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Supply Chain Management Problem Modelling in Hesitant Fuzzy Environment

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Abstract

Complex nature of the current market is often caused by uncertainties, data uncertainties, their manner of use, and differences in managers' viewpoints. To overcome these problems, Hesitant Fuzzy Sets (HFSs) can be useful as the extension of fuzzy set theory, in which the degree of membership of an element can be a set of possible values and provide greater flexibility in design and, thus, model performance. The power of this application becomes clear when different decision-makers tend to independently record their views. In most real-world situations, there are several goals for managers to achieve the desired performance. Therefore, in this study, a description of the solution of the Hesitant Fuzzy Linear Programming (HFLP) problem for solving hesitant fuzzy multi-objective problems is considered. In the following, the multi-objective and three-level supply chain management problem is modeled with the hesitant fuzzy approach. Then, with an example, the flexibility of the model responses is evaluated by the proposed method. The hesitant fuzzy model presented in this study can be extended to other supply chain management problems.

Keywords: Hesitant fuzzy, Supply chain, Multi-objective, Multi-level.

1 | Introduction

One of the key principles of various businesses to compete in the today's complex and turbulent markets is proper management of the Supply Chains (SCs) with the rapid changes in information and level of needs being met. Indeed, in the current wide market and in the presence of various levels of quality, price, service, and other factors affecting product delivery and satisfying customer satisfaction, if an SC fails to deliver superior customer service, products, and services to others, it will gradually be excluded from the competition market and lose its market share and, thus, its customers will be attracted by the competitors. Therefore, one of the best factors for staying profitable in these conditions is to be properly responsive to customer needs, have performance efficiency, and show greater adaptation to the environment. From Hughes's view, SC management is the coordination and

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transportation across SC units [1]. It is, therefore, necessary to consider two major points:

1. Improving all processes and actions in SC simultaneously.
2. Making models more compatible with the real world, due to the high level of uncertainty in the market.

It is clear that the decisions made in each sector can only lead to profitability and optimism in the same goals and do not provide the optimal global response for the whole chain. Therefore, the optimal problems in this area are modeled as multi-objective and multi-level to consider the optimal policy of all units in the overall structure.

The main performance of multi-level networks can be to supply, produce, and distribute goods to customers. One of the pioneers of multi-level models is Clark and Scarf [2], who examined the two-level inventory model in their research. In a review article, Gümüs and Güneri [3]extensively studied multi-level models.

Crisp numbers operate with limitations on their ability to perform mathematical modeling inefficiently. In the absence of comprehensive and accurate information, fuzzy execution is an effective tool for modeling complex systems. In fact, fuzzy set theory has the ability to represent many inaccurate and ambiguous concepts and systems in the mathematical form, thereby providing a basis for decision-making in an environment of uncertainty.

The complex structure of real-world problems is caused by uncertainty as well as some ambiguity in their meaning and definition. Nowadays, uncertainty has been the focus of many researchers on the way to better develop the models and adapt them to different domains, especially concerning the planning of SC management problems.

In 2006, Kumar et al. [5] used fuzzy goal programming to solve the problem of vendor selection in the SC with uncertain information. The hybrid problem of the three-objective fuzzy integers programming is used to solve the net costs of the network, number of network recurrences, and number of delayed sending and realistic constraints, in which the triangular fuzzy numbers are considered for objective function information [4]. Next, using a multi-objective fuzzy programming provided by Kumar et al. [5] solved a relatively similar problem for vendor selection in order to minimize cost and maximize quality and timely delivery of goods. This approach provides a decision-making tool, in which vendor selection and quota allocation under varying degrees of information uncertainty in the model decision parameters are facilitated. In their paper, Baykasoğlu and Goecken [6], while presenting a categorization of fuzzy mathematical programming problems, identified and presented methods for solving them including fuzzy ranking, fuzzy satisfaction criterion, meta-heuristic algorithms, and so on.

AmirKhan et al. [7] proposed a two-objective feasible linear programming model for solving the problem of multi-level, multi-commodity, and multi-period SC design considering uncertainties, time, and cost. They employed an interactive fuzzy approach.

Bashiri and Sherafati [8] introduced a two-objective model with the objective of minimizing cost and maximizing SC utility in order to design closed-loop SCs considering correlated indices under fuzzy conditions. They used the criterion as the principal component score to integrate and reduce the dimensions of the indices, eliminate the correlation between them in decision-making, and obtain the final answer using the metric LAP method. Pishvaee and Razmi [9] designed a two-objective model to minimize the total cost and environmental impact of an SC network with simultaneous inherent data uncertainties. Using the James' method, they applied a model of interactive approaches to solve the problem.

Bashiri et al. [10] employed a direct solution approach based on fuzzy ranking method and with a heuristic algorithm to balance the feasibility of constraints and optimality of the objective function in designing the three-level logistic network with fuzzy variables. In another study, a new mixed integer multi-objective linear programming model were applied for solving fully fuzzy multi-objective supplier selection problem as an important part in a SC by Nasseri et al. [11].

In 2022, the several sustainable objectives in the pharmaceutical SC optimization scheme under different uncertain constraints has extended by Ahmad et al. [12]. The trade-off between socio-economic and environmental objectives is investigated by ensuring the optimal assignment of various products among some levels and three robust techniques have presented to solve the main model.

Marzband [13], in order to obtain the performance of the SC in a manufacturing company applied the hierarchical analysis process for all suppliers were ordered and weighted based on each index in a fuzzy environment. Then, he evaluated all suppliers using the super efficiency data envelopment analysis. In 2020, Ghasempoor Anaraki et al. [14] determined reliable results for supplier selection model by combining three methods; simple multi-attribute rating technique, DEMATEL method and analytical network process in fuzzy state. Shafi Salimi and Edalatpanah [15], evaluate the suppliers by two methods of fuzzy hierarchical analysis with D-numbers. Then, as case study is different suppliers are ranked using two methods and then the findings are compared with each other.

The framework of a repurchase agreement related to the amount of good remaining in the two-echelon SC between the retailer and the manufacturer is evaluated by two (centralized and decentralized) scenarios in 2021, [16]. Recently, Nasiri et al. [17], by applying statistical methods of Kolmogorov-Smirnov, mean and Stepwise Weight Assessment Ratio Analysis (SWARA) approach, examined of effective factors of green SC management at famous Petrochemical Company.

In the past decade, various fuzzy researches and industrial fields have been observed and studied in some sciences by introducing hesitant fuzzy numbers [11], [12]. Ahmad et al. [20] constructed a multi-objective nonlinear programming problem in the manufacturing system. They gave a new approach based on single-valued neutrosophic hesitant fuzzy set to show the superiority of proposed method. To overcome the uncertainty and hesitation of the variables, Bharati [21], introduced two functions where called the hesitant fuzzy membership and non-membership functions and defined hesitant intuitionistic fuzzy pareto optimal solution. In another research, the definition of the neutrosophic hesitant fuzzy pareto optimal solution and two different optimization methods were given by Ahmad and John [22].

In this research, for the first time as far as the author's knowledge is concerned, a three-objective, three-level problem is modeled with the hesitant fuzzy approach. In this context, HFSs can be useful in modeling with ambiguity as an extension of fuzzy set theory where the element degree can be a set of possible values adopted by decision-makers. In this research, in addition to modeling, the hesitant fuzzy programming method for solving this model is developed and improved. To this end, the continuation of this paper is organized as follows:

Section 2 presents some of the prerequisites and concepts required for fuzzy sets and decision-making. In Section 3, with the overview of hesitant fuzzy programming problems, a model of multi-objective programming problems, in which objective functions and right values can be expressed as HFSs, is presented along with a method for its solving. In Section 4, the multi-objective and three-level SC management problem is presented under uncertain fuzzy conditions. Modeling with hesitant fuzzy approach is provided in Section 5. Via applying a practical example, the solution method outlined in Section 3 is evaluated in Section 6, and the findings and sensitive analysis with numerical results are proposed in Section 6. In Section 7, conclusions of the work are presented and suggestions are made for future research.

2 | Definitions and Concepts Related to Uncertain Fuzzy Sets (Hfss)

This article introduces the HFSs with respect to the issues that will be discussed in the next sections.

Definition 1. Consider the reference set X. An HFS is a set of values that, when apply on X, it returns a subset of [0,1]. Xia and Xu [23], described HFS using the following notation:

$$H = \{\langle x, h_H(x)\rangle | x \in X\},$$

where, $h_H(x)$ is a set of multiple values within [0,1] and represents the degree of possible membership for element $x \in X$ relative to set H. It is easier to call $h_H(x)$ the Hesitant Fuzzy Element (HFE).

Some operators on HFEs are listed below:

$$h_1(x) \cup h_2(x) = \bigcup_{\gamma_1 \in h_1(x), \gamma_2 \in h_2(x)} \max\{\gamma_1, \gamma_2\}.$$

$$h_1(x) \cap h_2(x) = \bigcap_{\gamma_1 \in h_1(x), \gamma_2 \in h_2(x)} \min\{\gamma_1, \gamma_2\}.$$

$$(h_1(x))^\lambda = \bigcup_{\gamma_1 \in h_1(x)} \{\gamma_1{}^\lambda\}.$$

$$\lambda(h_1(x)) = \bigcup_{\gamma_1 \in h_1(x)} \{1 - (1-\gamma_1)^\lambda\}.$$

We have a special case in HFS as the ordinary fuzzy sets, in which $h_H(x)$ is finite per $x \in X$. In this paper, HFS means that each member is a fuzzy number rather than a set of values within [0,1].

To solve the fuzzy programming problems from Bellman and Zadeh [24] view, G is assumed to be a fuzzy goal and C is a fuzzy constraint in the space of X. Then, C and G are combined to decide like D, which is the fuzzy decision of C and G. Symbolically, $D = G \cap C$ and, correspondingly, $\tau(h_G, h_c)$ where τ is used as the fuzzy operator in the fuzzy environment to compute the membership values of fuzzy elements sharing.

For the fuzzy multi-objective programming problem, we need to define a decision in the uncertain fuzzy environment. We employ this idea by extending the definition of decision-making in the fuzzy environment from Ranjbar and Effati [25] perspective:

Definition 2. Suppose $\tilde{\tilde{G}}$ is a hesitant fuzzy objective and $\tilde{\tilde{C}}$ is a hesitant fuzzy constraint in multiple choice space. In this case, decision $\tilde{\tilde{D}}$ from the combination of $\tilde{\tilde{C}}, \tilde{\tilde{G}}$ is called the fuzzy uncertain decision. Symbolically, we have $\tilde{\tilde{D}} = \tilde{\tilde{G}} \cap \tilde{\tilde{C}}$ and $h_{\tilde{\tilde{D}}} = \tau(h_{\tilde{\tilde{G}}}, h_{\tilde{\tilde{G}}})$ where τ as the T-norm in the environment hesitant fuzzy is used to compute membership values related to the HFEs subscription. We also have

$$h_{\tilde{\tilde{C}}} = \{h_{\tilde{\tilde{C}}}{}^1, h_{\tilde{\tilde{C}}}{}^2, \ldots, h_{\tilde{\tilde{C}}}{}^{P_C}\}, \qquad h_G = \{h_{\tilde{\tilde{G}}}{}^1, h_{\tilde{\tilde{G}}}{}^2, \ldots, h_{\tilde{\tilde{G}}}{}^{P_G}\}.$$

P_C *and* P_G represent a number of decision-makers who select different levels of the objective function and constraints, respectively.

In multi-objective problems, one can consider n objectives $\tilde{\tilde{G}}_1, \tilde{\tilde{G}}_2, \ldots, \tilde{\tilde{G}}_n$ and m constraints $\tilde{\tilde{C}}_1, \tilde{\tilde{C}}_2, \ldots, \tilde{\tilde{C}}_n$. In that case, the decision will lead to:

$$\tilde{\tilde{D}} = (\tilde{\tilde{G}}_1 \cap \tilde{\tilde{G}}_2 \cap \ldots \cap \tilde{\tilde{G}}_n) \cap (\tilde{\tilde{C}}_1 \cap \tilde{\tilde{C}}_2 \cap \ldots \cap \tilde{\tilde{C}}_n) = \tilde{\tilde{G}} \cap \tilde{\tilde{C}}.$$

Since T-norms use HFE intersection to calculate membership values for decision-making in the hesitant fuzzy environment as a concurrent operator, we provide the following definition adopted by Santos et al. [26] for T-norms on HFSs:

Definition 3. Suppose $\tau: H^{(m)} \times H^{(m)} \to H^{(m)}$where $H^{(m)}$ is an HFS of m members. In this case, τ is a common hesitant triangle (HT-norm). If for each $h_1, h_2, h_3 \epsilon H^{(m)}$ then, the following principles are satisfied:

$$\tau(h_1, h_2) = \tau(h_2, h_1); \text{Commutative},$$

$$\tau(h_1, \tau(h_2, h_3)) = \tau(\tau(h_1, h_2), h_3); \text{Associative},$$

$$\text{If } h_2 \leq_{H^{(m)}} h_3 \text{ then } \tau(h_1, h_2) \leq_{H^{(m)}} \tau(h_1, h_3); \text{monotony},$$

$$\tau\left(h_1, 1_{H^{(m)}}\right) = h_1; \text{Neutral member},$$

where $1_{H^{(m)}} = \{1,1, \dots ,1\}$ with m element is a complete HFE.

This definition depends on the comparison operator $\leq_{H^{(m)}}$. In this paper, we use the operator for HT-norm on HFE with fuzzy numerical members defined as follows:

Definition 4. Suppose $h \epsilon H^{(m)}$ is an HFE with m fuzzy member obtained using one of the ranking methods such as $\mathbb{R}$. Then, for every $h_1, h_2 \epsilon H^{(m)}$:

$$h_1{}^i <_{\sim\Re} h_2{}^i \quad \forall\ i = 1, \dots, m \quad \Leftrightarrow \quad h_1 <_{\approx\Re} h_2.$$

where $<_{\Re}$ with respect to the ranking function R indicates h_1 is less than h_2.

Remark 1. Suppose the number of values in HFEs can be different. The two HFEs must be of the same length in order to have the correct comparison. Then, for the two HFEs where $h_2 \epsilon H^{(n)}$ *and* $h_1 \epsilon H^{(m)}$, if $n < m$, then an expansion of h_1 by repeating the minimum value until being equal in length must be done. Choosing these values depends on the degree of risk in decision-makers' preferences. From the pessimistic view, expectation of undesirable results increases and, hence, can add minimal values, while optimistic prediction can give us more favorable results. Therefore, max values can be added.

A number of scoring functions for HFE are introduced as $S: [0,1]^n \longrightarrow [0,1]$, which establish the properties of boundary conditions and non-descending monotone. In this paper, in order to obtain the optimal solution for the hesitant fuzzy multi-objective fuzzy problem, we use a set of scoring functions defined as follows [27].

Definition 5. Suppose $h_H(x) = (h_H{}^1(x), \dots, h_H{}^m(x))$ be HFE. Then, we have following score functions:

$$S_{min}(h_H(x)) = \min\left\{h_H{}^1(x), \dots, h_H{}^m(x)\right\}; \text{Minimum scoring function},$$

$$S_{AM}(h_H(x)) = \frac{1}{m}\sum_{i=1}^{m} h_H{}^i(x); \text{Arithmetic mean},$$

$$S_{max}(h_H(x)) = \max\left\{h_H{}^1(x), \dots, h_H{}^m(x)\right\}; \text{Maximum scoring function}.$$

This definition introduces a suitable set of scoring functions appropriate to the decision-maker.

3 | Definitions Method for Solving Hesitant Fuzzy Multi-Objective

In this section, as an application of the HFSs, while introducing the HFMP, a method is presented for solving this kind of problem.

3.1 | HFMP

The HFLP can be expressed [25] as follows (HFLP):

$$\text{Max } z = \tilde{\tilde{c}}^T \tilde{\tilde{x}},$$
$$\text{s.t.} \quad \tilde{\tilde{A}}\tilde{\tilde{x}} <_{\approx} \tilde{\tilde{b}}, \qquad (1)$$
$$\tilde{\tilde{x}} >_{\approx} 0,$$

where, $\tilde{\tilde{A}}$ is a hesitant fuzzy matric and $\tilde{\tilde{c}}$, $\tilde{\tilde{b}}$ and $\tilde{\tilde{x}}$ are hesitant fuzzy vectors. In their work, they identified five categories of hesitant fuzzy programming:

1. Symmetric HFLP where the right-hand side values and objective function are fuzzy uncertain.
2. Asymmetric HFLP where only the right-hand side values are fuzzy uncertain.
3. The HFLP where the technological coefficients and right-hand side values are hesitant fuzzy.
4. The HFLP where the objective function coefficients are hesitant fuzzy.
5. The full HFLP where the objective function and the right-hand side values are hesitant fuzzy.

With the extension of models for multi-objective problems, we have

$$\text{(HFMP): Max } z = \left(\tilde{\tilde{c}}_1{}^T\tilde{\tilde{x}}, \tilde{\tilde{c}}_2{}^T\tilde{\tilde{x}}, \dots, \tilde{\tilde{c}}_r{}^T\tilde{\tilde{x}}\right),$$
$$\text{s.t.} \quad \tilde{\tilde{A}}\tilde{\tilde{x}} <_{\approx} \tilde{\tilde{b}}, \qquad (2)$$
$$\tilde{\tilde{x}} >_{\approx} 0.$$

Since the five proposed for HFLP modes are extensible to HFMOLP problems and given that the methods for solving different modes are different, here, an extension of the symmetrical HFLP is considered. In this concept, the right-hand side values and the objective functions of the problem can be expressed as hesitant fuzzy numbers; so, we have:

$$\text{(HFMP): Max } z = \left(\tilde{\tilde{c}}_1{}^T\tilde{\tilde{x}}, \tilde{\tilde{c}}_2{}^T\tilde{\tilde{x}}, \dots, \tilde{\tilde{c}}_r{}^T\tilde{\tilde{x}}\right) \geq \tilde{\tilde{z}}_o,$$
$$\text{s.t.} \quad \tilde{\tilde{A}}\tilde{\tilde{x}} <_{\approx} \tilde{\tilde{b}}, \qquad (3)$$
$$x \geq 0,$$

where $\tilde{\tilde{z}}_o = [\tilde{\tilde{z}}_1, \tilde{\tilde{z}}_2, \dots, \tilde{\tilde{z}}_r]^T$ is the hesitant fuzzy lower bound to maximize $(\tilde{\tilde{c}}_1{}^T\tilde{\tilde{x}}, \tilde{\tilde{c}}_2{}^T\tilde{\tilde{x}}, \dots, \tilde{\tilde{c}}_r{}^T\tilde{\tilde{x}})$ and $\tilde{\tilde{b}}$ is the HFEs components with fuzzy membership values. In this case, there is no distinction between goals and constraints. And several decision-makers can submit different views for the value of objective functions and constraints. The problem formulation can be transformed as follows:

$$\begin{aligned}
&\text{Find } x \\
&\text{s.t.} \\
&c_1{}^T x \geq \tilde{\tilde{z}}_1, \\
&c_2{}^T x \geq \tilde{\tilde{z}}_2, \\
&c_r{}^T x \geq \tilde{\tilde{z}}_r, \\
&Ax \leq \tilde{\tilde{b}}, \\
&x \geq 0.
\end{aligned} \qquad (4)$$

The current set of constraints includes the set of goals and hesitant fuzzy constraints.

If we have r goals and m constraints, then

$$B=\begin{bmatrix} -c_{11} & -c_{12} & \cdots & -c_{1n} \\ \vdots & & \ddots & \vdots \\ -c_{r1} & -c_{r2} & \cdots & -c_{rn} \\ a_{11} & a_{12} & \cdots & a_{rn} \\ \vdots & & \ddots & \vdots \\ a_{r1} & a_{r2} & \cdots & a_{mn} \end{bmatrix},\ \tilde{\tilde{d}}=\begin{bmatrix} -\tilde{\tilde{z}}_1 \\ \vdots \\ -\tilde{\tilde{z}}_r \\ \tilde{\tilde{b}}_1 \\ \vdots \\ \tilde{\tilde{b}}_m \end{bmatrix}.$$

All $m+r$ on row $\tilde{\tilde{d}}$ are specified below by HF elements:

$$\tilde{\tilde{d}}_i=\{h_i^{1},h_i^{2},\dots,h_i^{p_i}\},\qquad i=1,2,\dots,m+r,$$

where $\tilde{\tilde{d}}_i$ are fuzzy numbers and p_i are the number of decision-makers, satisfaction levels of which represent the values of the objective functions, and each constraint is based on the i^{th} line according to knowledge and experience. We consider h_i^{k} for $k_i=1,2,\dots,p_i$ with decreasing membership function as follows:

$$h_i^{k_i}(x)=\begin{cases} 1 & B_ix\le d_i^{k_i} \\ 1-\dfrac{B_ix-d_i^{k_i}}{q_i^{k_i}}, & d_i^{k_i}<B_ix\le d_i^{k_i}+q_i^{k_i} \\ 0 & B_ix\ge d_i^{k_i}+q_i^{k_i} \end{cases}. \tag{5}$$

That is, where B_i represents the i^{th} row of B ($i=1,2,\dots,m+r$), $d_i^{k_i}$is the constant value i^{th} of the selected row, and $q_i^{k_i}$ is an acceptable error corresponding to i^{th} row which is selected by the k_i^{th} decision-maker.

3.2 | HFMP Solving Method

First, in terms of the hesitant fuzzy decision definition of the model, we state:

$$h_D=\tau_M(h_1,h_2,\cdots,h_{m+r})=\bigcup_{\gamma_1\in h_1,\gamma_2\in h_2,\cdots,\gamma_{m+r}\in h_{m+r}}\min\{\gamma_1,\gamma_2,\cdots,\gamma_{m+r}\}.$$

In this case, $h_D=\{\ h_D^{1},h_D^{2},\dots,h_D^{p_1p_2\dots p_{m+r}}\}$ is a set of fuzzy numbers. Now, for the optimal solution to this problem, we can recommend the maximum of each member of h_D as follows:

$$\begin{aligned} &\max\ h_D^{s}(x^s),\\ &\text{s.t.}\quad x^s\ge 0,\qquad s=1,2,\cdots,(p_1p_2\cdots p_{m+r}). \end{aligned} \tag{6}$$

By introducing the variable λ^s that corresponds to $h_D^{s}(x^s)$ in the model, we have

$$\begin{aligned} &LP_s:\ \ \max\ \lambda^s\\ &\text{s.t}\quad \lambda^s q_i^{k_i}+B_ix^s\le q_i^{k_i},\qquad i=1,2,\dots,m+r\\ &0\le\lambda^s\le 1,\\ &x^s\ge 0. \end{aligned} \tag{7}$$

Then, after solving this model, λ^{*s} is the maximum degree corresponding to the level of satisfaction of the goals and constraints that can establish i^{th}. The $x^{*s}=(x_1^{*s},x_2^{*s},\cdots,x_n^{*s})$ is an HFMOLP problem solution. So, by solving $(p_1p_2\cdots p_{m+r})$, we have the LP problem as the following model:

$$h_D(x^*)=\{\lambda^{*1},\lambda^{*2},\cdots,\lambda^{*(p_1p_2\cdots p_{m+r})}\},$$

where $x^*=(x_1^{*},x_2^{*},\cdots,x_n^{*})$, such that

$$x_1^* = \{x_1^{*1}, x_1^{*2}, \cdots, x_1^{*(p_1p_2\cdots p_{m+r})}\}.$$

$$\vdots$$

$$x_n^* = \{x_n^{*1}, x_n^{*2}, \cdots, x_n^{*(p_1p_2\cdots p_{m+r})}\}.$$

Remark 2. If decision-makers state only some goals as hesitant fuzzy, that is to say, there exist crisp goal or goals in the model. Here, sub-problems are as MOLP where weighted average method can be used for their solving. In that case, only a weighty goal based on the decision-maker's priorities plays a role in the importance of the goals [28].

Remark 3. It is possible to examine responses at different levels of decision-makers' views with alpha levels in mind. In this case, in addition to the constraints presented in LP_s, we will have a constraint as ≥.

Remark 4. If the decision-makers are not interested in the hesitant fuzzy solution, then, the optimal solution of the problem can be found by using the scoring functions from different points of view, similar to those presented in *Table 1*, where l is the minimum membership index of $h_D(x^*)$ and u is the maximum index of $h_D(x^*)$.

Table 1. Optimal solutions to the MOHFLP problem from different perspectives.

λ^*	x^*	View
$\lambda^{*l} = S_{min}(h_D((x^*))$	$(x_1^{*l}, x_2^{*l}, \ldots, x_n^{*l})$	pessimistic
$\lambda^* = S_{AM}(h_D((x^*))$	$\left(\frac{\sum_{r=1}^{(p_1\cdots p_{m+r})} \lambda^* x_1^{*r}}{\sum_{r=1}^{(p_1\cdots p_{m+r})} \lambda^*}, \frac{\sum_{r=1}^{(p_1\cdots p_{m+r})} \lambda^* x_2^{*r}}{\sum_{r=1}^{(p_1\cdots p_{m+r})} \lambda^*}, \ldots, \frac{\sum_{r=1}^{(p_1\cdots p_{m+r})} \lambda^* x_n^{*r}}{\sum_{r=1}^{(p_1\cdots p_{m+r})} \lambda^*}\right)$	Normal
$\lambda^{*u} = S_{max}(h_D((x^*))$	$(x_1^{*u}, x_2^{*u}, \ldots, x_n^{*u})$	Optimistic

4 | Multi-Objective SC Problem with Hesitant Fuzzy Approach

In this section, with some limitations, we consider the multi-objective, three-level, single-product chain management model as *Fig. 1* in a form that should be considered by decision-makers for various purposes, some of which are conflicting. The following are the indices, parameters, decision variables, constraints, and goals.

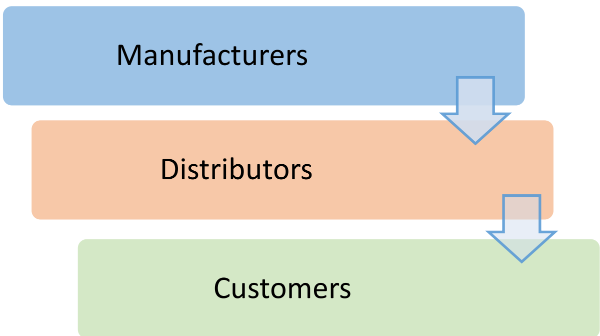


Fig. 1. Three-level supply chain structure.

Indices

Manufactures ($i \epsilon I$), $i = 1,2, \ldots, m$.

Distributors ($j \in J$), $j = 1,2, \ldots, n$.

Customers ($k \in O$), $k = 1,2, \ldots, o$.

Parameters

Q_i: Product quality produced by the i^{th} manufacturer.

$P_{ij}{}^T$: Cost of shipping the product from i^{th} manufacturer to j^{th} distributor.

$C_i{}^V$: Product shipping capacity from i^{th} manufacturer warehouse to warehouse distribution centers.

$P_j{}^H$: Cost of maintaining each unit of goods in the j^{th} distributor warehouse.

$P_{jk}{}^R$: Cost of payment for each unit of fine returned by the distributor j^{th} to the customer i^{th}.

$B_{jk}{}^R$: Return percentage of goods sold by distributor j^{th} to customer k^{th}.

$T_{jk}{}^S$: Delivery time from distributor j^{th} to customer k^{th}.

$C_j{}^V$: Freight forwarding capacity of distributor j^{th}.

S_{jk}: Sales price per unit of product from distributor j^{th} to customer k^{th}.

$U_i{}^P$: Maximum amount of product manufactured by i^{th} manufacturer to send to distribution center.

$L_j{}^D$: Minimum customer required demand for distributor.

Decision variables

x_{ij}: Quantity of product sent by manufacturer i^{th} to distributor j^{th}.

y_{jk}: Amount of customer demand k^{th} from distributor j^{th}.

Constraints

Product lack constraints: Obviously, one of the main reasons for developing and validating systems is to meet customer demand at the right time. Therefore, we need constraints that ensure that the amount of production is sufficient to meet the needs of the customers and does not increase warehousing costs. To this end, the following constraints may apply

$$\sum_{i=1}^{m} x_{ij} = \sum_{k=1}^{o} y_{jk}, \quad (j = 1,2, \ldots, n). \tag{8}$$

Maximum production capacity constraints: This type of constraint ensures that the amount of product produced by the i^{th} manufacturer to deliver to distributors has a certain maximum value. For this purpose, we have:

$$\sum_{i=1}^{m} x_{ij} \leq U_i^{P}, \qquad (i = 1,2, \dots, m). \tag{9}$$

Customer demand minimum constraints: This type of constraint ensures that the quantity of product requested by the distributor i^{th} is minimal. For this purpose, we have:

$$\sum_{k=1}^{n} y_{jk} \geq L_j^{D}, \qquad (j = 1,2, \dots, n). \tag{10}$$

In addition to the three types of *Constraints (8), (9)* and *(10)* mentioned above, we present the non-negative constraints of decision variables:

$$x_{ij}, \qquad y_{jk} \geq 0, \quad (i = 1,2, \dots, m), \qquad (j = 1,2, \dots, n), (k = 1,2, \dots, o). \tag{11}$$

Objective functions

Quality objective function: this objective function aims to maximize the quality of products sent by the manufacturer i^{th} to the distributor j^{th} in order to deliver more quality goods to distributors and, thus, to customers. For this purpose, we have the following objective function:

$$F_q = \sum_{i=1}^{m} \sum_{j=1}^{n} Q_i x_{ij}. \tag{12}$$

Total cost objective function: To minimize total system costs, including shipping, maintenance, and penalties for returning goods, it is formulated as follows:

$$\begin{aligned} F_p = \sum_{i=1}^{m} \sum_{j=1}^{n} P_{ij}^{T}(x_{ij} / C_i^{V}) + \sum_{i=1}^{m} \sum_{j=1}^{n} H_j(x_{ij}/2) \\ + \sum_{j=1}^{n} \sum_{k=1}^{o} P_{jk}(B_{ij}^{R} y_{jk}), \end{aligned} \tag{13}$$

which includes, respectively, the total shipping costs from the manufacturer i^{th} to the distributor j^{th}, the maintenance cost of the product shipped by the manufacturer i^{th} to the distributor j^{th}, and the return fine.

Delivery time objective function: it aims to minimize product delivery time by the distributor, as follows:

$$F_t = \sum_{j=1}^{n} \sum_{k=1}^{i} T_{jk}^{S}(y_{ij} / C_j^{V}). \tag{14}$$

Income objective function: to maximize revenue from product sales from the distributor i^{th} to the customer k^{th}, it will generate more revenue from selling the product to customers.

$$F_s = \sum_{j=1}^{n} \sum_{i=1}^{o} S_{jk} y_{jk}. \tag{15}$$

The objective functions presented in *Eqs. (12)-(15)* along with the deterministic model *Constraints (8)-(11)* form multi-objective SC management problem.

5 | Modeling with Hesitant Fuzzy Approach

Product quality, total cost, delivery time, and optimal revenue, which are considered definite goals in the model presented in the previous section, may be influenced by various factors such as management, competitor's status, inflation, and so on. Therefore, these goals may be desirable from the point of view of different decision-makers at a particular level and may allow a certain level of violation. For modeling the problem, the goals can be considered fuzzy by considering the decision-makers with the help of hesitant fuzzy numbers. This idea can be limited by constraints such as the amount of production capacity due to changes in the amount of raw materials available and overtime human force hours, limitation in the minimum amount of customer demand by product quality, relative satisfaction with after-sales service, manner of advertising develops status of competitors in the market, and so on. Hence, the model presented in the previous section can be modeled by the hesitant fuzzy approach:

$$F_q = \sum_{i=1}^{m} \sum_{j=1}^{n} Q_i x_{ij} \geq \tilde{\tilde{z}}_q.$$

$$F_p = \sum_{i=1}^{m} \sum_{j=1}^{n} P_{ij}{}^{T}(x_{ij} / C_i{}^{V}) + \sum_{i=1}^{m} \sum_{j=1}^{n} H_j(x_{ij}/2) + \sum_{j=1}^{n} \sum_{k=1}^{o} P_{jk}(B_{ij}{}^{R}\, y_{jk}) \leq \tilde{\tilde{z}}_p.$$

$$F_s = \sum_{j=1}^{n} \sum_{i=1}^{o} S_{jk} y_{jk} \leq \tilde{\tilde{z}}_s.$$

$$F_t = \sum_{j=1}^{n} \sum_{k=1}^{i} T_{jk}{}^{S}(y_{ij} / C_j{}^{V}) \geq \tilde{\tilde{z}}_t.$$

s. t.

$$\sum_{i=1}^{m} x_{ij} = \sum_{k=1}^{o} y_{jk} \quad (j = 1,2, \dots, n).$$

$$\sum_{i=1}^{m} x_{ij} \leq \tilde{\tilde{U}}_i{}^{P} \qquad (i = 1,2, \dots, m).$$

$$\sum_{k=1}^{n} y_{jk} \geq \tilde{\tilde{L}}_j{}^{D} \qquad (k = 1,2, \dots, o).$$

$$x_{ij}, y_{jk} \geq 0 \quad (i = 1,2, \dots, m), (j = 1,2, \dots, n), (k = 1,2, \dots, o).$$

Where fuzzy numbers are uncertain. A summary of the solution is given as flowchart in *Fig. 2* in accordance with the material presented in Section 3. The following section provides a numerical example to analyze the model and discuss and evaluate its results.

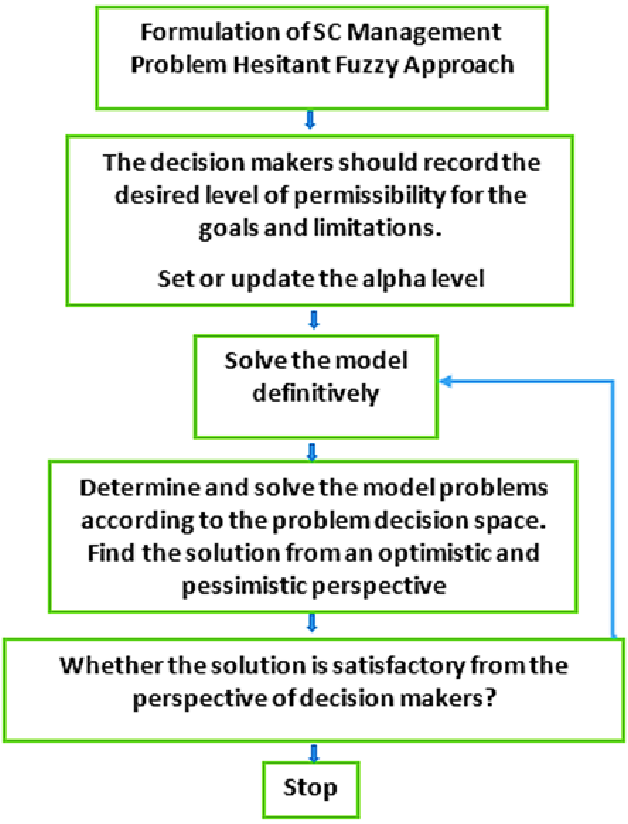


Fig. 2. Flowchart for fuzzy SC problem solving with hesitant approach.

i \ j		Distributor 1	Distributor 2
Distributer	1	5	3
	2	4	5

a.

i \ j		Distributor 1	Distributor 2
Distributer	1	3	4
	2	3	4

b.

j \ k		Customer 1	Customer 2	Customer 3	Customer 4
Distributer	1	2	3	3	2
	2	3	2	2	3

c.

j \ k		Customer 1	Customer 2	Customer 3	Customer 4
Distributer	1	40	48	48	40
	2	48	40	40	48

d.

j \ k		Customer 1	Customer 2	Customer 3	Customer 4
Distributer	1	20	24	24	20
	2	24	20	20	24

e.

j \ k		Customer 1	Customer 2	Customer 3	Customer 4
Distributer	1	0.03	0.02	0.02	0.03
	2	0.02	0.03	0.03	0.02

f.

j	1	2
C_j^V	50	45

g.

i	1	2
C_i^V	45	50

h.

j	1	2
L_j^D	800	700

i.

i	1	2
U_i^P	1200	1800

j.

Fig 3. SC problem data: a. shipping cost from manufacturer to distributor warehouse (in Currency); b. cost of keeping the manufacturer's goods in the distributor's warehouse (in currency); c. delivery time from the distributor's warehouse to the customer (in units of time); d. sales price per unit of distribution to customer (in units of time); e. amount of the fine paid by the distributor to the customer; f. distributor return percentage rate; g. capacity of carriers used by distribution center (in commodity units); h. capacity of carriers used by production center (in units of goods); i. minimum customer demand from distribution centers (in units); j. maximum production capacity (in units of commodity).

6 | Empirical Numerical Analysis

Consider the multi-objective, three-level problem of 2 manufacturers, 2 distributors, and 4 customers as in *Fig. 4*. Supplementary information is provided in *Figs. 3.a-3.j*. The return penalty per unit of commodity is half of its sales price. In addition, the quality percentages per unit of product produced by manufacturers 1 and 2 are 0.86 and 0.9, respectively. The two decision-makers record the desired values and the virtual violations for the second and third objective functions and the first and second constraints, whose views are presented in the *Table 2*.

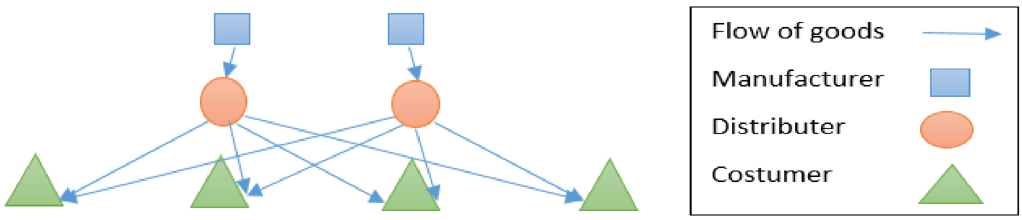


Fig. 4. Three-level diagram: with 2 manufacturers, 2 distributors, 4 customers.

Table 2. Desired values and permitted violations from the point of view of decision-makers for some objective and constraints.

DM	The desired value range and the permissible violation from the decision-maker's point of view					
	Obj-1	Obj-2	Obj-3	Obj-4	Cons-5	Cons-6
DM 1	(2800,1000)	(4000,900)	(70,8)	(150000,15000)	(700,140)	(650,250)
DM 2	(3200,800)	(4400,400)	(95,10)	(160000,20000)	(1250,50)	(1340,40)

According to the problem information, the following formulation is provided:

$$\widetilde{\widetilde{\max}}\, F_q = 0.86\, x_{11} + 0.86 x_{12} + 0.9 x_{21} + 0.9 x_{22},$$

$$\widetilde{\widetilde{\min}}\, F_p = ((\frac{5}{45})\, x_{11} + (\frac{3}{45})\, x_{12} + (\frac{4}{50})\, x_{21} + \left(\frac{5}{50}\right) x_{22} + 1.5(x_{11} + x_{21}) + 2(x_{12} + x_{22}) + 0.6\, y_{11} + 0.48\, y_{12} + 0.48\, y_{13} + 0.6\, y_{14} + 0.48\, y_{21} + 0.6\, y_{22} + 0.6\, y_{23} + 0.48\, y_{24},$$

$$\widetilde{\widetilde{\min}}\, F_t = (\frac{2}{50})\, y_{11} + (\frac{3}{50})\, y_{12} + \left(\frac{3}{50}\right) y_{13} + \left(\frac{2}{50}\right) y_{14} + \left(\frac{3}{50}\right) y_{21} + \left(\frac{2}{45}\right) y_{22} + \left(\frac{2}{45}\right) y_{23} + \left(\frac{3}{45}\right) y_{24},$$

$$\widetilde{\widetilde{max}}\, F_s = 40\, y_{11} + 48\, y_{12} + 48\, y_{13} + 40\, y_{14} + 48\, y_{21} + 40\, y_{22} + 40\, y_{23} + 48\, y_{24},$$

s. t.

$$x_{11} + x_{21} = y_{11} + y_{12} + y_{13} + y_{14},$$

$$x_{12} + x_{22} = y_{21} + y_{22} + y_{23} + y_{24},$$

$$x_{11} + x_{12} \le 1200,$$

$$x_{21} + x_{22} \le 1800,$$

$$y_{11} + y_{12} + y_{13} + y_{14} \ \ge \widetilde{\widetilde{800}},$$

$$y_{21} + y_{22} + y_{23} + y_{24} \ge \ \widetilde{\widetilde{700}},$$

$$x_{ij}\, , y_{jk} \ge 0 \quad (i = 1,2, \quad j = 1,2, \quad k = 1,2,3,4).$$

Results of the deterministic modeling with respect to the objectives are presented separately and together in *Table 3*.

Table 3. Example results considering objectives separately and multi-objectively.

Objective Variables	F_s 34984	F_q 2652	F_p 3433	F_t 60	F_s 144000	$f = 0.25 * (F_q + F_p + F_t + F_s)$ 34984
x_{11}	0	0	0	500	500	500
x_{12}	1200	700	700	700	700	700
x_{21}	1800	800	800	1800	1800	1800
x_{22}	0	0	0	0	0	0
y_{11}	0	0	0	0	0	0
y_{12}	0	0	0	0	0	0
y_{13}	0	800	0	2300	2300	2300
y_{14}	1800	0	800	0	0	0
y_{21}	0	0	0	0	700	700
y_{22}	0	0	0	0	0	0
y_{23}	0	0	700	0	0	0
y_{24}	12000	700	0	700	0	0

In this case, using the method presented in Section 1, we have 64 sub-problems; the results of solving each are given by selecting zero for alpha in *Table 4*.

Table 4. Example results considering objectives separately and multi-objectively.

$MOLP_r$	λ^*	x^*	f^*
$MOLP_1$	0.6	(1200,0,0,1800,0,0,1200,0,1800,0,0,0)	34830
$MOLP_2$	0.315	(1200,0,0,1800,0,0,665.6,534.4,1800,0,0,0)	33748
$MOLP_3$	0.6	(1200,0,0,1800,0,0,1200,0,1800,0,0,0)	34830
$MOLP_4$	0.6	(1200,0,0,1800,0,0,1200,0,1800,0,0,0)	34830
$MOLP_5$	0.2	(1200,0,0,1800,0,0,1200,0,1800,0,0,0)	34830
$MOLP_6$	0.6	(1200,0,30,1770,0,0,1230,0,1770,0,0,0)	34834
$MOLP_7$	0.6	(1200,0,476,1324,0,0,1676,0,1324,0,0,0)	34892
$MOLP_8$	0.315	(1200,0,0,1800,0,0,665.6,534.4,1800,0,0,0)	337.8
$MOLP_9$	0.315	(1200,0,0,1800,0,0,665.6,534.4,1800,0,0,0)	33748
$MOLP_{10}$	0.6	(1200,0,0,1800,0,0,1200,0,1800,0,0,0)	34830
$MOLP_{11}$	0.2	(1200,0,10,1790,0,0,1210,0,1790,0,0,0)	34831
$MOLP_{12}$	0.315	(1200,0,0,1800,0,0,665.6,534.4,1800,0,0,0)	33748
$MOLP_{13}$	0.2	(1200,0,0,1800,0,0,1200,0,1800,0,0,0)	34830
$MOLP_{14}$	0.315	(1200,0,15.8,1784.2,0,0,681.4,534.4,1784.2,0,0,0)	33750
$MOLP_{15}$	0.315	(1200,0,0,1800,0,0,665.6,534.4,1800,0,0,0)	33748
$MOLP_{16}$	0.6	(1200,0,0,1800,0,0,1200,0,1800,0,0,0)	34830
$MOLP_{17}$	0.2	(1200,0,0,1800,0,0,1200,0,1800,0,0,0)	34830
$MOLP_{18}$	0.6	(1200,0,30,1770,0,0,1230,0,1770,0,0,0)	34834
$MOLP_{19}$	0.6	(1200,0,184,1616,0,0,1384,0,1616,0,0,0)	34854
$MOLP_{20}$	0.2	(1200,0,0,1800,0,0,1200,0,1800,0,0,0)	34830
$MOLP_{21}$	0.6	(1200,0,30,1770,0,0,1230,0,1770,0,0,0)	34834
$MOLP_{22}$	0.6	(1200,0,476,1324,0,0,1676,0,1324,0,0,0)	34892
$MOLP_{23}$	0.2	(1200,0,10,1790,0,0,1210,0,1790,0,0,0)	34831
$MOLP_{24}$	0.2	(1200,0,492,1308,0,0,1692,0,1308,0,0,0)	34894
$MOLP_{25}$	0.2	(1200,0,0,1800,0,0,1200,0,1800,0,0,0)	34830
$MOLP_{26}$	0.315	(1200,0,15.8,1784.2,0,0,681.4,534.4,1784.2,0,0,0)	33750
$MOLP_{27}$	0.315	(1200,0,0,1800,0,0,665.6,534.4,1800,0,0,0)	33748
$MOLP_{28}$	0.2	(1200,0,0,1800,0,0,1200,0,1800,0,0,0)	34830
$MOLP_{29}$	0.315	(1200,0,15.7,1784.2,0,0,681.4,534.4,1784.2,0,0,0)	33750
$MOLP_{30}$	0.315	(1200,0,0,1800,0,0,665.6,534.4,1800,0,0,0)	33748
$MOLP_{31}$	0.2	(1200,0,10,1790,0,0,1210,0,1790,0,0,0)	34831
$MOLP_{32}$	0.2	(1200,0,0,1800,0,0,1200,0,1800,0,0,0)	34830
$MOLP_{33}$	0.2	(1200,0,10,1790,0,0,1210,0,1790,0,0,0)	34831
$MOLP_{34}$	0.2	(1200,0,10,1790,0,0,1210,0,1790,0,0,0)	34831
$MOLP_{35}$	0.2	(1200,0,10,1790,0,0,1210,0,1790,0,0,0)	34831
$MOLP_{36}$	0.2	(1200,0,10,1790,0,0,1210,0,1790,0,0,0)	34831

Farnam and Darehmiraki|J. Fuzzy. Ext. Appl. 3(4) (2022) 317-336

Table 4. Continued.

$MOLP_r$	λ^*	x^*	f^*
$MOLP_{37}$	0.315	(1200,0,15.7,1784.3,0,0,681.4,534.4,1784.3,0,0,0)	33751
$MOLP_{38}$	0.2	(1200,0,0,1800,0,0,1200,0,1800,0,0,0)	34830
$MOLP_{39}$	0.2	(1200,0,10,1790,0,0,1210,0,1790,0,0,0)	34831
$MOLP_{40}$	0.2	(1200,0,10,1790,0,0,1210,0,1790,0,0,0)	34831
$MOLP_{41}$	0.2	(1200,0,10,1790,0,0,1210,0,1790,0,0,0)	34831
$MOLP_{42}$	0.6	(1200,0,30,1770,0,1230,0,1770,0,0,0)	34834
$MOLP_{43}$	0.6	(1200,0,476,1324,0,0,1676,0,1324,0,0,0)	34892
$MOLP_{44}$	0.2	(1200,0,10,1790,0,0,1210,0,1790,0,0,0)	34831
$MOLP_{45}$	0.6	(1200,0,30,1770,0,0,1230,0,1770,0,0,0)	34834
$MOLP_{46}$	0.2	(1200,0,0,1800,0,0,1200,0,1800,0,0,0)	34830
$MOLP_{47}$	0.2	(1200,0,10,1790,0,0,1210,0,1790,0,0,0)	34831
$MOLP_{48}$	0.2	(1200,0,10,1790,0,0,1210,0,1790,0,0,0)	34831
$MOLP_{49}$	0.315	(1200,0,15.8,1784.2,0,0,681.4,534.4,1784.2,0,0,0)	33750
$MOLP_{50}$	0.2	(1200,0,0,1800,0,0,1200,0,1800,0,0,0)	34830
$MOLP_{51}$	0.2	(1200,0,10,1790,0,0,1210,0,1790,0,0,0)	34831
$MOLP_{52}$	0.315	(1200,0,15.8,1784.2,0,0,681.4,534.4,1784.2,0,0,0)	33750
$MOLP_{53}$	0.2	(1200,0,0,1800,0,0,1200,0,1800,0,0,0)	34830
$MOLP_{54}$	0.2	(1200,0,10,1790,0,0,1210,0,1790,0,0,0)	34831
$MOLP_{55}$	0.315	(1200,0,0,1800,0,0,665.6,534.4,1800,0,0,0)	33748
$MOLP_{56}$	0.315	(1200,0,15.8,1784.2,0,0,681.4,534.4,1784.2,0,0,0)	33750
$MOLP_{57}$	0.6	(1200,0,30,1770,0,0,1230,0,1770,0,0,0)	34834
$MOLP_{58}$	0.2	(1200,0,492,1308,0,0,1692,0,1308,0,0,0)	34894
$MOLP_{59}$	0.2	(1200,0,10,1790,0,0,1210,0,1790,0,0,0)	34831
$MOLP_{60}$	0.6	(1200,0,30,1770,0,0,1230,0,1770,0,0,0)	34834
$MOLP_{61}$	0.2	(1200,0,0,1800,0,0,1200,0,1800,0,0,0)	34830
$MOLP_{62}$	0.2	(1200,0,10,1790,0,0,1210,0,1790,0,0,0)	34831
$MOLP_{63}$	0.2	(1200,0,0,1800,0,0,1200,0,1800,0,0,0)	34830
$MOLP_{64}$	0.6	(1200,0,30,1770,0,0,1230,0,1770,0,0,0)	34834

If the decision-maker intends to obtain definite results, from the optimistic and pessimistic points of view, we obtain the values presented in *Table 5*.

Table 5. Results from the optimistic and pessimistic perspectives.

Views	λ^*	x^*	f^*
Pessimistic	0.2	(1200,0,10,1790,0,0,1210,0,1790,0,0,0)	34831
Optimist	0.6	(1200,0,476,1324,0,0,1676,0,1324,0,0,0)	34892

7 | Findings and Sensitive Analysis

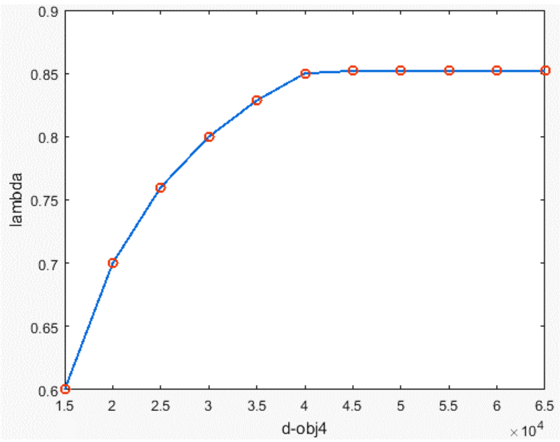
In the previous section, a multi-objective problem of the SC was solved extensively by presenting a practical example. According to the recording of the expected values and the acceptable violation from the point of view of two decision makers about objective *Function (4)* and *Constraints (5)* and *(6)*, 64 sub-problems were extracted. The results presented in *Table 4* have provided the optimal expectation level and the average objective function value in the λ^* and f^* columns for these 64 sub-problems. From the point of view of the decision maker with a higher level of expectation, $\lambda^* = 0.6$ the value of $f^* = 34892$ is obtained and from the point of view of a decision maker with a lower level of expectation, $\lambda^* = 0.2$ the value of $f^* = 34831$ is obtained (*Table 5*). Of course, considering the variety of solutions and changes caused by real world conditions, decision makers can finally use the results of one of the sub-problems in *Table 4* as the optimal solution.

For sensitivity analysis, since the expected value and the deviation depend on the opinion of the decision makers, it is clear that any increase or decrease in these values may cause a change in the final response (values of λ^* and f^*) for the related sub-problems. For example, if we keep the expected value for the fourth objective function constant and increase the acceptable deviation value, then the results in the sub-problems related to this value may change. For example, for sub-problem $MOLP_1$, results similar to those seen in *Table 6* and *Fig. 5* are obtained. As can be seen in *Table 6*, with the increase of the acceptable violation from 15000 to 65000, the value of λ^* increased from 0.6 to 0.852 and the value of f^* decreased from 34830 to 33914.

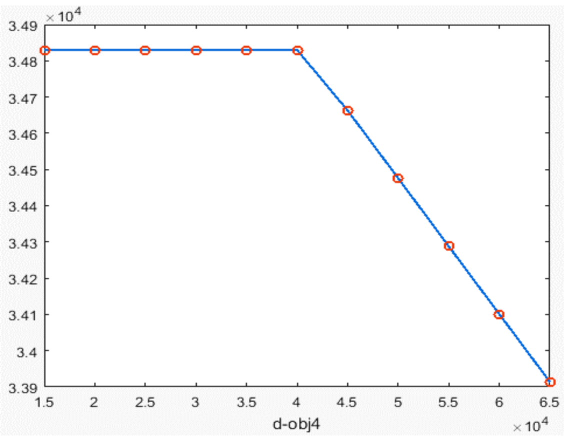
Table 6. Results of the changes in $\tilde{\tilde{d}}_{\tilde{\tilde{z}}_4}$ for the fourth objective of $MOLP_1$.

$(\tilde{\tilde{z}}_4, \tilde{\tilde{d}}_{\tilde{\tilde{z}}_4})$	λ^*	x^*	f^*
(150000,15000)	0.6000	(1200,0,0,1800,0,0,1200,0,1800,0,0,0)	34830
(150000,20000)	0.7000	(1200,0,0,1800,0,0,1200,0,1800,0,0,0)	34830
(150000,25000)	0.7600	(1200,0,0,1800,0,0,1200,0,1800,0,0,0)	34830
(150000,30000)	0.8000	(1200,0,0,1800,0,0,1200,0,1800,0,0,0)	34830
(150000,35000)	0.8286	(1200,0,0,1800,0,0,1200,0,1800,0,0,0)	34830
(150000,40000)	0.8500	(1200,0,0,1800,0,0,1200,0,1800,0,0,0)	34830
(150000,45000)	0.8520	(1200,0,0,1800,0,0,1117.5,82.5,1800,0,0,0)	34663
(150000,50000)	0.8520	(1200,0,0,1800,0,0,1025,175,1800,0,0,0)	34476
(150000,55000)	0.8520	(1200,0,0,1800,0,0,932.5,267.5,1800,0,0,0)	34288
(150000,60000)	0.8520	(1200,0,0,1800,0,0,840,360,1800,0,0,0)	34101
(150000,65000)	0.8520	(1200,0,0,1800,0,0,747.5,452.5,1800,0,0,0)	33914

In *Fig. 5*, the results obtained due to the increase of the acceptable violation for the fourth objective function and related to sub-problem $MOLP_1$ are depicted. The horizontal axis and the vertical axes show the acceptable violation and the amount of λ^* and f^*, respectively.



a.



b.

Fig. 5. a. Variations of λ^* by the changes in $\tilde{\tilde{d}}_{\tilde{\tilde{z}}_4}$ for fourth objective of $MOLP_1$; b. Variations of f^* by the changes in $\tilde{\tilde{d}}_{\tilde{\tilde{z}}_4}$ for fourth objective of $MOLP_1$.

As an example, for sensitivity analysis in constraints, if we increase the expected value for the 5th constraint and keep the acceptable violation value constant, then the results for the sub-problems related to this value may change. For example, for sub-problem $MOLP_{52}$, results similar to those seen in *Table*

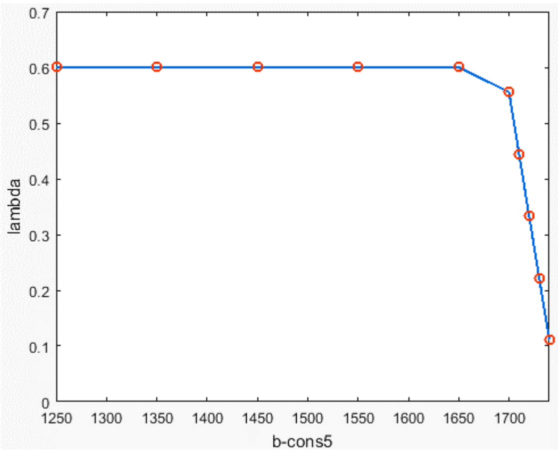
Farnam and Darehmiraki | J. Fuzzy. Ext. Appl. 3(4) (2022) 317-336

7 and *Fig. 6* are obtained. As can be seen in *Table 7*, by increasing the value on the right side from 1250 to 1740, the value of λ^* decreased from 0.6 to 0.1111 and the value of f^* increased from 34834 to 34886 and then decreased to 33038.

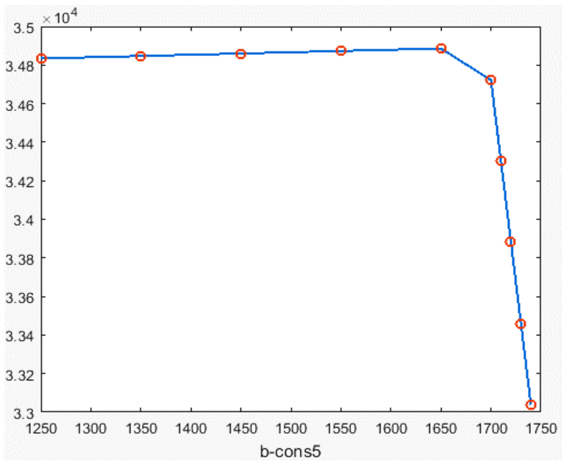
Table 7. Results of the changes in $\tilde{\tilde{b}}_5$ for the fifth constraint of MOLP$_{52}$.

$(\tilde{\tilde{b}}_5, \tilde{\tilde{d}}_{\tilde{\tilde{b}}_5})$	λ^*	x^*	f^*
(1250,50)	0.6000	(1200,0,30,1770,0,0,1230,0,1770,0,0,0)	34834
(1350,50)	0.6000	(1200,0,130,1670,0,0,1330,0,1670,0,0,0)	34847
(1450,50)	0.6000	(1200,0,230,1570,0,0,1430,0,1570,0,0,0)	34860
(1550,50)	0.6000	(1200,0,330,1470,0,0,1530,0,1470,0,0,0)	34873
(1650,50)	0.6000	(1200,0,430,1370,0,0,1630,0,1370,0,0,0)	34886
(1700,50)	0.5556	(1200,0,477.8,1322.2,0,0,1594.4,83.3,1322.2,0,0,0)	34722
(1710,50)	0.4444	(1200,0,482.2,1317.8,0,0,1390.6,291.7,1317.8,0,0,0)	34303
(1720,50)	0.3333	(1200,0,486.7,1313.3,0,0,1186.7,500,1313.3,0,0,0)	33881
(1730,50)	0.2222	(1200,0,491.1,1308.9,0,0,982.8,708.3,1308.9,0,0,0)	33460
(1740,50)	0.1111	(1200,0,495.6,1304.4,0,0,778.9,916.7,1304.4,0,0,0)	33038

In *Fig. 6*, the results obtained due to increasing the value on the right side for the fifth constraint and related to sub-problem $MOLP_{52}$ are depicted.



a.



b.

Fig. 6. a. Variations of λ^* by the changes in $\tilde{\tilde{b}}_5$ for the fifth constraint of MOLP$_{52}$; b. Variations of f^* by the changes in $\tilde{\tilde{b}}_5$ for the fifth constraint of MOLP$_{52}$.

8 | Conclusion

Uncertainties and their investigation manner in applied models are common research topics. In this study, an initial step was taken to apply the hesitant fuzzy programming to the uncertainties caused by these numbers in SC management problems. To this end, we extended and applied the method proposed by Ranjbar and Effati for symmetric and asymmetric HFLP problems for multi-objective fuzzy programming problems. Afterwards, we modeled a three-level four-objective SC problem in the hesitant fuzzy environment and provided an example to evaluate the efficiency of the proposed method. For 64 sub-problems, the results established in *Table 4* and shows the optimal expectation level, optimal point and the average objective function value. Due to the variety of responses, decision makers have a wide range of choices as the optimal response. The results presented in *Table 6* and *Fig. 5* showed that by keeping the expected value for the fourth objective function constant and increasing the acceptable violation value, the optimal response for sub-problem $MOLP_1$ has increasing or decreasing changes in the values of $\lambda^*$and f^* will be. Also, the results of increasing the expected value and keeping the acceptable violation value of the fifth constant illustrated differing changes in the values of $\lambda^*$and f^*. The exponential increase in the number of sub-problems with the increase in the number of opinions of decision makers and the increase in the number of goals and constraints are among the basic limitations in the application of this type of problems. Therefore, for problems with a higher volume, it is better to use more efficient algorithms such as heuristic and hybrid algorithms.

Among the benefits of this research were the opening up of a new view of applied research into SC management, group decision-making capability and, in addition, weight allocation for decision-makers.

Some of the innovations in this article are as follows:

- *Formulating the fuzzy symmetric multi-objective programming problem and its solution.*
- *Using weighted average objective function to solve sub-problems.*
- *Having ability to consider alpha cuts in each of the sub-problems and, hence, examine responses at different levels.*
- *Expressing a model of SC management with hesitant fuzzy approach and solving it using the proposed method.*
- *Providing strategies to improve model performance.*

To continue with the hesitant fuzzy number approach, the following points may be of interest for researchers:

1. Developing and interpreting uncertainties arising from uncertain fuzzy data for other SC-related areas.
2. Considering the uncertainties arising from fuzzy data over other model parameters.
3. Improving the solution methods presented in this paper to deal with uncertainties caused by fuzzy data.
4. Considering more goals or levels to solve the problem.
5. Implementing the method on higher-dimensional models, in particular solving them by combinatorial, heuristic, and meta-heuristic methods, and comparing responses with deterministic methods.
6. With the information obtained from the solutions presented, enabling managers to observe and identify the full range of outcomes, from the worst to best, for final decision-making.

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Farnam and Darehmiraki|J. Fuzzy. Ext. Appl. 3(4) (2022) 317-336