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Spherical Distance Measurement Method for Solving MCDM Problembs under Pythagorean Fuzzy Environment

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Abstract

Multiple Criteria Decision Analysis (MCDA) has been widely investigated and successfully applied to many fields,owing to its great capability of modeling the process of actual decision-making problems and establishing proper evaluation and assessment mechanisms. With the development of management and economics, real-world decision-making problem are becoming diversified and complicated to an increasing extent, especially within a changeable and unpredictable enviroment. Multi-criteria is a decision-making technique that explicitly evaluates numerous contradictory criteria. TOPSIS is a well-known multi-criteria decision-making process. The goal of this research is to use TOPSIS to solve MCDM problems in a Pythagorean fuzzy environment. The distance between two Pythagorean fuzzy numbers is utilized to create the model using the spherical distance measure. To construct a ranking order of alternatives and determine the best one,the revised index approach is utilized. Finally, we look at a set of MCDM problems to show how the proposed method and approach work in practice. In addition, it shows comparative data from the relative closeness and updated index methods.

Keywords: Multiple attribute decision making, TOPSIS, Pythagorean fuzzy sets, Score function, Spherical distance measurement, Revised index method.

1 | Introduction

Zadeh [1] introduced fuzzy set theory that provides a convenient and efficient tool for characterizing by mambership functionin [0,1] and managing Multiple Criteria Decision Analysis (MCDA) problems with vagueness and uncertainty. Nonetheless, in real decision situations, sometimes the membership function of an ordinary fuzzy set is not enough to depict the characters of assessment information because of the complexity of evaluation values and the ambiguity of human subjective judgments. Adak et al. [2], [3] had been extentd the MCDA problems in generalized form.

However in reality, it may not always be true that the degree of non-membership of an element in a fuzzy set is equal to 1 minus the membership degree because there may be some hesitation degree. To overcome this situation, Atanassov [4], [5] introduced the concept of Intuitionistic Fuzzy Sets (IFSs), which is a generalization of fuzzy sets and incorporate with the membership degree(α), non-

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membership degree (β) and hesitation degree(γ) (defined as 1 minus the sum of membership and non-membership). The notion of intutionistic fuzzy set is quite interesting and useful in many application areas. The knowledge and semantic representation of IFS become more meaningful, resourceful and applicable since it includes the degree of belongingness, degree of non-belongingness and the hesitation margin. Several research work done in the field of IFSs [6]-[10].

In IFSs, the pair of membership grades and non-membership grades are denoted by (α, β) satisfying the condition $0 \leq \alpha + \beta \leq 1$. Recently, Yager and Abbasov [11] and Yager [12], [13] extended the condition $\alpha + \beta \leq 1$ to $\alpha^2 + \beta^2 \leq 1$ and then introduced a class of Pythagorean Fuzzy Sets (PFSs) whose membership values are ordered pairs (α, β) that fulfill the required condition $\alpha^2 + \beta^2 \leq 1$. The space of all intuitionistic fuzzy values is also Pythagorean Fuzzy Values (PMVs), but converse is not necessary true. For instance, consider the situation when $\alpha = 0.7$ and $\beta = 0.4$, we can use PFSs, but IFSs cannot be used since $\alpha + \beta \leq 1$, but $\alpha^2 + \beta^2 \leq 1$ PFSs are wider than IFSs so that they can tackle more daily life problems under imprecision and uncertainty cases. Due to this relaxed condition, PF sets are more general than other nonstandard sets, such as IFSs, making the PF theory more powerful and useful than other nonstandard fuzzy models. Furthermore, Zhang and Xu [14] presented the detailed mathematical expression for PF sets and put forward the concept of PF numbers. In recent years, various results have been introduced in PFSs [15]-[17].

How to measure the distance between two PFSs is still an open issue. Different methods had been proposed to present the question in former researches. However, not all existing methods can accurately manifest differences among PFSs and satisfy the property of similarity. And some other kinds of methods neglect the relationshiop among three variables of PFS.

Some researchers have extended the distance measure of IF sets. Zhang and Xu [14] considered three parameters of PFSs, namely, the membership degree, the non-memership degree, and the hesitation degree, while ignoring the direction of commitment, the strength of commitment, and the radian. Li and Zeng [18] considered four basic parameters (the membership degree, the non-memership degree, and the hesitation degree, the strength of commitment, the direction of commitment) of PF sets in the distance measure equation. Wang et al. [20] introduced a distance measure that is based on the long distance and angular distance and angular distance in a bidirectional projection model under the PF environment. Yu et al. [21] proposed a new distance formula that employs Induced Ordered Weighted Averaging (IOWA) with PF information; however, this basic distance formula considers only three parameters, which are the same as the parameters that are considered in the method of Zhang and Xu [14]. Peng and Li [22] proposed a new distance measure for IVPF sets that has two parameters (the membership degree and the non-membership degree) for resolving the counter-intuitive situation.

To address the problem, a new method of measuring distance is proposed that meets the requirements of all axioms of distance measurements and is able to indicate the degree of distinction of PFSs well. For a Pythagorean fuzzy number, membership (α) and non-membership (β) degree satisfying the condition $0 \leq \alpha + \beta \leq 1$ and hesitation degree is $\gamma = \sqrt{1 - \alpha^2 - \beta^2}$, i.e, $\alpha^2 + \beta^2 + \gamma^2 = 1$. Utilizing the relation $\alpha^2 + \beta^2 + \gamma^2 = 1$, we may assume that the triplet (α, β, γ) lies on the spherical surface of unit radius and centre at the origin. This interpretation encourages defining the spherical distance between two Pythagorean fuzzy numbers on restricted spherical surface.

The purpose of this paper is to propose a spherical distance measurement and use to find the distance between two Pythagorean fuzzy numbers and applied in Pythagorean fuzzy TOPSIS method. This measurement is essential to determine distances for both the Pythagorean fuzzy positive ideal solution and Pythagorean fuzzy negative ideal solution. The score function of Pythagorean fuzzy number is used to determine PFPIS and PFNIS in this approch. Revised index and relative closeness are used to rank the alternatives.

The remainder of this paper is organized as follows. Section 2 briefly introduces some basic concepts of PF sets. Section 3 formulates spherical distance measurement method for Pythagorean fuzzy numbers.

Moreover, some comparative discussions with other measurement method are conducted to demonstrate the effectiveness and advantages of the devloped method. Section 4 develops TOPSIS for solving MCDM problems in Pythagorean fuzzy environment. Section 5 applies the proposed methodology to a real-life problem to demonstrate its feasibility and practically. Finally, section 6 presents the conclusions and scope for the future work.

2 | Preliminaries and Definitions

In this section, we recall some basic notions such as the IFSs and the PFSs. Also,we include some elementary aspects that are necessary for this paper.

Definition 1 ([4]). IFS let X be a set of finite universal sets. An IFS I in X is an expression having the form

$$I=\{\langle x,\alpha_I(x),\beta_I(x)\rangle : x\in X\}.$$

Where $\alpha_I(x)$ and $\beta_I(x)$ are the degree of membership and the degree of non-membership of element of $x\in X$ respectively. Also $\alpha_I : X\to[0,1]$, $\beta_I : X\to[0,1]$ and $0\le\alpha_I(x)+\beta_I(x)\le 1,$ for all $x\in X$. The degree of indeterminacy $\pi_I(x)=1-\alpha_I(x)-\beta_I(x)$.

In practice for some reson, the condition $0\le\alpha+\beta\le 1$, is not always true. For instance, $0.6+0.7=1.3>1$ but $0.6^2+0.7^2<1$,or $0.7+0.7=1.4>1$ but $0.7^2+0.7^2<1$. To overcome this situations, in 2013, Yager [12] introduced the concept of PFS.

Definition 2 ([12]). PFS A Pythagorean fuzzy set P in a finite universe of discourse X is given by:

$$P=\{\langle x,\alpha_p(x),\beta_p(x)\rangle \mid x\in X\},$$

Where $\alpha_p : X\to[0,1]$ denotes the degree of membership and $\beta_p : X\to[0,1]$ denotes the degree of non-membership of the element of $x\in X$ to the set A respectively with the condition that $0\le(\alpha_p(x))^2+(\beta_p(x))^2\le 1$.The degree of indeterminacy, $\gamma_p(x)=\sqrt{1-(\alpha_p(x))^2-(\beta_p(x))^2}$.

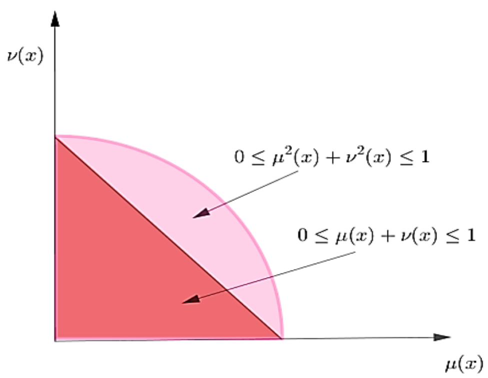

Fig. 1. Comparison space for IFSs and PFSs.

Zhang and Xu [14] introduced the concept of Pythagorean fuzzy numbers. A PFS $P=\langle \alpha_p(x),\beta_p(x)\rangle$ can be expressed as Pythagorean fuzzy numbers $\langle \alpha_p,\beta_p\rangle$, where $\alpha_p,\beta_p \in [0,1], \gamma_p=(1-(\alpha_p)^2-(\beta_p)^2)^{\frac{1}{2}}$ and $0 \le (\alpha_p)^2+(\beta_p)^2 \le 1$.

Yager [12] proposed another PFN formulation, namely, $p=\langle r,d\rangle$,where r denotes the strength of commitment and $r\in[0,1]$. The larger value of r is the stronger commitment and the lower the uncertainty of the commitment. D denote the direction of commitment. r and d associates with the membership grade and non-membership grade; $\alpha=r\cos\theta$ and $\beta=r\sin\theta$. Therefore, $d=1-\frac{2}{\pi}\theta$.

Example 1. Let us consider a Pythagorean fuzzy number $p=\langle 0.7,0.4\rangle$, then we have:

$$\alpha_p=0.7,\beta_p=0.4,\gamma_p=\sqrt{1-(0.7)^2-(0.4)^2}=0.592, r=\sqrt{(0.7)^2+(0.4)^2}=0.806,$$

$$\theta=\arctan\left(\frac{0.4}{0.7}\right)=0.519 \text{ and } d=1-2\frac{0.519}{\pi}=0669.$$

2.1 | Some Operations on Pythagorean Fuzzy Numbers

Here we discussed some operations on Pthagorean fuzzy numbers and PFSs and operations are used in the rest of the paper.

Given three PFNs $p=\langle\alpha,\beta\rangle$, $p_1=\langle\alpha_1,\beta_1\rangle$ and $p_2=\langle\alpha_2,\beta_2\rangle$. The basic operations can be defined as follows:

- $\overline{p}=\langle\beta,\alpha\rangle$.
- $p_1\cup p_2=\langle \max\{\alpha_1,\alpha_2\},\min\{\beta_1,\beta_2\}\rangle$.
- $p_1\cap p_2=\langle \min\{\alpha_1,\alpha_2\},\max\{\beta_1,\beta_2\}\rangle$.

Example 2. Let $p_1=\langle 0.5,0.2\rangle$ and $p_2=\langle 0.6,0.4\rangle$,then $p_1\cup p_2=\langle 0.6,0.2\rangle$, $p_1\cap p_2=\langle 0.5,0.4\rangle$ and $\overline{p_1}=\langle 0.2,0.5\rangle$.

Definition 3 ([23]). Let $p=\langle\alpha,\beta\rangle$ be a Pythagorean fuzzy number. The score function of p is defined as $s(p)=(\alpha)^2-(\beta)^2$, where $s(p)\in[-1,1]$.

Example 3. Let $p_1=\langle 0.7,0.3\rangle$ and $p_2=\langle 0.4,0.6\rangle$, then $s(p_1)=0.40$ and $s(p_1)=-0.20$.

There are some situations, the score function is not sufficient for magnitude comparison of Pythagorean fuzzy numbers.Using the concept of score function Peng and Yang [22] developed accuracy function for magnitude comparison of Pythagorean fuzzy numbers.

Definition 4. Let $p=\langle\alpha,\beta\rangle$ be a Pythagorean fuzzy number. The accuracy function of p is defined as $h(p)=(\alpha)^2+(\beta)^2$, where $h(p)\in[0,1]$.

Let $p_1 = \langle \alpha_1, \beta_1 \rangle$ and $p_2 = \langle \alpha_2, \beta_2 \rangle$ be two PFNs; $s(p_1) = \alpha_1^{\,2} - \beta_1^{\,2}$ and $s(p_2) = \alpha_2^{\,2} - \beta_2^{\,2}$ be their score functions; $h(p_1) = \alpha_1^{\,2} + \beta_1^{\,2}$ and $h(p_2) = \alpha_2^{\,2} + \beta_2^{\,2}$ be the accuracy functions of p_1 and p_2, then yager and abbasov [11] defined the following:

I. If $s(p_1) < s(p_2)$, then p_1 is smaller than p_2, that is $p_1 < p_2$.

II. If $s(p_1) > s(p_2)$, then $p_1 > p_2$.

III. If $s(p_1) = s(p_2)$, then:

- *If* $h(p_1) < h(p_2)$, *then* $p_1 < p_2$.
- *If* $h(p_1) > h(p_2)$, *then* $p_1 > p_2$.
- *If* $h(p_1) = h(p_2)$, *then* p_1 *and* p_2 *represent the same information, that is* $p_1 = p_2$.

3 | Spherical Distance Measurement Method for Pythagorean Fuzzy Numbers

In this section, we analyze Hamming distance measurement method and Euclidean distance measurement method. Then, we propose a spherical distance measurement method of PFNs.

3.1 | Distance Measurement Method for PFNs

Zhang and Xu [14] presented the Hamming distance measurement method for Pythagorean fuzzy numbers with help of membership,non-membership and indeterminacy degree.

Definition 5. Let $p_i = \langle \alpha_i, \beta_i \rangle, i = 1,2$ be two Pythagorean fuzzy numbers. Then the Hamming distance between p_1 and p_2 can be denied as follows:

$$D_{ZH}(p_1, p_2) = \frac{1}{2}(|(\alpha_1)^2 - (\alpha_2)^2| + |(\beta_1)^2 - (\beta_2)^2| + |(\gamma_1)^2 - (\gamma_2)^2|). \tag{1}$$

Li and Zeng [18] proposed a new distance measurement method containing four parameters α, β, γ and d of Pythagorean fuzzy numbers.

Definition 6. Let $p_i = \left\langle \alpha_i, \beta_i \right\rangle, i = 1,2$ be two Pythgorean fuzzy numbers. Then the normalized Hamming distance and the normalized Euclidean distance measure between p_1 and p_2 can be denied as follow:

$$D_{LH}(p_1, p_2) = \frac{1}{4}(|\alpha_1 - \alpha_2| + |\beta_1 - \beta_2| + |r_1 - r_2| + |d_1 - d_2|). \tag{2}$$

$$D_{LE}(p_1, p_2) = \left[\frac{1}{4}(\alpha_1 - \alpha_2)^2 + (\beta_1 - \beta_2)^2 + (r_1 - r_2)^2 + (d_1 - d_2)^2\right]^{\frac{1}{2}}. \tag{3}$$

New distance measurement method using five the parameters membership (α), non-membership (β), hesitation (γ), strength of commitment (r) and direction of commitment (d) of Pythagorean fuzzy numbers.

Definition 7. Let $p_i = \left\langle \alpha_i, \beta_i \right\rangle, i = 1,2$ be two Pythgorean fuzzy numbers. Then the normalized Hamming distance and the normalized Euclidean distance measure between p_1 and p_2 can be denied as follows:

$$D_{LH}(p_1,p_2)=\frac{1}{5}(|\alpha_1-\alpha_2|+|\beta_1-\beta_2|+|\gamma_1-\gamma_2|+|r_1-r_2|+|d_1-d_2|). \quad (4)$$

$$D_{LE}(p_1,p_2)=\left[\frac{1}{4}(\alpha_1-\alpha_2)^2+(\beta_1-\beta_2)^2+(\gamma_1-\gamma_2)^2+(r_1-r_2)^2+(d_1-d_2)^2\right]^{\frac{1}{2}}. \quad (5)$$

Example 4. Let $p_1=\langle 0.6,0.5\rangle$ and $p_2=\langle 0.8,0.2\rangle$ be two Pythagorean fuzzy numbers. Then utilize the *Eq.* (*1*) to (*5*), the different types of distance between p_1 and p_2 and are as follows:

$$D_{ZH}(p_1,p_2)=0.280, D_{LH}(p_1,p_2)=0.206, D_{LE}(p_1,p_2)=0.232, D_{LH}(p_1,p_2)=0.178,$$
$$D_{LE}(p_1,p_2)=0.209.$$

3.2 | Spherical Distance Measurement Method for PFNs

Let $p=\langle\alpha,\beta\rangle$ be a Pythagorean fuzzy number satisfying the condition $0\le\alpha^2+\beta^2\le 1$ and hesitation function is $\gamma=\sqrt{1-\alpha^2-\beta^2}$, i.e., $\alpha^2+\beta^2+\gamma^2=1$.

From this relation we may assume that the triplet (α,β,γ) lies on the spherical surface of unit radius and centre at origin. This interpretation encourage defining the spherical distance between two Pythagorean fuzzy numbers on restricted spherical surface.

On spherical surface the shortest distance is the length arc of the great circle passing through both points.

Definition 8. Let A and C be two points on the spherical surface with co-ordinate (x_1,y_1,z_1) and (x_2,y_2,z_2), then the spherical distance between these two points is defined as:

$$D_{SP}(A,C)=\arccos\{1-\frac{1}{2}[(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2]\}. \quad (6)$$

Incorporated this expression, the spherical distance between two Pythagorean fuzzy numbers defined as follows:

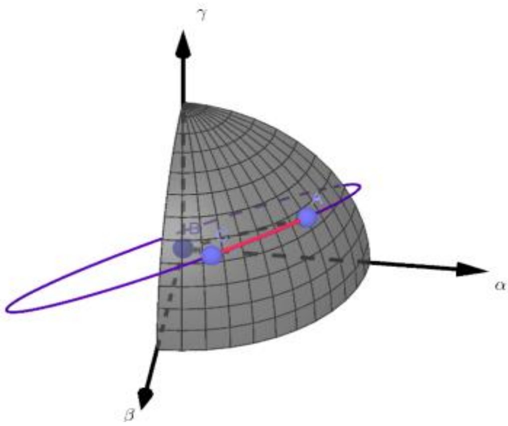

Fig. 2. Spherical distance between points A and B.

Definition 9. Let $p_1=\langle\alpha_1,\beta_1\rangle$ and $p_2=\langle\alpha_2,\beta_2\rangle$ be two Pythagorean fuzzy numbers with hesitation function γ_1 and γ_2 respectively. Then the spherical distance between these two Pythagorean fuzzy numbers is:

$$D_s(p_1,p_2)=\frac{2}{\pi}\arccos\{1-\frac{1}{2}[(\alpha_1-\alpha_2)^2+(\beta_1-\beta_2)^2+(\gamma_1-\gamma_2)^2]\}. \quad (7)$$

To get the distance value in between $[0,1]$ the factor $\frac{2}{\pi}$ is introduced.

Since, $\alpha_1^2+\beta_1^2+\gamma_1^2=1$ and $\alpha_2^2+\beta_2^2+\gamma_2^2=1$, so after simplifying the *Eq. (7)*, we have:

$$D_s(p_1,p_2)=\frac{2}{\pi}\arccos[\alpha_1\alpha_2+\beta_1\beta_2+\gamma_1\gamma_2]. \quad (8)$$

Now, we define the spherical and normalized distances between two PFSs.

Definition 10. Let $P=\{x_i,\langle\alpha_p(x_i),\beta_p(x_i)\rangle:x_i\in X\}$ and $Q=\{x_i,\langle\alpha_Q(x_i),\beta_Q(x_i)\rangle:x_i\in X\}$ of the universe of discourse $X=\{x_1,x_2,.....x_n\}$,then their spherical and normalized spherical distances are;

I. Spherical distance:

$$D_s(P,Q)=\frac{2}{\pi}\sum_{i=1}^{n}\arccos[\alpha_P(x_i)\alpha_Q(x_i)+\beta_P(x_i)\beta_Q(x_i)+\gamma_P(x_i)\gamma_Q(x_i)]. \quad (9)$$

Where $0\le D_{NSP}(P,Q)\le n$.

II. Normalized spherical distance:

$$D_{NS}(P,Q)=\frac{2}{n\pi}\sum_{i=1}^{n}\arccos[\alpha_P(x_i)\alpha_Q(x_i)+\beta_P(x_i)\beta_Q(x_i)+\gamma_P(x_i)\gamma_Q(x_i)]. \quad (10)$$

Where $0\le D_{NSP}(P,Q)\le 1$.

Example 5. Let $p_1=\langle 0.9,0.2\rangle$ and $p_2=\langle 0.7,0.3\rangle$ be two Pythagorean fuzzy numbers. Then the spherical distance between p_1 and p_2 is:

$$D_s(p_1,p_2)=\frac{2}{\pi}\arccos[(0.9\times 0.7)+(0.2\times 0.3)\times(\sqrt{1-0.9^2-0.2^2}\times\sqrt{1-0.7^2-0.3^2})]=0.2198.$$

Definition 11. Let $p_1=(\alpha_{1j},\beta_{1j})$, $p_2=(\alpha_{2j},\beta_{2j})$, $j=1,2,3,.....,n$, be two Pythagorean fuzzy numbers. w_j is the weight of j, i.e., $w=(w_1,w_2,...,w_n)^T$, where $0\le w_i\le 1$ and $\sum_{i=1}^{n}w_i=1$.

Then the weighted normalized spherical distance between p_1 and p_2 is defined as;

$$D_{NS}(p_1,p_2)=\frac{2}{n\pi}\sum_{i=1}^{n}\arccos[\alpha_{1j}\alpha_{2j}+\beta_{1j}\beta_{2j}+\gamma_{1j}\gamma_{2j}]. \quad (11)$$

Example 6. Let $p_1=\{\langle 0.6,0.3\rangle,\langle 0.8,0.2\rangle,\langle 0.5,0.4\rangle\}$ and $p_2=\{\langle 0.7,0.2\rangle,\langle 0.7,0.3\rangle,\langle 0.9,0.1\rangle\}$ be two PFSs with weights $w=\{0.2,0.5,0.3\}$. Then, the weighted spherical distance between P_1 and P_2 is calculated as:

$$\begin{aligned}D_{NS}=&\frac{2}{3\pi}[0.2\times\arccos((0.6\times 0.7)+(0.3\times 0.2)+(\sqrt{1-0.6^2-0.3^2}\times\sqrt{1-0.7^2-0.2^2}))\\&+0.5\times\arccos((0.8\times 0.7)+(0.2\times 0.3)+(\sqrt{1-0.8^2-0.2^2}\times\sqrt{1-0.7^2-0.3^2}))\\&+0.5\times\arccos((0.5\times 0.9)+(0.4\times 0.1)+(\sqrt{1-0.5^2-0.4^2}\times\sqrt{1-0.9^2-0.1^2}))]=0.0499.\end{aligned}$$

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4 | Proposed Pythagorean Fuzzy TOPSIS Method for MCDM Problems

In this section, we introduce multi-criteria Decision-making problem where the information has been taken in the form of the fuzzy numbers and apply spherical distance measurement method to solve this problems.

Let $A=\{A_1,A_2,\ldots,A_n\},(m\geq 2)$ be a set alternatives and $C=\{C_1,C_2,\ldots,C_n\},(n\geq 2)$ be a set of criterion, $w=(w_1,w_2,\ldots w_n)^T$ where $0\leq w_i\leq 1$ and $\sum_{j=1}^{n} w_i=1,$ be the weight vector for each criteria.

Let the Pythagorean fuzzy numbers $\langle\alpha_{ij},\beta_{ij}\rangle$ denotes the assessment value of the i-th alternative for the j-th criteria, viz, $C_j(v_i)=\langle\alpha_{ij},\beta_{ij}\rangle$ and $R=(C_j(v_i))_{m\times n}$ denotes Pythagorean fuzzy decision matrix, where

$$\begin{bmatrix} \langle\alpha_{11},\beta_{11}\rangle & \langle\alpha_{12},\beta_{12}\rangle & \cdots & \langle\alpha_{1n},\beta_{1n}\rangle \\ \langle\alpha_{21},\beta_{21}\rangle & \langle\alpha_{22},\beta_{22}\rangle & \cdots & \langle\alpha_{2n},\beta_{2n}\rangle \\ \cdots & \cdots & \cdots & \cdots \\ \langle\alpha_{m1},\beta_{m1}\rangle & \langle\alpha_{m2},\beta_{m2}\rangle & \cdots & \langle\alpha_{mn},\beta_{mn}\rangle \end{bmatrix}.$$

4.1 | Process of the Proposed Method

To solve MCDM problems in Pythagorean fuzzy environment, we present Pythagorean fuzzy TOPSIS system. The primery concept of the TOPSIS approach is that the most preferred alternative should not only have shortest distance from the positive ideal solution but also have the furthest distance from the negative ideal solution.

We start this method by computing PFPIS and PFNIS. Let j_i be the set of benefit criteria and j_2 be the set of cost cretria. PFPIS and PFNIS were determined by score function. Let v^+ and v^- denote PFPIS and PFNIS respectively. These values are calculated using the following formula:

$$v^+=\{C_j,\max(S(C_j(v_i)))\,|\,j=1,2\ldots,n\}. \quad (12)$$

$$v^-=\{C_j,\min(S(C_j(v_i)))\,|\,j=1,2\ldots,n\}. \quad (13)$$

Next, we calculate normalized spherical distance from each alternative to the PFPIS $D_{NS}(v_i,v^-)$. Now, we obtain weighted normalized spherical distance of alternative v_i from PFPIS v^+ based on (11) which can be defined as follows:

$$D_{NS}(v_i,v^+)=\sum_{j=1}^{n}D_{NS}(C_j(v_i),C_j(v_i)^+)=\frac{2}{n\pi}\sum_{j=1}^{n}w_j\arccos((\alpha_{ij}\alpha_j^{+}+\beta_{ij}\beta_j^{+}+\gamma_{ij}\gamma^{+}). \quad (14)$$

Where $i=1,2\ldots,n$.

According to the principle of TOPSIS, the smaller $D_{NS}(v_i,v^+)$ is the better alternative x_i.

Let

$$D_{\min}(x_i,x^+)=\min_i D_{NS}(v_i,v^+), i=1,2\ldots,n.$$

Similarly, the weighted normalized spherical distance of alternative v_i from PFNIS v^- calculated as follows:

$$D_{NS}(v_i, v^-) = \sum_{j=1}^{n} D_{NS}(C_j(v_i), C_j(v^-)) = \frac{2}{n\pi}\sum_{j=1}^{n} w_j \arccos((\alpha_{ij}\alpha_j^- + \beta_{ij}\beta_j^- + \gamma_{ij}\gamma^-). \quad (15)$$

Where $i = 1, 2..., n$.

According to the principle of TOPSIS, the greater $D_{NS}(v_i, v^-)$ is the better alternative v_i. Let $D_{max}(v_i, v^+) = \max_i D_{NS}(v_i, v^+), i = 1, 2, ..., n$.

Now, we calculate relative closeness co-efficient of the alternative x_i with respect to PFPIS (x^+) and PFNIS (x^-) with the help of basic principle of classical TOPSIS method.

The formula for $RC(x_i)$ is as follows:

$$RC(v_i) = \frac{D_{NS}(v_i, v^-)}{D_{NS}(v_i, v^+) + D_{NS}(v_i, v^-)}. \quad (16)$$

According to the Hadi Venecheh [12], the optimal solution is the shortest distance from positive ideal solution and farthest distance from negative ideal solution. Consequently, Zhang and Xu [14] utilized revised index, which is denoted by $\xi(v_i)$ to determine the ranking order. The index formula is expressed as follow:

$$\xi(v_i) = \frac{D_{NS}(v_i, v^-)}{D_{max}(v_i, v^-)} - \frac{D_{NS}(v_i, v^+)}{D_{min}(v_i, v^-)}. \quad (17)$$

According to $RC(v_i)$ or $\xi(v_i)$, we obtain the rank of the alternatives x_i, which is used to determine the optimal solution according to the maximum value of $RC(v_i)$ or $\xi(v_i)$.

4.2 | Algorithm for Proposed Method

The traditional TOPSIS introduced by Yoon and Hwang [23] is a classic and useful method to solve the MCDM problems with crisp numbers. Zhang and Xu [14] developed a revised TOPSIS method to deal effectively with Pythagorean fuzzy information. The algorithm involves the following steps:

Step 1. For MCDM problem with PFNs, we construct the decision matrix $R = (C_j(v_i))_{m\times n}$, where the elements $C_j(v_i), i = 1, 2, ..., m,\ j = 1, 2, ..., n$ are the assessments of alternative v_i with respect to the criterion C_j.

Step 2. Utilize the score function to determine the Pythagorean fuzzy positive ideal solution (v^-).

Step 3. Use *Eq. (14)* and *(15)* to calculate the weighted spherical distances of each alternative v_i from the Pythagorean fuzzy PIS (v^+) and Pythagorean fuzzy NIS (v^-).

Step 4. Utlize *Eq. (16)* and *(17)* to calculate relative closeness $RC(v_i)$ and revised closeness $\xi(v_i)$ of the alternative v_i.

Step 5. Rank the alternative and select the best one(s) according to the decreasing relative closeness $RC(v_i)$ and revised closeness $\xi(v_i)$ obtained from Step 4.

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The bigger the $RC(v_i)$ the more desirable the v_i, $(i = 1, 2, ..., m)$ will be.

5 | Illustrative Example

In this section, we consider a decision-making problem that concerns daily life problems to illustrate the proposed approach.

A decision maker want to buy a car. There are more than one branded cars with their criterion. Decision-maker considers only five banded cars v_1, v_2, v_3, v_4 and v_5 among these he/she want to buy a particular with his/her availability. In order to buy the cars four criterion viz., cost (C_1), fuel consumption (C_2), comfort (C_3) and attractiveness (C_4) are considered as avaluation factor. According to the assessment of attributes and criterion, Pythagorean fuzzy decision matrix are considered as follows:

$$
\begin{array}{ccccc}
 & C_1 & C_2 & C_4 & C_5 \\
v_1 & \langle 0.7, 0.3 \rangle & \langle 0.5, 0.4 \rangle & \langle 0.7, 0.6 \rangle & \langle 0.9, 0.2 \rangle \\
v_2 & \langle 0.6, 0.4 \rangle & \langle 0.6, 0.3 \rangle & \langle 0.5, 0.3 \rangle & \langle 0.7, 0.4 \rangle \\
v_3 & \langle 0.5, 0.6 \rangle & \langle 0.7, 0.4 \rangle & \langle 0.5, 0.4 \rangle & \langle 0.6, 0.4 \rangle \\
v_4 & \langle 0.4, 0.7 \rangle & \langle 0.5, 0.8 \rangle & \langle 0.6, 0.2 \rangle & \langle 0.5, 0.3 \rangle \\
v_5 & \langle 0.5, 0.8 \rangle & \langle 0.7, 0.2 \rangle & \langle 0.6, 0.5 \rangle & \langle 0.6, 0.2 \rangle
\end{array}.
$$

Where $C_1(v_1) = \langle 0.7, 0.3 \rangle$ represents the degree to which alternative v_1 satisfies criteria C_1 is 0.7 and degree to which satisfies alternative v_1 dissatisfies criterion C_1 is 0.3.

Considering that fuel consumption, comfort and attractiveness of the cars as benefit criteria, $j_1 = \{C_2, C_3, C_4\}$ and cost of the car is the cost criterion $j_2 = \{C_1\}$.

To calculate score type Pythagorean fuzzy positive ideal solution (v^+) and Pythagorean fuzzy negative ideal solution (v^-), we utilize the *Eq. (12)* and *(13)*. We get the result as follows:

$$v^+ = \{\langle 0.5, 0.8 \rangle, \langle 0.7, 0.2 \rangle, \langle 0.6, 0.2 \rangle, \langle 0.9, 0.2 \rangle\}.$$

$$v^- = \{\langle 0.7, 0.3 \rangle, \langle 0.5, 0.8 \rangle, \langle 0.5, 0.4 \rangle, \langle 0.5, 0.3 \rangle\}.$$

Next, utilize *Eq. (14)* and *(15)* to calculate the weighted spherical distances of each alternatives v_i from Pythagorean fuzzy positive ideal solution and Pythagorean fuzzy negative ideal solutions;

	$D_{NS}(v_i, v^+)$	$D_{NS}(v_i, v^-)$
v_1	0.0590	0.0554
v_2	0.0593	0.0475
v_3	0.0561	0.0475
v_4	0.0712	0.0359
v_5	0.0290	0.0742

We utilize *Eq. (16)* and *(17)* to compute the $RC(v_i)$ and $\xi(v_i)$ for each alternative v_i and results are listed below:

	$RC(v_i)$(Rank)	$\xi(v_i)$(Rank)
v_1	0.4842(2)	–1.2878(3)
v_2	0.4447(4)	–1.4046(4)
v_3	0.4642(3)	–1.2794(2)
v_4	0.3352(5)	–1.9713(5)
v_5	0.7189(1)	0(1)

According to $RC(v_i)$ rank of the alternatives are $v_5 \succ v_1 \succ v_3 \succ v_2 \succ v_4$ among which v_5 v_5 is the best alternative. However, according to the revised index $\xi(v_i)$ the ranking of the alternatives are $v_5 \succ v_3 \succ v_1 \succ v_2 \succ v_4$. Here also, the best alternative is v_5.

6 | Conclusion and Future Work

In this paper, the spherical distance measurements method has been introduced and are applied in TOPSIS method for MCDM approach with with Pythagorean fuzzy is developed. The main advantage of this method is that it is able to reflect the importance of the degree of membership, non-membership and hesitancy of decision-maker. Moreover, it provides a more complete representation of the decision process because the decision makers can consider many different scenarios depending on his interest by dealing with Pythagorean fuzzy environment. The spherical distance measurement method combined with the TOPSIS method by Pythagorean fuzzy data has enormous chance of success for MCDM problems. Ordering of the alternative by utilizing relative closeness and revised index method.

In future research, we expect to devlop further developments by using spherical distance measurement methods in the environment of intuitiontic fuzzy,picture fuzzy,fermatean fuzzy etc. This approach will be considered, especially in supply chain management and logistic, engineering, manufacturing system, business and marketing, human resources, and water resource management.

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Conflicts of Interest

No potential conflict of interest was reported by the authors.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authers.

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