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Global Domination in Fuzzy Graphs Using Strong Arcs

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Abstract

Sampathkumar [7] introduces the notion of global domination in graphs. Nagoorgani and Hussain [24] introduced the concept of global domination in fuzzy graphs using effective arcs. This paper presents global domination in fuzzy graphs using strong arcs. The strong global domination number of different classes of fuzzy graphs is obtained. An upper bound for the strong global domination number of fuzzy graphs is obtained. Strong global domination in fuzzy trees is studied. It is established that every node of a strong global dominating set of a fuzzy tree is either a fuzzy cut node or a fuzzy end node. It is proved that in a fuzzy tree, each node of a strong global dominating set is incident on a fuzzy bridge. Also, the characteristic properties of the existence of a strong global dominating set for a fuzzy graph and its complement are established.

Keywords: Fuzzy graph, Strong arcs, Weight of arcs, Strong domination, Strong global domination.

1 | Introduction

Fuzzy graphs were introduced by Rosenfeld [26]. Rosenfeld [26] has described the fuzzy analog of several graph theoretic concepts like paths, cycles, trees, and connectedness and established some of their properties. Bhutani and Rosenfeld [9] have introduced the concept of strong arcs. Akram [1] also does several works on fuzzy graphs, Akram and Dudek [2], Akram et al. [3], Manjusha and Sunitha [16], Narayan and Sunitha [28], Mathew and Sunitha [14], and Rashmanlou et al. [5]. Global domination in graphs was discussed by Sampathkumar [7]. Somasundaram and Somasundaram [25] discussed domination in fuzzy graphs. They defined domination using effective edges in fuzzy graphs [19], [25]. Nagoorgani and Chandrasekharan [21] explained domination in fuzzy graphs using strong arcs. Manjusha and Sunitha [15], [16] discussed some concepts of domination and total domination in fuzzy graphs using strong arcs. Akram [1] did related works on bipolar fuzzy graphs, Akram et al. [3], Akram and Waseem [4], and Nagoorgani et al. [22]. This paper discusses global domination in fuzzy graphs using strong arcs.

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2 | Preliminaries

It is pretty well-known that graphs are simply models of relations. A graph is a convenient way of representing information involving the relationship between objects. Vertices and relations by edges represent the objects. When there is vagueness in the description of the objects, their relationships, or both, it is natural that we must design a 'fuzzy graph model.' It briefly summarises some basic definitions in fuzzy graphs presented in [8], [9], [12], [15], [17], [20], [21], [23], [25], [26].

A fuzzy graph is denoted by $G:(V,\sigma,\mu)$, where V is a node-set, σ and μ are mappings defined as $\sigma:V\to[0,1]$ and $\mu:V\times V\to[0,1]$, where σ and μ represent the membership values of a node and an arc respectively. For any fuzzy graph, $\mu(x,y)\le\sigma(x)\wedge\sigma(y)$. We consider fuzzy graph G with no loops and assume that V is finite and nonempty, μ is reflexive (i.e. $\mu(x,x)\le\sigma(x)$, for all x) and symmetric (i.e. $\mu(x,y)=\mu(y,x)$, for all (x, y)). In all the examples, σ is chosen suitably. Also, we denote the underlying crisp graph by G*: (σ*, μ*) where σ* = $\{u\in V:\sigma(u)>0\}$ and $\{(u,v)\in V\times V:\mu(u,v)>0\}$. Throughout, we assume that $\sigma^*=V$. The fuzzy graph $H:(\tau,\nu)$ is said to be a partial fuzzy subgraph of $G:(V,\sigma,\mu)$ if $\nu\subseteq\mu$ and $\tau\subseteq\sigma$. In particular, we call $H:(\tau,\nu)$ a fuzzy subgraph of $G:(V,\sigma,\mu)$ if $\tau(u)=\sigma(u)$ for all $u\in\tau^*$ and $\nu(u,v)=\mu(u,v)$ for all $(u,v)\in\nu^*$. A fuzzy graph $G:(V,\sigma,\mu)$ is called trivial if $\sigma^*=1$. Two nodes u and v in a fuzzy graph G are said to be adjacent (neighbors) if $\mu(u,v)>0$. The set of all neighbors of u is denoted by $N(u)$. An arc (u,v) of a fuzzy graph $G:(V,\sigma,\mu)$ with $\mu(u,v)>0$ is called a the weakest arc of G if (u,v) is an arc with a minimum $\mu(u,v)$. A path P of length n is a sequence of distinct nodes $u_o,u_1,\dots\dots,u_n$ such that $\mu(u_{i-1},u_i)>0, i=1,2,\dots\dots n$ and the degree of membership of the weakest arc is defined as its strength. If $u_0=u_n$, and $n\ge3$, then P is called a cycle, and P is called a fuzzy cycle if it contains more than one weakest arc. A cycle's strength is the strength of its weakest arc. The strength of connectedness between two nodes, x and y, is defined as the maximum of the strengths of all paths between x and y and is denoted by $CONN_G(x,y)$. A fuzzy graph $G:(V,\sigma,\mu)$ is connected if for every x, y in σ^*, $CONN_G(x,y)>0$. An arc (u,v) of a fuzzy graph is called an effective arc if $\mu(u,v)=\sigma(u)\wedge\sigma(v)$. neighbors of u are called the effective neighborhood of u and are denoted by $EN(u)$.

A fuzzy graph $G:(V,\sigma,\mu)$ is said to be complete if μ $\mu(u,v)=\sigma(u)\wedge\sigma(v)$, for all $u, v \in \sigma^*$.

The order p and size q of a fuzzy graph $G:(V,\sigma,\mu)$ is defined to be $p=\sum_{x\in V}\sigma(x)$ and $q=\sum_{(x,y)\in V\times V}\mu(x,y)$.

Let $G:(V,\sigma,\mu)$ be a fuzzy graph and $S\subseteq V$. Then the scalar cardinality of S is defined to be $\sum_{v\in S}\sigma(v)$, and it is denoted by $|S|_s$. Let p denotes the scalar cardinality of V, also called the order of G.

The complement of a fuzzy graph $G:(V,\sigma,\mu)$, denoted by $\overline{G}$, is defined to be G = (V, σ, μ) where $\overline{\mu(x,y)}=\sigma(x)\wedge\sigma(y)-\mu(x,y)$ for all x, y ∈ V [13]. An arc of a fuzzy graph G: (V, σ, μ) is called strong if its weight is at least as great as the strength of connectedness of its end nodes when it is deleted. A fuzzy graph G is called a strong fuzzy graph if each arc in G is a strong arc. Depending on $CONN_G(x, y)$ of an arc (x, y) in a fuzzy graph G, Mathew and Sunitha [17] defined three different types of arcs. Note that

$CONN_{G-(x,y)}(x, y)$ is the strength of connectedness between x and y in the fuzzy graph obtained from G by deleting the arc (x, y). An arc (x, y) in G is α−strong if $\mu(x, y) > CONN_{G-(x,y)}(x, y)$.

An arc (x, y) in G is β−strong if $\mu(x, y) = CONN_{G-(x,y)}(x, y)$. An arc (x, y) in G is δ−arc if $\mu(x, y) > CONN_{G-(x,y)}(x, y)$.

Thus, an arc (x, y) is strong if it is either α strong or β strong. Also, y is called a strong neighbor of x if the arc (x, y) is strong. The set of all strong neighbors of x is called the strong neighborhood of x and is denoted by $N_s(x)$. The closed strong neighborhood $N_s[x]$ is defined as $N_s[x] = N_s(x) \cup \{x\}$ A path P is called a strong path if P contains only strong arcs.

A fuzzy graph G: (V, σ, μ) is said to be bipartite [25] if the vertex set V can be partitioned into two non-empty sets V_1 and V_2 such that $\mu(v_1, v_2) = 0$ if $V_1, V_2 \in V_1$ or $V_1, V_2 \in V_2$. Further if $\mu(x, y) = \sigma(x) \wedge \sigma(y)$ for all u ∈ V_1 and v ∈ V_2, then G is called a complete bipartite graph and is denoted by $K_{\sigma 1, \sigma 2}$, where σ_1 and σ_2 are respectively the restrictions of σ to V_1 and V_2.

A node u is said to be isolated if μ(u, v) = 0 for all $v \neq u$.

3 | Strong Global Domination in Fuzzy Graphs

This section introduces the concept of global domination in fuzzy graphs using strong arcs. Recall the notion of global domination in graphs introduced by Sampathkumar [7]. According to him, a dominating set S of G is a global dominating set of G if S is also a dominating set of the complement of G. The minimum number of vertices in a global dominating set of G is the global domination number $\gamma_g(G)$ of G.

Nagoorgani and Hussain [24] introduced the concept of global domination in fuzzy graphs using effective arcs. According to then a set $D \subseteq V$ is a dominating set of a fuzzy graph G if every vertex in V\ D is effective adjacent to some vertex in D. A fuzzy dominating set D of an effective fuzzy graph G is a global fuzzy dominating set if D is also a fuzzy dominating set of the complement of a fuzzy graph G. The global fuzzy domination number is the minimum fuzzy cardinality of a fuzzy global dominating set. These concepts have motivated researchers to reformulate the concept of global domination more effectively. The studies in [24] are the main motivation of this article, and it is introduced the definition of global domination of a fuzzy graph using strong arcs. This introduction is essential because the parameter 'global domination number' defined by Nagoorgani and Hussain [24] is inclined more towards graphs than fuzzy graphs. Using the new definition of global domination number, it is possible to reduce the value of the old global domination number and extract classic results in a fuzzy graph.

Definition 1 ([21]). A node v in a fuzzy graph G : (V, σ, μ) is said to strongly dominate itself and each of its strong neighbors; that is, v strongly dominates the nodes in $N_s[v]$. A set D of nodes of G is a strong dominating G set if every node of V(G) −D is a strong neighbor of some node in D.

Definition 2. A strong dominating set D of a fuzzy graph G : (V, σ, μ) is called a strong global dominating set of G if D is also a strong dominating set of the complement of the fuzzy graph G.

Definition 3. The weight of a strong global dominating set D is defined as W (D) = u∈D μ(u, v), where μ(u, v) is the minimum of the membership values (weights) of the strong arcs incident on u. The strong global domination number of a fuzzy graph G is defined as the minimum weight of strong global dominating sets of G, and it is denoted by $\gamma_{sg}(G)$ or simply γ_{sg}. A minimum strong global dominating set in a fuzzy graph G is a strong global dominating set of minimum weight.

Let $\gamma_{sg}(G)$ or γ_{sg} denote the strong global domination number of the complement of a fuzzy graph G.

Remark 1. Note that in any undirected fuzzy graph $G:(V, \sigma, \mu)$, for any $x, y \in V$, if (x, y) is a strong arc in G, then (x, y) need not be a strong arc in $\overline{G}$. That is, if x strongly dominates y in G, then x need not strongly dominate y in $\overline{G}$.

Remark 2. If all the nodes are isolated, then V is the only strong global dominating set of G of order p and $\gamma_{sg} = 0$. That is, $N_s(u) = \phi$ for each $u \in V$.

Example 1. Consider the fuzzy graph given in *Fig. 1*.

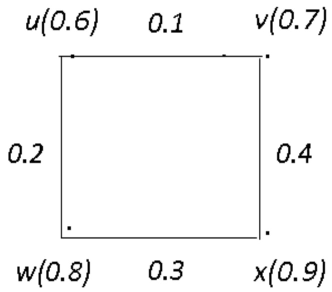

Fig. 1. Illustration of strong global domination in fuzzy graphs.

In this fuzzy graph, strong arcs are (u, w), (w, x), and (x, v). The strong global dominating sets are $D_1 = \{u, x\}$, $D_2 = \{u, v\}$, $D_3 = \{w, x\}$, and $D_4 = \{v, w\}$. Among these, the minimum strong global dominating sets are D_1 and D_3 where

$$W(D_1) = 0.2 + 0.3 = 0.5 \text{ and } W(D_3) = 0.2 + 0.3 = 0.5.$$

Hence $\gamma_{sg} = 0.5$.

4 | Strong Global Domination Number for Classes of Fuzzy Graphs

This section determines the strong global domination number of a complete fuzzy graph, complete bipartite fuzzy graphs, and fuzzy cycles.

Proposition 1. If $G:(V, \sigma, \mu)$ is a complete fuzzy graph, then

$$\gamma_{sg}(G) = n\mu(u, v).$$

Where n is the number of nodes in G and $\mu(u, v)$ is the weight of the weakest arc in G.

Proof. Since G is a complete fuzzy graph, all arcs are strong [25] and each node is adjacent to all other nodes. Also, all nodes in $\overline{G}$ are isolated nodes hence the set of all nodes of G is the strong global dominating set of G. Hence the result follows.

Proposition 2. In any fuzzy graph $G:(V, \sigma, \mu)$, the number of elements in any minimum strong global dominating set of both G and $\overline{G}$ are the same.

Proof. By definition, a strong global dominating set is a dominating set of both G and $\overline{G}$. Hence the proposition.

Proposition 3. $\gamma_{sg}(K_{\sigma1,\sigma2}) = 2\mu(u, v)$, where $\mu(u, v)$ is the weight of the weakest arc in $K_{\sigma1,\sigma2}$ and $u \in V_1$, $v \in V_2$.

Proof. In $K_{\sigma1,\sigma2}$, all arcs are strong. Also, each node in V_1 is adjacent to all nodes in V_2. Hence in $K_{\sigma1,\sigma2}$, the strong global dominating set is any set containing precisely two nodes, one in V_1 and the other in V_2. The end nodes say $\{u, v\}$ of any weakest arc (u, v) in $K_{\sigma1,\sigma2}$ forms the minimum strong global dominating set of G. Hence $\gamma_{sg}(K_{\sigma1,\sigma2}) = \mu(u, v) + \mu(u, v) = 2\mu(u, v)$. So the proposition is proved.

Theorem 1. Let $G:(V, \sigma, \mu)$ be a fuzzy cycle where G^* is a cycle. Then, $\gamma_{sg}(G) = \vee\left\{W\left(D\right): D \text{ is a strong global dominating set in } G \text{ with } |D| \geq \left\lfloor \frac{n}{3} \right\rfloor\right\}$, where n is the number of nodes in G.

Proof. In a fuzzy cycle, every arc is strong. Also, the number of nodes in a strong global dominating set of both G and G^* are the same because each arc in both graphs is strong. In graph G^*, the strong global domination number of G^* is obtained as $\lfloor\frac{n}{3}\rfloor$ [27]. Hence the minimum number of nodes in a strong global dominating set of G is $\lfloor\frac{n}{3}\rfloor$. Therefore the result follows.

Proposition 4. Let $G:(V, \sigma, \mu)$ be a non trivial fuzzy graph of size q. Then $\gamma_{sg}(G) = q$ if and only if all arcs are strong and each node is either isolated or has a unique, strong neighbor.

Proof. If all arcs are strong and each node is either an isolated node or has a unique, strong neighbor, then the minimum strong global dominating set of G is a set D containing nodes, each of which is either an isolated node or an end node of each unique, strong arc. Hence the weight of D is exactly

$$W\left(D\right) = \sum_{u \in D} \mu\left(u, v\right) = q.$$

Hence $\gamma_{sg} = q$.

Conversely, suppose that $\gamma_{sg} = q$. To prove that all arcs are strong and each node is either isolated or has a unique, strong neighbor. If possible, let (u, v) be an arc of G, which is not strong. Then the weight of this arc is not counted for getting γ_{sg}. Hence $\gamma_{sg} < q$, a contradiction. Hence all arcs are strong.

Let x be any node of G. If x is an isolated node, then clearly, x is contained in the minimum strong global dominating set. If possible, suppose x has two strong neighbors say v and w. Then exactly one of the weights of the arcs (x,v) and (x,w) contribute to the weight of the minimum strong global dominating set. Hence $\gamma_{sg} < q$, a contradiction. Hence each node has a unique, strong neighbor.

Remark 3. In proposition 4, G is exactly $S \cup N$ where S is a set of isolated nodes, may be empty, and N is a union of K_2 s.

Remark 4. In any fuzzy graph $G:(V, \sigma, \mu)$, $\gamma_{sg} < p$ always holds since $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$ for all $x, y \in \sigma^*$, [p is the scalar cardinality of G, which is obtained by using the node weights, and γ_{sg} is the weight of the minimum strong global dominating set, which is obtained by using the arc weights].

For the strong global domination number γ_{sg}, the following theorem gives a Nordhaus-Gaddum type result.

Theorem 2. For any fuzzy graph $G:(V, \sigma, \mu), \gamma_{sg} + \gamma_{sg} < 2p$.

Proof. Since $\gamma_{sg} < p$ and $\gamma_{sg} < p$ by remark 4, we have $\gamma_{sg} + \gamma_{sg} < p + p = 2p$.

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Definition 4. A strong global dominating set D of a fuzzy graph G : (V, σ, μ) is called a minimal strong global dominating set if no proper subset of D is a strong global dominating set.

Example 2. In *Fig. 1* of example 1, $D=\{u,v\}$ is a minimal strong global dominating set.

Definition 5 ([18]). A strong dominating set D of a fuzzy graph G : (V, σ, μ) is a strongly connected dominating set of G if the induced fuzzy sub graph < D > is connected.

Remark 5 ([18]). Note that a fuzzy graph G : (V, σ, μ) contains a strong, connected dominating set if and only if G is connected.

Definition 6 ([18]). The weight of a strong, connected dominating set D is defined as $W\ D) = \sum_{u\in D} \mu(u,v)$, where μ(u, v) is the minimum of the membership values(weights) of strong arcs incident on u. The strong connected domination number of a fuzzy graph G is defined as the minimum weight of strong, connected dominating sets of G, and it is denoted by $\gamma_{sc}(G)$ or simply γ_{sc}. A minimum strong, connected dominating set in a fuzzy graph G is a strong, connected dominating set of minimum weight.

Let $\gamma_{sc}(\overline{G})$ or $\overline{\gamma_{sc}}$ denote the strong connected domination number of the complement of a fuzzy graph G.

Remark 6. Let D be a minimum strong global dominating set of a fuzzy graph G : (V, σ, μ). Then D induces a connected subgraph in G or G^c. Hence D is a strongly connected dominating set of G or G^c. Thus the following proposition is established.

Proposition 5. For any fuzzy graph G : (V, σ, μ), at least one of the following holds;

- $\gamma_{sc} \leq \gamma_{sg}$.
- $\overline{\gamma_{sc}} \leq \overline{\gamma_{sg}}$.

5 | Strong Global Domination in Complement of Fuzzy Graphs

Sunitha and Vijayakumar [13] have defined the present notion of the complement of a fuzzy graph. Sandeep and Sunitha [28] have studied the connectivity concepts in a fuzzy graph and its complement. The complement of a fuzzy graph G, denoted by $\overline{G}$ or G^c is defined to be $\overline{G}=(V,\sigma,\overline{\mu})$ where $\overline{\mu}(x,y)=\sigma(x)\wedge\sigma(y)-\mu(x,y)$ for all $x,y\in V$ [13]. Bhutani [8] has defined the isomorphism between fuzzy graphs.

Consider the fuzzy graphs $G_1 : (V_1, \sigma_1, \mu_1)$ and $G_2 : (V_2, \sigma_2, \mu_2)$ with $\sigma_1^* = V_1$ and $\sigma_2^* = V_2$. An isomorphism [8] between two fuzzy graphs G_1 and G_2 is a bijective map h: $V_1 \rightarrow V_2$ that satisfies;

- $\sigma_1(u)=\sigma_2(h(u))$ *for all* $u \in V_1$.
- $\mu_1(u, v) = \mu_2(h(u), h(v))$ *for all* $u, v \in V_1$ *and we write* $G_1 \approx G_2$.

A fuzzy graph G is self-complementary [13] if $G \approx \overline{G}$.

Theorem 3 ([13]). If G is an M-strong fuzzy graph, then G^c is also an M-strong fuzzy graph.

Theorem 4. If G is an M-strong fuzzy graph, then G and G^c have the same strong global dominating set.

Proof. By theorem 3, if G is an M-strong fuzzy graph, then G^c is also an M-strong fuzzy graph. Then the end nodes of the M-strong arcs in G are isolated nodes in G^c, and isolated nodes in G are the end nodes of M-strong arcs in G^c. Hence every strong global dominating set of G is a strong global dominating set of G^c and vice-versa. Thus the theorem follows.

Theorem 5. Every non-trivial self-complementary connected fuzzy graph G has a strong global dominating set D whose complement $V \setminus D$ is also a strong global dominating set.

Proof. Every non-trivial connected fuzzy graph G has a strong dominating set D whose complement V − D is also a strong dominating set [16]. Since G is self-complementary, G ≅ G^c. Hence G and G^c are connected. Hence the theorem follows by using the result in [16].

Theorem 6. For any self-complementary connected fuzzy graph $G:(V, \sigma, \mu)$, $\gamma_{sg} \leq p/2$.

Proof. Let D be a minimal strong global dominating set of G. Then, by theorem 3, V − D is a strong global dominating set of G. Then $\gamma_{sg} \leq W(D)$ and $\gamma_{sg} \leq W(V-D)$.

Therefore $2\gamma_{sg} \leq W(D) + W(V-D) \leq p$ implies $\gamma_{sg} \leq p/2$. Hence the proof.

Corollary 1. Let G be a self complimentary connected fuzzy graph. Then $\gamma_{sg} + \overline{\gamma_{sg}} \leq p$ further equality holds if and only if $\gamma_{sg} = \overline{\gamma_{sg}} = p/2$.

Proof. By theorem 6, $\gamma_{sg} \leq p/2, \overline{\gamma_{sg}} \leq p/2 \Rightarrow \gamma_{sg} + \overline{\gamma_{sg}} \leq p/2 + p/2 = p$, that is $\gamma_{sg} + \overline{\gamma_{sg}} \leq p$. If $\gamma_{sg} = \overline{\gamma}_{sg} = p/2$, then obviously $\gamma_{sg} + \overline{\gamma}_{sg} = p$. Conversely, suppose $\gamma_{sg} + \overline{\gamma}_{sg} = p$. Then, by theorem 6, we have $\gamma_{sg} \leq p/2$, $\overline{\gamma_{sg}} \leq p/2$. If either $\gamma_{sg} < p/2$ or $\overline{\gamma_{sg}} < p/2$, then $\gamma_{sg} + \overline{\gamma}_{sg} < p$, which is a contradiction. Hence the only possibility is that $\gamma_{sg} = \overline{\gamma}_{sg} = p/2$.

Theorem 7. In any fuzzy graph, $G:(V, \sigma, \mu)$, $\gamma_{sg} = p/2$ if and only if the following conditions hold.

I. The graph is a self-complementary fuzzy graph.
II. All nodes have the same weight.
III. All arcs are M-strong arcs.
IV. For every minimum strong global dominating set D of G, $|D| = n/2$, where n is the number of nodes of G, and n is even.

Proof. If all the above conditions hold, then obviously $\gamma_{sg} = p/2$.

Conversely, suppose $\gamma_{sg} = p/2$. If the graph is not self-complementary, then clearly $\gamma_{sg} < p/2$. If some nodes say u and v have different weights, then the arc weight corresponding to these nodes is $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$.

If $\mu(x, y) < \sigma(x) \wedge \sigma(y)$, then obviously $\gamma_{sg} < p/2$, a contradiction.

If $\mu(x, y) = \sigma(x) \wedge \sigma(y)$, then (x,y) is a M-strong arc.

If $|D| < n/2$, then clearly $\gamma_{sg} < p/2$, a contradiction.

Hence all the conditions are sufficient.

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6 | Strong Global Domination in Fuzzy Trees

A fuzzy subgraph H : (τ, ν) spans the fuzzy graph G : (V, σ, μ) if τ = σ [20]. A connected fuzzy graph G = (V, σ, μ) is called a fuzzy tree (f-tree) if it has a fuzzy spanning subgraph F : (σ, ν), which is a tree, where for all arcs (x, y) not in F there exists a path from x to y in F whose strength is more than μ(x, y) [26]. Note that here F is a tree that contains all nodes of G and hence is a spanning tree of G. Also, note that F is the unique Maximum Spanning Tree (MST) of G [11], where a MST of a connected fuzzy graph G: (V, σ, μ) is a fuzzy spanning subgraph T : (σ, ν), such that T* is a tree, and for which $\sum_{u=v} \nu(u,v)$ is maximum [20].

An arc is called a fuzzy bridge (f-bridge) of a fuzzy graph G: (V, σ, μ) if its removal reduces the strength of connectedness between some pair of nodes in G [26].

Similarly, a fuzzy cut node (f-cut node) w is a node in G whose removal from G reduces the strength of connectedness between some pair of nodes other than w [26].

A node z is called a fuzzy end node (f-end node) if it has precisely one strong neighbor in G [10].

A non-trivial fuzzy tree G contains at least two fuzzy end nodes, and every node in *G* is either a fuzzy cut node or a fuzzy end node [10].

In an f-tree G, an arc is strong if and only if it is an arc of F, where F is the associated unique MST of G [9], [11]. Note that these strong arcs are α-strong, and there are no β-strong arcs in an f-tree [17]. Also, note that in an f-tree G, an arc (x, y) is α-strong if and only if (x, y) is an f-bridge of G [17].

Theorem 8 ([12]). The strong arc incident with a fuzzy end node is a fuzzy bridge in any non-trivial fuzzy graph G : (V, σ, μ).

Corollary 2 ([12]). In a non-trivial fuzzy tree G : (V, σ, μ) except K_2, the strong neighbor of a fuzzy end node is a fuzzy cut node of G.

Theorem 9. In a non-trivial fuzzy tree G : (V, σ, μ), every node of a strong global dominating set is either a fuzzy cut node or a fuzzy end node.

Proof. A non-trivial fuzzy tree G contains at least two fuzzy end nodes, and every node in G is either a fuzzy cut node or a fuzzy end node [10]. Hence the theorem.

Theorem 10. In a non-trivial fuzzy tree G : (V, σ, μ), each node of a strong global dominating set is incident on a fuzzy bridge of G.

Proof. Let D be a strong global dominating set of G. Let $u \in D$. Since D is a strong global dominating set, there exists $v \in V \setminus D$ such that (u, v) is a strong arc. Also, D is a strong global dominating set of $\overline{G}$. Then (u, v) is an arc of the unique MST F of G [9], [11]. Hence (u, v) is an f-bridge of G [26]. Since u is arbitrary, this is true for every node of the strong global dominating set of G. This completes the proof.

7 | Practical Application

Let G be a graph that represents the road network connecting various hospitals. Let the vertices denote the hospitals, and the edges denote the roads connecting the hospitals. Suppose we need to navigate patients in between hospitals during busy hours. The membership functions σ and μ on the vertex set and the edge set of G's can be constructed from the statistical data that represents the number of ambulances going to various hospitals and the number of ambulances passing through multiple roads during a busy hour. Now the term' busy' is vague in nature. It depends on the availability of ambulances, time of journey, hospital's demands, special requirements of patients, etc. Thus, we get a fuzzy graph model. Some of the roads may be too traffic during the busy hour. So, we must think of taking patients to various hospitals through secret roads. In this fuzzy graph, a strong global dominating set D can be interpreted as a set of busy hospitals in the sense that every hospital not in D is connected to a hospital in D by a road or a secret road in which the traffic flow is full.

8 | Conclusion

Global fuzzy domination yields specific, adaptable, and conformable results compared to classical domination and fuzzy domination. Hence it introduced global domination in fuzzy graphs using strong arcs and found some results using the newly defined parameter' global domination number.' It is established that every node of a strong global dominating set of a fuzzy tree is either a fuzzy cut node or a fuzzy end node. It is proved that in a fuzzy tree, each node of a strong global dominating set is incident on a fuzzy bridge. Also, the characteristic properties of the existence of a strong global dominating set for a fuzzy graph and its complement are established.

Reference

[1] Akram, M. (2011). Bipolar fuzzy graphs. *Information sciences*, *181*(24), 5548-5564.

[2] Akram, M., & Dudek, W. A. (2012). Regular bipolar fuzzy graphs. *Neural computing and applications*, *21*, 197-205.

[3] Akram, M., Waseem, N., & Davvaz, B. (2017). Certain types of domination in m–polar fuzzy graphs. *Journal of multiple-valued logic & soft computing*, *29*(6), 619–646.

[4] Akram, M., & Waseem, N. (2017). Novel decision making method based on domination in m-polar fuzzy graphs. *Communications of the Korean mathematical society*, *32*(4), 1077-1097.

[5] Rashmanlou, H., Samanta, S., Pal, M., & Borzooei, R. A. (2015). A study on bipolar fuzzy graphs. *Journal of intelligent & fuzzy systems*, *28*(2), 571-580.

[6] Manjusha, O. T., & Sunitha, M. S. (2015). Strong domination in fuzzy graphs. *Fuzzy information and engineering*, *7*(3), 369-377.

[7] Sampathkumar, E. (1989). The global domination number of a graph. *J. math. phys. sci*, *23*(5), 377-385.

[8] Bhutani, K. R. (1989). On automorphisms of fuzzy graphs. *Pattern recognition letters*, *9*(3), 159-162.

[9] Bhutani, K. R., & Rosenfeld, A. (2003). Strong arcs in fuzzy graphs. *Information sciences*, *152*, 319-322.

[10] Bhutani, K. R., & Rosenfeld, A. (2003). Fuzzy end nodes in fuzzy graphs. *Information sciences*, *152*, 323-326.

[11] Sunitha, M. S., & Vijayakumar, A. (1999). A characterization of fuzzy trees. *Information science*, *113*, 293–300.

[12] Kalathodi, S., & Sunitha, M. S. (2012). *Distance in fuzzy graphs*. Lap Lambert Academic Publishing. https://books.google.com/

[13] Sunitha, M. S., & Vijayakumar, A. (2002). Complement of a fuzzy graph. *Indian journal of pure and applied mathematics*, *33*(9), 1451-1464

[14] Mathew, S., & Sunitha, M. S. (2010). Node connectivity and arc connectivity of a fuzzy graph. *Information sciences*, *180*(4), 519-531.

[15] Manjusha, O. T., & Sunitha, M. S. (2014). Notes on domination in fuzzy graphs. *Journal of intelligent & fuzzy systems*, *27*(6), 3205-3212.

[16] Manjusha O. T., & Sunitha M. S. (2015). Total domination in fuzzy graphs using strong arcs. *Annals of pure and applied mathematics*, *9*(1), 23–33.

[17] Mathew, S., & Sunitha, M. S. (2009). Types of arcs in a fuzzy graph. *Information sciences*, *179*(11), 1760-1768.

[18] Manjusha, O. T., & Sunitha, M. S. (2015). Connected domination in fuzzy graphs using strong arcs. *Annals of fuzzy mathematics and informatics*, *10*(6), 979-994.

[19] Somasundaram, A. (2005). Domination in fuzzy graphs-II. *Journal of fuzzy mathematics*, *13*(2), 281.

[20] Mordeson J. N., & Nair P. S. (2000). *Fuzzy graphs and fuzzy hypergraphs*. Physica Heidelberg. https://doi.org/10.1007/978-3-7908-1854-3

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[21] Nagoorgani, A., & Chandrasekaran, V. T. (2006). Domination in fuzzy graph. *Advances in fuzzy sets and systems*, *1*(1), 17-26.

[22] Nagoorgani, A., Akram, M., & Anupriya, S. (2016). Double domination on intuitionistic fuzzy graphs. *Journal of applied mathematics and computing*, *52*, 515-528.

[23] Gani, A. N., & Vijayalakshmi, P. (2011). Insensitive arc in domination of fuzzy graph. *Int. j. contemp. math. sciences*, *6*(26), 1303-1309.

[24] Nagoorgani, A., & Hussain, R. J. (2008). Connected and global domination of fuzzy graph. *Bull pure appl sci*, 27(2), 1-11.

[25] Somasundaram, A., & Somasundaram, S. (1998). Domination in fuzzy graphs–I. *Pattern recognition letters, 19*(9), 787-791.

[26] Rosenfeld, A. (1975). Fuzzy graphs, fuzzy sets and their applications to cognitive and decision processes. *Acad. Press, New York*, 77-95.

[27] Sampathkumar, E., & Walikar, H. B. (1979). The connected domination number of a graph. *J. math. phys. sci*, *13*(6), 607-613.

[28] Narayan, K. S., & Sunitha, M. S. (2012). Connectivity in a fuzzy graph and its complement. *General mathematics notes, 9*(1), 38-43.