)^2}{2\sigma_B{}^2}\right]$, respectively.

Solving the α–cut for the two fuzzy set we have, $A_\alpha = m_A + \sigma_A\sqrt{-2\ln\alpha}$ and $B_\alpha = m_B + \sigma_B\sqrt{-2\,ln\,\alpha}$ respectively for fuzzy set A and B. Dividing we have

$$\frac{A_\alpha}{B_\alpha} = \frac{m_A+\sigma_A\sqrt{-2\ln\alpha}}{m_B+\sigma_B\sqrt{-2\ln\alpha}}. \tag{14}$$

$$\text{Let } x = \frac{m_A+\sigma_A\sqrt{-2\ln\alpha}}{m_B+\sigma_B\sqrt{-2\ln\alpha}}.$$

Then, $x\left(m_B + \sigma_B\sqrt{-2\,ln\,\alpha}\right) = m_A + \sigma_A\sqrt{-2\,ln\,\alpha}$

Simplifying and making ∝ the subject we have,

$$\mu_{\frac{A}{B}}(x) = \exp\left(\frac{-(xm_B - m_A)^2}{(\sigma_A - \sigma_B)^2}\right). \tag{15}$$

For example, if we choose arbitrary the specific values for $\sigma_A = 1$, $\sigma_B = 2$, $m_A = 10$ and $m_B = 20$, therefore we will have the membership as, $\mu_A(x) = exp\left(\frac{-(x-10)^2}{2(1)^2}\right)$, $\mu_B(x) = exp\left(\frac{-(x-20)^2}{2(2)^2}\right)$ and $\mu_{\frac{A}{B}}(x) = exp\left(\frac{-(20x-10)^2}{(1)^2}\right)$. On the same axes their graphs will be see as in the *Fig. 5* below $\mu_A(x)$ in blue, $\mu_B(x)$ in Green and $\mu_{\left(\frac{A}{B}\right)}(x)$ in Red).

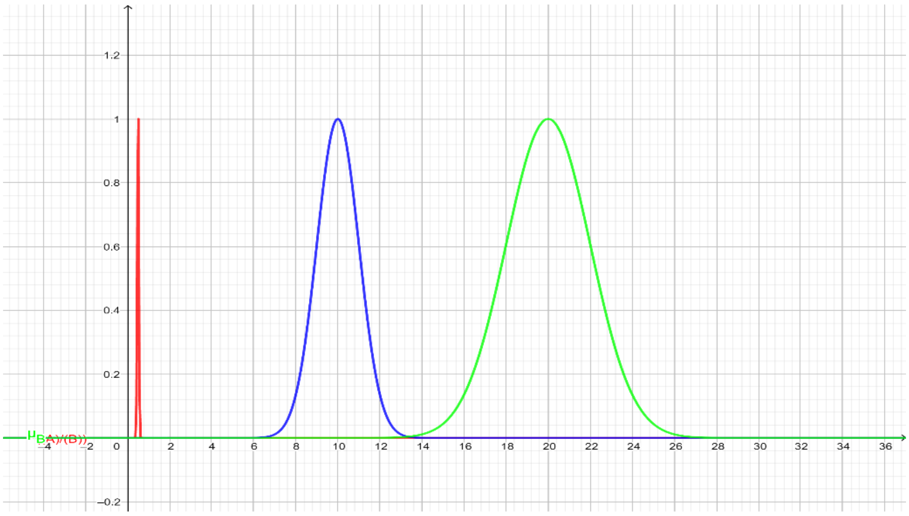

Fig. 5. Graphs of $\mu_A(x)$, $\mu_B(x)$ and $\mu_{(A/B)}(x)$.

3.5 | Reciprocal

For the fuzzy sets A with membership as $\mu_A(x) = exp\left[\frac{-(x-m_A)^2}{2\sigma_A{}^2}\right]$, by solving the α – cut for the given fuzzy set we have, $A_\alpha = m_A + \sigma_A\sqrt{-2\ln\alpha}$.

The reciprocal membership can be found as $\frac{1}{A_\alpha} = \frac{1}{m_A+\sigma_A\sqrt{-2\ln\alpha}}$. Then let $x = \frac{1}{m_A+\sigma_A\sqrt{-2\ln\alpha}}$,

$$\mathrm{x}m_A + \mathrm{x}\sigma_A\sqrt{-2\ln\alpha} = 1. \tag{16}$$

Making α the subject we have

$$\alpha = \exp\left(-\frac{1}{2}\left(\frac{\mathrm{x}m_A - 1}{\mathrm{x}\sigma_A}\right)^2\right). \tag{17}$$

Therefore, the reciprocal membership will be written as

$$\mu_{A^{-1}} = \exp\left(-\frac{1}{2}\left(\frac{\mathrm{x}m_A - 1}{\mathrm{x}\sigma_A}\right)^2\right). \tag{18}$$

For example, if we choose arbitrary the specific values for $\sigma_A = 1$ and $m_A = 10$ therefore we will have the membership as $\mu_A(x) = exp\left(\frac{-(x-10)^2}{2(1)^2}\right)$, and $\mu_{A^{-1}} = exp\left(-\frac{1}{2}\left(\frac{10x-1}{x}\right)^2\right)$.

On the same axes their graphs will be see as in the *Fig. 6* below $\mu_A(x)$ in Green and $\mu_{A^{-1}}$ in Blue.

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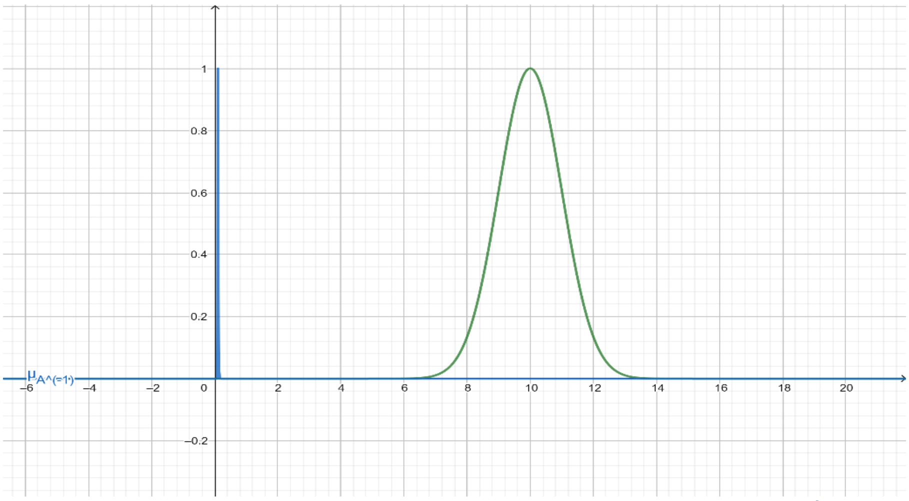

Fig. 6. Graphs of , $\mu_A(x) = \exp\left(\frac{-(x-10)^2}{2(1)^2}\right)$ and $\mu_{A^{-1}} = \exp\left(-\frac{1}{2}\left(\frac{10x-1}{x}\right)^2\right)$.

3.6 | Exponential

For the fuzzy sets A with membership as $\mu_A(x) = \exp\left[\frac{-(x-m_A)^2}{2\sigma_A{}^2}\right]$, by solving the α – cut for the given fuzzy set we have, $A_\alpha = m_A + \sigma_A\sqrt{-2\ln\alpha}$. Then apply exponential throughout, that is $exp\,(A_\alpha) = exp\,(m_A + \sigma_A\sqrt{-2\,ln\,\alpha})$.

Let $x = \exp\,(m_A + \sigma_A\sqrt{-2\ln\alpha})$.

Solve for α, that is $ln\,x = m_A + \sigma_A\sqrt{-2\,ln\,\alpha}$, then we have the member ship as

$$\mu_{e^A}(x) = \exp\left(-\frac{1}{2}\left(\frac{\ln x - m_A}{\sigma_A}\right)^2\right). \tag{19}$$

For example, if we choose arbitrary the specific values for $\sigma_A = 1$ and $m_A = 5$, therefore we will have the membership as, $\mu_A(x) = exp\left(\frac{-(x-5)^2}{2(1)^2}\right)$ and $\mu_{e^A}(x) = exp\left(-\frac{1}{2}\left(\frac{(ln\,x) - 5}{1}\right)^2\right)$.

On the same axes their graphs will be see as in the *Fig. 7*, $\mu_A(x)$ in Green and $\mu_{e^A}(x)$ in Red color.

3.7 | Logarithmic

For the fuzzy sets A with membership as $\mu_A(x) = exp\left[\frac{-(x-m_A)^2}{2\sigma_A{}^2}\right]$, by solving the α–cut for the given fuzzy set we have, $A_\alpha = m_A + \sigma_A\sqrt{-2\ln\alpha}$. Apply logarithm throughout, we have $ln\ A_\alpha = ln\left(m_A + \sigma_A\sqrt{-2\ln\alpha}\right)$.

Then, let x = $ln\left(m_A + \sigma_A\sqrt{-2\ln\alpha}\right)$, from this make α the subject.

Apply exponential throughout we have

$$e^x = \left(m_A + \sigma_A\sqrt{-2\ln\alpha}\right). \tag{20}$$

$$-2\ln\alpha = \left(\frac{e^x - m_A}{\sigma_A}\right)^2. \tag{21}$$

Then,

$$\alpha = \exp\left(-\frac{1}{2}\left(\frac{e^x - m_A}{\sigma_A}\right)^2\right). \tag{22}$$

Therefore, the membership function for logarithm is given by

$$\mu_{\ln(A)}(x) = \exp\left(-\frac{1}{2}\left(\frac{e^x - m_A}{\sigma_A}\right)^2\right). \tag{23}$$

For example, if we choose arbitrary the specific values for $\sigma_A = 1$ and $m_A = 10$, therefore we will have the membership as, $\mu_A(x) = exp\left(\frac{-(x-10)^2}{2(1)^2}\right)$ and $\mu_{ln(A)}(x) = exp\left(-\frac{1}{2}\left(\frac{e^x-10}{1}\right)^2\right)$.

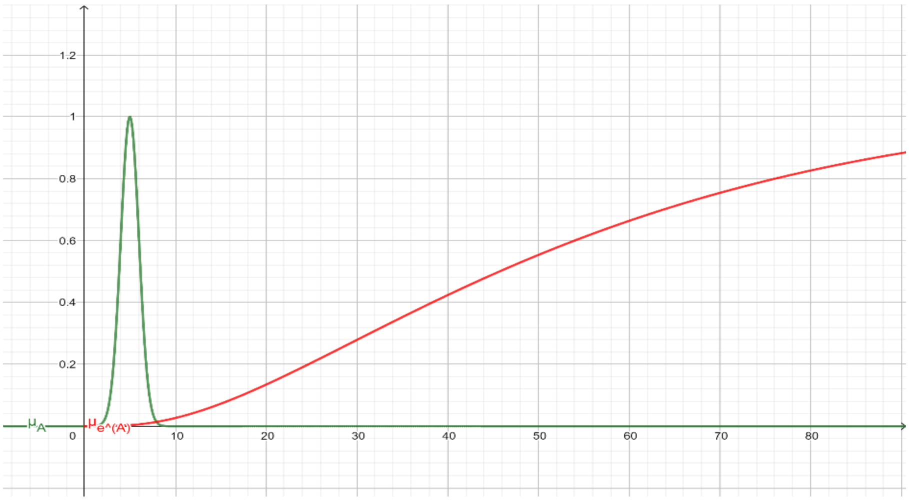

Fig. 7. Graphs of $\mu_A(x) = \exp\left(\frac{-(x-5)^2}{2(1)^2}\right)$ and $\mu_{e^A}(x) = \exp\left(-\frac{1}{2}\left(\frac{\ln x - 5}{1}\right)^2\right)$.

On the same axes their graphs will be see as in the *Fig. 8* below $\mu_A(x)$ in Green and $\mu_{ln\,A}(x)$ in Red.

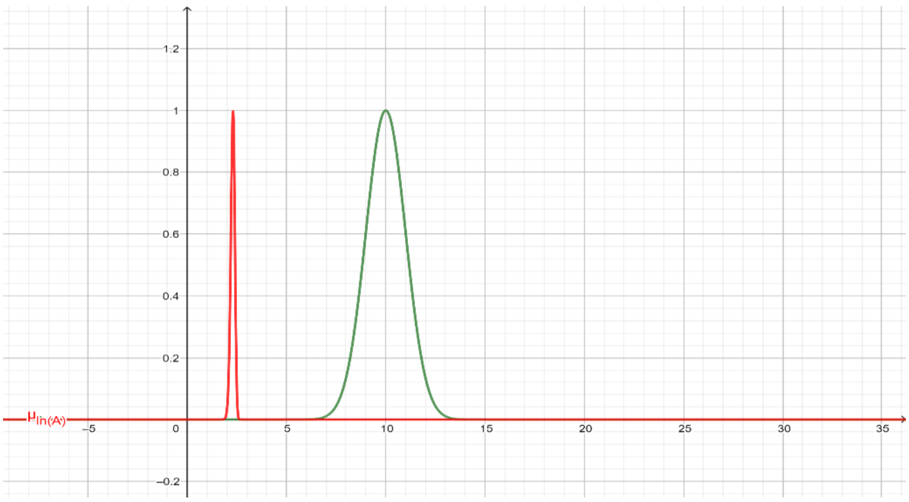

Fig. 8. Graphs of $\mu_A(x) = \exp\left(\frac{-(x-10)^2}{2(1)^2}\right)$ and $\mu_{\ln(A)}(x) = \exp\left(-\frac{1}{2}\left(\frac{e^x - 10}{1}\right)^2\right)$.

3.8 | The nth power

The n[th] power can be used to solve the fuzzy sets A with membership as $\mu_A(x) = \exp\left[\frac{-(x-m_A)^2}{2\sigma_A{}^2}\right]$,

Solving the α–cut for the given fuzzy set we have, $A_\alpha = m_A + \sigma_A\sqrt{-2\,ln\,\alpha}$.

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For nth power we have, $A_\alpha{}^n = \left(m_A + \sigma_A\sqrt{-2\ln\alpha}\right)^n \ \forall n \epsilon \mathbb{R}$.

Letting $x = A_\alpha{}^n$, that is

$$x = \left(m_A + \sigma_A\sqrt{-2\ln\alpha}\right)^n \ \forall n \epsilon \mathbb{R}. \tag{24}$$

Simplifying we have, $\sqrt[n]{x} = \left(m_A + \sigma_A\sqrt{-2\ln\alpha}\right) \ \forall n \epsilon \mathbb{R}$.

Then making α the subject we get

$$\mu_{(A^n)}(x) = \exp\left(-\frac{1}{2}\left(\frac{\sqrt[n]{x} - m_A}{\sigma_A}\right)^2\right) \ \forall n \epsilon \mathbb{R}. \tag{25}$$

For example, if we choose arbitrary the specific values for $\sigma_A = 1$, $m_A = 10$ and $n = 1.3$ therefore we will have the membership as, $\mu_A(x) = exp\left(\frac{-(x-10)^2}{2(1)^2}\right)$ and $\mu_{(A^n)}(x) = exp\left(-\frac{1}{2}\left(\frac{\sqrt[1.3]{x} - 10}{1}\right)^2\right)$.

On the same axes their graphs will be see as in the *Fig. 9* below $\mu_A(x)$ in Green and $\mu_{(A^n)}(x)$ in Red.

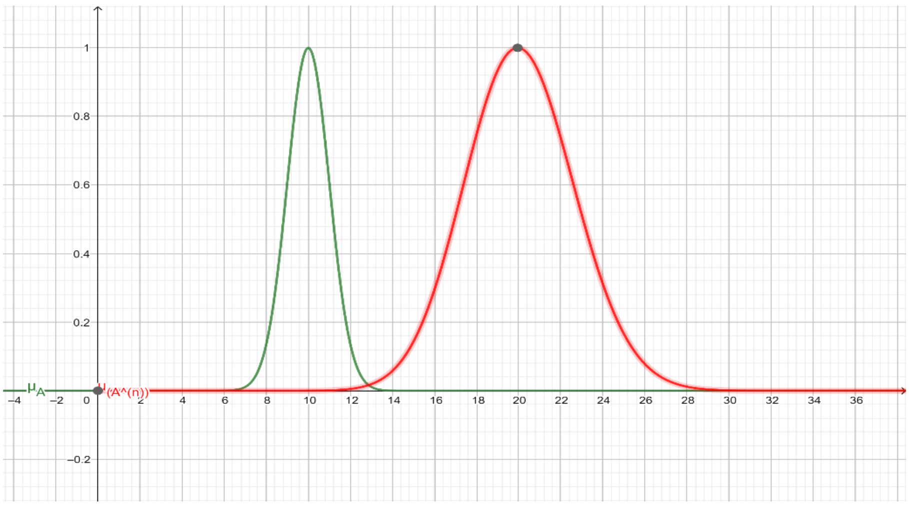

Fig. 9. Graphs of $\mu_A(x) = \exp\left(\frac{-(x-10)^2}{2(1)^2}\right)$ and $\mu_{(A^{1.3})}(x) = \exp\left(-\frac{1}{2}\left(\frac{\sqrt[1.3]{x} - 10}{1}\right)^2\right)$.

4 | Conclusions

We have explored the basic operations which are, addition, subtraction, multiplication, division, logarithm, reciprocal, exponential and nth power for the fuzzy set with Gaussian membership function by using the alpha–cut. The sightseen operations can further be used in analysis of fuzzy sets with Gaussian Membership and using the alpha-cut method make things easier for the calculations of all the basic operations. Therefore, we propose this approach to be used when analyzing the fuzzy problem with Gaussian Membership.

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