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Single-Valued Neutrosophic Uncertain Linguistic Set Based on Multi-Input Relationship and Semantic Transformation

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Abstract

This study introduces an approach for Multiple Attribute Decision-Making (MADM) that deals with the complexity of Single-Valued Neutrosophic Uncertain Linguistic Variables (SVNULVs). This method is engineered to grasp the interconnectedness of multiple inputs and to meet the diverse requirements for semantic transformations. Due to the shortcomings of existing operational rules in terms of closeness and flexibility, this paper proposes a novel set of operational rules and a ranking process for SVNULVs, integrating the concept of a Linguistic Scale Function (LSF). We propose an innovative operator along with its weighted counterpart to amalgamate SVNULVs, thereby characterizing the dynamics among various inputs through these new operations. Concurrently, we scrutinize and discuss the unique cases and favorable properties of these proposed operators. Building upon this new operator, the paper also unveils a fresh MADM methodology leveraging SVNULVs. To validate the effectiveness of this proposed methodology, an illustrative example is employed, demonstrating the precision of the method and its advantages over existing MADM techniques.

Keywords: Multiple attribute decision-making, Hamy mean operator, Neutrosophic uncertain linguistic set, Linguistic scale function, Neutrosophic set.

1 | Introduction

Multiple Attribute Decision-Making (MADM) involves the organization and choice of limited alternatives using values attributed by experts [1]–[7]. The nuanced nature of certain attributes and the opaque reasoning of experts often necessitate qualitative over quantitative descriptions. Following Zadeh's [8] proposition for linguistic descriptors in qualitative assessment, a spectrum of representational methods for linguistic information has emerged, including Hesitant fuzzy linguistic terms, multi-granular fuzzy linguistic sets, and the 2-tuple fuzzy linguistic model, among others [9]–[28]. These models have broadened the scope of traditional LVs, enhancing fuzzy data communication and garnering extensive attention and practical deployment. Nevertheless, real-world decision-making often grapples with partial, uncertain, and conflicting information. To address this, Smarandache's [29] neutrosophic concept, which introduces hesitation as a distinct form of uncertainty, presents a promising means of representing such data. This approach has gained

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considerable recognition in scientific and engineering research, with diverse implementations as noted in the works of various scholars [10]-[44].

In the realm of linguistic variable representation, Ye [45] expanded upon the concept of Interval Neutrosophic Linguistic Variables (INLVs), introducing rule-based systems and a series of Aggregation Operators (AOs), such as the arithmetic and geometric weighted averages, while also exploring the properties of these operators. However, the INLVs, as an extension of the Intuitionistic Linguistic Set (ILS), primarily focused on Linguistic Variables (LVs), which limited their capacity to convey qualitative data robustly. Recognizing this limitation, Ye [46] advanced the field by proposing the integration of interval neutrosophic sets with Uncertain Linguistic Variables (ULVs), formulating operational rules for Interval Neutrosophic Uncertain Linguistic Variables (INULVs) that enhance precision and efficiency in dealing with uncertain, ambiguous, and inconsistent data. Building upon this foundation, a Multi-Attribute Decision-Making (MADM) method utilizing INULVs was developed. Broumi et al. [47] adapted the TOPSIS method, which employs the Hamming distance for INULVs, to refine the decision-making process in MADM scenarios. Similarly, Dey et al. [48] innovated a grey relational analysis for MADM where attribute weights are not fully determined. Focusing on prioritization within MADM contexts, Tian et al. [49] addressed the representation of attributes as simplified neutrosophic uncertain linguistic elements. Liu and Tang [50] merged INULVs with the Choquet integral, proposing variants of arithmetic and geometric AOs. Khan et al. [51] introduced a set that reconciles hesitant fuzzy sets with single-valued neutrosophic uncertain linguistic set (SVNULS), defining functions for certainty, score, and accuracy. Additionally, Liu and Teng [52] presented novel AOs that combine linguistic sets and interval neutrosophic hesitant sets with the Bonferroni mean operator, further contributing to the field as evidenced in subsequent studies [53].

Single-Valued Neutrosophic Uncertain Linguistic Variables (SVNULVs) enhance traditional linguistic terms by incorporating concepts from ULVs and single-valued neutrosophic numbers. This fusion allows for the expression of linguistic assessments, coupled with the confidence and indeterminacy levels of the decision-makers' evaluations, offering a more holistic representation. Such variables more accurately mirror the nuanced perspectives of decision-makers, making them user-friendly and relevant. The exploration into MADM methods that utilize SVNULS is thus of considerable practical importance. Liu and Shi [54] expanded on the Neutrosophic Uncertain Linguistic Set (NULS) with the inclusion of the Heronian Mean (HM). They formulated the SVNULS and developed HM operators based on this model, applying them to MADM contexts. Fan et al. [55] delved into the innovative expressions and operations of SVNULS and presented a novel variant, along with weighted arithmetic and geometric operators derived from this new version. Yang and Li [56] pioneered the definition of uncertain linguistic sets within a multi-valued framework and established fundamental rules for employing Dombi norms. Additionally, Song and Geng [57] combined the Maclaurin Symmetric Mean (MSM) operator with SVNULS and introduced MSM operators tailored to this integration.

Despite the fact that these ensemble operators may capture the relationship between qualities, the mentioned research has the following limitations:

I. The existing neutrosophic and uncertain linguistic variable integration algorithms lack closure and are difficult to adapt to the different semantic transformation needs of decision makers. It is pointed out that the existing algorithms are algebraic operations directly based on linguistic subscripts. Although they are simple and easy to use, there is a phenomenon of out-of-bounds.

For example, if $\tilde{\alpha}_1 = \prec [s_{\theta_1}, s_{\tau_1}], [t_1, i_1, f_1] \succ$ and $\tilde{\alpha}_2 = \prec [s_{\theta_2}, s_{\tau_2}], [t_2, i_2, f_2] \succ$ be two Single-Valued Neutrosophic Uncertain Linguistic Numbers (SVNULNs), then based on existing arithmetic, $\tilde{\alpha}_1 \oplus \tilde{\alpha}_2 = \prec [s_{\theta_1+\theta_2}, s_{\tau_1+\tau_2}], [t_1 + t_2 - t_1 t_2, i_1 i_2, f_1 f_2] \succ$ [54]. Now, for a seven-granularity linguistic term set, let $\tilde{\alpha}_1 = \prec [s_2, s_4], [0.7, 0.2, 0.3] \succ$ and $\tilde{\alpha}_2 = \prec [s_5, s_6], [0.5, 0.4, 0.2] \succ$ are two SVNULNs, according to the existing algorithm, we can get $\tilde{\alpha}_1 \oplus \tilde{\alpha}_2 = \prec [s_7, s_{10}], [0.85, 0.08, 0.06] \succ$. The calculated result definitely

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surpasses the top limit of the selected collection of linguistic terms. Clearly, this is contrary to common sense and logic. In addition, current algorithms presume that the semantic deviations of nearby language phrases are always equal. In practice, however, when linguistic subscripts grow from the center to both ends, decision makers may believe that the semantic bias of nearby linguistic phrases will either rise or decrease, but not always in a proportional manner. So, the existing ensemble methods cannot satisfy decision makers' similar semantic transformation requirements.

II. Existing MADM techniques for uncertain neutrosophic linguistics are mostly based on the weighted arithmetic operator, geometric average operator, Heronian average operator, and Bonferroni ordered weighted average operator, etc. All of these integration operators presume attribute independence or can only represent the association between two input parameters. However, there may be many correlations between input variables in many decision-making circumstances. For instance, there is a correlation between the product's cost, pricing, and quality while selecting a product scheme. Existing neutrosophic uncertain linguistic AOs and MADM approaches cannot tackle such issues at this moment.

Therefore, this paper proposes a new neutrosophic uncertain linguistic variable algorithm based on the Linguistic Scale Function (LSF), which will solve the algorithm's lack of closure and failure to fulfill the semantic transformation requirements. Second, a neutrosophic uncertain linguistic Hamy average operator is suggested in order to evaluate the correlation between multivariate input parameters. In addition, a MADM approach for SVNULSs with a broader application scope is offered.

2 | Preparatory Concepts

This section summarizes some basic definitions and concepts that would help expand the theory of SVNULVs.

2.1 | Single-Valued Neutrosophic Uncertain Linguistic Variables

Let $S=\{s_0,s_1,\dots s_{2n}\}$ represent a collection of non-empty discrete linguistic phrases. s_0 and s_{2n} represent the lowest and upper limits of S, whereas 2n+1 represents the granularity of S. For example, if n = 3, the set of seven-granularity linguistic terms S can be represented as $S=\{s_0,s_1,\dots s_6\}$ = {very poor, poor, slightly poor, fair, slightly good, good, very good}. Xu [58] expands the discrete linguistic term set S into a continuous linguistic term set $\bar{S}=\{s_\rho \mid \rho\in R^+\}$ in order to decrease the loss of information during the computation process.

Definition 1 ([58]). Assume $\tilde{S}=\left[s_x,s_y\right], s_x,s_y\in\bar{S}, 0<s_x\le s_y$, then $\tilde{s}$ are termed as ULVs. Also, for any ULVs $\tilde{S}_1=\left[s_{x_1},s_{y_1}\right]$ and $\tilde{S}_2=\left[s_{x_2},s_{y_2}\right]$ we have the following rules:

I. $\tilde{S}_1\oplus\tilde{S}_2=\left[s_{x_1},s_{y_1}\right]\oplus\left[s_{x_2},s_{y_2}\right]=\left[s_{x_1+x_2},s_{y_1+y_2}\right]$,

II. $\kappa\tilde{s}_1=\kappa\left[s_{x_1},s_{y_1}\right]=\left[s_{\kappa.x_1},s_{\kappa.y_1}\right],\kappa\ge 0$,

III. $\tilde{S}_1\otimes\tilde{S}_2=\left[s_{x_1},s_{y_1}\right]\otimes\left[s_{x_2},s_{y_2}\right]=\left[s_{x_1\times x_2},s_{y_1\times y_2}\right]$,

IV. $(\tilde{s}_1)^\kappa=\left[s_{x_1},s_{y_1}\right]^\kappa=\left[s_{x^\kappa_1},s_{y^\kappa_1}\right],\kappa\ge 0.$

Definition 2 ([54]). Suppose X is a given universe of discourse, and then:

$$A=\{\prec x\mid[s_{\theta(x)},s_{\tau(x)}],[t(x),i(x),f(x)]\succ\mid x\in X\}, \tag{1}$$

is referred to as a SVNULS, where $[t(x), i(x), f(x)]$ is the single value neutrosophic part of the element x to A.

Definition 3 ([54]). Let $\tilde{\alpha}_1 = \prec [s_{\theta_1}, s_{\tau_1}], [t_1, i_1, f_1] \succ$ and $\tilde{\alpha}_2 = \prec [s_{\theta_2}, s_{\tau_2}], [t_2, i_2, f_2] \succ$ both be SVNULNs. The operational laws are thereafter specified as follows:

I. $\tilde{\alpha}_1 \oplus \tilde{\alpha}_2 = \prec [s_{\theta_1+\theta_2}, s_{\tau_1+\tau_2}], [t_1 + t_2 - t_1 t_2, i_1 i_2, f_1 f_2] \succ$.

II. $\tilde{\alpha}_1 \otimes \tilde{\alpha}_2 = \prec \left[s_{\theta_1\times\theta_2}, s_{\tau_1\times\tau_2}\right], [t_1 t_2, i_1 + i_2 - i_1 i_2, f_1 + f_2 - f_1 f_2] \succ$.

III. $\lambda\tilde{\alpha}_1 = < \left[s_{\lambda\times\theta_1}, s_{\lambda\times\tau_1}\right], [1-(1-t_1)^{\lambda}, (i_1)^{\lambda}, (f_1)^{\lambda}] >$.

IV. $\tilde{\alpha}_1^{\lambda} = < \left[s_{\theta_1^{\lambda}}, s_{\tau_1^{\lambda}}\right], [(t_1)^{\lambda}, 1-(1-i_1)^{\lambda}, 1-(1-f_1)^{\lambda},] >$.

2.2 | Hamy Mean Operator

Among the extensively researched AOs that can imprisonment and portray interrelationships among aggregated arguments, the Hamy Mean (HM) is a significant option. HM is utilized extensively in information fusion. From a mathematical standpoint, it may be viewed as a simpler version of the MSM and its extensions. This operator is distinguished by its ability to capture the overall interrelationships between arguments and to represent interaction phenomena.

Definition 4 ([59]). Suppose a_i (i =0,1,…, p) be a set of non-negative real numbers. HM is defined as follows:

$$HM^{(k)}(a_1, a_2, ..., a_p) = \sum_{1\le i_1 < ... < i_k \le p} (\prod_{j=1}^{k} a_{i_j})^{\frac{1}{k}} / \binom{p}{k}, \quad (2)$$

where, k (k =1,2,…, p) is the aggregation parameter of HM operator, $(i_1, ... < i_p)$ crosses all the p-tuple combination of (1,2,…, p), and $\binom{p}{k}$ represents the binomial coefficient.

The most essential characteristic of the Hamy average operator, as defined in *Definition 4*, is that it may reflect the correlation between several input parameters, not only the correlation between two values. To capture the association between multivariate characteristics, decision-makers might therefore choose a range of values based on the specifics of the decision-making circumstance. In addition, Hamy operator has the following properties:

I. If $a_i = 0 (i = 1, .., p)$, then $HM^{(k)} = 0$.

II. If $a_i = \alpha (i = 1, .., p)$, then $HM^{(k)} = \alpha$.

III. If $a_i \le \tilde{a}_i (i = 1, .., p)$, then $HM^{(k)}(a_1, a_2, ..., a_p) \le HM^{(k)}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_p)$.

IV. $\min_i(a_i) \le HM^{(k)}(a_1, a_2, ..., a_p) \le \max_i(a_i)$.

3 | Single-Valued Neutrosophic Uncertain New Operations on Linguistic Variables

3.1 | The New Approach for SVNULVs

In the existing algorithms for neutrosophic uncertain linguistic variables, there are two shortcomings: the absence of closure and the phenomena of out-of-bounds. In conjunction with the notion of the LSF, a novel method for SVNULVs is proposed in this study.

Definition 5 ([60]). Given a predetermined collection of linguistic terms denoted as S, with each term s_i within S corresponding to the i-th term, and γ_i, a non-negative real number within the range $[0,+\infty)$ for $i=0,1,\ldots,2n$, the LSF *f* is the mapping that assigns each s_i to a specific γ_i, as depicted in *Eq. (3)*:

$$f: s_i \rightarrow \gamma_i (i=0,1,\ldots,2n), \tag{3}$$

In this context, the function *f* is strictly monotonically increasing with respect to the index i. Here, γ_i for $i=0,1,\ldots,2n$ conveys the semantic preference of the decision maker when opting for the i-th linguistic term, indicating a progression of preference or priority. Here, some different kinds of LSFs are introduced to express the different semantic transformation needs of decision makers:

$$f_1(s_i) = \gamma_i = \frac{i}{2n} (i=0,1,2,\ldots,2n). \tag{4}$$

$$f_2(s_i) = \gamma_i = \begin{cases} \dfrac{\xi^n - \xi^{n-i}}{2\xi^n - 2} (i=0,1,2,\ldots,n), \\ \dfrac{\xi^n - \xi^{n-i}}{2\xi^n - 2} (i=n+1,n+2,\ldots,2n). \end{cases} \tag{5}$$

$$f_3(s_i) = \gamma_i = \begin{cases} \dfrac{n^\alpha - (n-i)^\alpha}{2n^\alpha} (i=0,1,2,\ldots,n), \\ \dfrac{n^\beta - (n-i)^\beta}{2n^\alpha} (i=n+1,n+2,\ldots,2n). \end{cases} \tag{6}$$

The aforementioned three LSFs have distinct descriptors. *Eq. (4)* suggests that the absolute deviation of neighboring linguistic phrases is always the same, which is simple and straightforward to use, but lacks a theoretical foundation. *Eq. (5)* illustrates a semantic scenario in which the absolute divergence between neighboring linguistic phrases grows steadily from the center to the ends. For example, the semantic deviation between "very good" and "good" is greater than the semantic deviation between "good" and "slightly good". Among them, the value of parameter ζ can be obtained by experimental method or subjective method. Using survey data, the experimental technique estimates the value space of ζ to be [1.36, 1.4].

Eq. (6) suggests that the difference in value between successive linguistic terms diminishes as one moves from the central term outward in either direction. Within this framework, α and β are coefficients that lie in the interval [0,1]. To illustrate, the semantic gap between terms such as "slightly good" and "good" would be wider than that between "good" and "very good". It is important to highlight that in the special case where both α and β are set to 1, *Eq. (6)* simplifies and becomes identical to *Eq. (4)*.

Also, in order to reduce information loss and facilitate computation, the LSF is further extended to continuous space: $f^*: \bar{s} \rightarrow R^+$ satisfying $f^*(s_i) = \gamma_i$. f^* is a continuous and strictly monotonically increasing function whose inverse is denoted as f^{*-1} [60].

For the seven-granularity linguistic term set S, the inverse functions of the above three LSFs are defined as follows:

I. If $f_1^*(s_i)=\gamma_i=\frac{i}{6}(i=0,1,2,...,2n)$, then: $f_1^{*-1}(\gamma_i)=s_{6\gamma_i}(\gamma_i\in[0,1])$.

II. If $f_2^*(s_i)=\gamma_i=\begin{cases}\frac{\xi^3-\xi^{3-i}}{2\times\xi^3-2}(i=0,1,2,3)\\ \frac{\xi^3+\xi^{i-3}-2}{2\times\xi^3-2}(i=4,5,6)\end{cases}$, then: $f_2^{*-1}(\gamma_i)=\begin{cases}s_{3-\log_\xi(\xi^3-(2\times\xi^3-2)\gamma_i)},\gamma_i\in[0,0.5]\\ s_{3+\log_\xi((2\times\xi^3-2)\gamma_i-\xi^3+2)},\gamma_i\in[0.5,1]\end{cases}$.

III. If $f_3^*(s_i)=\gamma_i=\begin{cases}\frac{3^\alpha-(3-i)^\alpha}{2\times3^\alpha}(i=0,1,2,3)\\ \frac{3\beta+(i-3)^\beta}{2\times3^\beta}(i=4,5,6)\end{cases}$, then $f_3^{*-1}(\gamma_i)=\begin{cases}s_{3-(3^\alpha-2\times3^\alpha\times\gamma_i)^{1/\alpha}},\gamma_i\in[0,0.5]\\ s_{3+(2\times3^\beta\times\gamma_i-3^\beta)^{1/\beta}},\gamma_i\in(0.5,1]\end{cases}$. .

Next, we propose a novel neutrosophic uncertain linguistic variable algorithm to address the deficiencies of existing algorithms, which lack closure and cannot fulfill the demands of decision makers for semantic transformation.

Definition 6. Suppose and both be SVNULVs, and $\lambda\geq0$. The operational laws are thereafter specified as follows:

I. $\tilde{\alpha}_1\oplus\tilde{\alpha}_2=\prec[f^{*-1}(f^*(s_{\theta(a_1)})+f^*(s_{\theta(a_2)})-f^*(s_{\theta(a_1)})f^*(s_{\theta(a_2)})),\ f^{*-1}(f^*(s_{\tau(a_1)})+f^*(s_{\tau(a_2)})-f^*(s_{\tau(a_1)})f^*(s_{\tau(a_2)}))],$ $[(u(a_1)+u(a_2)-u(a_1)u(a_2),\ v(a_1)v(a_2),\ f(a_1)f(a_2)]\succ,$

II. $\tilde{\alpha}_1\otimes\tilde{\alpha}_2=\prec[f^{*-1}(f^*(s_{\tau(a_1)})\times f^*(s_{\tau(a_2)})),f^{*-1}(f^*(s_{\tau(a_1)})\times f^*(s_{\tau(a_2)}))],$ $[(u(a_1)u(a_2),v(a_1)+v(a_2)-v(a_1)v(a_2),f(a_1)+f(a_2)-f(a_1)f(a_2)]\succ,$

III. $\lambda\tilde{\alpha}_1=\prec[f^{*-1}(1-(1-f^*(s_{\theta(a_1)}))^\lambda,\ f^{*-1}(1-(1f^*(s_{\tau(a_1)}))^\lambda],$ $[(1-(1-u(a_1))^\lambda,\ (v(a_1))^\lambda,(f(a_1))^\lambda]\succ,$

IV. $\tilde{\alpha}_1^\lambda=<[f^{*-1}((f^*(s_{\theta(a_1)}))^\lambda),f^{*-1}((f^*(s_{\tau(a_1)}))^\lambda],$ $[((u(a_1))^\lambda,1-(1-v(a_1))^\lambda),1-(1-f(a_1))^\lambda)]\succ.$

Example 1. Let S be a seven-granularity linguistic term set $\tilde{\alpha}_1=\prec[s_3,s_4],[0.4,0.6,0.3]\succ$, and $\tilde{\alpha}_2=\prec[s_4,s_5],[0.2,0.3,0.5]\succ$. Using the new approach, the above three different LSFs are introduced, and the scheming outcomes shown in *Table 1*. Here, ξ in *Formula (5)* is taken as 1.4, and both α and β in *Formula (6)* are taken as 0.5.

Table 1. Calculation results obtained using different LFSs.

LFS	Operation	Results
Eq. (4)	Addition	$\prec[s_5,s_{5.7}],[0.52,0.18,0.15]\succ$
	Multiplication	$\prec[s_2,s_{3.3}],[0.08,0.72,0.65]\succ$
Eq. (5)	Addition	$\prec[s_{5.16},s_{5.65}],[0.52,0.18,0.15]\succ$
	Multiplication	$\prec[s_{1.47},s_{2.77}],[0.08,0.72,0.65]\succ$
Eq. (6)	Addition	$\prec[s_{4.17},s_{4.29}],[0.52,0.18,0.15]\succ$
	Multiplication	$\prec[s_{2.4},s_{3.87}],[0.08,0.72,0.65]\succ$

From *Table 1*, we can see that the calculation results do not exceed the upper boundary of the linguistic term set, and the calculation results obtained under different semantic preferences are also different, that is to say, the new algorithm can adapt to different semantic scenarios, and decision makers can the semantic preference of choosing different LSFs. Therefore, compared with the existing algorithm, the new algorithm is closed and flexible.

3.2 | Comparison Method

The common neutrosophic uncertain linguistic variable comparison method in the existing literature is to compare the size by defining the expected value and the accuracy value, but it does not consider the semantic needs of different decision makers. Therefore, combined with the LSF, this paper proposes a new neutrosophic uncertain linguistic variable comparison method to meet different decision-making situations.

Definition 7. Let $\tilde{\alpha}_1 = \prec [s_{\theta(a_1)}, s_{\tau(a_1)}], [u(a_1), \upsilon(a_1), f(a_1)] \succ$ be the SVNULV, then the score value $S(\tilde{\alpha}_1)$, and the accuracy value $A(\tilde{\alpha}_1)$ is defined as:

$$S(\tilde{\alpha}_1) = (2 + u(a_1) - \upsilon(a_1) - f(a_1)) \times \frac{(f^*(s_{\theta(a_1)}) + f^*(s_{\tau(a_1)}))}{4}, \tag{7}$$

$$A(\tilde{\alpha}_1) = (u(a_1) + \upsilon(a_1) + f(a_1)) \times \frac{(f^*(s_{\theta(a_1)}) + f^*(s_{\tau(a_1)}))}{2}. \tag{8}$$

Among them, for linguistic term sets S of different granularities, $f^*(s_{\theta(a_1)})$ and $f^*(s_{\tau(a_1)})$ are calculated according to the expansions of the three mentioned LSFs in *Definition 5*.

Definition 8. Suppose $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ be any two SVNULVs. Then:

I. If $S(\tilde{\alpha}_1) > S(\tilde{\alpha}_2)$, then we say that $\tilde{\alpha}_1 > \tilde{\alpha}_2$;

II. If $S(\tilde{\alpha}_1) = S(\tilde{\alpha}_2)$, then:

- *If* $A(\tilde{\alpha}_1) > A(\tilde{\alpha}_2)$ *we say that* $\tilde{\alpha}_1 > \tilde{\alpha}_2$.
- *If* $A(\tilde{\alpha}_1) = A(\tilde{\alpha}_2)$ *we say that* $\tilde{\alpha}_1 \approx \tilde{\alpha}_2$.

3.3 | Single Valued Neutrosophic Uncertain Linguistic Hamy Mean Operator

Definition 9. Let is a set of SVNULVs, then the Single Valued Neutrosophic Uncertain Linguistic Hamy Mean Operator (SVNULHM) is defined as follows:

$$\mathrm{SVNULHM}^{(k)}(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_p) = \sum_{1 \le l_1 < \ldots < l_k \le p} \left(\prod_{j=1}^{k} \tilde{\alpha}_{i_j}\right)^{\frac{1}{k}} \Big/ \binom{p}{k}, \tag{9}$$

where, parameters are similar to *Definition 4*.

Theorem 1. Let $\tilde{\alpha}_i = \prec [s_{\theta(a_i)}, s_{\tau(a_i)}], [u(a_i), \upsilon(a_i), f(a_i)] \succ (i = 1, \ldots, p)$ is a set of SVNULVs, then the aggregated outcome of the SVNULHM operator is also SVNULVs, and we have:

$$\begin{aligned}&SVNULHM^{(k)}(\tilde{\alpha}_1,\tilde{\alpha}_2,\ldots,\tilde{\alpha}_p)=\\&\prec\left[\begin{array}{l}f^{*-1}\left(1-\prod_{1\le i<\ldots<i_k\le p}\left(1-\prod_{j=1}^{k}(f^{*}(s_{\theta(\alpha_1)}))^{1/k}\right)^{\frac{1}{\binom{p}{k}}}\right),f^{*-1}\left(1-\prod_{1\le i<\ldots<i_k\le p}\left(1-\prod_{j=1}^{k}(f^{*}(s_{\tau(\alpha_1)}))^{1/k}\right)^{\frac{1}{\binom{p}{k}}}\right)\end{array}\right],\\&\left[1-\prod_{1\le i<\ldots<i_k\le p}\left(1-\prod_{j=1}^{k}(u(\alpha_{ij}))^{1/k}\right)^{\frac{1}{\binom{p}{k}}},\prod_{1\le i<\ldots<i_k\le p}\left(1-\prod_{j=1}^{k}(1-\upsilon(\alpha_{ij}))^{1/k}\right)^{\frac{1}{\binom{p}{k}}},\prod_{1\le i<\ldots<i_k\le p}\left(1-\prod_{j=1}^{k}(1-f(\alpha_{ij}))^{1/k}\right)^{\frac{1}{\binom{p}{k}}}\right]\succ\end{aligned}\tag{10}$$

Proof: According to *Definitions 4* and *6*, it is simple to get.

In addition, based on *Theorem 1*, $SVNULHM^{(k)}$ has the following properties:

Property 1 (Idempotency). Let $\tilde{\alpha}_i=\prec[s_{\theta(a_i)},s_{\tau(a_i)}],[u(a_i),\upsilon(a_i),f(a_i)]\succ(i=1,\ldots,p)$ is a set of SVNULVs, and $\tilde{\alpha}_i=\tilde{\alpha}(i=1,\ldots,p)$. Then $SVNULHM^{(k)}(\tilde{\alpha}_1,\tilde{\alpha}_2,\ldots,\tilde{\alpha}_p)=\tilde{\alpha}$.

Property 2 (Commutativity). Let $\tilde{\alpha}_i=\prec[s_{\theta(a_i)},s_{\tau(a_i)}],[u(a_i),\upsilon(a_i),f(a_i)]\succ(i=1,\ldots,p)$ is a set of SVNULVs, if $\tilde{\alpha}_i$ is any permutation of $\tilde{b}_i(i=1,\ldots,p)$, we have:

$$SVNULHM^{(k)}(\tilde{\alpha}_1,\tilde{\alpha}_2,\ldots,\tilde{\alpha}_p)=SVNULHM^{(k)}(\tilde{b}_1,\tilde{b}_2,\ldots,\tilde{b}_p).$$

Property 3 (Monotonicity). Suppose $\tilde{\alpha}_i=\prec[s_{\theta(a_i)},s_{\tau(a_i)}],[u(a_i),\upsilon(a_i),f(a_i)]\succ$ and $\tilde{b}_i=\prec[s_{\theta(b_i)},s_{\tau(b_i)}],[u(b_i),\upsilon(b_i),f(b_i)]\succ$ be two collections of SVNULVs. If for any i, $\tilde{\alpha}_i\le\tilde{b}_i$, then we have: $SVNULHM^{(k)}(\tilde{\alpha}_1,\tilde{\alpha}_2,\ldots,\tilde{\alpha}_p)\le SVNULHM^{(k)}(\tilde{b}_1,\tilde{b}_2,\ldots,\tilde{b}_p)$.

Property 4 (Boundary). Let $\tilde{\alpha}_i=\prec[s_{\theta(a_i)},s_{\tau(a_i)}],[u(a_i),\upsilon(a_i),f(a_i)]\succ(i=1,\ldots,p)$ is a set of SVNULVs, if, $\tilde{\alpha}^{+}=\prec[s_{\max\theta(a_i)},s_{\max\tau(a_i)}],[\max(u(a_i)),\min(\upsilon(a_i)),\min(f(a_i))]\succ(i=1,\ldots,p)$ and $\tilde{\alpha}^{-}=\prec[s_{\min\theta(a_i)},s_{\min\tau(a_i)}],[\min(u(a_i)),\max(\upsilon(a_i)),\max(f(a_i))]\succ(i=1,\ldots,p)$, then:

$$\tilde{\alpha}^{-}\le SVNULHM^{(k)}(\tilde{\alpha}_1,\tilde{\alpha}_2,\ldots,\tilde{\alpha}_p)\le\tilde{\alpha}^{+}.$$

Property 5. Let $\tilde{\alpha}_i=\prec[s_{\theta(a_i)},s_{\tau(a_i)}],[u(a_i),\upsilon(a_i),f(a_i)]\succ(i=1,\ldots,p)$ is a set of SVNULVs, then the SVNULHM operator is a monotonically decreasing function of the associated parameter k.

Special Case 1: If $k=1$, that is, when there is no correlation between attributes, the SVNULHM operator degenerates into a neutrosophic uncertain linguistic average operator combined with the LSF:

$$\begin{aligned}&SVNULHM^{(k)}(\tilde{\alpha}_1,\tilde{\alpha}_2,\ldots,\tilde{\alpha}_p)=<[f^{*-1}\left(1-\prod_{1\le i<\ldots<i_k\le p}\left(1-\prod_{j=1}^{k}(f^{*}(s_{\theta(\alpha_1)}))^{1/k}\right)^{\frac{1}{\binom{p}{k}}}\right),\\&f^{*-1}\left(1-\prod_{1\le i<\ldots<i_k\le p}\left(1-\prod_{j=1}^{k}(f^{*}(s_{\tau(\alpha_1)}))^{1/k}\right)^{\frac{1}{\binom{p}{k}}}\right)],\\&[1-\prod_{1\le i<\ldots<i_k\le p}\left(1-\prod_{j=1}^{k}(u(\alpha_{ij}))^{1/k}\right)^{\frac{1}{\binom{p}{k}}},\prod_{1\le i<\ldots<i_k\le p}\left(1-\prod_{j=1}^{k}(1-\upsilon(\alpha_{ij}))^{1/k}\right)^{\frac{1}{\binom{p}{k}}}]>.\end{aligned}$$

Special Case 2: If $k=n$, the SVNULHM operator degenerates into a neutrosophic uncertain linguistic geometric mean operator combined with LSFs, as follows:

$$SVNULHM^{(k)}(\tilde{\alpha}_1,\tilde{\alpha}_2,\ldots,\tilde{\alpha}_p) =\prec [f^{*-1}(1-\prod_{1\le i<\ldots<i_k\le p}(1-\prod_{j=1}^{k}(f^{*}(s_{\theta(\alpha_1)})^{1/k})^{\frac{1}{\binom{p}{k}}}),$$
$$f^{*-1}(1-\prod_{1\le i<\ldots<i_k\le p}(1-\prod_{j=1}^{k}(f^{*}(s_{\tau(\alpha_1)}))^{1/k})^{\frac{1}{\binom{p}{k}}})],$$
$$[1-\prod_{1\le i<\ldots<i_k\le p}(1-\prod_{j=1}^{k}(u(\alpha_{ij}))^{1/k})^{\frac{1}{\binom{p}{k}}}), \prod_{1\le i<\ldots<i_k\le p}(1-\prod_{j=1}^{k}(1-\upsilon(\alpha_{ij}))^{1/k})^{\frac{1}{\binom{p}{k}}}]\succ.$$

Definition 10. If $\tilde{\alpha}_i=\prec[s_{\theta(a_i)},s_{\tau(a_i)}],[u(a_i),\upsilon(a_i),f(a_i)]\succ(i=1,\ldots,p)$ is a set of SVNULVs and $\omega=(\omega_1, \omega_2,\ldots,\omega_p)^T$ is an ALFA weight vector fulfilling $\sum\omega_i=1$ and $\omega_i\in[0,1]$, the Single Valued Neutrosophic Uncertain Linguistic Weighted Hamy Mean Operator (SVNULWHM) is stated as follows:

$$SVNULWHM^{(k)}(\tilde{\alpha}_1,\tilde{\alpha}_2,\ldots,\tilde{\alpha}_p)=\begin{cases}\sum_{1\le i_1<\ldots<i_k\le p}(1-\sum_{j=1}^{k}\omega_{ij})(\prod_{j=1}^{k}\tilde{\alpha}_{ij})^{1/k}\Big/\binom{p-1}{k}, & 1\le k\le p,\\ \prod_{i=1}^{k}\tilde{\alpha}_i^{\frac{1-\omega_i}{p-1}}, & k=p,\end{cases} \tag{11}$$

where, parameters are similar to *Definition 4*.

Theorem 2. In *Definition 10*:

I. When $1\le k<p$, then

$$\begin{aligned}&SVNULWHM^{(k)}(\tilde{\alpha}_1,\tilde{\alpha}_2,\ldots,\tilde{\alpha}_p)=\prec[f^{*-1}(1-\prod_{1\le i<\ldots<i_k\le p}(1-\prod_{j=1}^{k}(f^{*}(s_{\theta(\alpha_{ij})}))^{1/k})^{(1-\sum_{j=1}^{k}\omega_{ij})})^{1/\binom{p-1}{k}}],\\&f^{*-1}(1-(\prod_{1\le i<\ldots<i_k\le p}(1-\prod_{j=1}^{k}(f^{*}(s_{\theta(\alpha_{ij})}))^{1/k})^{(1-\sum_{j=1}^{k}\omega_{ij})})^{1/\binom{p-1}{k}})],\\&[1-(\prod_{1\le i<\ldots<i_k\le p}(1-\prod_{j=1}^{k}(u(\alpha_{ij}))^{1/k})^{(1-\sum_{j=1}^{k}\omega_{ij})})^{1/\binom{p-1}{k}}),\\&(\prod_{1\le i<\ldots<i_k\le p}(1-\prod_{j=1}^{k}(1-\upsilon(\alpha_{ij}))^{1/k})^{(1-\sum_{j=1}^{k}\omega_{ij})})^{1/\binom{p-1}{k}}]\succ.\end{aligned} \tag{12}$$

II. When $k=p$, then

$$\begin{aligned}&SVNULWHM^{(k)}(\tilde{\alpha}_1,\tilde{\alpha}_2,\ldots,\tilde{\alpha}_p)=\prec[f^{*-1}(\prod_{i=1}^{k}(f^{*}\ s_{\theta(\alpha_i)}))^{\frac{1-\omega_i}{p-1}}),\\&f^{*-1}(\prod_{i=1}^{k}(f^{*}\ s_{\tau(\alpha_i)}))^{\frac{1-\omega_i}{p-1}})],\\&(\prod_{i=1}^{k}(u(\alpha_i))^{\frac{1-\omega_i}{p-1}},1-\prod_{i=1}^{k}(1-\upsilon(\alpha_i))^{\frac{1-\omega_i}{p-1}})\succ.\end{aligned} \tag{13}$$

The SVNULWHM operator also possesses idempotency, boundary, and monotonicity qualities, which are ignored here. In addition, we will go through several SVNULWHM operator specific instances.

Special Case 1: When $k=1$, the SVNULWHM operator degenerates into an integrated operator that ignores the relationship between attributes, and we get:

$$\begin{aligned}&SVNULWHM^{(k)}(\tilde{\alpha}_1,\tilde{\alpha}_2,\ldots,\tilde{\alpha}_p)=\prec[f^{*-1}(\prod_{i=1}^{p}(1-f^{*}\ s_{\theta(\alpha_i)}))^{\frac{1-\omega_i}{p-1}}),\\&f^{*-1}(\prod_{i=1}^{p}(1-f^{*}\ s_{\tau(\alpha_i)}))^{\frac{1-\omega_i}{p-1}})],\\&[1-\prod_{i=1}^{k}(1-u(\alpha_i))^{\frac{1-\omega_i}{p-1}},\prod_{i=1}^{k}(\upsilon(\alpha_i))^{\frac{1-\omega_i}{p-1}}]\succ.\end{aligned} \tag{14}$$

Special Case 2: The SVNULWHM operator is simplified to the SVNULHM operator if $w_i = 1/p$ (i=1,2, ⋯, p).

4 | Novel MADM Framework Utilizing SVNULHM Operators

In addressing MADM problems with attributes assessed as SVNULVs, consider a set of p alternatives Z = { z_1 , z_2 ,..., z_p }, and n evaluative attributes C ={ c_1 , c_2,... , c_n }. The vector of attribute weights is denoted as ω ={ ω_1 , ω_2 ,…, ω_n }^{T} , where the sum of weights ω_j equals 1, with each weight satisfying $0 \le \omega_j \le 1$. The evaluative scores are represented by SVNULVs, and the resultant Single-Valued Neutrosophic Uncertain Linguistic Decision Matrix (SVNULDM) is formulated as $S = [\alpha_{ij}]_{p \times n}$. The linguistic term set within the matrix is $S = \{s_0, s_1, \dots s_{2n}\}$. To resolve the decision-making quandaries presented, the subsequent procedural steps are proposed:

Step 1 . Convert the SVNULDM $S = [\alpha_{ij}]_{p \times n}$ to the normalized decision matrix $\tilde{S} = [\tilde{\alpha}_{ij}]_{p \times n}$.

Step 2. To obtain the complete evaluation value $\tilde{\alpha}_i$ (i=1, 2,…, p) of the option z_i, use the SVNULWHM operation to aggregate the attribute values of the i-th row of the normalized decision matrix.

Step 3. Compute the score value $S(\tilde{\alpha}_1)$ and the accuracy value $A(\tilde{\alpha}_1)$ for each alternative z_i in accordance with *Definition 7*.

Step 4. Rank the alternatives z_i (i = 1, 2, . . . , p) as per *Definition 8* and pinpoint the most suitable choice.

5 | Example Analysis

5.1 | Decision-Making Process

Example 2. A company plans to establish branches in different places according to its actual development. After rigorous screening in the early stage, the board of directors has identified four alternatives {z_1, z_2, z_3, z_4}. At the same time, the company evaluates the alternatives according to the following four attributes: c_1 geographic location, c_2 traffic conditions, c_3 labor costs and c_4 land prices. Among them, c_1 and c_2 are benefit attributes, and c_3 and c_4 are cost attributes. ω =(0.32,0.26,0.18,0.24)^T is the feature weight vector. Experts employ SVNULVs with a seven-granularity linguistic term set to describe the evaluation information. Constructed SVNULDM $S = [\alpha_{ij}]_{4 \times 4}$ considered in *Table 2*.

Table 2. SVNULV form of decision matrix.

	c_1	c_2	c_3	c_4
z_1	$\prec [s_5, s_5], [0.5, 0.1, 0.4] \succ$	$\prec [s_2, s_3], [0.4, 0.3, 0.5] \succ$	$\prec [s_4, s_4], [0.3, 0.5, 0.4] \succ$	$\prec [s_2, s_3], [0.6, 0.4, 0.1] \succ$
z_2	$\prec [s_5, s_5], [0.4, 0.2, 0.6] \succ$	$\prec [s_5, s_5], [0.4, 0.3, 0.5] \succ$	$\prec [s_1, s_2], [0.8, 0.3, 0.1] \succ$	$\prec [s_2, s_2], [05, 0.2, 0.5] \succ$
z_3	$\prec [s_3, s_4], [0.3, 0.3, 0.5] \succ$	$\prec [s_4, s_4], [0.2, 0.4, 0.6] \succ$	$\prec [s_1, s_2], [0.5, 0.2, 0.3] \succ$	$\prec [s_1, s_2], [0.6, 0.5, 0.2] \succ$
z_4	$\prec [s_3, s_5], [0.5, 0.3, 0.4] \succ$	$\prec [s_2, s_3], [0.2, 0.3, 0.8] \succ$	$\prec [s_2, s_3], [0.6, 0.2, 0.2] \succ$	$\prec [s_3, s_3], [0.6, 0.4, 0.3] \succ$

We sort the alternatives using the new MADM technique based on the SVNULHM operator (k = 2). In order to facilitate the calculation, *Formula (4)* is used to participate in the calculation. According to *Step 1* to *Step 4*, the score values of the alternative z_i (i = 1, 2 , 3, 4) can be obtained as:

$$S(\tilde{\alpha}_1)=0.2576,\ S(\tilde{\alpha}_2)=0.2712,\ S(\tilde{\alpha}_3)=0.2556,\ S(\tilde{\alpha}_4)=0.1796.$$

Therefore, $z_2 \succ z_1 \succ z_3 \succ z_4$, so the company's optimal choice is z_2.

Below, *Table 3* considers the impact of the correlation parameter k on the sorting results. *Table 3* shows that different association factors k have a little influence on the ranking outcomes. Although the optimal schemes obtained by the three k values are all z_2, and the worst schemes are all z_4, there are differences in the ordering of other schemes. As a result, unlike previous MADM approaches for neutrosophic uncertain linguistics, the method suggested in this work may represent the relationship between several input parameters, rather than only the binary relationship. The decision maker in the real decision-making problem might pick the right value of k based on the unique relational structure of the actual situation.

Table 3. Sorting results obtained with different parameters k.

k	Expected Values	Sorting
k = 1	$S(\tilde{\alpha}_1)=0.2882,\ S(\tilde{\alpha}_2)=0.2995,\ S(\tilde{\alpha}_3)=0.2686,\ S(\tilde{\alpha}_4)=0.1998$	$z_2 \succ z_1 \succ z_3 \succ z_4$
k = 2	$S(\tilde{\alpha}_1)=0.2576,\ S(\tilde{\alpha}_2)=0.2712,\ S(\tilde{\alpha}_3)=0.2556,\ S(\tilde{\alpha}_4)=0.1796$	$z_2 \succ z_1 \succ z_3 \succ z_4$
k = 3	$S(\tilde{\alpha}_1)=0.2461,\ S(\tilde{\alpha}_2)=0.2578,\ S(\tilde{\alpha}_3)=0.2533,\ S(\tilde{\alpha}_4)=0.1736$	$z_2 \succ z_3 \succ z_1 \succ z_4$

6 | Conclusion

The application of MADM processes to neutrosophic uncertain linguistics is extensive. This paper introduces an innovative algorithm and ranking method to overcome prevalent issues in the field, specifically addressing the existing algorithms' lack of closure regarding neutrosophic uncertain linguistic variables and their inflexibility towards the varying semantic conversion requirements of decision-makers. Furthermore, to enhance the integration of neutrosophic uncertain linguistic variables, this research proposes a new neutrosophic uncertain linguistic Hamy mean operator along with its weighted version. This advanced operator is capable of representing the connections among multiple inputs and caters to the semantic transformation needs of distinct decision-makers, yielding more precise and equitable results. Building on this development, the paper also suggests an MADM technique utilizing the neutrosophic uncertain linguistic weighted Hamy mean operator. The practicality of this method is demonstrated through a numerical example, and its superiority is established through comparative analysis with various parameters. The MADM approach for neutrosophic uncertain linguistics devised herein offers a cutting-edge resolution to the challenges of linguistic MADM.

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