n-Cylindrical Fuzzy Neutrosophic Topological Spaces

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Abstract

The objective of this study is to incorporate topological space into the realm of n- cylindrical fuzzy neutrosophic sets (n-CyFNS), which are the most novel type of fuzzy neutrosophic sets. In this paper, we introduce n-cylindrical fuzzy neutrosophic topological spaces (n-CyFNTS), n-CyFN Open sets, and n-CyFN Closed sets. We also defined the n-CyFN base, n-CyFN subbase, and some related theorems here.

Keywords: n-Cylindrical fuzzy neutrosophic sets, n-Cylindrical fuzzy Neutrosophic open sets, n-Cylindrical fuzzy Neutrosophic closed sets, n-cylindrical fuzzy neutrosophic base.

1 | Introduction

In 1965, L.A. Zadeh[27] laid the stepping stone to the field of uncertainties called fuzzy sets. The prime field of mathematics where the concepts and ideas of fuzzy sets drew a parallel was Topology. In 1968, Chang [5] enlivened the concept of fuzzy topological spaces using Zadeh's definition. Since then the various notions in classical topology have been extended to fuzzy topological spaces. Subsequently in the second half of 1970 and the beginning of 1980, many authors contributed a lot to this new field. Later in 1986, Atanassov[3-4] introduced a new set called intuitionistic fuzzy set (IFS) in which the sum of both acceptance degree and rejection degree grades does not exceed 1. Later, intuitionistic fuzzy topological spaces via intuitionistic fuzzy sets were obtained by Coker et al. [7] In intuitionistic fuzzy topological spaces, Lee and Lee [18] discovered the properties of continuous, open, and closed maps. Yager proposed the Pythagorean fuzzy set (PyFS) as a generalisation of IFS in 2013, which ensures that the value of the square sum of its membership degrees is less than or equal to 1. The concept of Pythagorean fuzzy topological space was introduced by Olgun et al. [20].Cuong and Kreinovich (2014)[6] initiated the idea of the picture fuzzy set (PFS). He utilized three indices (membership degree P (x), neutral-membership degree I (x), and non-membership degree N(x) in PFS with the condition that is $0 \leq P(x) + I(x) + N(x) \leq 1$. Obviously PFSs is more suitable than IFS and PyFS to deal with fuzziness and vagueness.The idea of picture fuzzy topological spaces was first initiated by Abdul Rasaq et.al[22].Later Spherical fuzzy sets (SFS) have been proposed by Gündogdu and Kahraman[17]. SFS should satisfy the condition that the squared sum of membership degree and non-membership degree and hesitancy degree should be equal to or less than one .In 2019, Princy R et.al[21] introduced spherical fuzzy topological spaces.

https://doi.org/10.22105/jfea.2022.365810.1234

The Neutrosophic set was introduced by F. Smarandache[8] and Neutrosophic set is a generalization of Intuitionistic fuzzy set. In 2012, Salama, Alblowi [1], introduced the concept of Neutrosophic topological spaces. They introduced Neutrosophic topological space as a generalization of Intuitionistic fuzzy topological space and a Neutrosophic set besides the degree of membership, the degree of indeterminacy and the degree of non- membership of each element. F.Smarandache introduced the dependence degree of (also, the independence degree of) the fuzzy components, as well as the neutrosophic components, for the first time in 2006[7]. I. Arokiarani and et.al,[14] initiated the notion of fuzzy neutrosophic set as the sum of all the three membership functions does not exceed 3. Fuzzy Neutrosophic topological space and basic operations on it was proposed by Y Veereswari [26] in 2017. Sarannya et al. [24] recently introduced n-Cylindrical fuzzy neutrosophic sets (n-CyFNS), which have T and F as dependent components and I as independent components. Except for fuzzy neutrosophic sets, the n-CyFNS is the largest extension of fuzzy sets. In this case, the degree to which positive, neutral, and negative membership functions satisfy the condition, $0 \leq \beta_A(\mathbf{x}) \leq 1$ and $0 \leq \alpha_A{}^{\mathbf{n}}(\mathbf{x}) + \gamma_A{}^{\mathbf{n}}(\mathbf{x}) \leq 1$, $n>1$, is an integer. They also defined the distance between two n- CyFNS, as well as their properties and basic operations.

In this paper, we introduce topological space in n-CyFNS environment. This is a new type of fuzzy neutrosophic sets in which T and F are dependent components and I independent components. Here we defined n-CyFN topological space, n-CyFN open sets. We also initiated n-CyFN base, n-CyFN subbase and some related results.

2. Preliminaries

This section covers some basic definitions and examples that will be useful in subsequent discussions.

Throughout this paper, U denotes the universe of discourse.

Definition: 2.1 [27]

A fuzzy set A in U is defined by membership function μ_A: A⟶ [0, 1] whose membership value μ_A (x) shows the degree to which x∈ U includes in the fuzzy set A, for all x∈ U.

Definition: 2.2 [5]

A fuzzy topological space is a pair (X, T), where X is any set and T is a family of fuzzy sets in X satisfying following axioms

- **i)** Φ,X ∈ T
- **ii)** If A,B ∈ T, then A∩B ∈T
- **iii)** If A_i ∈T for each i ∈ I, then $\cup_i A_i$ ∈T

Definition:2.3 [3]

An intuitionistic fuzzy set A on U is an object of the form

$A = \{(x, \alpha_A(x), \gamma_A(x) \mid x \in U)\}$ where $\alpha_A(x) \in [0,1]$ is called the degree of membership of x in A, $\gamma_A(x) \in [0,1]$ is called the degree of non-membership of x in A, and where α_A and γ_A satisfy $(\forall x \in U)\ (\alpha_A(x) + \gamma_A(x) \leq 1)$ IFS (U) denote the set of all the intuitionistic fuzzy sets (IFSs) on a universe U.

Definition: 2.4 [8]

A Neutrosophic set A on U is $A = < x, T_A(x), I_A(x), F_A(x) >$; $x \in U$, where T_A, , I_A , F_A: $A \rightarrow\ ]^-0,1^+[$ and

$^-0 < T_A(x) + I_A(x) + F_A(x) < 3^+$

Definition: 2.5 [14]

A fuzzy Neutrosophic set A on U is $A = < x, T_A(x), I_A(x), F_A(x) >$; $x \in U$, where T_A, , I_A , F_A: $A \rightarrow [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$

Definition: 2.6 [8]

A neutrosophic set A on U is an object of the form:

$A = \{(x, u_A(x), \zeta_A(x), v_A(x)) : x \in U\}$, where $u_A(x), \zeta_A(x), v_A(x) \in [0,1]$, $0 \leq u_A(x) + \zeta_A(x) + v_A(x) \leq 3$, for all $x \in U$. $u_A(x)$ is the degree of truth membership, $\zeta_A(x)$ is the degree of indeterminacy and $v_A(x)$ is the degree of non-membership. Here $u_A(x)$ *and* $v_A(x)$ are dependent components and $\zeta_A(x)$ is an independent component.

Definition: 2.7 [1]

A neutrosophic topology (*NT* for short) on a non-empty set X is a family τ of neutrosophic subsets in X satisfying the following axioms

(NT1) $0_N, 1_N \in \tau$

(NT2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(NT3) $\cup G_i \in \tau, \forall \{G_i : i \in J\} \subseteq \tau$

In this case the pair (X, τ) is called a neutrosophic topological space (*NTS* for short) and any neutrosophic set in τ is known as neutrosophic open set (*NOS* for short) in X. The elements of τ are. called open neutrosophic sets, A neutrosophic set F is closed if and only if it C (F) is neutrosophic open

Definition: 2.8 [26]

A fuzzy neutrosophic topology (FNT for short) a non-empty set X is a family τ of fuzzy neutrosophic subsets in X satisfying the following axioms.

(FNT1) $0_N, 1_N \in \tau$

(FNT2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(FNT3) $\cup G_i \in \tau, \forall \{G_i : i \in J\} \subseteq \tau$

In this case the pair (X, τ) is called a Fuzzy neutrosophic topological space (FNTS for short) and any fuzzy neutrosophic set in τ is known as fuzzy neutrosophic open set (FNOS for short) in X. The elements of τ are called open fuzzy neutrosophic sets.

Definition: 2.9 [24]

An n- cylindrical fuzzy neutrosophic set (n-CyFNS) A on U is an object of the form A= $\{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle \mid x \in U\}$ where $\alpha_A(x) \in [0, 1]$, called the degree of positive membership of x in A, $\beta_A(x) \in [0,1]$, called the degree of neutral membership of x in A and $\gamma_A(x) \in [0, 1]$, called the degree of negative membership of x in A, which satisfies the condition, $(\forall x \in U)$ $(0 \le \beta_A(x) \le 1$ and $0 \le \alpha_A{}^n(x) + \gamma_A{}^n(x) \le 1$. , n>1, is an integer. Here T and F are dependent Neutrosophic components and I is 100 % independent.

For the convenience, $\langle \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle$ is called as n-Cylindrical fuzzy Neutrosophic Number (n-CyFNN) and is denoted as A= $\langle \alpha_A, \beta_A, \gamma_A\rangle$

Definition:2.10 The Basic Connectives:[24]

Let C_N (U) denote the family of all n-cylindrical fuzzy neutrosophic sets on U.

Definition:2.10.1: Inclusion: For every two A, B $\in C_N$ (U),

A $\subseteq$ B iff $(\forall x \in U, \alpha_A(x) \le \alpha_B(x)$ and $\beta_A(x) \le \beta_B(x)$ and $\gamma_A(x) \ge \gamma_B(x))$ and

A = B iff (A $\subseteq$ B and B $\subseteq$ A)

Definition: 2.10..2 Union: For every two A, B $\in C_N$ (U), the union of two n-CyFNSs A and B is

$A \cup B(x) = \{\langle x, \max(\alpha_A(x), \alpha_B(x)), \max(\beta_A(x), \beta_B(x)), \min(\gamma_A(x), \gamma_B(x))\rangle \mid x \in U\}$

Definition: 2.10..3 Intersection: For every two A, B $\in C_N$ (U), the intersection of two n- CyFNSs A and B is

$A \cap B(x) = \{\langle x, \min(\alpha_A(x), \alpha_B(x)), \min(\beta_A(x), \beta_B(x)), \max(\gamma_A(x), \gamma_B(x))\rangle \mid x \in U\}$

Definition: 2.10.4 Complementation: For every A $\in C_N$ (U), the complement of an n-CyFNS A is

$A^c = \{\langle x, \gamma_A(x), \beta_A(x), \alpha_A(x)\rangle \mid x \in U\}$

Definition: 2.10.5 Sum: For every two A, B $\in C_N$ (U), the sum of two n- CyFNSs A and B is

$A\oplus B(x) = \{\langle x, (\frac{\alpha_A(x).\alpha_B(x)}{\alpha_A(x)+\alpha_B(x)}$, max $(\beta_A(x), \beta_B(x))$, min $(\gamma_A(x), \gamma B(x))\rangle \mid x \in \mho\}$

Definition: 2.10.6 Difference: For every two A, B $\in \mathcal{C}_N$ (Ʊ), the difference of two n-CyFNSs A and B is

$A\ominus B(x) = \{\langle x$, max $(\alpha_A(x), \alpha_B(x))$, mini $(\beta_A(x), \beta_B(x)), \frac{\gamma_A(x).\gamma_B(x)}{\gamma_A(x)+\gamma_B(x}\rangle \mid x \in \mho\}$

Definition: 2.10.7 Product: For every two A, B $\in \mathcal{C}_N$ (Ʊ), the product of two n-CyFNSs A and B is

$A\otimes B(x) = \{\langle x, (\alpha_A(x).\alpha_B(x), \beta_A(x).\ \beta_B(x), \gamma_A(x).\gamma_B(x))\rangle \mid x \in \mho\}$

Definition: 2.10.8 Division: For every two A, B $\in \mathcal{C}_N$ (Ʊ), A$\oslash$ B is A$\oslash$ B(x) = $\{\langle x$, min $(\alpha_A(x), \alpha_B(x)), \beta_A(x).\ \beta_B(x)$, max $(\gamma_A(x).\gamma_B(x))\rangle \mid x \in \mho\}$

Results:2.11 [24]

a) If A ⊆ B and B ⊆ C then A ⊆ C;
b) A⋃B=B⋃A & A⋂B=B⋂A
c) (A⋃B) ⋃C=A⋃ (B⋃C) & (A⋂B) ⋂C=A⋂ (B⋂C)
d) (A⋃B) ⋂C= (A⋂C) ⋃ (B⋂C) & (A⋂B) ⋃C= (A⋃C) ⋂ (B⋃C)
e) A⋂A=A & A⋃A=A
f) De Morgan's Law for A& B ie, $(A\cup B)^{c}=A^{c}\cap B^{c}$ & $(A\cap B)^{c} = A^{c} \cup B^{c}$
g) (A⊕B) = (B⊕A)
h) (A⊗ B) = (B ⊗A)

3. n-Cylindrical Fuzzy Neutrosophic Topological Spaces

Definition: 3.1 Let $\{A_i : i \in I\}$ be an arbitrary family of n-CyFNS in Ʊ. Then

$\{\langle x$, inf $(\alpha_{Ai}(x))$, inf $(\beta_{Ai}(x))$, sup $((\gamma_{Ai}(x))\rangle \mid x \in \mho\} \cap A_i =$

$\cup A_i = \{\langle x$, sup $(\alpha_{Ai}(x))$, sup $(\beta_{Ai}(x))$, inf $((\gamma_{Ai}(x))\rangle \mid x \in \mho\}$

Definition: 3.2 $0_{CyN} = \{\langle x, 0, 0, 1\rangle \mid x \in \mho\}$ and $1_{CyN} = \{\langle x, 1, 1, 0\rangle \mid x \in \mho\}$

n-Cylindrical Fuzzy Neutrosophic Topological Spaces

In this part, we give a definition of n-cylindrical fuzzy neutrosophic topology and its related properties according to Chang's FTS.

Definition:3.3

An n-cylindrical fuzzy neutrosophic topology (n-CyFNT) on a non-empty set X is a family,τ_X, of n-cylindrical fuzzy neutrosophic sets in X which satisfies the following conditions:

i) 0_{cyN} , 1_{cyN} $\in \tau_X$
ii) $A_1 \cap A_2 \in \tau_X$
iii) $\cup A_i \in \tau_X$, for any arbitrary family $A_i \in \tau_X$, i$\in I$

The pair (X, τ_X) is called an n-cylindrical fuzzy neutrosophic topological space n-CyFNTS and any n-CyFNS belongs to τ_X is called an n-cylindrical fuzzy neutrosophic open set (n-CyFNOS) and the complement of n-CyFNOS is called n-cylindrical fuzzy neutrosophic closed set (n-CyFNCS) in X. Like classical topological spaces and fuzzy topological spaces, the family {0_{cyN}, 1_{cyN}} is called indiscrete n-CyFNTS and the topology containing all the n-CyFN subsets is called Discrete n-CyFNTs.

Remark :3.4

Obviously any fuzzy topological space or intuitionistic fuzzy topological space or pythagorean fuzzy topological space is an n-CyFN topological space as any subsets of the fuzzy space, intuitionistic fuzzy space, and pythagorean fuzzy space can be viewed as n-CyFN subsets. But the converse of the above doesn't follow and it can be evident from the following example.

Example: 3.5

Let X ={ p, q} and τ_X ={ 1_{cyN} , 0_{cyN} ,A,B,C,D}, where

A= {<p; 0.5, 0.5, 0.7>, <q; 0.2,05, 0.4>}, B= {<p; 0.6, 0.5, 0.5>, <q; 0.3, 0.5, 0.9>}

C= {<p; 0.6, 0.5, 0.5>, <q; 0.3, 0.5, 0.4>} D= {<p; 0.5, 0.5, 0.7>, <q; 0.2, 0.5, 0.9>}

is clearly an n-CyFNTS.

Definition: 3.6

Let (X, τ_{X1}) & (X, τ_{X2}) be n-CyFNTSs

i)τ_{X2} is finer than τ_{X1} if $\tau_{X2} \supseteq \tau_{X1}$

if $\tau_{X2} \supset \tau_{X1} ii)\tau_{X2}$ is strictly finer than τ_{X1}

τ_{X2} & τ_{X1} are said to be comparable if it holds $\tau_{X2} \supseteq \tau_{X1}$ or $\tau_{X1} \supseteq \tau_{X2}$.

Example: 3.7

Consider the example 3.5

X ={ p, q}, τ_X ={ 1_{cyN} , 0_{cyN} ,A,B,C,D} and τ_{X1}= {1_{cyN} , 0_{cyN} ,A} are two n-CyFN topologies on X. Clearly we can see that $\tau_X \supset \tau_{X1}$

Definition: 3.8

Let (X, τ_X) be a CyFNTS on X

i) $\mathcal{B} \subseteq \tau_X$, a sub family of τ_X is called an n-CyFN base for (X, τ_X), if each member of τ_X may be expressed as the union of members in $\mathcal{B}$.

a sub family of τ_X is called a n-CyFN sub-base for (X, τ_X),If the family of all $ii)\ \mathcal{S} \subseteq \tau_X$, finite intersections of $\mathcal{S}$ forms a base for (X, τ_X). Here it can be said that $\mathcal{S}$ $generates$ (X, τ_X).

Theorem: 3.9 Let (X, τ_X) be an n-CyFNTS and $\mathcal{B} \subseteq \tau_X$, be a n-cylindrical fuzzy neutrosophic base for τ_X. Then τ_X is the collection of all union of members of $\mathcal{B}$.

Proof: The definition of the base of an n-CyFNTS clearly proves the theorem.

Theorem: 3.10

Let (X, τ_X) be an n-CyFNTS and $\mathcal{B} \subseteq \tau_X$, Then $\mathcal{B}$ is an n-cylindrical fuzzy neutrosophic base for τ_X if and only if for any x∈X and any G∈ τ_X containing x, there exists B∈ $\mathcal{B}$ such that x∈B⊆ G

Proof: Suppose $\mathcal{B}$ is an n-cylindrical fuzzy neutrosophic base for τ_X.

Let G∈ τ_X and x∈G. Now G $=\bigcup_i B_i$, $B_i \in \mathcal{B}$.

x∈G ⇒x∈ $\bigcup_i B_i$
⇒x∈ B_i for some B_i and let $B_i = B$
That is, x∈B $= B_i \subseteq \bigcup_i B_i$ ⊆G.
Hence x∈B⊆ G.

Conversely suppose the given condition holds, ie, Let G ∈ τ_X. For each x∈G, there exists B_x∈ $\mathcal{B}$ such that x∈B_x⊆ G

B_x⊆ G for all x. Then,

B_x ⊆G............**(1)**$\bigcup_{x\in G}$

But from the assumption G∈ τ_X and , x∈B_x for all x∈G and B_x⊆ G

Since G is n-CyFNOS in X, G can be expressed as
G ⊆$\bigcup_{x\in G}$ B_x where B_x∈ $\mathcal{B} \subseteq \tau_X$ ....**(2)**
From **1 & 2**
G = $\bigcup_{x\in G}$ B_x ; B_x∈ $\mathcal{B}$ Thus $\mathcal{B}$ is an n-CyFN base for τ_X.

Definition: 3.11

Let (X, τ_X) be an n-CyFNTS and Y⊆ X. Then the collection $\tau_Y = \{X_i \cap Y: X_i \in \tau_X, i \in I\}$ is called n-cylindrical fuzzy neutrosophic subspace topology on Y. Hence (Y, τ_Y) is called n-cylindrical fuzzy neutrosophic topological subspace of (X, τ_X).

Theorem: 3.12

Let (X, τ_X) be an n-CyFNTS and Y⊆ X, then τ_Y, an n-CyFN subspace topology on Y is an n-CyFNTS.

Proof: Certainly 0_{cy}, $1_{cy} \in \tau_Y$ since $0_{cx} \cap Y = 0_{cy}$ and $1_{cx} \cap Y = 1_{cy}$.

Also $\tau_X = \{X_i \subseteq X, \ i \in I\}$.

Hence it is closed under arbitrary n-cylindrical fuzzy neutrosophic union

$\cup_i (X_i \cup Y) = (\cup_i X_i) \cup Y$.

Also it is closed under finite n-cylindrical fuzzy neutrosophic intersection.

$$\bigcap_{i=1}^{n} (X_i \cap Y) = (\bigcap_{i=1}^{n} X_i) \cap Y$$

Hence the theorem follows.

Example: 3.13 Let X be **the set of all integers.** Consider f ∈ n-CyFNS such that

f(x) = < 1, $\frac{1}{x}$,0 > ; x≥ 1 and x ∈ X

= < 0, $-\frac{1}{x}$, 1 >; x≤ -1

= < 1, 1, 0 > ; x =0

Then (X, τ_X) is an n-CyFNTS with τ_X= {1_{cyN} , 0_{cyN} , f }

Let Y denote set of all even integers ie, y=2x ∈Y

g(y) = < 1, $\frac{1}{y}$,0 > ; y≥ 1

= < 0, $-\frac{1}{y}$, 1> ; y≤ -1

= <1, 1, 0>; y =0

Clearly (Y, τ_Y) is a sub space topology, τ_Y = {{1_{cyN} , 0_{cyN} , g}

Theorem: 3.14

If $\mathcal{B}$ is an n-CyFN base for (X, τ_X) and $Y \subseteq X$, then $\mathcal{B}_Y = \{B \cap Y \mid B \in \mathcal{B}\}$ is an n-CyFN base for (Y, τ_Y)

Proof: Let G is n-CyFN open in X and $y \in G \cap Y$. Now choose $B \in \mathcal{B}$ such that $y \in B \subseteq G$.

Thus $y \in B \cap Y \subseteq G \cap Y$. Hence $\mathcal{B}_Y$ an n-CyFN base for (Y, τ_Y) by theorem 3.10

Theorem: 3.15 Let (X, τ_X) be a CyFNTS and (Y, τ_Y) be an n-cylindrical fuzzy neutrosophic topological subspace. If $Z \subseteq Y$ is n-cylindrical fuzzy neutrosophic open in Y then Z is n-cylindrical fuzzy neutrosophic open in X.

Proof: It is evident from the definition of n-cylindrical fuzzy topological subspace.

Conclusion

Our goal with this paper is to broaden the scope of n-Cylindrical fuzzy neutrosophic sets to topological spaces. Here we introduce the fundamental definitions of n-CyFNTS, n-CyFN open sets, and n-CyFN closed sets, as well as examples. The terms n-CyFN base, n-CyFN sub base, and related theorems were also defined. This paper is the first to investigate n-Cylindrical fuzzy neutrosophic topological spaces. This research will undoubtedly be the basis for the further development of n-Cylindrical fuzzy neutrosophic topological spaces and their applications in various fields .Evidently, these ideas have the potential to inspire additional research in the future.

Funding

"This research received no external funding"

Conflicts of Interest

"The authors declare no conflict of interest."

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