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A Cubic Set Discussed in Incline Algebraic Sub-Structure

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Abstract

An effective and flexible method for encoding ambiguous data is using cubic sets. The concept of incline algebraic sub-structure is considered and is interlinked with the notation of the cubic set to define cubic subincline. The sense of cubic sub incline of algebra is established with relevant results. Additionally, the results such as homomorphic image, preimage, cartesian product and level sets of cubic sub incline are worked out in this study, and several of its associated findings were looked into.

Keywords: Incline algebra, Cubic sets, Cubic subincline, Image and preimage, Cartesian product.

1 | Introduction

Zadeh [16] worked on a new set called as fuzzy set where the concept of uncertainty in many real applications are analyzed and this fuzzy set plays a vital role in the recent research. And the concept of interval valued fuzzy set were brought out that is membership function in terms of a collection of closed sub-interval of [0, 1]. Later, Atanassov [18] extended the fuzzy set into an intuitionistic fuzzy set by adding non membership for every element. Further, Jun et al. [4] explored a notation in which the first term is of interval valued fuzzy and the second term as fuzzy and is named as a cubic set. Many research is moving a long way with these types of sets and from those work this paper is motivated to work on this topic [1], [2], [8]-[15].

All the structure of incline algebra is introduced by Coa et al. [3] which is a generalization of both Boolean and Fuzzy algebras, which is associative, commutative under addition and multiplication is distributive over addition with $x_0 + x_0 = x_0$, $x_0 + x_0\, y_0 = x_0, y_0 + x_0\;\; y_0 = y_0\;\; for\ all\ x_0, y_0$. It has both a semiring structure and a poset structure and this incline algebra deals with different fields such as the graph theory, decision making, matrices etc.

Many algebraic structures are fused with fuzzy set was started by Rosenfeld [17]. Further many algebras like BF, BCK, BCI and B are successfully correlated with various kinds of fuzzy sets and the

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incline algebra was merged with fuzzy and its generalization by Jun et al. [5]-[7]. Here, this paper is also merging the concept of incline algebra with cubic set and introduces the notion of cubic subincline. Moreover, it deals about the cartesian product, homomorphic image and some related results of the structure introduced.

This work is characterized as the following sections: Section 1 introduces about the work. Section 2 defines the needed definition of this paper and Section 3 deals with the definition of cubic subincline and at last Section 4 concludes the work.

2 | Preliminaries

This part provides some essential definitions on incline and fuzzy sets.

Definition 1. A non-empty set $(\daleth, +, *)$ is an incline algebra if for all $x_0, y_0, z_0 \in \daleth$ the following hold:

- $+$ *is commutative and associative.*
- $*$ *is associative and distributive (both left and right) under* $+$.
- $x_0 + x_0 = x_0$.
- $x_0 + (x_0 * y_0) = x_0$.
- $y_0 + (x_0 * y_0) = y_0$.

Definition 2. A subincline of an incline $\daleth$ is a non-empty subset M of $\daleth$ which is closed under addition and multiplication.

Definition 3. A fuzzy set in a universal set χ is defined as $\gamma : \chi \to [0,1]$.

Definition 4. An interval valued fuzzy set on χ is defined by $C = \{x_0, \overline{\gamma}_C(x_0)\}$ *for all* $x_0 \in \chi$ where $\overline{\gamma}_C : \chi \to D[0,1]$; $D[0,1]$ denotes the family of all closed subintervals of [0, 1]. Here $\overline{\gamma}_C(x_0) = [\gamma_C^L(x_0), \gamma_C^U(x_0)]$ *for all* $x_0 \in \chi$ *with* $\gamma_C^L \leq \gamma_C^U$ *and* γ_C^L, γ_C^U are fuzzy sets.

Definition 5. Let χ be a non-empty set and a cubic fuzzy set in χ is of the form $C = \{x_0, \overline{\gamma}_C(x_0), \nu_C(x_0)\}$ *for all* $x_0 \in \chi$ where $\overline{\gamma}_C : \chi \to D[0,1]$ and $\nu_C : \chi \to [0,1]$, where $\overline{\gamma}_C(x_0)$ is an interval valued fuzzy set and $\nu_C(x_0)$ is a fuzzy set.

Definition 6. A non-empty set C in an incline is called a fuzzy subincline if $\wedge\left(C(x_0 + y_0), C(x_0 * y_0)\right) \geq \wedge\left(C(x_0), C(y_0)\right)$ *for all* $x_0, y_0 \in \daleth$.

Here the notation for min is represented as $\wedge$, rmin is represented as $\overline{\wedge}$ and max as $\vee$.

3 | Cubic Subincline of Incline Algebra

This section deals the structure of cubic subincline and some related results.

Definition 7. Let $\daleth$ be an incline algebra and a cubic set C in $\daleth$ is said to be a cubic subincline of incline algebra if it satisfies:

- $\overline{\wedge}\left(\overline{\gamma}(x_0 + y_0), \overline{\gamma}(x_0 * y_0)\right) \geq \overline{\wedge}\left(\overline{\gamma}(x_0), \overline{\gamma}(y_0)\right)$,
- $\vee\left(\nu(x_0 + y_0), \nu(x_0 * y_0)\right) \leq \vee\left(\nu(x_0), \nu(y_0)\right)$ *for all* $x_0, y_0 \in \daleth$.

Theorem 1. Let C_1 and C_2 be two cubic subinclines of $\daleth$, then so is $C_1 \cap C_2$.

Proof: Let C_1 and C_2 be two cubic subinclines of $\daleth$, *for all* $x_0, y_0 \in \daleth$.

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Since $\overline{\gamma}_{(C_1)}$ & $\overline{\gamma}_{(C_2)}$ are interval valued fuzzy sets on C_1 and C_2

$$\barwedge\ (\barwedge\,(\overline{\gamma}_{(C_1\cap C_2)}\,(x_0+y_0),\overline{\gamma}_{(C_1\cap C_2)}\,(x_0*y_0)))$$

$$=\barwedge\,(\barwedge\,(\overline{\gamma}_{(C_1)}\,(x_0+y_0),\overline{\gamma}_{(C_2)}(x_0+y_0)),\barwedge\,(\overline{\gamma}_{(C_1)}\,(x_0*y_0),\overline{\gamma}_{(C_2)}\,(x_0*y_0)))$$

$$=\barwedge\,(\barwedge\,(\overline{\gamma}_{(C_1)}(x_0+y_0),\overline{\gamma}_{(C_1)}\,(x_0*y_0))\,,\ \barwedge\,(\overline{\gamma}_{(C_2)}\,(x_0+y_0),\overline{\gamma}_{(C_2)}\,(x_0*y_0)))$$

$$\geq\barwedge\left(\barwedge\left(\overline{\gamma}_{(C_1)}\,(x_0),\overline{\gamma}_{(C_1)}\,(y_0)\right),\barwedge\left(\overline{\gamma}_{(C_2)}\,(x_0),\overline{\gamma}_{(C_2)}\,(y_0)\right)\right)$$

$$\geq\barwedge\ \left(\barwedge\left(\overline{\gamma}_{(C_1)}\,(x_0),\overline{\gamma}_{(C_2)}\,(x_0)\right)\,,\ \barwedge\left(\overline{\gamma}_{(C_1)}\,(y_0)\ ,\overline{\gamma}_{(C_2)}(y_0)\right)\right)$$

$$=\barwedge\,(\overline{\gamma}_{(C_1\cap C_2)}\,(x_0),\overline{\gamma}_{(C_1\cap C_2)}\,(y_0))$$

$$\barwedge\ (\barwedge\,(\overline{\gamma}_{(C_1\cap C_2)}\,(x_0+y_0),\overline{\gamma}_{(C_1\cap C_2)}\,(x_0*y_0)))\geq\barwedge\,(\overline{\gamma}_{(C_1\cap C_2)}\,(x_0),\overline{\gamma}_{(C_1\cap C_2)}\,(y_0)).$$

Similarly, $\upsilon_{(C_1)}$ *and* $\upsilon_{(C_2)}$ *are fuzzy sets on* C_1 *and* C_2.

$$\curlyvee\ (\curlyvee\,(\upsilon_{(C_1\cap C_2)}\,(x_0+y_0),\upsilon_{(C_1\cap C_2)}\,(x_0*y_0)))$$

$$=\curlyvee\,(\curlyvee\left(\upsilon_{(C_1)}\,(x_0+y_0),\upsilon_{(C_2)}(x_0+y_0)\right),\curlyvee\,(\upsilon_{(C_1)}\,(x_0*y_0),\upsilon_{(C_2)}\,(x_0*y_0)))$$

$$=\curlyvee\,(\curlyvee\,(\upsilon_{(C_1)}(x_0+y_0),\upsilon_{(C_1)}\,(x_0*y_0))\,,\ \curlyvee\,(\upsilon_{(C_2)}\,(x_0+y_0),\upsilon_{(C_2)}\,(x_0*y_0)))$$

$$\leq\curlyvee\left(\curlyvee\left(\upsilon_{(C_1)}\,(x_0),\upsilon_{(C_1)}\,(y_0)\right),\curlyvee\left(\upsilon_{(C_2)}\,(x_0),\upsilon_{(C_2)}\,(y_0)\right)\right)$$

$$\leq\curlyvee\ \left(\curlyvee\left(\upsilon_{(C_1)}\,(x_0),\upsilon_{(C_2)}\,(x_0)\right)\,,\curlyvee\left(\upsilon_{(C_1)}\,(y_0)\ ,\upsilon_{(C_2)}(y_0)\right)\right)$$

$$=\curlyvee\,(\upsilon_{(C_1\cap C_2)}\,(x_0),\upsilon_{(C_1\cap C_2)}\,(y_0))$$

$$\curlyvee\left(\curlyvee\left(\upsilon_{(C_1\cap C_2)}\,(x_0+y_0),\upsilon_{(C_1\cap C_2)}\,(x_0*y_0)\right)\right)\leq\curlyvee\left(\upsilon_{(C_1\cap C_2)}\,(x_0),\upsilon_{(C_1\cap C_2)}\,(y_0)\right).$$

Thus $C_1\cap C_2$ is a cubic subincline of $\daleth$.

Theorem 2. Let the cubic set $\mathscr{C}=\{\overline{\gamma}_{\mathscr{C}},\ \nu_{\mathscr{C}}\}$ be a cubic subincline of $\daleth\Leftrightarrow\overline{\gamma}_{\mathscr{C}}$ is an interval valued fuzzy and $\nu_{\mathscr{C}}$ are anti fuzzy subincline of $\daleth$.

Proof: Let the cubic set $\mathscr{C}=\{\ \overline{\gamma}_{\mathscr{C}},\ \nu_{\mathscr{C}}\}$ be a cubic subincline of $\daleth$ and take $x_0\,,\ y_0\ \in\ \daleth$.

Then to prove $\barwedge\left(\overline{\gamma}_{\mathscr{C}}(x_0+y_0),\overline{\gamma}_{\mathscr{C}}(x_0*y_0)\right)\ \geq\barwedge\left(\overline{\gamma}_{\mathscr{C}}(x_0),\overline{\gamma}_{\mathscr{C}}(y_0)\right)$:

$$\left(\overline{\gamma}_{\mathscr{C}}(x_0+y_0)\right)=\left[\ \gamma^L_{\mathscr{C}}(x_0+y_0)\ ,\gamma^U_{\mathscr{C}}(x_0+y_0)\right]$$

$$\geq\ [\ \wedge[\gamma^L_{\mathscr{C}}(x_0),\gamma^L_{\mathscr{C}}(y_0)],\ \wedge\left[\gamma^U_{\mathscr{C}}(x_0),\gamma^U_{\mathscr{C}}(y_0)\right]]$$

$$=\ [\ \wedge[\gamma^L_{\mathscr{C}}(x_0),\gamma^U_{\mathscr{C}}(x_0)],\ \wedge\left[\gamma^L_{\mathscr{C}}(y_0),\gamma^U_{\mathscr{C}}(y_0)\right]]$$

$$\geq\barwedge\left(\overline{\gamma}_{\mathscr{C}}(x_0),\overline{\gamma}_{\mathscr{C}}(y_0)\right)$$

$\barwedge\left(\bar{\gamma}_{\mathscr{C}}(x_0+y_0)\right) \geq \barwedge\left(\bar{\gamma}_{\mathscr{C}}(x_0), \bar{\gamma}_{\mathscr{C}}(y_0)\right)$.

Similarly, $\barwedge\left(\bar{\gamma}_{\mathscr{C}}(x_0 * y_0)\right) \geq \barwedge\left(\bar{\gamma}_{\mathscr{C}}(x_0), \bar{\gamma}_{\mathscr{C}}(y_0)\right)$

$\barwedge\left(\bar{\gamma}_{\mathscr{C}}(x_0+y_0), \bar{\gamma}_{\mathscr{C}}(x_0 * y_0)\right) \geq \barwedge\left(\barwedge\left(\bar{\gamma}_{\mathscr{C}}(x_0), \bar{\gamma}_{\mathscr{C}}(y_0)\right), \barwedge\left(\bar{\gamma}_{\mathscr{C}}(x_0), \bar{\gamma}_{\mathscr{C}}(y_0)\right)\right)$

$\barwedge\left(\bar{\gamma}_{\mathscr{C}}(x_0+y_0), \bar{\gamma}_{\mathscr{C}}(x_0 * y_0)\right) \geq \barwedge\left(\bar{\gamma}_{\mathscr{C}}(x_0), \bar{\gamma}_{\mathscr{C}}(y_0)\right)$.

To prove $\upsilon_{\mathscr{C}}$ is an anti-fuzzy subincline.

$$\vee\left(\upsilon_{\mathscr{C}}(x_0+y_0), \upsilon_{\mathscr{C}}(x_0 * y_0)\right) = \vee\left((1-\upsilon'_{\mathscr{C}}(x_0+y_0)), (1-\upsilon'_{\mathscr{C}}(x_0 * y_0))\right)$$

$$= \vee\left((1-(\upsilon'_{\mathscr{C}}(x_0+y_0), \upsilon'_{\mathscr{C}}(x_0 * y_0))\right)$$

$$= 1-\vee\left((\upsilon'_{\mathscr{C}}(x_0+y_0), \upsilon'_{\mathscr{C}}(x_0 * y_0))\right)$$

$$\geq 1-\vee\left((\upsilon'_{\mathscr{C}}(x_0), \upsilon'_{\mathscr{C}}(y_0))\right)$$

$$\geq \vee\left((1-\upsilon'_{\mathscr{C}}(x_0), 1-\upsilon'_{\mathscr{C}}(y_0))\right)$$

$$= \vee\left(\upsilon_{\mathscr{C}}(x_0), \upsilon_{\mathscr{C}}(y_0)\right).$$

Therefore, $\upsilon_{\mathscr{C}}$ is an anti-fuzzy subincline of $\daleth$.

Conversely, $\bar{\gamma}_{\mathscr{C}}$ is aninterval valued fuzzy subincline of $\daleth$ and $\upsilon_{\mathscr{C}}$ is an anti-fuzzy subincline of $\daleth$.

To prove $(\bar{\gamma}_{\mathscr{C}}, \upsilon_{\mathscr{C}})$ is a cubic subincline of $\daleth$.

Therefore, the proof is obvious by the definition of a cubic subincline.

$\mathscr{C} = (\bar{\gamma}_{\mathscr{C}}, \upsilon_{\mathscr{C}})$ is a cubic subincline of $\daleth$.

Definition 8. Let $\mathscr{C}_1$ and $\mathscr{C}_2$ be two cubic fuzzy sets of $\daleth$ and $\beth$ respectively. The direct product of $\mathscr{C}_1 \times \mathscr{C}_2 : \daleth \times \beth \to [0,1]$ of $\mathscr{C}_1$ and $\mathscr{C}_2$ is defined by $\bar{\gamma}_{\mathscr{C}_1 \times \mathscr{C}_2}(x_0, y_0) = \barwedge\left(\bar{\gamma}_{\mathscr{C}_1}(x_0), \bar{\gamma}_{\mathscr{C}_2}(y_0)\right)$ and $\upsilon_{\mathscr{C}_1 \times \mathscr{C}_2}(x_0, y_0) = \vee\left(\upsilon_{\mathscr{C}_1}(x_0), \upsilon_{\mathscr{C}_2}(y_0)\right)$.

Lemma 1. Let $(\daleth, +, *)$ and $(\beth, +, *)$ be two incline algebra, for all $(x_1, y_1), (x_2, y_2) \in \daleth \times \beth$, define $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2), (y_1 + y_2)$; $(x_1, y_1) * (x_2, y_2) = (x_1 * x_2), (y_1 * y_2)$. Then $(\daleth \times \beth, +, *)$ is also an incline algebra.

Theorem 3. Let $\mathscr{C}_1$ & $\mathscr{C}_2$ be two cubic subincline of $\daleth$ and $\beth$ respectively, then $\mathscr{C}_1 \times \mathscr{C}_2$ is a cubic subincline of $\daleth \times \beth$.

Proof: Let $\bar{\gamma}_{\mathscr{C}_1}, \bar{\gamma}_{\mathscr{C}_2}$ be two interval valued fuzzy cubic subincline of $\daleth$ and $\beth$ respectively.

Now, $\barwedge\left(\bar{\gamma}_{\mathscr{C}_1 \times \mathscr{C}_2}(x_1, y_1) + (x_2, y_2), \bar{\gamma}_{\mathscr{C}_1 \times \mathscr{C}_2}(x_1, y_1) * (x_2, y_2)\right)$

$$= \barwedge\left(\bar{\gamma}_{\mathscr{C}_1 \times \mathscr{C}_2}(x_1 + x_2), (y_1 + y_2), \bar{\gamma}_{\mathscr{C}_1 \times \mathscr{C}_2}(x_1 * x_2), (y_1 * y_2)\right)$$

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$$= \barwedge\Big(\big(\,\overline{\gamma}_{\mathscr{C}_1}(x_1+x_2),\ \overline{\gamma}_{\mathscr{C}_2}(y_1+y_2)\,\big)\ ,\big(\,\overline{\gamma}_{\mathscr{C}_1}(x_1*x_2),\ \overline{\gamma}_{\mathscr{C}_2}(y_1*y_2)\big)\Big)$$

$$= \barwedge\Big(\barwedge\big(\,\overline{\gamma}_{\mathscr{C}_1}(x_1+x_2),\ \overline{\gamma}_{\mathscr{C}_1}(x_1*x_2)\,\big)\ ,\barwedge\ \big(\,\overline{\gamma}_{\mathscr{C}_2}(y_1+y_2)\ ,\ \overline{\gamma}_{\mathscr{C}_2}(y_1*y_2)\big)\Big)$$

$$\geq \barwedge\Big(\barwedge\big(\,\overline{\gamma}_{\mathscr{C}_1}(x_1),\ \overline{\gamma}_{\mathscr{C}_1}(x_2)\,\big)\ ,\ \barwedge\ \big(\,\overline{\gamma}_{\mathscr{C}_2}(y_1)\ ,\ \overline{\gamma}_{\mathscr{C}_2}(y_2)\big)\Big)$$

$$= \barwedge\Big(\barwedge\big(\,\overline{\gamma}_{\mathscr{C}_1}(x_1),\overline{\gamma}_{\mathscr{C}_2}(y_1)\ \big)\ ,\ \barwedge\ \big(\,\overline{\gamma}_{\mathscr{C}_1}(x_2)\ ,\ \overline{\gamma}_{\mathscr{C}_2}(y_2)\big)\Big)$$

$$= \barwedge\ \big(\,\overline{\gamma}_{\mathscr{C}_1\times\mathscr{C}_2}(x_1,y_1)\ ,\ \overline{\gamma}_{\mathscr{C}_1\times\mathscr{C}_2}(x_2,y_2)\big)$$

$$\barwedge\big(\overline{\gamma}_{\mathscr{C}_1\times\mathscr{C}_2}(x_1,y_1)+\ (x_2,y_2)\ ,\overline{\gamma}_{\mathscr{C}_1\times\mathscr{C}_2}(x_1,y_1)*\ (x_2,y_2)\big)\geq \barwedge\ \big(\,\overline{\gamma}_{\mathscr{C}_1\times\mathscr{C}_2}(x_1,y_1)\ ,\ \overline{\gamma}_{\mathscr{C}_1\times\mathscr{C}_2}(x_2,y_2)\big).$$

Let $\upsilon_{\mathscr{C}_1}$ and $\upsilon_{\mathscr{C}_2}$be fuzzy subincline of $\urcorner$ and $\sqsupset$ respectively.

$$\curlyvee\big(\upsilon_{\mathscr{C}_1\times\mathscr{C}_2}(x_1,y_1)+\ (x_2,y_2)\ ,\upsilon_{\mathscr{C}_1\times\mathscr{C}_2}(x_1,y_1)*\ (x_2,y_2)\big)$$

$$= \curlyvee\big(\ \upsilon_{\mathscr{C}_1\times\mathscr{C}_2}(x_1+x_2),\ \ (y_1+y_2)\ \ ,\upsilon_{\mathscr{C}_1\times\mathscr{C}_2}(x_1*x_2),(y_1*y_2)\big)$$

$$=\curlyvee\Big(\big(\upsilon_{\mathscr{C}_1}(x_1+x_2),\ \upsilon_{\mathscr{C}_2}(y_1+y_2)\,\big)\ ,\big(\,\upsilon_{\mathscr{C}_1}(x_1*x_2),\upsilon_{\mathscr{C}_2}(y_1*y_2)\big)\Big)$$

$$= \curlyvee\Big(\barwedge\big(\upsilon_{\mathscr{C}_1}(x_1+x_2),\upsilon_{\mathscr{C}_1}(x_1*x_2)\,\big)\ ,\curlyvee\ \big(\upsilon_{\mathscr{C}_2}(y_1+y_2)\ ,\upsilon_{\mathscr{C}_2}(y_1*y_2)\big)\Big)$$

$$\leq \curlyvee\Big(\curlyvee\big(\upsilon_{\mathscr{C}_1}(x_1),\ \upsilon_{\mathscr{C}_1}(x_2)\,\big)\ ,\ \curlyvee\ \big(\upsilon_{\mathscr{C}_2}(y_1)\ ,\upsilon_{\mathscr{C}_2}(y_2)\big)\Big)$$

$$=\curlyvee\Big(\curlyvee\big(\upsilon_{\mathscr{C}_1}(x_1),\upsilon_{\mathscr{C}_2}(y_1)\ \big)\ ,\ \curlyvee\ \big(\,\upsilon_{\mathscr{C}_1}(x_2)\ \ ,\upsilon_{\mathscr{C}_2}(y_2)\big)\Big)$$

$$= \curlyvee\big(\,\upsilon_{\mathscr{C}_1\times\mathscr{C}_2}(x_1,y_1)\ ,\ \upsilon_{\mathscr{C}_1\times\mathscr{C}_2}(x_2,y_2)\big)$$

$$\curlyvee\big(\upsilon_{\mathscr{C}_1\times\mathscr{C}_2}(x_1,y_1)+(x_2,y_2)\ ,\upsilon_{\mathscr{C}_1\times\mathscr{C}_2}(x_1,y_1)*\ (x_2,y_2)\big)\leq \curlyvee\ \big(\,\upsilon_{\mathscr{C}_1\times\mathscr{C}_2}(x_1,y_1)\ ,\ \upsilon_{\mathscr{C}_1\times\mathscr{C}_2}(x_2,y_2)\big).$$

Thus, $\mathscr{C}_1\ \times\ \mathscr{C}_2$ is also a cubic subincline of $\urcorner\ \times\sqsupset$.

Definition 9. A mapping $f:\urcorner\rightarrow\ \sqsupset$. Let $\mathscr{C}$ is a cubic subincline of $\urcorner$ then the image of a cubic subincline of $\sqsupset$ is defined as $f(\mathscr{C})=(\overline{\gamma}_f\,,\upsilon_f)$ where $\overline{\gamma}_f(y_0)=\barwedge\ \{\,\overline{\gamma}_{\mathscr{C}}(\,x_0\,)/\,f(x_0)\ =y_0\}$, $\upsilon_f(y_0)=\curlyvee\ \{\,\upsilon_{\mathscr{C}}(\,x_0\,)/\ f(x_0)\ =y_0\}$.

Theorem 4. Let $f:\ \urcorner\rightarrow\ \sqsupset$ be a mapping and $\mathscr{C}=\{\ \overline{\gamma}_{\mathscr{C}},\ \nu_{\mathscr{C}}\}$ is an cubic subincline of $\urcorner$, then $f(\mathscr{C})$ is also an cubic subincline of $\sqsupset$ where $f(\mathscr{C})=(\overline{\gamma}_f\,,\upsilon_f)$ satisfies $\overline{\gamma}_f(y_0)=\barwedge\ \{\,\overline{\gamma}_{\mathscr{C}}(\,x_0\,)/\,f(x_0)=y_0\}$ and $\upsilon_f(y_0)=\curlyvee\ \{\,\upsilon_{\mathscr{C}}(\,x_0\,)/\,f(x_0)\ =y_0\}$.

Proof: Given $f:\ \urcorner\rightarrow\ \sqsupset$ be a mapping and $x_1,x_2\in\ \sqsupset$ such that $f(x_1)=y_1\,,\ f(x_2)=y_2\ for\ all\ \ y_1,y_2\in\sqsupset$.

Since $\mathscr{C}$ is a cubic subincline of $\urcorner$, $\barwedge\big(\overline{\gamma}(x_1+x_2),\overline{\gamma}(x_1*x_2)\big)\ \geq\barwedge\big(\overline{\gamma}(x_1),\overline{\gamma}(x_2)\big)$.

$\overline{\barwedge}=\barwedge\ \{\,\overline{\gamma}_{\mathscr{C}}(\,x_0\,)/\,f(x_0)\ =y_1+\ y_2\}$ where $x_0=\ x_1+x_2$,

$\overline{\gamma}_f(y_1+\ y_2)=\barwedge\ \{\,\overline{\gamma}_{\mathscr{C}}(\,x_1+x_2\,)/\,f(x_1+x_2)\ =y_1+\ y_2\}$,

$\overline{\gamma}_f(y_1+\ y_2)=\barwedge\ \{\,\overline{\gamma}_{\mathscr{C}}(\,x_1+x_2\,)/\,f(x_1)+f(x_2)\ =y_1+\ y_2\}$,

$\overline{\gamma}_f(y_1 * y_2) = \barwedge \{ \overline{\gamma}_{\mathscr{C}}(x_0)/ f(x_0) = y_1 * y_2\}$ where $x_0 = x_1 * x_2$,

$\overline{\gamma}_f(y_1 * y_2) = \barwedge \{ \overline{\gamma}_{\mathscr{C}}(x_1 * x_2)/ f(x_1 * x_2) = y_1 * y_2\}$,

$\overline{\gamma}_f(y_1 * y_2) = \barwedge \{ \overline{\gamma}_{\mathscr{C}}(x_1 * x_2)/ f(x_1) * f(x_2) = y_1 * y_2\}$,

$\barwedge (\overline{\gamma}_f(y_1 + y_2), \overline{\gamma}_f(y_1 * y_2)) \geq \barwedge (\barwedge \{ \overline{\gamma}_{\mathscr{C}}(x_1 + x_2)/ f(x_1) + f(x_2) = y_1 + y_2\},$

$\barwedge \{ \overline{\gamma}_{\mathscr{C}}(x_1 * x_2)/ f(x_1) * f(x_2) = y_1 * y_2\})$

$= \barwedge (\barwedge \{ \overline{\gamma}_{\mathscr{C}}(x_1 + x_2), \overline{\gamma}_{\mathscr{C}}(x_1 * x_2)/ f(x_1) + f(x_2) = y_1 + y_2, f(x_1) * f(x_2) = y_1 * y_2\})$

$= \barwedge (\barwedge \{ \overline{\gamma}_{\mathscr{C}}(x_1), \overline{\gamma}_{\mathscr{C}}(x_2)/ f(x_1) + f(x_2) = y_1 + y_2, f(x_1) * f(x_2) = y_1 * y_2\})$

$= \barwedge (\barwedge \{ \overline{\gamma}_{\mathscr{C}}(x_1), \overline{\gamma}_{\mathscr{C}}(x_2)/ f(x_1) = y_1, f(x_2) = y_2\})$

$= \barwedge (\barwedge \{ \overline{\gamma}_{\mathscr{C}}(x_1)/ f(x_1) = y_1\}, \barwedge \{ \overline{\gamma}_{\mathscr{C}}(x_2)/, f(x_2) = y_2\})$

$= \barwedge \{\overline{\gamma}_f(y_1), \overline{\gamma}_f(y_2)\}$

$\barwedge (\overline{\gamma}_f(y_1 + y_2), \overline{\gamma}_f(y_1 * y_2)) \geq \barwedge \{\overline{\gamma}_f(y_1), \overline{\gamma}_f(y_2)\}$.

The same procedure is followed for the falsity membership function.

Definition 10. A mapping f on $\daleth$, if $\mathscr{C}_0 = \left\{ \overline{\gamma}_{\mathscr{C}_0}, \nu_{\mathscr{C}_0}\right\}$ is a cubic fuzzy set in $f(\daleth)$, $\mathscr{C} = \{ \overline{\gamma}_{\mathscr{C}}, \nu_{\mathscr{C}}\}$ is a cubic fuzzy set in $\daleth$ then cubic fuzzy set $\mathscr{C} = \{ \overline{\gamma}_{\mathscr{C}}, \nu_{\mathscr{C}}\} = \mathscr{C}_0 \circ f$, $\overline{\gamma}_{\mathscr{C}}(x_0) = (\overline{\gamma}_{\mathscr{C}_0} \circ f)(x_0) = \overline{\gamma}_{\mathscr{C}_0}(f(x_0))$ and $\nu_{\mathscr{C}}(x_0) = (\nu_{\mathscr{C}_0} \circ f)(x_0) = \nu_{\mathscr{C}_0}(f(x_0))$ in $\daleth$ is called preimage of $\mathscr{C} = \{ \overline{\gamma}_{\mathscr{C}}, \nu_{\mathscr{C}}\}$ under f.

Theorem 5. An epimorphism preimage of a cubic subincline of $\daleth$ is a cubic subincline.

Proof: Let $f: \daleth \rightarrow \beth$ be an epimorphism ($\daleth, \beth$ are inclines) $\mathscr{C}_0 = \left\{ \overline{\gamma}_{\mathscr{C}_0}, \nu_{\mathscr{C}_0}\right\}$ is a cubic fuzzy set in $f(\daleth)$ and $\mathscr{C} = \{ \overline{\gamma}_{\mathscr{C}}, \nu_{\mathscr{C}}\}$ be an inverse image of $\beth$ under f. For $x_1, x_2 \in \daleth$,

$\overline{\gamma}_{\mathscr{C}}(x_1 + x_2) = (\overline{\gamma}_{\mathscr{C}_0} \circ f)(x_1 + x_2)$

$= \overline{\gamma}_{\mathscr{C}_0}(f(x_1 + x_2))$

$= \overline{\gamma}_{\mathscr{C}_0}(f(x_1) + f(x_2))$

$\overline{\gamma}_{\mathscr{C}}(x_1 * x_2) = (\overline{\gamma}_{\mathscr{C}_0} \circ f)(x_1 * x_2)$

$= \overline{\gamma}_{\mathscr{C}_0}(f(x_1 * x_2))$

$= \overline{\gamma}_{\mathscr{C}_0}(f(x_1) * f(x_2))$

$\barwedge (\overline{\gamma}_{\mathscr{C}}(x_1 + x_2), \overline{\gamma}_{\mathscr{C}}(x_1 * x_2)) = \barwedge (\overline{\gamma}_{\mathscr{C}_0}(f(x_1) + f(x_2)), \overline{\gamma}_{\mathscr{C}_0}(f(x_1) * f(x_2)))$

$\geq \barwedge (\overline{\gamma}_{\mathscr{C}_0}(f(x_1)), \overline{\gamma}_{\mathscr{C}_0}(f(x_2)))$

$\geq \barwedge (\overline{\gamma}_{\mathscr{C}}(x_1), \overline{\gamma}_{\mathscr{C}}(x_2))$

$\nu_{\mathscr{C}}(x_1 + x_2) = (\nu_{\mathscr{C}_0} \circ f)(x_1 + x_2)$

$= \nu_{\mathscr{C}_0}(f(x_1 + x_2))$

$= \nu_{\mathscr{C}_0}(f(x_1) + f(x_2))$

$\nu_{\mathscr{C}}(x_1 * x_2) = (\nu_{\mathscr{C}_0} \circ f)(x_1 * x_2)$

$= \nu_{\mathscr{C}_0}(f(x_1 * x_2))$

$= \nu_{\mathscr{C}_0}(f(x_1) * f(x_2))$

$\vee(\nu_{\mathscr{C}}(x_1 + x_2), \nu_{\mathscr{C}}(x_1 * x_2)) = \vee(\nu_{\mathscr{C}_0}(f(x_1) + f(x_2)), \nu_{\mathscr{C}_0}(f(x_1) * f(x_2)))$

$\leq \vee(\nu_{\mathscr{C}_0}(f(x_1)), \nu_{\mathscr{C}_0}(f(x_2)))$

$\leq \vee(\nu_{\mathscr{C}}(x_1), \overline{\gamma}_{\mathscr{C}}(x_2))$.

Definition 11. Let $\mathscr{C} = \{\overline{\gamma}_{\mathscr{C}}, \nu_{\mathscr{C}}\}$ be cubic set of $\urcorner$ and the cubic level subset is defined as $\mathscr{C}_{(\bar{\alpha},\beta)} = \{x_0 \in \urcorner / \overline{\gamma}_{\mathscr{C}}(x_0) \geq \bar{\alpha}, \nu_{\mathscr{C}}(x_0) \leq \beta\}$.

Theorem 6. For a cubic set $\mathscr{C} = \{\overline{\gamma}_{\mathscr{C}}, \nu_{\mathscr{C}}\} \in \mathscr{C}(\urcorner)$, the following are equivalent ($\mathscr{C}(\urcorner)$ is the family of cubic sets in a set $\urcorner$).

I. $\mathscr{C} = \{\overline{\gamma}_{\mathscr{C}}, \nu_{\mathscr{C}}\}$ is a cubic subincline of $\urcorner$.
II. The non-empty cubic level set of $\mathscr{C} = \{\overline{\gamma}_{\mathscr{C}}, \nu_{\mathscr{C}}\}$ is a subincline of $\urcorner$.

Proof: Assume that $\mathscr{C} = \{\overline{\gamma}_{\mathscr{C}}, \nu_{\mathscr{C}}\}$ is a cubic subincline of $\urcorner$.

Let $x_0, y_0 \in \mathscr{C}(\mathscr{C}, (\bar{\alpha}, \beta))$ for all $\bar{\alpha} \in D[0, 1]$; $\beta \in [0, 1]$.

Then $\overline{\gamma}_{\mathscr{C}}(x_0) \geq \bar{\alpha}, \overline{\gamma}_{\mathscr{C}}(y_0) \geq \bar{\alpha}$; $\nu_{\mathscr{C}}(x_0) \leq \beta, \nu_{\mathscr{C}}(y_0) \leq \beta$.

By the definition of subincline

$\barwedge(\overline{\gamma}_{\mathscr{C}}(x_0 + y_0), \overline{\gamma}_{\mathscr{C}}(x_0 * y_0)) \geq \barwedge(\overline{\gamma}_{\mathscr{C}}(x_0), \overline{\gamma}_{\mathscr{C}}(y_0))$

$= \barwedge(\bar{\alpha}, \bar{\alpha}) = \bar{\alpha}$

$\vee(\nu_{\mathscr{C}}(x_0 + y_0), \nu_{\mathscr{C}}(x_0 * y_0)) \leq \vee(\nu_{\mathscr{C}}(x_0), \nu_{\mathscr{C}}(y_0))$

$= \vee(\beta, \beta) = \beta$.

So that $x_0 + y_0$ & $x_0 * y_0 \in \mathscr{C}\left(\mathscr{C}, (\bar{\alpha}, \beta)\right)$.

Therefore, the non-empty cubic level subset of $\mathscr{C} = \{\overline{\gamma}_{\mathscr{C}}, \nu_{\mathscr{C}}\}$ is a subincline of $\urcorner$.

Conversely, assume that $\mathscr{C}\left(\mathscr{C}, (\bar{\alpha}, \beta)\right)$ is a subincline of $\urcorner$ for all $\beta \in [0, 1]$; $\bar{\alpha} \in D[0, 1]$, with $\mathscr{C}\left(\mathscr{C}, (\bar{\alpha}, \beta)\right) \neq \emptyset$:

I. Suppose that $\mathscr{C}$ is not a cubic set and to prove $\mathscr{C}$ is a cubic subincline of $\urcorner$. There exists $\bar{\alpha}' \in D[0, 1]$ and $x_0, y_0 \in \urcorner$ such that

$\barwedge(\overline{\gamma}_{\mathscr{C}}(x_0 + y_0), \overline{\gamma}_{\mathscr{C}}(x_0 * y_0)) < \bar{\alpha}'$

$\leq \bar{\wedge} \ (\bar{\gamma}_{\mathscr{C}}\ (x_0)\ ,\ \bar{\gamma}_{\mathscr{C}}\ (y_0))$

$\vee \ (\nu_{\mathscr{C}}\ (x_0 + y_0)\ , \nu_{\mathscr{C}}\ (\ x_0 * y_0)) \leq \vee \ (\nu_{\mathscr{C}}\ (x_0)\ ,\ \nu_{\mathscr{C}}\ (y_0)),$

which implies that $x_0\ , y_0 \in \mathscr{C}\ \ (\mathscr{C}\ ; \bar{\alpha}', \vee\ (\nu_{\mathscr{C}}(x_0)\ , \nu_{\mathscr{C}}\ (y_0))),$

but $(x_0 + y_0)\ , (x_0 * y_0) \notin \mathscr{C}(\mathscr{C}\ ; \bar{\alpha}', \vee\ (\nu_{\mathscr{C}}\ (x_0)\ , \nu_{\mathscr{C}}\ (y_0)))$.

This is a contradiction.

II. Now assume that $\mathscr{C}$ is a cubic set but $\mathscr{C}$ is not a cubic subincline, then

$\bar{\wedge}\ (\bar{\gamma}_{\mathscr{C}}\ (x_0 + y_0)\ , \bar{\gamma}_{\mathscr{C}}\ (x_0 * y_0)) \leq \bar{\wedge}\ (\bar{\gamma}_{\mathscr{C}}\ (x_0)\ ,\ \bar{\gamma}_{\mathscr{C}}\ (y_0))$ and $\vee\ (\nu_{\mathscr{C}}\ (x_0 + y_0)\ , \nu_{\mathscr{C}}\ (x_0 * y_0)) > \beta' > \vee\ (\nu_{\mathscr{C}}\ (x_0)\ ,\ \nu_{\mathscr{C}}\ (y_0))$ for some $\beta' \in [0,1]$ and $x_0\ , y_0 \in \urcorner$.

Thus, $x_0\ , y_0 \in \mathscr{C}\ \ (\mathscr{C}\ ;\ (\bar{\gamma}_{\mathscr{C}}\ (x_0)\ ,\ \bar{\gamma}_{\mathscr{C}}\ (y_0)), \beta'\ ),$

but $(x_0 + y_0)\ , (x_0 * y_0) \notin \mathscr{C}\ \ (\mathscr{C}\ ;\ (\bar{\gamma}_{\mathscr{C}}\ (x_0)\ ,\ \bar{\gamma}_{\mathscr{C}}\ (y_0)), \beta'\ )$ which is a contradiction.

III. Assume that there exists $\bar{\alpha}' \in D[0, 1]\ , \beta' \in [0,1]$ and $x_0\ , y_0 \in \urcorner$ such that

$\bar{\wedge}\ (\bar{\gamma}_{\mathscr{C}}\ (x_0 + y_0)\ , \bar{\gamma}_{\mathscr{C}}\ (\ x_0 * y_0)) < \bar{\alpha}' \leq \bar{\wedge}\ (\bar{\gamma}_{\mathscr{C}}\ (x_0)\ ,\ \bar{\gamma}_{\mathscr{C}}\ (y_0))$ and

$\vee\ (\nu_{\mathscr{C}}\ (x_0 + y_0)\ , \nu_{\mathscr{C}}\ (x_0 * y_0)) > \beta' > \vee\ (\nu_{\mathscr{C}}\ (x_0)\ ,\ \nu_{\mathscr{C}}\ (y_0)).$

Then $x_0\ , y_0 \in \mathscr{C}\ \ (\mathscr{C}\ ; \bar{\alpha}', \beta')$ but $(x_0 + y_0)\ , (x_0 * y_0) \notin \mathscr{C}\ \ (\mathscr{C}\ ; \bar{\alpha}', \beta')$.

This is a contradiction.

Hence $\mathscr{C}$ is a cubic set and $\mathscr{C}$ is a cubic subincline of $\urcorner$.

Therefore, $\mathscr{C} = \{\bar{\gamma}_{\mathscr{C}},\ \nu_{\mathscr{C}}\}$ is a cubic subincline of $\urcorner$.

Theorem 7. For a subset $\mathscr{M}$ of $\urcorner$, let $\mathscr{C} = \{\ \bar{\gamma}_{\mathscr{C}},\ \nu_{\mathscr{C}}\} \in \mathscr{C}\ (\urcorner)$ be defined by

$$\bar{\gamma}_{\mathscr{C}}\ (x_0) = \begin{cases} \bar{\alpha}, & \text{if } x_0 + y_0, \quad x_0 * y_0 \in \mathscr{M}, \\ \bar{0}, & \text{otherwise,} \end{cases}$$

and

$$\nu_{\mathscr{C}}\ (x_0) = \begin{cases} 1, & \text{if } x_0 + y_0, \quad x_0 * y_0 \in \mathscr{M}, \\ \beta, & \text{otherwise,} \end{cases}$$

where $\bar{\alpha} \in D\ [0, 1]\ , \beta \in [0, 1]$ with $<\ .\ \alpha_2$ Then

I. If $\mathscr{M}$ is a subincline of $\urcorner$ then $\mathscr{C} = \{\ \bar{\gamma}_{\mathscr{C}},\ \nu_{\mathscr{C}}\}$ is a cubic subincline of $\urcorner$ and $\mathscr{C}\left(\mathscr{C}\ , (\bar{\alpha}\ , \beta)\right) = \mathscr{M}$.

II. If $\mathscr{C} = \{\ \bar{\gamma}_{\mathscr{C}},\ \nu_{\mathscr{C}}\}$ is a cubic subincline of $\urcorner$, then $\mathscr{M}$ is a subincline of $\urcorner$.

Proof: Assume $\mathscr{M}$ is a subincline of $\urcorner$.

Obviously, $\mathscr{C}\left(\mathscr{C}\ , (\bar{\alpha}\ , \beta)\right) = \mathscr{M}$.

Let $x_0, y_0 \in \urcorner$ if $x_0\ ,\ y_0 \in \mathscr{M}$ then $x_0 + y_0\ ,\ x_0 * y_0 \in \mathscr{M}$ and so

$\bar{\wedge}\ (\bar{\gamma}_{\mathscr{C}}\ (x_0 + y_0)\ , \bar{\gamma}_{\mathscr{C}}\ (x_0 * y_0)) = \bar{\alpha}$

$= \barwedge \{\bar{\alpha}, \bar{\alpha}\}$

$\leq \barwedge (\bar{\gamma}_{\mathscr{C}}(x_0), \bar{\gamma}_{\mathscr{C}}(y_0))$

$\curlyvee (\nu_{\mathscr{C}}(x_0 + y_0), \nu_{\mathscr{C}}(x_0 * y_0)) = 1 = \curlyvee (\nu_{\mathscr{C}}(x_0), \nu_{\mathscr{C}}(y_0))$.

I. If $x_0 + y_0$, $x_0 * y_0 \notin \mathscr{M}$ then

$\bar{\gamma}_{\mathscr{C}}(x_0 + y_0) = \bar{0} = \bar{\gamma}_{\mathscr{C}}(x_0 * y_0)$ and

$(\nu_{\mathscr{C}}(x_0 + y_0) = \beta = \nu_{\mathscr{C}}(x_0 * y_0))$.

Hence, $\barwedge (\bar{\gamma}_{\mathscr{C}}(x_0 + y_0), \bar{\gamma}_{\mathscr{C}}(x_0 * y_0)) \geq \bar{0}$

$= \barwedge (\bar{0}, \bar{0})$

$= \barwedge (\bar{\gamma}_{\mathscr{C}}(x_0), \bar{\gamma}_{\mathscr{C}}(y_0))$

$\curlyvee (\nu_{\mathscr{C}}(x_0 + y_0), \nu_{\mathscr{C}}(x_0 * y_0)) \geq \beta$

$= \curlyvee (\beta, \beta)$

$= \curlyvee (\nu_{\mathscr{C}}(x_0), \nu_{\mathscr{C}}(y_0))$.

II. If $x_0 + y_0$, $x_0 * y_0 \notin \mathscr{M}$ then

$\bar{\gamma}_{\mathscr{C}}(x_0 + y_0) = \bar{\alpha}$, $\bar{\gamma}_{\mathscr{C}}(x_0 * y_0) = \bar{0}$ and

$(\nu_{\mathscr{C}}(x_0 + y_0) = 1$; $\nu_{\mathscr{C}}(x_0 * y_0)) = \beta$

$\barwedge (\bar{\gamma}_{\mathscr{C}}(x_0 + y_0), \bar{\gamma}_{\mathscr{C}}(x_0 * y_0)) \geq \bar{0}$

$= \barwedge (\bar{0}, \bar{0})$

$= \barwedge (\bar{\gamma}_{\mathscr{C}}(x_0), \bar{\gamma}_{\mathscr{C}}(y_0))$

$\curlyvee (\nu_{\mathscr{C}}(x_0 + y_0), \nu_{\mathscr{C}}(x_0 * y_0)) \leq \beta$

$= \curlyvee (1, \beta)$

$= \curlyvee (\nu_{\mathscr{C}}(x_0), \nu_{\mathscr{C}}(y_0))$.

III. If $x_0 + y_0$, $x_0 * y_0 \notin \mathscr{M}$ then

$\bar{\gamma}_{\mathscr{C}}(x_0 + y_0) = \bar{0}$, $\bar{\gamma}_{\mathscr{C}}(x_0 * y_0) = \bar{\alpha}$ and

$(\nu_{\mathscr{C}}(x_0 + y_0) = \beta$; $\nu_{\mathscr{C}}(x_0 * y_0)) = 1$

$\barwedge (\bar{\gamma}_{\mathscr{C}}(x_0 + y_0), \bar{\gamma}_{\mathscr{C}}(x_0 * y_0)) \geq \bar{0}$

$= \barwedge (\bar{0}, \bar{0})$

$\leq \barwedge (\bar{\gamma}_{\mathscr{C}}(x_0), \bar{\gamma}_{\mathscr{C}}(y_0))$

$\curlyvee (\nu_{\mathscr{C}}(x_0 + y_0), \nu_{\mathscr{C}}(x_0 * y_0)) \leq \beta$

$= \curlyvee (1, \beta)$

$= \curlyvee \ (\nu_{\mathscr{C}}\ (x_0)\ , \nu_{\mathscr{C}}\ (y_0)\ ).$

Therefore, $\mathscr{C} = \{\ \overline{\gamma}_{\mathscr{C}},\ \nu_{\mathscr{C}}\}$ is a cubic subincline of $\urcorner$.

Suppose that $\mathscr{C} = \{\ \overline{\gamma}_{\mathscr{C}},\ \nu_{\mathscr{C}}\}$ is a cubic subincline of $\urcorner$.

$x_0 +\ y_0\ ,\ x_0 * y_0\ \notin\ \mathscr{M}$ then $\overline{\gamma}_{\mathscr{C}}\ (x_0 + y_0\ ) = \ \overline{\alpha}\ =\ \overline{\gamma}_{\mathscr{C}}\ (x_0 * y_0\ ),$

And $\ (\ \nu_{\mathscr{C}}\ (\ x_0 +\ y_0\ ) = 1 =\ \nu_{\mathscr{C}}\ (x_0 * y_0))$

$\overline{\curlywedge}\ (\ \overline{\gamma}_{\mathscr{C}}\ (\ x_0 +\ y_0\ )\ ,\ \overline{\gamma}_{\mathscr{C}}\ (\ x_0 *\ y_0\ )\ ) \geq\ (\overline{\gamma}_{\mathscr{C}}\ (\ x_0)\ , \overline{\gamma}_{\mathscr{C}}\ (\ y_0)\ )$

$= \overline{\curlywedge}\ \{\overline{\alpha}\ , \overline{\alpha}\ \}\ \geq\ \overline{\alpha}$

$\curlyvee\ (\ \nu_{\mathscr{C}}\ (\ x_0 +\ y_0\ ), \nu_{\mathscr{C}}\ (\ x_0 *\ y_0\ )\ ) \leq\ \curlyvee\ (\ \nu_{\mathscr{C}}\ (\ x_0\ ), \nu_{\mathscr{C}}\ (\ y_0\ )\ )$

$= \curlyvee\ (1{,}1)\ = 1.$

Thus, $x_0 +\ y_0\ ,\ x_0 * y_0\ \notin\ \mathscr{M}$ and therefore, $\mathscr{M}$ is a subincline of $\urcorner$.

4 | Conclusion

The structure of cubic subincline was introduced in this paper as an extension of the interval valued fuzzy subincline of incline algebra and analyzed the study of cubic subincline using homomorphic image, preimage, cartesian product and the level subset. The same idea can also be applied and extended to many other substructures like regular, filter of an incline algebra for a future scope.

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