E-ISSN: 2717-3453 | P-ISSN: 2783-1442

Journal of Fuzzy Extension and Applications

www.journal-fea.com

J. Fuzzy. Ext. Appl. Vol. 4, No. 2 (2023) 125–135.

Paper Type: Research Paper

Pythagorean and Fermatean Fuzzy Sub-Group Redefined in Context of $\mathscr{T}$-Norm and $\mathscr{S}$-Conorm

Vishnu Narayan Mishra[1,*], **Tarun Kumar**[2], **Mukesh Kumar Sharma**[2], **Laxmi Rathour**[3],

[1] Department of Mathematics, Indira Gandhi National Tribal University, Lalpur, Amarkantak, Anuppur, Madhya Pradesh 484 887, India; vishnunarayanmishra@gmail.com

[2] Department of Mathematics, Chaudhary Charan Singh University, Meerut, India-250004; tkvats3@gmail.com; mukeshkumarsharma@ccsuniversity.ac.in.

[3] Ward Number – 16, Bhagatbandh, Anuppur 484 224, Madhya Pradesh, India; laxmirathour817@gmail.com.

Citation:

Mishra, V. N., Kumar, T., Sharma, M. K., & Rathour, L. (2023). Pythagorean and fermatean fuzzy sub-group redefined in context of $\mathscr{T}$-norm and $\mathscr{S}$-conorm. *Journal of fuzzy extension and applications*, 4(2), 125-135.

Received: 10/03/2023 Reviewed: 11/04/2023 Revised: 19/05/2023 Accepted: 26/05/2023

Abstract

This paper aims to study Pythagorean and Fermatean Fuzzy Subgroups (FFSG) in the context of $\mathscr{T}$-norm and $\mathscr{S}$-conorm functions. The paper examines the extensions of fuzzy subgroups, specifically "Pythagorean Fuzzy Subgroups (PFSG)" and "FFSG", along with their properties. In the existing literature on Pythagorean and FFSG, the standard properties for membership and non-membership functions are based on the "min" and "max" operations, respectively. However, in this work, we develop a theory that utilizes the $\mathscr{T}$-norm for "min" and the $\mathscr{S}$-conorm for "max", providing definitions of Pythagorean and FFSG with these functions, along with relevant examples. By incorporating this approach, we introduce multiple options for selecting the minimum and maximum values. Additionally, we prove several results related to Pythagorean and FFSG using the $\mathscr{T}$-norm and $\mathscr{S}$-conorm, and discuss important properties associated with them.

Keywords: Fuzzy sets, Pythagorean fuzzy subgroups, Fermatean fuzzy subgroups, $\mathscr{T}$-Norm, $\mathscr{S}$-Conorm.

1 | Introduction

In the fields of mathematical analysis and information sciences, it's crucial to investigate and manage uncertainty. This inherent uncertainty extends even to symmetries in various objects, requiring the conceptual tools of fuzzy logic and group theory for its characterization. Zadeh [1] ignited the spark of this exploration with the introduction of fuzzy set theory, opening up avenues for dealing with nuanced classifications and gradations in sets. Rosenfeld [2] brought this concept into the domain of group theory, significantly contributing to our understanding of fuzzy groups. As the field matured, Anthony and Sherwood [3] added another layer of depth to fuzzy sets by redefining fuzzy groups with respect to t-norm. Meanwhile, Bhattacharya and Mukherjee [4] expanded upon the interaction between sets and groups by introducing the concept of fuzzy relations and fuzzy groups. Atanassov

Corresponding Author: vishnunarayanmishra@gmail.com

https://doi.org/10.22105/jfea.2023.396751.1262

[5] proposed Intuitionistic Fuzzy Sets (IFSs), further refining the concept of fuzzy sets. Around the same time, Das [6] introduced the notion of fuzzy groups and level subgroups, paving a new direction for fuzzy set theory. Ajmal and Prajapati [7] and Ajmal and Thomas [8] contributed significantly to the evolution of this field. Their introduction of fuzzy cosets, fuzzy normal subgroups, and the lattice structure of these subgroups provided a fresh perspective on the structure of fuzzy groups. Adding complexity to fuzzy set theory, Dixit et al. [9] delved into level subgroups and the union of fuzzy subgroups. Complementing this work, Gau and Buehrer [10] presented vague sets, adding a new dimension to the understanding of fuzziness. Fast forwarding to the new millennium, Khan et al. [11] brought the vague sets concept into the fold of groups, contributing to the formation of vague groups. Yager [12] then stretched the boundaries of classical fuzzy set theory with the introduction of Pythagorean fuzzy sets. Rasuli [13–15] expanded the application of IFSs to intuitionistic fuzzy subgroups with respect to norms and introduced fuzzy equivalence relation, fuzzy congruence relation, and fuzzy normal subgroups on group G over t-norms.

Simultaneously, the 21st century has seen the advent of innovative fuzzy set categories. Gayen et al. [16] proposed the interval-valued neutrosophic subgroup based on interval-valued triple t-norm, while Senapati and Yager [17] introduced Fermatean fuzzy sets. In a similar vein, Bejines et al. [18] and Ardanza-Trevijano et al. [19] explored the aggregation of fuzzy and T-subgroups respectively. Kumar et al. [20] redefined vague groups with respect to t-norms, while Bhunia et al. [21] and Razaq et al. [22] investigated the characterization of Pythagorean Fuzzy Subgroups (PFSG) and normal subgroups. Adding to the complexity of the fuzzy logic framework, Boixader and Recasens [23] presented the concepts of vague and fuzzy t-norms and t-conorms. Moreover, Silambarasan [24] applied the Fermatean fuzzy sets concept to subgroups, leading to the exploration of Fermatean Fuzzy Subgroups (FFSG).

These works demonstrate the application of fuzzy logic and its extensions in the realm of crisp abstract algebra, equipping researchers with powerful tools to address uncertainty in group theory and related fields. The $\mathscr{T}$-norm function, serving as a key component, can be defined in various ways, including standard intersection, algebraic product, bounded difference, and drastic intersection. Similarly, the $\mathscr{S}$-conorm function, another crucial element, offers multiple definitions, such as standard union, algebraic sum, bounded sum, and drastic union. In this paper, we take a significant step by replacing the traditional "minimum" and "maximum" properties in the definitions of Pythagorean and FFSG, respectively, with the $\mathscr{T}$-norm and $\mathscr{S}$-conorm functions. By doing so, we establish a novel framework and explore its implications. Furthermore, we provide proof for several propositions associated with these modified definitions, deepening our understanding of Pythagorean and FFSG in the context of the $\mathscr{T}$-norm and $\mathscr{S}$-conorm functions.

This research paper is structured into six sections, each contributing to the exploration of Pythagorean and FFSG. Section 1 serves as an introduction, providing an overview of previous work in the field. It establishes the foundation upon which our research builds. In Section 2, we present the definitions of key notions and concepts that underpin our study. These fundamental definitions lay the groundwork for our subsequent analyses. Section 3 focuses on the core definitions, delving into the specifics of PFSG. We establish their formal definitions, employing the $\mathscr{T}$ norm and $\mathscr{S}$-conorm functions. Moving forward, Section 4 unveils significant findings derived from our exploration of PFSG. We present and discuss the outcomes, shedding light on their implications. Section 5 introduces the Fermatean fuzzy subgroup, defined in relation to the $\mathscr{T}$-norm and $\mathscr{S}$-conorm functions. Additionally, we provide a collection of results that are closely associated with this new concept. Finally, the research paper concludes in the sixth section, summarizing our findings and providing insights into the significance of our contributions.

2 | Notations

$R = non - empty\ set.$

$R_f = Fuzzy\ set.$

$*= binary\ operation\ defined\ on\ set\ R.$

$(R,*) = A\ group\ equipped\ with\ binary\ operation\ *.$

3 | Preliminaries

3.1 | Fuzzy Set

Assume that R is a set that is not void. A fuzzy set R_f of R is described as $R_f = \{(v, \gamma_{R_f}\ v)) : v \in R\}$. Where γ_{R_f} is a membership function defined as $\gamma_{R_f}: R \to [0,1]\quad for\ all\ v \in R$.

3.2 | Fuzzy Subgroup

Suppose $(R,*)$ be a group. If a fuzzy set R_f of $(R,*)$ satisfies the requirements listed below, then R_f is a fuzzy subgroup.

I. $\gamma_{R_f}\ v_1 * v_2) \geq min(\gamma_{R_f}\ v_1), \gamma_{R_f}\ v_2))\quad for\ all\ v_1, v_2\ \in R.$
II. $\gamma_{\tilde{R}_f}(\tilde{v}^{-1}) \geq \gamma_{\tilde{R}_f}(\tilde{v})\ \ for\ all\ \tilde{v}\ \in \tilde{R}.$

Where γ_{R_f} is a membership function defined as $\gamma_{R_f}: R \to [0,1]\ for\ all\ v \in R.$

3.3 | Intuitionistic Fuzzy Set

A definition of IFS $\widetilde{IR}_f$ of a non-void set R is $\widetilde{IR}_f = \{(v, \gamma_{\widetilde{IR}_f}\ v), \vartheta_{\widetilde{IR}_f}\ v)) : v \in R\}$. Where $\gamma_{\widetilde{IR}_f}$ is a membership function defined as $\gamma_{\widetilde{IR}_f}: R \to [0,1]$ and where $\vartheta_{\widetilde{IR}_f}$ is a non-membership function defined as $\vartheta_{\widetilde{IR}_f}: R \to [0,1]\ \ for\ all\ v \in R.$ Also, $0 \leq \gamma_{\widetilde{IR}_f}\ v) + \vartheta_{\widetilde{IR}_f}\ v) \leq 1$ and the degree of hesitation is given by $1 - \gamma_{\widetilde{IR}_f}\ v) - \vartheta_{\widetilde{IR}_f}\ v)\ for\ all\ v \in R.$

3.4 | Intuitionistic Fuzzy Subgroup

Suppose $(R,*)$ be a group. If IFS $\widetilde{IR}_f$ of $(R,*)$ satisfies the characteristics listed below, then $\widetilde{IR}_f$ is an IFSG of $(R,*)$:

I.

$$\gamma_{\widetilde{IR}_f}\ v_1 * v_2) \geq \min(\gamma_{\widetilde{IR}_f}\ v_1), \gamma_{\widetilde{IR}_f}\ v_2))\ \text{and}\ \vartheta_{\widetilde{IR}_f}\ v_1 * v_2) \leq \max(\vartheta_{\widetilde{IR}_f}\ v_1), \vartheta_{\widetilde{IR}_f}\ v_2))\ \text{for all}\ v_1, v_2\ \in R.$$

II.

$$\gamma_{\widetilde{IR}_f}(v^{-1}) \geq \gamma_{\widetilde{IR}_f}(v) \text{ and } \vartheta_{\widetilde{IR}_f}(v^{-1}) \leq \vartheta_{\widetilde{IR}_f}(v) \text{ for all } v \in R,$$

where $\gamma_{\widetilde{IR}_f}$ is a membership function defined as $\gamma_{\widetilde{IR}_f}: R \to [0,1]$ and $\vartheta_{\widetilde{IR}_f}$ is a non-membership function defined as $\vartheta_{\widetilde{IR}_f}: R \to [0,1]$ $for\ all\ v \in R$.

3.5 | Pythagorean Fuzzy Set

Let R be a non-void set. The definition of Pythagorean fuzzy set $\widetilde{PR}_f$ of $R$is: $\widetilde{PR}_f = \left\{\left(v, \gamma_{\widetilde{PR}_f}(v), \vartheta_{\widetilde{PR}_f}(v)\right) : v \in R\right\}$. Where $\gamma_{\widetilde{PR}_f}$ is a membership function defined as $\gamma_{\widetilde{PR}_f}: R \to [0,1]$ and, $\vartheta_{\widetilde{PR}_f}$ is a non-membership function defined as $\vartheta_{\widetilde{PR}_f}: R \to [0,1]$ $for\ all\ v \in R$. Also, $0 \leq \left(\gamma_{\widetilde{PR}_f}(v)\right)^2 + \left(\vartheta_{\widetilde{PR}_f}(v)\right)^2 \leq 1$ $for\ all\ v \in R$.

3.6 | Pythagorean Fuzzy Subgroup

If a Pythagorean fuzzy set $\widetilde{PR}_f$ of group $(R,*)$ satisfies the following criteria, it supposedly is a Pythagorean fuzzy subgroup.

I.

$$\gamma^2_{\widetilde{PR}_f}(v_1 * v_2) \geq \min\left(\gamma^2_{\widetilde{PR}_f}(v_1), \gamma^2_{\widetilde{PR}_f}(v_2)\right) \text{ and } \vartheta^2_{\widetilde{PR}_f}(v_1 * v_2) \leq \max\left(\vartheta^2_{\widetilde{PR}_f}(v_1), \vartheta^2_{\widetilde{PR}_f}(v_2)\right) \text{ for all } v_1, v_2 \in R.$$

II.

$$\gamma^2_{\widetilde{PR}_f}(v^{-1}) \geq \gamma^2_{\widetilde{PR}_f}(v) \text{ and } \vartheta^2_{\widetilde{PR}_f}(v^{-1}) \leq \vartheta^2_{\widetilde{PR}_f}(v) \text{ for all } v \in R,$$

where γ_{R_f} is a membership function defined as $\gamma_{R_f}: R \to [0,1]$ and $\vartheta_{\widetilde{IR}_f}$ is a non-membership function defined as $\vartheta_{\widetilde{IR}_f}: R \to [0,1]$ $for\ all\ v \in R$ and $\gamma^2_{\widetilde{PR}_f}(v) = \left(\gamma_{\widetilde{IR}_f}(v)\right)^2$ and $\vartheta^2_{\widetilde{PR}_f}(v) = \left(\vartheta_{\widetilde{IR}_f}(v)\right)^2$ $for\ all\ v \in R$.

3.7 | Fermatean Fuzzy Set

Let R be a non-void set. The definition of Fermatean fuzzy set $\widetilde{FR}_f$ of R is $\widetilde{FR}_f = \left\{\left(v, \gamma_{\widetilde{FR}_f}(v), \vartheta_{\widetilde{FR}_f}(v)\right) : v \in R\right\}$. Where $\gamma_{\widetilde{FR}_f}$ is a membership function defined as $\gamma_{\widetilde{FR}_f}: R \to [0,1]$ and, $\vartheta_{\widetilde{FR}_f}$ is a non-membership function defined as $\vartheta_{\widetilde{FR}_f}: R \to [0,1]$ $for\ all\ v \in R$. Also, $0 \leq \left(\gamma_{\widetilde{FR}_f}(v)\right)^3 + \left(\vartheta_{\widetilde{FR}_f}(v)\right)^3 \leq 1$ $for\ all\ v \in R$.

3.8 | Fermatean Fuzzy Subgroup

If a Fermatean fuzzy set $\widetilde{FR}_f$ of group $(R,*)$ satisfies the following criteria, then it is Fermatean fuzzy subgroup.

Mishra et al. |J. Fuzzy. Ext. Appl. 4(2) (2023) 125-135

I.

$$\gamma^3_{\widetilde{FR}_f}(v_1 * v_2) \geq \min\left(\gamma^3_{\widetilde{FR}_f}(v_1), \gamma^3_{\widetilde{FR}_f}(v_2)\right) \text{ and } \vartheta^3_{\widetilde{FR}_f}(v_1 * v_2) \leq \max\left(\vartheta^3_{\widetilde{FR}_f}(v_1), \vartheta^3_{\widetilde{FR}_f}(v_2)\right) \text{ for all } v_1, v_2 \in \widetilde{R}.$$

II.

$$\gamma^3_{\widetilde{FR}_f}\left(v^{-1}\right) \geq \gamma^3_{\widetilde{FR}_f}(v) \text{ and } \vartheta^3_{\widetilde{FR}_f}\left(v^{-1}\right) \leq \vartheta^3_{\widetilde{FR}_f}(v) \text{ for all } v \in \widetilde{R},$$

where γ_{R_f} is a membership function defined as $\gamma_{R_f}: R \to [0,1]$ and $\vartheta_{\widetilde{IR}_f}$ is a non-membership function defined as $\vartheta_{\widetilde{IR}_f}: R \to [0,1]$ $for\ all\ v \in R$ and $\gamma^3_{\widetilde{FR}_f}(v) = \left(\gamma_{\widetilde{FR}_f}(v)\right)^3$ and $\vartheta^3_{\widetilde{PR}_f}(v) = \left(\vartheta_{\widetilde{IR}_f}(v)\right)^3$ $for\ all\ v \in R$.

3.9 | $\mathcal{T}$-Norm

Definition 1. A function $\mathcal{T} : [0,1] \times [0,1] \to [0,1]$ is referred to as $\mathcal{T}$- norm if "$\mathcal{T}$" meets the criteria listed below:

I. $\mathcal{T}(0,0) = 0, \mathcal{T}(\rho',1) = \gamma' = \mathcal{T}(1,\rho')$.
II. $\mathcal{T}(\alpha',\mathfrak{b}') \leq \mathcal{T}(\rho',\tau')\ if\ \alpha' \leq \rho'\ and\ \mathfrak{b}' \leq \tau'$.
III. $\mathcal{T}(\alpha',\tau') = \mathcal{T}(\tau',\alpha')$.
IV. $\mathcal{T}(\alpha',\mathcal{T}(\tau',\rho')) = \mathcal{T}(\mathcal{T}(\alpha',\tau'),\rho')\ for\ all\ \alpha',\mathfrak{b}',\tau',\rho' \in R$.

Some various types of $\mathcal{T}$-norm function are as follows:

I. Standard intersection: $\mathcal{T}(\alpha',\mathfrak{b}') = min(\alpha',\mathfrak{b}')$.
II. Algebraic product: $\mathcal{T}(\alpha',\mathfrak{b}') = \alpha'.\mathfrak{b}'$.
III. Bounded difference: $\mathcal{T}(\alpha',\mathfrak{b}') = max(0, \alpha' + \mathfrak{b}' - 1)$.
IV. Drastic intersection: $\mathcal{T}(\alpha',\mathfrak{b}') = \begin{cases} \alpha', & when\ \mathfrak{b}' = 1, \\ \mathfrak{b}', & when\ \alpha' = 1, \\ 0, & otherwise. \end{cases}$

The relation between these four are $T_{min}(\alpha',\mathfrak{b}') \leq max(0, \alpha' + \mathfrak{b}' - 1) \leq \alpha'.\mathfrak{b}' \leq min(\alpha',\mathfrak{b}')$.

3.10 | $\widetilde{\mathcal{S}}$-Conorm

Definition 2. A function $\mathcal{S}: [0,1] \times [0,1] \to [0,1]$ is referred to as $\mathcal{S}$-conorm if "$\mathcal{S}$" meets the criteria listed below:

I. $\mathcal{S}(\alpha',0) = \alpha'$.
II. $\mathfrak{b}' \leq \rho'\ implies\ \mathcal{S}(\alpha',\mathfrak{b}') \leq \mathcal{S}(\alpha',\rho')$.
III. $\mathcal{S}(\alpha',\tau') = \mathcal{S}(\tau',\alpha')$.
IV. $\mathcal{S}(\alpha', S(\tau',\rho')) = \mathcal{S}(\mathcal{S}(\alpha',\tau'),\rho')\ for\ all\ \alpha',\mathfrak{b}',\rho',\tau' \in R$.

Some various types of $\mathcal{S}$-conorm function are as follows:

I. Standard union: $\mathcal{S}(\alpha',\mathfrak{b}') = max(\alpha',\mathfrak{b}')$.
II. Algebraic sum: $\mathcal{S}(\alpha',\mathfrak{b}') = \alpha' + \mathfrak{b}' - \alpha'\mathfrak{b}'$.
III. Bounded sum: $\mathcal{S}(\alpha',\mathfrak{b}') = min(1, \alpha' + \mathfrak{b}')$.
IV. Drastic union: $\mathcal{S}(\alpha',\mathfrak{b}') = \begin{cases} \alpha', & when\ \mathfrak{b}' = 0, \\ \mathfrak{b}', & when\ \alpha' = 0, \\ 1, & otherwise. \end{cases}$

The relation between these four are:

$$max\ \mathfrak{a}', \mathfrak{b}') \leq \mathfrak{a}' + \mathfrak{b}' - \mathfrak{a}'\mathfrak{b}' \leq \min 1, \mathfrak{a}' + \mathfrak{b}') \leq \mathrm{S}_{\max}\ \mathfrak{a}', \mathfrak{b}').$$

Definition 3. A fuzzy set R_f of a group $\left(R,*\right)$ is a Fuzzy Subgroup (FSG) in terms of the t-norm "$\mathscr{T}$" if R_f meets the criteria listed below:

I. $\gamma_{R_f}\ v_1 * v_2) \geq T\left(\gamma_{R_f}\ v_1), \gamma_{R_f}\ v_2)\right)\ for\ all\ v_1, v_2\ \in R.$

II. $\gamma_{R_f}\left(v^{-1}\right) \geq \gamma_{R_f}\ v)\ for\ all\ v\ \in R.$

Where γ_{R_f} is a membership function defined as $\gamma_{R_f}: R \rightarrow\ [0,1]\ for\ all\ v \in R$.

4 | Pythagorean Fuzzy Subgroup in Context of $\widetilde{\mathcal{T}}$-Norm and $\widetilde{\mathcal{S}}$-Conorm

In this section, we redefine the concept of PFSG within the framework of the $\mathscr{T}$-norm and $\mathscr{S}$-conorm, providing a fresh perspective and enhancing our understanding of this fundamental concept. We delve into the intricate properties associated with these redefined PFSGs, shedding light on their unique characteristics and implications.

Bhunia et al. [21] made significant contributions to the study of PFSG. His work was instrumental in defining the concept and establishing crucial conditions for the membership and non-membership functions. The conditions, as provided below, play a vital role in characterizing PFSG. Bhunia's work laid the foundation for our exploration, paving the way for the redefinition and analysis of PFSGs within the $\mathscr{T}$-norm and $\mathscr{S}$-conorm framework. By leveraging Bhunia's insights, we build upon the existing knowledge to further deepen our understanding of PFSGs and their properties.

I. $\gamma^2_{\widetilde{PR}_f}\ v_1 * v_2) \geq min\left(\gamma^2_{\widetilde{PR}_f}\ v_1), \gamma^2_{\widetilde{PR}_f}\ v_2)\right)\ and\ \vartheta^2_{\widetilde{PR}_f}\ v_1 * v_2) \leq max\left(\vartheta^2_{\widetilde{PR}_f}\ v_1), \vartheta^2_{\widetilde{PR}_f}\ v_2)\right)$
$for\ all\ t_1, t_2\ \in R.$

II. $\gamma^2_{\widetilde{PR}_f}\left(v^{-1}\right) \geq \gamma^2_{\widetilde{PR}_f}\ v)\ and\ \vartheta^2_{\widetilde{PR}_f}\left(v^{-1}\right) \leq \vartheta^2_{\widetilde{PR}_f}\ v)\ \ for\ all\ v\ \in R.$

In Section 2, an in-depth exploration of the $\mathscr{T}$-norm and $\mathscr{S}$-conorm was undertaken, highlighting their diverse types and characteristics. Upon closer examination of the definition of a Pythagorean fuzzy subgroup, a compelling insight emerged: the traditional "minimum" and "maximum" operations can be effectively replaced by the $\mathscr{T}$-norm and $\mathscr{S}$-conorm, respectively. This revelation implies that we are no longer confined to a single option for selecting the minimum and maximum values. Harnessing this newfound flexibility, we proceed to define the Pythagorean fuzzy subgroup, leveraging the power of the $\mathscr{T}$-norm and $\mathscr{S}$-conorm. By adopting this novel approach, we expand the possibilities and refine our understanding of PFSG within the context of these operations.

Definition 4. Suppose $R,*)$ be a group. A Pythagorean fuzzy set $\widetilde{PR}_f$ of $\left(R,*\right)$ is a PFSG in context of t-norm '$\mathscr{T}$' and s-conorm '$\mathscr{S}$' if $\widetilde{PR}_f$ satisfies the following conditions:

I. $\gamma^2_{\widetilde{PR}_f}\ v_1 * v_2) \geq \mathscr{T}\left(\gamma^2_{\widetilde{PR}_f}\ v_1), \gamma^2_{\widetilde{PR}_f}\ v_2)\right)\ and\ \vartheta^2_{\widetilde{PR}_f}\ v_1 * v_2) \leq \mathscr{S}\left(\vartheta^2_{\widetilde{PR}_f}\ v_1), \vartheta^2_{\widetilde{PR}_f}\ v_2)\right)\ for\ all\ v_1, v_2\ \in R.$

II. $\gamma^2_{\widetilde{PR}_f}(\tilde{v}^{-1}) \geq \gamma^2_{\widetilde{PR}_f}(\tilde{v})\ and\ \vartheta^2_{\widetilde{PR}_f}(\tilde{v}^{-1}) \leq \vartheta^2_{\widetilde{PR}_f}(\tilde{v})\ \ for\ all\ \tilde{v}\ \in \tilde{R}.$

Where $\gamma_{\widetilde{PR}_f}$ is a membership function defined as $\gamma_{\widetilde{PR}_f}: R \to [0,1]$ and $\vartheta_{\widetilde{PR}_f}$ is a non-membership function defined as $\vartheta_{\widetilde{PR}_f}: R \to [0,1]$ $for\ all\ t \in R$ and $\gamma^2_{\widetilde{PR}_f}\ v) = \left(\gamma_{\widetilde{PR}_f}\ v)\right)^2$ and $\vartheta^2_{\widetilde{PR}_f}\ v) = \left(\vartheta_{\widetilde{PR}_f}\ v)\right)^2$ $for\ all\ v \in R$.

Example 1. Let $z_3,*)$ be a group of addition modulo 3 and let Pythagorean fuzzy set $\widetilde{PR}_f$ of $z_3,*)$ is $\widetilde{P\ z_3,*)}_f = \{<0,[0.8,0.5]>,<1,[0.7,0.6]>,<2,[0.7,0.6]>\}$. Then $\widetilde{P\ z_3,*)}_f$ is a Pythagorean fuzzy subgroup in context of t-norm '$\mathscr{T}$'and s-conorm '$\mathscr{S}$'. Where $\mathscr{T} = min\left(\gamma^2_{\widetilde{PR}_f}\ v_1), \gamma^2_{\widetilde{PR}_f}\ v_2)\right)$ and $\mathscr{S} = max\left(\vartheta^2_{\widetilde{PR}_f}\ v_1), \vartheta^2_{\widetilde{PR}_f}\ v_2)\right)$.

Proposition 1. If $\widetilde{PR}_f$ is a PFSG of a group $(R,*)$ in context of t-norm '$\mathscr{T}$' and s-conorm '$\mathscr{S}$', then $(R,*)_1 = \{v \in (R,*) : \widetilde{PR}_f(v) = 1\ i.e.\ \gamma^2_{\widetilde{PR}_f}\ v) = 1\ and\ \vartheta^2_{\widetilde{PR}_f}\ v) = 1\}$ is either empty or is a subgroup of $(R,*)$.

Proof: If $v_1, v_2 \in (R,*)_1$ then $\gamma^2_{\widetilde{PR}_f}\left(v_1 * v_2{}^{-1}\right) \geq \mathscr{T}\left(\gamma^2_{\widetilde{PR}_f}\ v_1), \gamma^2_{\widetilde{PR}_f}\left(v_2{}^{-1}\right)\right) = \mathscr{T}\left(\gamma^2_{\widetilde{PR}_f}\ v_1), \gamma^2_{\widetilde{PR}_f}\ v_2)\right) = \mathscr{T}\ 1,1) = 1$. Therefore, $\gamma^2_{\widetilde{PR}_f}\left(v_1 * v_2{}^{-1}\right) = 1$ implies $v_1 * v_2{}^{-1} \in (R,*)_1$. And if $v_1, v_2 \in (R,*)_1$ then $\vartheta^2_{\widetilde{PR}_f}\left(v_1 * v_2{}^{-1}\right) \leq \mathscr{S}\left(\vartheta^2_{\widetilde{PR}_f}\ v_1), \vartheta^2_{\widetilde{PR}_f}\left(v_2{}^{-1}\right)\right) = \mathscr{S}\left(\vartheta^2_{\widetilde{PR}_f}\ v_1), \vartheta^2_{\widetilde{PR}_f}\ v_2)\right) = \mathscr{S}\ 1,1) = 1$. Therefore, $\vartheta^2_{\widetilde{PR}_f}\left(v_1 * v_2{}^{-1}\right) = 1$ implies $v_1 * v_2{}^{-1} \in (R,*)_1$.

Consequently, $(R,*)_1$ is a subgroup of group $(R,*)$.

Proposition 2. Let $\widetilde{PR}_f$ be a PFSG of a group $(R,*)$ in context of t-norm '$\mathscr{T}$'and s-conorm '$\mathscr{S}$'. If $\gamma^2_{\widetilde{PR}_f}\left(v_1 * v_2{}^{-1}\right) = 1$ then $\gamma^2_{\widetilde{PR}_f}\ v_1) = \gamma^2_{\widetilde{PR}_f}\ v_2)$, and if $\vartheta^2_{\widetilde{PR}_f}\left(v_1 * v_2{}^{-1}\right) = 1$ then $\vartheta^2_{\widetilde{PR}_f}\ v_1) = \vartheta^2_{\widetilde{PR}_f}\ v_2)\ for\ all\ v \in (R,*)$.

Proof: Consider $for\ all\ v_1, v_2 \in (R,*)$.

$\gamma^2_{\widetilde{PR}_f}\ v_1) = \gamma^2_{\widetilde{PR}_f}((v_1 * v_2{}^{-1})v_2) \geq \mathscr{T}\left(\gamma^2_{\widetilde{PR}_f}\left(v_1 * v_2{}^{-1}\right), \gamma^2_{\widetilde{PR}_f}\ v_2)\right) = \mathscr{T}\left(1, \gamma^2_{\widetilde{PR}_f}\ v_2)\right) = \gamma^2_{\widetilde{PR}_f}\ v_2) = \gamma^2_{\widetilde{PR}_f}(v_2{}^{-1})$. [Since $\widetilde{PR}_f$ is a Pythagorean fuzzysubgroup]. Again consider, $\gamma^2_{\widetilde{PR}_f}\ v_2) = \gamma^2_{\widetilde{PR}_f}(v_1(v_1 * v_2{}^{-1})) \geq \mathscr{T}\left(\gamma^2_{\widetilde{PR}_f}\ v_1), \gamma^2_{\widetilde{PR}_f}\left(v_1 * v_2{}^{-1}\right)\right) = \mathscr{T}\left(\gamma^2_{\widetilde{PR}_f}\ v_1), 1\right) = \gamma^2_{\widetilde{PR}_f}\ v_2) = \gamma^2_{\widetilde{PR}_f}(v_2{}^{-1})$. [Since $\widetilde{PR}_f$is a Pythagorean fuzzy subgroup].

$\Rightarrow \gamma^2_{\widetilde{PR}_f}\ v_1) = \gamma^2_{\widetilde{PR}_f}\ v_2)$.

In the same manner, we can prove that $\vartheta^2_{\widetilde{PR}_f}\ v_1) = \vartheta^2_{\widetilde{PR}_f}\ v_2)$.

Proposition 3. Let $\widetilde{PR}_f$ be a PFS of a group $(R,*)$ in context of thet-norm '$\mathscr{T}$'and s-conorm '$\mathscr{S}$'. If $\widetilde{PR}_f(e) = 1$ i.e., $\gamma^2_{\widetilde{PR}_f}\ e) = 1\ and\ \vartheta^2_{\widetilde{PR}_f}\ e) = 1$ and $\gamma^2_{\widetilde{PR}_f}\left(v_1 * v_2{}^{-1}\right) \geq \mathscr{T}\left(\gamma^2_{\widetilde{PR}_f}\ v_1), \gamma^2_{\widetilde{PR}_f}\ v_2)\right)$ $for\ all\ v_1, v_2 in (R,*)$ and $\vartheta^2_{\widetilde{PR}_f}\left(v_1 * v_2{}^{-1}\right) \leq$

$S\left(\vartheta^2_{\widetilde{PR}_f}\ v_1),\vartheta^2_{\widetilde{PR}_f}\ v_2)\right)$ *for all* $v_1, v_2 in\left(R,*\right)$. Then $\widetilde{PR}_f$ is a PFSG of group $\left(R,*\right)$ in context of t-norm '$\mathscr{T}$' and s-conorm '$\mathscr{S}$'.

Proof: Consider, *for all* $v_1, v_2 \in \left(R,*\right), \gamma^2_{\widetilde{PR}_f}\left(v^{-1}\right) = \gamma^2_{\widetilde{PR}_f}\left(ev^{-1}\right) \geq \mathscr{T}\left(\gamma^2_{\widetilde{PR}_f}\ e), \gamma^2_{\widetilde{PR}_f}\ v)\right) = \mathscr{T}\left(1, \gamma^2_{\widetilde{PR}_f}\ v)\right) = \gamma^2_{\widetilde{PR}_f}(v)$. And similarly, $\gamma^2_{\widetilde{PR}_f}(v) \geq \gamma^2_{\widetilde{PR}_f}(v^{-1})$ so that $\gamma^2_{\widetilde{PR}_f}\ v) = \gamma^2_{\widetilde{PR}_f}(v^{-1})$. Moreover, $\gamma^2_{\widetilde{PR}_f}\ v_1 * v_2) \geq \gamma^2_{\widetilde{PR}_f}\left(v_1 * {v_2}^{-1}\right) \geq \mathscr{T}\left(\gamma^2_{\widetilde{PR}_f}\ v_1), \gamma^2_{\widetilde{PR}_f}\left({v_2}^{-1}\right)\right) = \mathscr{T}\left(\gamma^2_{\widetilde{PR}_f}\ v_1), \gamma^2_{\widetilde{PR}_f}\ v_2)\right)$. Thus $\gamma^2_{\widetilde{PR}_f}$ is a PFSG of group $\left(R,*\right)$ in context of t-norm '$\mathscr{T}$'.

Likewise, we can demonstrate $\vartheta^2_{\widetilde{PR}_f}$ is a PFSG of group $\left(R,*\right)$ in context of s-conorm '$\mathscr{S}$'. Consequently, $\widetilde{PR}_f$ is a PFSG of group $\left(R,*\right)$ in context of t-norm '$\mathscr{T}$'and s-conorm '$\mathscr{S}$'.

5 | Fermatean Fuzzy Subgroup in Context of $\mathscr{T}$-Norm and $\mathscr{S}$-Conorm

Silambarasan [24] introduced the concept of FFSG. Building upon this pioneering work, we further advance the field by redefining the notion of a "Fermatean fuzzy subgroup" in the context of the t-norm function denoted as '$\mathscr{T}$' and the s-conorm function denoted as '$\mathscr{S}$'. This redefinition provides a novel perspective and deepens our understanding of FFSG. Furthermore, we explore and present a collection of properties associated with these redefined subgroups, shedding light on their unique characteristics and implications within the broader context of fuzzy subgroup theory.

Definition 5. Suppose$\left(R,*\right)$ be a group. AFFS $\widetilde{FR}_f$ of $\left(R,*\right)$ is FFSG in context of t-norm '$\mathscr{T}$' and s-conorm '$\mathscr{S}$' if $\widetilde{FR}_f$ satisfies the following conditions:

I.

$$\gamma^3_{\widetilde{FR}_f}\ v_1 * v_2) \geq \mathscr{T}\left(\gamma^3_{\widetilde{FR}_f}\ v_1), \gamma^3_{\widetilde{FR}_f}\ v_2)\right) \text{ and } \vartheta^3_{\widetilde{FR}_f}\ v_1 * v_2) \leq \mathscr{S}\left(\vartheta^3_{\widetilde{FR}_f}\ v_1), \vartheta^3_{\widetilde{FR}_f}\ v_2)\right) \text{ for all } v_1, v_2\ \in R.$$

II.

$$\gamma^3_{\widetilde{FR}_f}\left(v^{-1}\right) \geq \gamma^3_{\widetilde{FR}_f}\ v) \text{ and } \vartheta^3_{\widetilde{FR}_f}\left(v^{-1}\right) \leq \vartheta^3_{\widetilde{FR}_f}\ v)\ \text{ for all } v\ \in R,$$

where $\gamma_{\widetilde{FR}_f}$ is a membership function defined as $\gamma_{\widetilde{FR}_f}: R \rightarrow\ [0,1]$ and $\vartheta_{\widetilde{FR}_f}$ is a non-membership function defined as: $\vartheta_{\widetilde{FR}_f}: R \rightarrow\ [0,1]$ *for all* $v \in R$ and $\gamma^3_{\widetilde{FR}_f}\ v) = \left(\gamma_{\widetilde{FR}_f}\ v)\right)^3$ and $\vartheta^3_{\widetilde{FR}_f}\ v) = \left(\vartheta_{\widetilde{FR}_f}\ v)\right)^3$ *for all* $v \in R$.

Example 2. Let $z_3,*)$ be a group of addition modulo 3 and let FFS $\widetilde{FR}_f$ of $z_3,*)$ is $\widetilde{F\ z_3,*)}_f = \{< 0, [0.9,0.5] >, < 1, [0.9,0.6] >, < 2, [0.9,0.6] >\}$.Then $\widetilde{P\ z_3,*)}_f$ is a Fermatean fuzzy subgroupin context of t-norm '$\mathscr{T}$' and s-norm '$\mathscr{S}$'. Where $\mathscr{T} = min\left(\gamma^3_{\widetilde{PR}_f}\ v_1), \gamma^3_{\widetilde{PR}_f}\ v_2)\right)$ and $\mathscr{S} = max\left(\vartheta^3_{\widetilde{PR}_f}\ v_1), \vartheta^3_{\widetilde{PR}_f}\ v_2)\right)$.

Proposition 4. If $\widetilde{FR}_f$ is a FFSG of a group $\left(R,*\right)$ in context of t-norm '$\mathscr{T}$', and s-conorm '$\mathscr{S}$' then $\left(R,*\right)_1 = \{v \in \left(R,*\right):\ \widetilde{FR}_f(v)\ = 1\, i.e. \gamma^3_{\widetilde{FR}_f}\ v) = 1 \text{ and } \vartheta^3_{\widetilde{FR}_f}\ v) = 1\}$ is either empty or is a subgroup of $\left(R,*\right)$.

Proof: If $v_1, v_2 \in (R,*)_1$ then $\gamma^3_{\widetilde{FR}_f}(v_1 * v_2^{-1}) \geq \mathcal{T}\left(\gamma^3_{\widetilde{FR}_f}\ v_1), \gamma^3_{\widetilde{FR}_f}(v_2^{-1})\right) = \mathcal{T}\left(\gamma^3_{\widetilde{FR}_f}\ v_1), \gamma^3_{\widetilde{FR}_f}\ v_2)\right) = \mathcal{T}\ 1,1) = 1$. Therefore, $\gamma^3_{\widetilde{FR}_f}(v_1 * v_2^{-1}) = 1$ implies $v_1 * v_2^{-1} \in (R,*)_1$. And if $v_1, v_2 \in (R,*)_1$ then $\vartheta^3_{\widetilde{FR}_f}(v_1 * v_2^{-1}) \leq \mathcal{S}\left(\vartheta^3_{\widetilde{FR}_f}\ v_1), \vartheta^3_{\widetilde{FR}_f}(v_2^{-1})\right) = \mathcal{S}\left(\vartheta^3_{\widetilde{FR}_f}\ v_1), \vartheta^3_{\widetilde{FR}_f}\ v_2)\right) = \mathcal{S}\ 1,1) = 1$. Therefore, $\vartheta^3_{\widetilde{FR}_f}(v_1 * v_2^{-1}) = 1$ implies $v_1 * v_2^{-1} \in (R,*)_1$.

Consequently, $(R,*)_1$ is a subgroup of group $(R,*)$.

Proposition 5. Let $\widetilde{FR}_f$ be a FFSG of a group $(R,*)$ in context of t-norm $\mathcal{T}$ "and s-conorm '$\mathcal{S}$'. If $\gamma^3_{\widetilde{FR}_f}(v_1 * v_2^{-1}) = 1$ then $\gamma^3_{\widetilde{FR}_f}\ v_1) = \gamma^3_{\widetilde{FR}_f}\ v_2)$, and if $\vartheta^3_{\widetilde{FR}_f}(v_1 * v_2^{-1}) = 1$ then $\vartheta^3_{\widetilde{FR}_f}\ v_1) = \vartheta^3_{\widetilde{FR}_f}\ v_2)\ for\ all\ v \in (R,*)$.

Proof: Consider $for\ all\ v_1, v_2 \in (R,*)$, $\gamma^3_{\widetilde{FR}_f}\ v_1) = \gamma^3_{\widetilde{FR}_f}((v_1 * v_2^{-1})v_2) \geq \mathcal{T}\left(\gamma^3_{\widetilde{FR}_f}(v_1 * v_2^{-1}), \gamma^3_{\widetilde{FR}_f}\ v_2)\right) = \mathcal{T}\left(1, \gamma^3_{\widetilde{FR}_f}\ v_2)\right) = \gamma^3_{\widetilde{FR}_f}\ v_2) = \gamma^3_{\widetilde{FR}_f}(v_2^{-1})$. [Since $\widetilde{FR}_f$ is a Fermatean fuzzy subgroup].

Again consider $\gamma^3_{\widetilde{FR}_f}\ v_2) = \gamma^3_{\widetilde{FR}_f}(v_1(v_1 * v_2^{-1})) \geq \mathcal{T}\left(\gamma^3_{\widetilde{FR}_f}\ v_1), \gamma^3_{\widetilde{FR}_f}(v_1 * v_2^{-1})\right) = \mathcal{T}\left(\gamma^3_{\widetilde{FR}_f}\ v_1), 1\right) = \gamma^3_{\widetilde{FR}_f}\ v_2) = \gamma^3_{\widetilde{FR}_f}(v_2^{-1})$. [Since $\widetilde{FR}_f$ is a Fermatean fuzzy subgroup].

$\Rightarrow \gamma^3_{\widetilde{FR}_f}\ v_1) = \gamma^3_{\widetilde{FR}_f}\ v_2)$.

In a similar way, we may demonstrate that $\vartheta^3_{\widetilde{FR}_f}\ v_1) = \vartheta^3_{\widetilde{FR}_f}\ v_2)$.

Proposition 6. Let $\widetilde{FR}_f$ be a Fermatean fuzzy set of a group $(R,*)$ in context of t-norm '$\mathcal{T}$' and s-conorm '$\mathcal{S}$'. If $\widetilde{FR}_f(e) = 1$ i.e., $\gamma^3_{\widetilde{FR}_f}\ e) = 1\ and\ \vartheta^3_{\widetilde{FR}_f}\ e) = 1$ and $\gamma^3_{\widetilde{FR}_f}(v_1 * v_2^{-1}) \geq \mathcal{T}\left(\gamma^3_{\widetilde{FR}_f}\ v_1), \gamma^3_{\widetilde{FR}_f}\ v_2)\right)\ for\ all\ v_1, v_2\ in\ (R,*)$ and $\vartheta^3_{\widetilde{FR}_f}(v_1 * v_2^{-1}) \leq S\left(\vartheta^3_{\widetilde{FR}_f}\ v_1), \vartheta^3_{\widetilde{FR}_f}\ v_2)\right)\ for\ all\ v_1, v_2\ in\ (R,*)$. Then $\widetilde{PR}_f$ is a FFSG of group $(R,*)$ in context of t-norm '$\mathcal{T}$' and s-conorm '$\mathcal{S}$'.

Proof: Consider $for\ all\ v_1, v_2 \in (R,*)$, $\gamma^3_{\widetilde{FR}_f}(v^{-1}) = \gamma^3_{\widetilde{FR}_f}(ev^{-1}) \geq \mathcal{T}\left(\gamma^3_{\widetilde{FR}_f}\ e), \gamma^3_{\widetilde{FR}_f}\ v)\right) = \mathcal{T}\left(1, \gamma^3_{\widetilde{FR}_f}\ v)\right) = \gamma^3_{\widetilde{FR}_f}(v)$. And similarly, $\gamma^3_{\widetilde{FR}_f}(v) \geq \gamma^3_{\widetilde{FR}_f}(v^{-1})$ so that $\gamma^3_{\widetilde{FR}_f}\ v) = \gamma^3_{\widetilde{FR}_f}(v^{-1})$. Moreover, $\gamma^3_{\widetilde{FR}_f}\ v_1 * v_2) \geq \gamma^3_{\widetilde{FR}_f}(v_1 * v_2^{-1}) \geq \mathcal{T}\left(\gamma^3_{\widetilde{FR}_f}\ v_1), \gamma^3_{\widetilde{FR}_f}(v_2^{-1})\right) = \mathcal{T}\left(\gamma^3_{\widetilde{FR}_f}\ v_1), \gamma^3_{\widetilde{FR}_f}\ v_2)\right)$. thus $\gamma^3_{\widetilde{FR}_f}$ is a FFSG of group $(R,*)$ in context of t-norm '$\mathcal{T}$'.

Likewise, we can demonstrate $\vartheta^3_{\widetilde{FR}_f}$ is a FFSG of group $(R,*)$ in context of s-conorm '$\mathcal{S}$'. Consequently, $\widetilde{FR}_f$ is a FFSG of group $(R,*)$ in context of '$\mathcal{T}$'and s-conorm '$\mathcal{S}$'.

6 | Conclusion

The presence of symmetry in our environment is undeniable, yet it is not always flawlessly precise. Symmetry often exhibits a degree of vagueness. To address this inherent vagueness, fuzzy logic has found its application in the field of group theory. Over time, extensions of fuzzy sets have emerged, introducing concepts such as Pythagorean and Fermatean fuzzy sets. This research paper has aimed to redefine PFSG and FFSG by leveraging the $\mathscr{T}$-norm and $\mathscr{S}$-conorm functions. The $\mathscr{T}$-norm function offers diverse definitions, including standard intersection, algebraic product, bounded difference, and drastic intersection. Similarly, the $\mathscr{S}$-conorm function encompasses various definitions, such as standard union, algebraic sum, bounded sum, and drastic union. In this study, we have utilized the $\mathscr{T}$-norm and $\mathscr{S}$-conorm in the existing definitions of Pythagorean and FFSG, resulting in their redefinition within the context of these functions. Concrete examples have been provided to elucidate these redefined subgroups. Moreover, we have established and proven several propositions pertinent to these newly defined subgroups.

In conclusion, the research conducted in this paper sets the stage for further promotion and dissemination of our findings. It paves the way for future investigations and advancements in the field of fuzzy subgroups, PFSG and Fermatean fuzzy subgroup in context of norm and conorm, fostering a deeper understanding of symmetry in the presence of uncertainty.

Acknowledgements

The University Grants Commission's financial support is appreciated by the first author.

Conflicts of Interests

There are no conflicts of interest for authors.

References

[1] Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, *8*(3), 338–353. DOI:10.1016/S0019-9958(65)90241-X

[2] Rosenfeld, A. (1971). Fuzzy groups. *Journal of mathematical analysis and applications*, *35*(3), 512–517. DOI:10.1016/0022-247X(71)90199-5

[3] Anthony, J. M., & Sherwood, H. (1979). Fuzzy groups redefined. *Journal of mathematical analysis and applications*, *69*(1), 124–130. DOI:10.1016/0022-247X(79)90182-3

[4] Bhattacharya, P., & Mukherjee, N. P. (1985). Fuzzy relations and fuzzy groups. *Information sciences*, *36*(3), 267–282. DOI:10.1016/0020-0255(85)90057-X

[5] Atanassov, A. (1986). Intuitionistic fuzzy sets. *Fuzzy sets and systems*, *20*(1), 87-96. DOI:10.5555/1708507.1708520

[6] Das, P. S. (1981). Fuzzy groups and level subgroups. *Journal of mathematical analysis and applications*, *84*(1), 264–269. DOI:10.1016/0022-247X(81)90164-5

[7] Ajmal, N., & Prajapati, A. S. (1992). Fuzzy cosets and fuzzy normal subgroups. *Information sciences*, *64*(1–2), 17–25. DOI:10.1016/0020-0255(92)90107-J

[8] Ajmal, N., & Thomas, K. V. (1994). The lattices of fuzzy subgroups and fuzzy normal subgroups. *Information sciences*, *76*(1–2), 1–11. DOI:10.1016/0020-0255(94)90064-7

[9] Dixit, V. N., Kumar, R., & Ajmal, N. (1990). Level subgroups and union of fuzzy subgroups. *Fuzzy sets and systems*, *37*(3), 359–371. DOI:10.1016/0165-0114(90)90032-2

[10] Gau, W. L., & Buehrer, D. J. (1993). Vague Sets. *IEEE transactions on systems, man and cybernetics*, *23*(2), 610–614. DOI:10.1109/21.229476

[11] Khan, H., Ahmad, M., & Biswas, R. (2007). On vague groups. *International journal of computational cognition*, *5*(1), 27–30.

[12] Yager, R. R. (2013). *Pythagorean fuzzy subsets* [presentation]. Proceedings of the 2013 joint ifsa world congress and nafips annual meeting, ifsa/nafips 2013 (pp. 57–61). DOI: 10.1109/IFSA-NAFIPS.2013.6608375

[13] Rasuli, R. (2017). Fuzzy subgroups on direct product of groups over a \$t\$-norm. *Journal of fuzzy set valued analysis*, *2017*(3), 96–101. DOI:10.5899/2017/jfsva-00339

[14] Rasuli, R. (2019). Fuzzy equivalence relation, fuzzy congrunce relation and fuzzy normal subgroups on group G over T-Norms. *Asian journal of fuzzy and applied mathematics*, *7*(2). DOI:10.24203/ajfam.v7i2.5736

[15] Rasuli, R. (2020). Intuitionistic fuzzy subgroups with respect to norms (T , S). *Engineering and applied science letters*, *3*(2), 40–53. DOI:10.30538/psrp-easl2020.0040

[16] Gayen, S., Smarandache, F., Jha, S., & Kumar, R. (2020). Interval-valued neutrosophic subgroup based on interval-valued triple T-Norm. In *Neutrosophic sets in decision analysis and operations research* (pp. 215–243). IGI Global. DOI: 10.4018/978-1-7998-2555-5.ch010

[17] Senapati, T., & Yager, R. R. (2020). Fermatean fuzzy sets. *Journal of ambient intelligence and humanized computing*, *11*(2), 663–674. DOI:10.1007/s12652-019-01377-0

[18] Bejines, C., Chasco, M. J., & Elorza, J. (2021). Aggregation of fuzzy subgroups. *Fuzzy sets and systems*, *418*, 170–184. DOI:10.1016/j.fss.2020.05.017

[19] Ardanza-Trevijano, S., Chasco, M. J., Elorza, J., de Natividade, M., & Talavera, F. J. (2023). Aggregation of T-subgroups. *Fuzzy sets and systems*, *463*, 108390. DOI:10.1016/j.fss.2022.08.022

[20] Kumar, T., Dhiman, N., Vandana, Vashistha, S., Sharma, M. K., & Mishra, V. N. (2020). Vague groups redefined with respect to t-norm. *Advances in mathematics: scientific journal*, *9*(10), 8475–8483. DOI:10.37418/amsj.9.10.76

[21] Bhunia, S., Ghorai, G., & Xin, Q. (2021). On the characterization of pythagorean fuzzy subgroups. *AIMS mathematics*, *6*(1), 962–978. DOI:10.3934/math.2021058

[22] Razaq, A., Alhamzi, G., Razzaque, A., & Garg, H. (2022). A comprehensive study on Pythagorean fuzzy normal subgroups and Pythagorean fuzzy isomorphisms. *Symmetry*, *14*(10), 2084. DOI:10.3390/sym14102084

[23] Boixader, D., & Recasens, J. (2022). Vague and fuzzy t-norms and t-conorms. *Fuzzy sets and systems*, *433*, 156–175. DOI:10.1016/j.fss.2021.07.008

[24] Silambarasan, I. (2021). Fermatean fuzzy subgroups. *Journal of the international mathematical virtual institute*, *11*(1), 1–16.