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Bridge Domination in Fuzzy Graphs

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Abstract

In communication networks, strong connectivity between nodes is critical. The failure of strong connectivity between nodes may jeopardize the network's stability. In fuzzy graphs, various dominating sets using strong edges are identified to avoid network stability. In this paper, the concept of bridge domination set and bridge domination number γ_B G) in fuzzy graphs is introduced. A few prominent properties of bridge domination numbers are chosen and analyzed using relevant examples. The bridge domination number of fuzzy trees, constant fuzzy cycles, and complete fuzzy and bipartite fuzzy graphs are identified. The use of bridge domination in a partial mesh topology to ensure network continuity is demonstrated in the event of a node failure.

Keywords: Fuzzy graph, Domination set, Domination number, Connected bridge domination, Mesh topology.

1 | Introduction

In real-life situations, most of the problems related to marketing, economics, technology, medical diagnosis, environment, etc., can be represented as a network, and if the network size grows, it becomes challenging to maintain track of all of its nodes and connections. So, identifying the nodes dominating the entire network is crucial. Domination has become one of the emerging areas of graph theory. Domination has many applications in neural networks, communication networks, image compression, and parallel computing. In graph theory, Ore [1] and Berge [2] presented the concept of dominating sets and dominance numbers. Sampathkumar and Walikar [3] discussed connected domination in graphs. Banerjee [4] introduced the spectrum of hypergraphs and their applications.

Rosenfeld [5] introduced fuzzy graphs based on fuzzy relations and explored some of their properties in 1975. The concept of dominating set and domination number in fuzzy graphs was introduced by [4]. Somasundaram and Somasundaram [6] obtained several bounds for the domination number in fuzzy graphs. Nagoorgani and Chandrasekaran [7] discussed dominance in fuzzy graphs using strong Arcs and investigated some of their properties. Binu et al. [8], [9] introduced connectivity status in fuzzy graphs using strong edges and discussed cyclic connectedness in a fuzzy network. Nazeer et al. [10] introduced and studied the dominance of fuzzy incidence graphs and derived a relationship

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between strong fuzzy and weak fuzzy incidence domination for complete fuzzy incidence graphs. Chen et al. [11] introduced total efficient domination in fuzzy graphs and studied some of their properties.

Strong connectivity between nodes is crucial in information networks. The stability of the network may be at risk if components are not connected strongly. Different dominating sets [12]–[14] with strong edges are found to prevent network stability in fuzzy networks. Sunitha and Manjusha [15], [16] introduced the global dominating set, connected and strong domination number using strong edges in fuzzy graphs. Motivated by the concept of a connected, strong dominating number and its applicability ([10], [17]–[24]), we focused on introducing bridge dominating set and bridge domination number in fuzzy graphs.

This paper is structured as follows; the preliminary definitions are given in Section 2, and Section 3 introduces the bridge domination number in fuzzy graphs. Section 4 discusses the application of bridge domination in fuzzy graphs in a partial mesh topology, and the paper is concluded in Section 5.

2 | Preliminaries

Definition 1 ([25]). Let V be a non-empty fuzzy set. A fuzzy graph is G=(σ,μ), where σ (vertex membership) is a fuzzy subset of V and μ (edge membership) is a symmetric fuzzy relation on σ, i.e., σ: V→ [0,1] and μ: V×V→ [0,1] such that $\mu(u,v)\le\sigma(u) \wedge \sigma(v)$, for all $u,v \in V$.

Definition 2 ([14]). A fuzzy graph G is said to be complete if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$, for all $u,v \in V$.

Definition 3 ([14]). A fuzzy graph G is said to be strong if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$, for all $(u,v) \in E$.

Definition 4 ([14]). An Arc (u,v) of a fuzzy graph is called a strong Arc if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$, for all $u,v \in V$.

Definition 5 ([6]). Let G=(σ,μ) be a fuzzy graph. A subset S of V is called a dominating set in G if every vertex in V\S is adjacent to some vertex in S.

Definition 6 ([6]). The minimum value of all dominating sets of a fuzzy graph G and it is given by $\gamma(G)$.

Definition 7 ([14]). A connected fuzzy graph $G = (\sigma,\mu)$ is said to be a fuzzy tree if it is spanning fuzzy subgraph $F = (\sigma,\nu)$ is a tree where for all Arcs (x,y) not in F, $\mu(x,y) < \mu^{\infty}(x,y)$.

Definition 8 ([12]). A fuzzy graph G is known as bipartite if V is divided into two non-empty sets V_1 and V_2 such that

$$\mu(v_1,v_2) = \begin{cases} 0, & v_1,v_2 \in V_1 \text{ or } v_1,v_2 \in V_2, \\ \sigma(u) \wedge \sigma(v), & u \in V_1 \text{ and } v \in V_2. \end{cases}$$

Then G is called a complete bipartite fuzzy graph and is denoted by $K_{m,n}$.

Definition 9 ([15]). If the deletion of an Arc (u,v) reduces the strength of connectedness between some pair of nodes, then (u,v) is a fuzzy bridge of $G = (\sigma,\mu)$.

Definition 10 ([15]). A dominating set D of a fuzzy graph $G = (\sigma,\mu)$ is a strongly connected dominating set of G if the induced fuzzy subgraph $< D >$ is strongly connected.

Definition 11 ([15]). The strength of connectedness between two nodes u and v is the maximum of the strengths of all paths between u and v and is denoted by $CONN_G(\boldsymbol{u},\boldsymbol{v})$.

Definition 12 ([14]). Let $G = (\sigma,\mu)$ be a fuzzy graph such that G^* is a cycle. Then G is called a fuzzy cycle if it has more than one weakest Arc.

3 | Bridge Domination in Fuzzy Graphs

Definition 13 ([8]). "Consider $G = (\sigma, \mu)$ to be a fuzzy graph. Let $u, v \in V$, and we say that u dominates v in G if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$. In G, a subset S of V is called a dominating set if there exists $u \in S$ such that u dominates v for every $v \in V - S$. The domination number $\gamma(G)$ of G is denoted by the minimum fuzzy cardinality among all dominating sets in G.

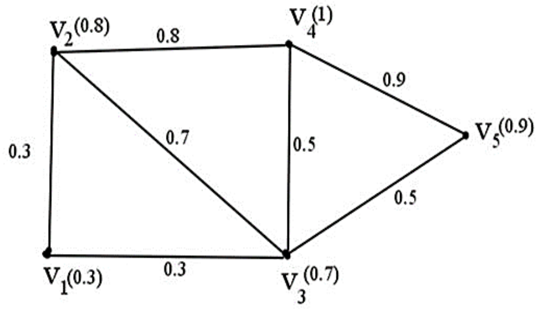

Fig. 1. Domination of a fuzzy graph.

Here, $\{v_1, v_2, v_3, v_4, v_5\}$, $\{v_1, v_2, v_4, v_5\}$, $\{v_1, v_3, v_4, v_5\}$, $\{v_1, v_4, v_5\}$, $\{v_2, v_4, v_5\}$, $\{v_1, v_4\}$, $\{v_2, v_4\}$, $\{v_2, v_5\}$ are some dominating sets of G for *Fig. 1* and its γ(G)=1.3.

Definition 14. Consider $G = (\sigma, \mu)$ to be a fuzzy graph. If the induced fuzzy subgraph $\langle S \rangle$ is connected, and each Arc in S is a fuzzy bridge, a dominating set S of V is a bridge dominating set in G. The bridge domination number of G is defined by $\sum_{(u,v)\in S} \mu(u, v)$, which is the minimum of all bridge-dominating sets of G and is denoted by $\gamma_B(G)$.

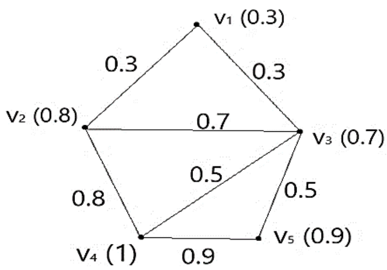

Fig. 2. Domination of a fuzzy graph.

From *Fig. 2*, $\{v_1, v_3, v_2, v_4, v_5\}$, $\{v_1, v_2, v_4, v_5\}$, $\{v_1, v_3, v_2, v_4\}$, $\{v_1, v_2, v_4\}$, $\{v_2, v_4, v_5\}$, $\{v_2, v_4\}$ are some bridge dominating sets of G and $\gamma_B(G)$ =0.8.

Definition 15. Let $G = (\sigma, \mu)$ represent a complete fuzzy graph. The bridge domination number of a complete fuzzy graph is defined by $\gamma_B(G) = \Lambda\{\mu(u, v)/u, v \in V\}$.

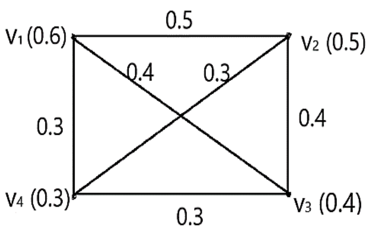

Fig. 3. Complete fuzzy graph.

The bridge domination number of *Fig. 3*, $\gamma_B(G)$ is 0.3.

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Definition 16. Let $G = \sigma, \mu)$ be a fuzzy tree. The bridge domination number of a G is defined by $\bigwedge \sum_{(u,v)\in T} \mu\ u, v)$.

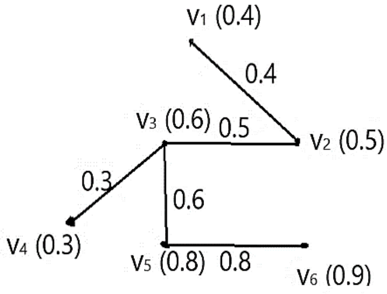

Fig. 4. Fuzzy tree.

The bridge domination number of *Fig. 4*, $\gamma_B\ G)$ is 1.1.

Definition 17. Let $G = \sigma, \mu)$ be a complete bipartite fuzzy graph. The bridge domination number of a complete bipartite fuzzy graph is defined by $\gamma_B\ G) = \{ \sum \mu\left(v_i, v_j\right) / \mu\left(v_i, v_j\right)$ is minimum for every $v_i \in V_1$ *and* $v_j \in V_2$ }.

Theorem 1. For a complete fuzzy graph, $\gamma_B\ G) = \gamma(G)$.

Proof: Let $G = \sigma, \mu)$ be a complete fuzzy graph. Every Arc in a complete fuzzy graph is a strong Arc and $\mu\ u, v) = \sigma\ u) \bigwedge \sigma(v)$. The minimal dominating set S contains only one vertex, i.e., $S = \{v_i\}$, where v_i has the minimum membership value. The minimum cardinality of S is denoted by $\gamma(G)$. The bridge domination number of a K_n is defined by $\gamma_B\ G) = \bigwedge\left\{\mu(u, v)/(u, v) \in G\right\}$. The minimum edge membership value is associated with v_i and hence $\mu(u, v_i)$ is minimum for some $u \in V$ and since G is complete, $\mu\ u, v_i) = \sigma\ v_i)$. Thus $\gamma_B\ G) = \gamma\ G)$.

Theorem 2. If S is a bridge-dominating set of a fuzzy graph $G = \sigma, \mu)$ corresponds to the bridge domination number, then it is a fuzzy tree.

Proof: Let S be the bridge dominating set of a fuzzy graph G. By definition, every Arc (u, v) in S is a fuzzy bridge and is connected. We must show that S contains no fuzzy cycles. Suppose S contains a cycle C. Since each Arc is a fuzzy bridge, they are strong Arcs of C, and S is the dominating set with bridge domination number, so it will not have an additional Arc in the cycle C. So, S is an acyclic graph which is a contradiction. Thus, S is a connected acyclic fuzzy graph, and hence S is a fuzzy tree.

Theorem 3. If $G = \sigma, \mu)$ is a constant fuzzy graph, then $\gamma_B\ G) = \gamma_{SC}\ G)$.

Proof: Let $G = \sigma, \mu)$ be a constant fuzzy graph. The membership values of vertices σ and the edges μ are the same. Let S be the connected bridge dominating set G. Every Arc in S is connected and is a fuzzy bridge. The minimum among all connected bridge dominating sets S and its connected domination number is $\gamma_B\ G)$. Let D be a strongly connected dominating set of a fuzzy graph G. Then every Arc in D is connected, and the minimum among all strongly connected dominating sets D and its strongly connected domination number is $\gamma_{SC}\ G)$.

Here S and D are both minimal dominating and connected. Thus, $\gamma_B\ G) = \gamma_{SC}\ G)$.

Theorem 4. If $K_{n,n}$ is a complete bipartite fuzzy graph, then

$$\gamma_B\left(K_{n,n}\right) = \begin{cases} n\,\sigma + (n-1)\,\sigma^*, & \sigma\ u) \le \sigma^*\ v), \\ n\,\sigma^* +\ n-1)\sigma, & \sigma\ u) > \sigma^*\ v), \end{cases}$$

where σ and σ^* is the minimum membership value of V_1 and V_2 respectively.

Proof: In $K_{n,n}$, all edges are strong edges, and each vertex in V_1 is adjacent to all vertices in V_2. Hence in $K_{n,n}$, the dominating sets are V_1, V_2. The bridge domination number of $K_{n,n}$ is defined by γ_B G) = $\{\sum \mu(v_i, v_j)/\mu(v_i, v_j)$ is minimum for every $v_i \in V_1$ *and* $v_j \in V_2\}$. $K_{n,n}$ is complete, then $\mu(v_i, v_j)$ is either σ v_i) or $\sigma(v_j)$. If σ v_i) is the minimum membership value of V_1, then $\mu(v_i, v_j)$ is the sum of n times of σ v_i) and $(n-1)$ times of $\sigma^*(v_j)$. Also, if $\sigma^*(v_j)$ is the minimum membership value of V_2, then $\mu(v_i, v_j)$ is the sum of n times of $\sigma^*(v_j)$and $(n-1)$ times of $\sigma(v_i)$. Thus,

$$\gamma_B(K_{n,n}) = \begin{cases} n\,\sigma + (n-1)\,\sigma^*, & \sigma\ u) \le \sigma^*\ v), \\ n\,\sigma^* +\ n-1)\sigma\,, & \sigma\ u) > \sigma^*\ v). \end{cases}$$

Theorem 5. If C_n is a constant fuzzy cycle, then the bridge domination number is

$$\gamma_B(C_n) = \begin{cases} \sigma, & n = 3,4, \\ (n-2)\sigma, & n \ge 5. \end{cases}$$

Proof: Since C_n is a constant fuzzy cycle with σ and μ as constant functions, then $\mu = \sigma$.

When $n = 3,4$, the result is trivial.

If $n \ge 5$, then the cycle C_n must contain at least $n-2$) Arcs so that cycle C_n is connected, and each Arc is a bridge. Since $\mu = \sigma$, the minimum among all bridge-dominating sets C_n will contain $n-2$) Arcs with $n-1$) vertices which dominate C_n and its bridge domination number is

$$\gamma_B\ C_n) = \begin{cases} \sigma, & n = 3,4, \\ n-2)\sigma, & n \ge 5. \end{cases}$$

4 | Application of Bridge Domination in Fuzzy Graphs in Partial Mesh Topology

A mesh topology is a basic network topology connection between network devices or computers to share data amongst all the devices. The idea of having this type of network connection is to avoid the breakdown of the entire network in case one of the devices or networks fails. The components of this network are partially interlinked with each other. There is no single centralized control.

About two or three network devices/computers are connected to most devices (not all). These devices are dominating. This means that there is a connection between every device to at least one of the dominating networks. Due to this network topology, it is highly unlikely to lose data.

Let us consider the following network shown below in *Table 1*. to demonstrate the application of bridge domination in partial-mesh topology. Let nodes A, B, C, D, and E be:

Table 1. Nodes of the network.

Node	System
A	Data warehouses
B	Information management systems
C	Transaction processing systems
D	Executive information systems
E	Decision support systems

Assume that each system represents a vertex and the relationship between each system represents the edges. Assigning edge values prioritizes the amount of information received from the systems.

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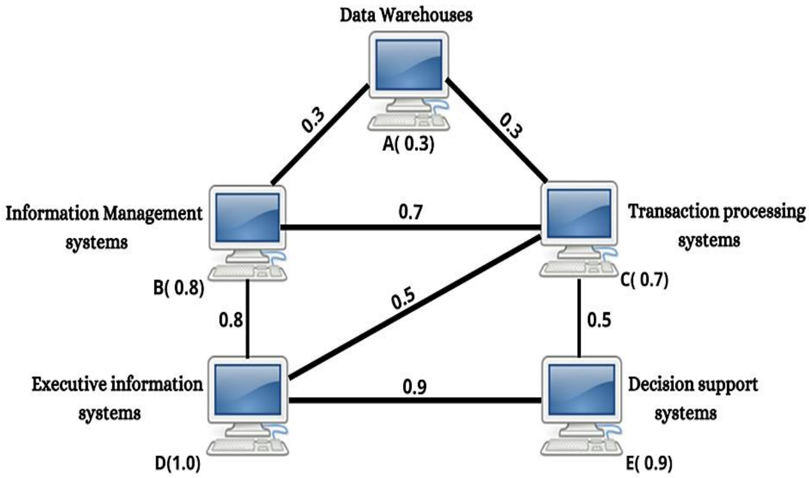

Fig. 5. Representation of computer networks using partial mesh topology.

From the *Fig. 5*, {A, B, C, D, E}, {A, B, D, E}, {A, B, D, C}, {A, B, D}, {B, D, E}, {B, D} are some dominating sets of G and γ_B G) = 0.8. Further, (A, C), (B, C), (B, D), and (D, E) are the fuzzy bridges, and B, C, and D are the fuzzy cut nodes of G.

In this application, the bridge dominating set with induced fuzzy subgraph is {B, D}.

If suppose one of the devices A, C, or E breaks down, then the rest of the networks remain intact due to their connection with the strongly connected dominating elements B and D through the fuzzy bridge. The same procedure applies to complex network connections.

5 | Conclusion

This study introduces the idea of the bridge domination set of a fuzzy graph with strong edges, and several properties of the bridge domination number are investigated. Moreover, the bridge domination set and the related bridge domination number are identified for complete fuzzy graphs, fuzzy trees, and complete bipartite fuzzy graphs. An application of bridge domination sets in partial-mesh topology to find the strong connectivity of a network is demonstrated. The concept of bridge domination can be applied in networking for retrieval of data from systems/servers even in case of a breakdown is propounded. In future work, we can find the bridge domination number in a fuzzy graph using an algorithmic approach.

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Conflicts of Interest

The authors declare that they have no conflict of interest.

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