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Enhancing Decision Accuracy in DEMATEL using Bonferroni Mean Aggregation under Pythagorean Neutrosophic Environment

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Abstract

DEMATEL serves as a tool for addressing multi-criteria decision-making problems, primarily by identifying critical factors that exert the most significant influence on a specific system. To enhance its capabilities in handling contextual decision problems, DEMATEL has been further developed through integration with various other MCDM methods. The inherent reliance on direct input from experts for initial decision information in DEMATEL raises concerns about the potential limitations imposed by experts' domain knowledge and bounded rationalities. The effectiveness of decision-making can be compromised if the initial information provided by experts is deemed unreliable, leading to debatable outcomes. To address these challenges, this study proposes the incorporation of a Bonferroni mean aggregation operator within a Pythagorean neutrosophic environment, illustrated through a numerical example applied to DEMATEL. This integration is intended to fortify decision accuracy by introducing a more enhanced decision framework by developing a new normalized weighted Bonferroni mean operator for Pythagorean neutrosophic set aggregation (PN-NWBM). By integrating this operator, this study aims to alleviate the impact of unreliable initial information and enhance the overall reliability of decision outcomes thereby contributing to its improvement in decision making. Through the implementation of the Bonferroni mean aggregation operator, the study anticipates achieving a more comprehensive and accurate representation of decision factors as illustrated in the numerical example. This research includes a comparative and sensitivity analysis to thoroughly examine the implications and effectiveness of the proposed integration.

Keywords: Aggregation operator, Bonferroni mean, Pythagorean neutrosophic set, DEMATEL.

1 | Introduction

In Multi-Criteria Decision-Making (MCDM), the Decision-Making Trial and Evaluation Laboratory (DEMATEL) method, initially introduced by Gabus and Fontela in 1972 has emerged as a valuable tool for identifying critical factors that exert substantial influence on specific systems and unraveling their interdependence [1]. Unlike alternative methodologies, DEMATEL uniquely unveils causal relationships and comprehensively gauges the collective impact of theoretical and empirical factors. Its application extends to solving contextual decision problems, where it has been further refined through integration with diverse MCDM methods [2]-[4]. However, a critical aspect of DEMATEL lies in the subjective nature of the initial decision information provided by experts, which raises concerns about potential limitations stemming from experts' domain knowledge and bounded rationalities. The consequences of relying on inherent limitations of expert input become apparent

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in the form of debatable decision outcomes. Nevertheless, the active involvement of decision makers from diverse fields is imperative, fostering a multifaceted understanding of issues and enriching decision-making through varied perspectives [5]. As the complexity and uncertainty in practical decision-making scenarios escalate, efforts to address these challenges have led to the integration of DEMATEL with fuzzy set theory and other MCDM methods [6]-[8]. While these endeavors have shown promise in improving decision outcomes, a critical gap remains in the consideration of other extension theory. Neutrosophic sets have emerged as an extension of fuzzy sets, designed to reflect the existence of uncertain, imprecise, incomplete, and inconsistent information that is present in the real world [9]. In this framework, each element within the set is assigned degrees of truth, falsity, and indeterminacy membership [10]. A novel extension of neutrosophic set theory is the Pythagorean Neutrosophic Set (PNS) where PNS has dependent truth and false components where the sum of the truth square, falsity square, and indeterminacy square must be less than or equal to two [11]. This novel extension of neutrosophic set addresses a common challenge encountered in real-life scenarios [11]–[16]. PNS is a relatively new concept and its applications are still being explored. A recent study conducted by [17] has innovatively integrated PNS with DEMATEL but a significant challenge surfaces in decision-making problems when experts from diverse professional backgrounds provide evaluations influenced by their unique status, experience, and academic influence. These discrepancies in preferences present a critical challenge in MCDM. To effectively tackle this issue, this study proposes the incorporation of a Bonferroni Mean (BM) aggregating operator, aiming to harmonize diverse evaluations and enhance the decision-making processes. The BM introduced by [18] is a significant aggregation operator that effectively captures the interrelationship among individual arguments. While many existing aggregation operators assume that the criteria used in decision making are mutually independent, BM takes into account the interactions between these criteria. This allows BM to provide a more comprehensive representation of the relationships and dependencies among the individual arguments, making it a valuable tool in decision making and analysis. Since aggregation process is essential as it combines the evaluations of all decision makers into a single collective evaluation before proceeding with further steps, we utilize BM aggregation operator under Pythagorean neutrosophic environment within the DEMATEL method. Motivated by the concept normalized weighted BM operators which has the ability to deal with inter-related criteria [19], [20], this paper aims to develop a new normalized weighted BM operator for PNS aggregation (PN-NWBM) thereby contributing to its improvement in decision making. This study stands as one of the pioneering efforts to introduce a BM aggregation operator within DEMATEL under PNS environment. This paper is organized into the following sections: fundamental PNS theories are introduced in Section 2. The proposed approach is fully explained in Section 3. Numerical example is presented is Section 4 and Section 5 provides findings and discussions.

2 | Preliminaries

The foundational theories that are important for the development of PN-NWBM are presented in this section.

Definition 1 ([18]). Consider a set of positive real numbers denoted by p and q. Let $(a_1,\ldots,a_n)$ be a collection of values such that $a_i \in [0,1]$ and $(i=1,2,3,\ldots\ldots,n)$, then the BM is defined as

$$BM^{p,q}(a_1,\ldots,a_n)=\left(\frac{1}{n(n-1)}\sum_{\substack{i,j=1\\ i\neq j}}^{n} a_i^p a_j^q\right)^{\frac{1}{p+q}} \quad (1)$$

This BM operator consider interrelationship between ai and aj while disregarding the individual weights of the aggregated arguments. The importance of each argument, however, differs depending on the scenario, necessitating different weightings. As a result, the definitions of the Weighted Bonferroni Mean (WBM) and Normalized Weighted Bonferroni Mean (NWBM) are given in *Definition 2* below.

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Definition 2 ([21]). Consider a set of positive real numbers denoted by p and q. Let $(a_1,...,a_n)$ be a collection of values such that $a_i \in [0,1]$ and $(i=1,2,3,.....,n)$ with weight vector $\omega=(\omega_1,\omega_2,...,\omega_n)$ where $\omega_i>0$ and $\sum_{i=1}^{n}\omega_i=1$ represents the importance degree of a_i, then WBM is defined as:

$$WBM^{p,q}(a_1,...,a_n)=\left(\frac{1}{n(n-1)}\sum_{\substack{i,j=1\\ i\neq j}}^{n}(\omega_i a_i)^p(\omega_j a_j)^q\right)^{\frac{1}{p+q}}. \tag{2}$$

Definition 3 ([22]). Consider a set of positive real numbers denoted by p and q. Let $(a_1,...,a_n)$ be a collection of values such that $a_i \in [0,1]$ and $(i=1,2,3,.....,n)$ with weight vector $\omega=(\omega_1,\omega_2,...,\omega_n)$ where $\omega_i>0$ and $\sum_{i=1}^{n}\omega_i=1$ represents the importance degree of a_i, then NWBM is defined as:

$$NWBM^{p,q}(a_1,...,a_n)=\left(\sum_{\substack{i,j=1\\ i\neq j}}^{n}\frac{\omega_i\omega_j}{1-\omega_i}(a_i)^p(a_j)^q\right)^{\frac{1}{p+q}}. \tag{3}$$

Definition 4 ([23]). Let X be a universe or non-empty set. A Pythagorean fuzzy set A is defined:

$$A=\{(x,\psi_A(x),\kappa_A(x))\mid x\in X\}, \tag{4}$$

where $\psi,\kappa\in[0,1]$ denote the truth membership and false membership respectively of each element $x \in X$ to the set A, and $0\le\psi^2+\kappa^2\le 1$ for each $x \in X$. The indeterminacy membership is given by $\varsigma_A(x)=\sqrt{1-\psi_A{}^2(x)-\kappa_A{}^2(x)}$.

Definition 5 ([24]). Let X be a universe or non-empty set. A single valued neutrosophic set X in β is defined as:

$$\beta=\{(x,\psi_\beta(x),\varsigma_\beta(x),\kappa_\beta(x))\mid x\in X\}, \tag{5}$$

where $\psi_\beta,\varsigma_\beta,\kappa_\beta\in[0,1]$ and $0\le\psi_\beta+\varsigma_\beta+\kappa_\beta\le 3$.

Definition 6 ([11]). Let X be a universe or non-empty set. A PNS with ψ and κ as dependent membership is defined as:

$$A=\{\langle x,\psi_A(x),\varsigma_A(x),\kappa_A(x)\rangle\mid x\in X\}, \tag{6}$$

where ψ_A, ς_A and κ_A are the truth, indeterminacy and false membership respectively such that $\psi,\varsigma,\kappa\in[0,1]$ and satisfying

$$0\le\psi^2+\kappa^2\le 1. \tag{7}$$

$$0\le\psi^2+\varsigma^2+\kappa^2\le 2. \tag{8}$$

Definition 7 ([25]). Let $x_1=(\psi_{x_1},\varsigma_{x_1},\kappa_{x_1})$, $x_2=(\psi_{x_2},\varsigma_{x_2},\kappa_{x_2})$ and $x=(\psi_x,\varsigma_x,\kappa_x)$ are any two PNSs, then the operational rules for PNSs are defined as follows:

$$x_1\oplus x_2=\left(\sqrt{\psi_{x_1}{}^2+\psi_{x_2}{}^2-\psi_{x_1}{}^2\psi_{x_2}{}^2},\varsigma_{x_1}\varsigma_{x_2},\kappa_{x_1}\kappa_{x_2}\right). \tag{9}$$

$$x_1 \otimes x_2 = \left(\psi_{x_1}\psi_{x_2}, \varsigma_{x_1} + \varsigma_{x_2} - \varsigma_{x_1}\varsigma_{x_2} \sqrt{\kappa_{x_1}^{\ 2} + \kappa_{x_2}^{\ 2} - \kappa_{x_1}^{\ 2}\kappa_{x_2}^{\ 2}} \right). \tag{10}$$

$$\mu x = \left(\sqrt{1-\left(1-\psi_x^{\ 2}\right)^{\mu}}, \varsigma_x^{\ \mu}, \kappa_x^{\ \mu} \right) \text{ where } \mu \in \Re \text{ and } \mu \geq 0. \tag{11}$$

$$x^{\mu} = \left(\psi_x^{\ \mu}, 1-\left(1-\varsigma_x\right)^{\mu}, \sqrt{1-\left(1-\kappa_x^{\ 2}\right)^{\mu}} \right) \text{ where } \mu \in \Re \text{ and } \mu \geq 0. \tag{12}$$

3 | Proposed Method

In this section, we aim to expand the capability of NWBM operator's in *Definition 3* to account for circumstances in which PNS are utilized as input arguments. Thus, we propose the PN-NWBM operator under Pythagorean neutrosopic environment.

Definition 8. Let $x_i = \left(\psi_i(x), \varsigma_i(x), \kappa_i(x)\right)$ where $(i = 1, 2, \dots, n)$ be a set of PNS and $p, q \geq 0$, then PNBM is defined as:

$$\text{PNBM}(x_1, x_2, \dots\dots, x_n)^{p,q} = \left(\frac{1}{n(n-1)} \bigoplus_{\substack{i,j=1 \\ i\neq j}}^{n} \left(x_i^{\ p} \otimes x_j^{\ q} \right) \right)^{\frac{1}{p+q}}. \tag{13}$$

Based on the algebraic operations defined for PNSs in *Definition 7* and PNBM defined in *Definition 8*, we have the following propositions.

Proposition 1. Let $x_i = \left(\psi_i(x), \varsigma_i(x), \kappa_i(x)\right)$ where $(i = 1, 2, \dots, n)$ be a set of PNS and $p, q \geq 0$, then for any i, j and $i \neq j$:

$$x_i^{\ p} \otimes x_j^{\ q} = \left(\psi_i^{\ p}\psi_j^{\ q}, 1-\left(1-\varsigma_i\right)^p\left(1-\varsigma_j\right)^q, \sqrt{1-\left(1-\kappa_i^{\ 2}\right)^p\left(1-\kappa_j^{\ 2}\right)^q} \right). \tag{14}$$

Proof: Let two PNS numbers $x_i = \left(\psi_i(x), \varsigma_i(x), \kappa_i(x)\right)$ and $x_j = \left(\psi_j(x), \varsigma_j(x), \kappa_j(x)\right)$. By the *Operational Law (12)* in *Definition 7*, we have $x_i^{\ p} = \left(\psi_i^{\ p}, 1-\left(1-\varsigma_i\right)^p, \sqrt{1-\left(1-\kappa_i^{\ 2}\right)^p} \right)$, and $x_j^{\ q} = \left(\psi_j^{\ q}, 1-\left(1-\varsigma_j\right)^q, \sqrt{1-\left(1-\kappa_j^{\ 2}\right)^q} \right)$. Then based on *Operational Law (10)* in *Definition 7*, we have:

$$x_i^{\ p} \otimes x_j^{\ q} = \left(\psi_i^{\ p}, 1-\left(1-\varsigma_i\right)^p, \sqrt{1-\left(1-\kappa_i^{\ 2}\right)^p} \right) \otimes \left(\psi_j^{\ q}, 1-\left(1-\varsigma_j\right)^q, \sqrt{1-\left(1-\kappa_j^{\ 2}\right)^q} \right)$$

$$= \left(\begin{array}{c} \psi_i^{\ p}\psi_j^{\ q}, 1-\left(1-\varsigma_i\right)^p + 1-\left(1-\varsigma_j\right)^q - \left(1-\left(1-\varsigma_i\right)^p\right)\left(1-\left(1-\varsigma_j\right)^q\right), \\ \sqrt{1-\left(1-\kappa_i^{\ 2}\right)^p + 1-\left(1-\kappa_j^{\ 2}\right)^q - \left(1-\left(1-\kappa_i^{\ 2}\right)^p\right)\left(1-\left(1-\kappa_j^{\ 2}\right)^q\right)} \end{array} \right)$$

$$= \left(\psi_i^{\ p}\psi_j^{\ q}, 1-\left(1-\varsigma_i\right)^p\left(1-\varsigma_j\right)^q, \sqrt{1-\left(1-\kappa_i^{\ 2}\right)^p\left(1-\kappa_j^{\ 2}\right)^q} \right).$$

Thus, *Proposition 1* holds.

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Proposition 2. Let $x_i=\left(\psi_i(x),\varsigma_i(x),\kappa_i(x)\right)$ where $(i=1,2,\ldots,n)$ be a set of PNS and $p,q\geq 0$, then for any i, j and $i\neq j$:

$$\left(x_i^p\otimes x_j^q\right)\oplus\left(x_j^p\otimes x_i^q\right)=\left(\begin{array}{l}\sqrt{1-\left(1-\psi_i^{2p}\psi_j^{2q}\right)\left(1-\psi_j^{2p}\psi_i^{2q}\right)},\\ \left(1-\left(1-\varsigma_i\right)^p\left(1-\varsigma_j\right)^q\right)\left(1-\left(1-\varsigma_j\right)^p\left(1-\varsigma_i\right)^q\right),\\ \sqrt{\left(1-\left(1-\kappa_i^2\right)^p\left(1-\kappa_j^2\right)^q\right)\left(1-\left(1-\kappa_j^2\right)^p\left(1-\kappa_i^2\right)^q\right)}\end{array}\right). \tag{15}$$

Proof: Based on the result from *Proposition 1*, we have:

$$x_i^p\otimes x_j^q=\left(\psi_i^p\psi_j^q,1-\left(1-\varsigma_i\right)^p\left(1-\varsigma_j\right)^q,\sqrt{1-\left(1-\kappa_i^2\right)^p\left(1-\kappa_j^2\right)^q}\right) \text{ and}$$

$$x_j^p\otimes x_i^q=\left(\psi_j^p\psi_i^q,1-\left(1-\varsigma_j\right)^p\left(1-\varsigma_i\right)^q,\sqrt{1-\left(1-\kappa_j^2\right)^p\left(1-\kappa_i^2\right)^q}\right).$$

Then based *Operational Law (9)* in *Definition 7*, we have:

$$\left(x_i^p\otimes x_j^q\right)\oplus\left(x_j^p\otimes x_i^q\right)=\left(\begin{array}{l}\sqrt{\left(\psi_i^p\psi_j^q\right)^2+\left(\psi_j^p\psi_i^q\right)^2-\left(\psi_i^p\psi_j^q\right)^2\left(\psi_j^p\psi_i^q\right)^2},\\ \left(1-\left(1-\varsigma_i\right)^p\left(1-\varsigma_j\right)^q\right)\left(1-\left(1-\varsigma_j\right)^p\left(1-\varsigma_i\right)^q\right),\\ \sqrt{1-\left(1-\kappa_i^2\right)^p\left(1-\kappa_j^2\right)^q}\sqrt{1-\left(1-\kappa_j^2\right)^p\left(1-\kappa_i^2\right)^q}\end{array}\right)$$

$$=\left(\begin{array}{l}\sqrt{1-\left(1-\left(\psi_i^p\psi_j^q\right)^2\right)\left(1-\left(\psi_j^p\psi_i^q\right)^2\right)},\\ \left(1-\left(1-\varsigma_i\right)^p\left(1-\varsigma_j\right)^q\right)\left(1-\left(1-\varsigma_j\right)^p\left(1-\varsigma_i\right)^q\right),\\ \sqrt{\left(1-\left(1-\kappa_i^2\right)^p\left(1-\kappa_j^2\right)^q\right)\left(1-\left(1-\kappa_j^2\right)^p\left(1-\kappa_i^2\right)^q\right)}\end{array}\right)$$

$$=\left(\begin{array}{l}\sqrt{1-\left(1-\left(\psi_i^{2p}\psi_j^{2q}\right)\right)\left(1-\left(\psi_j^{2p}\psi_i^{2q}\right)\right)},\\ \left(1-\left(1-\varsigma_i\right)^p\left(1-\varsigma_j\right)^q\right)\left(1-\left(1-\varsigma_j\right)^p\left(1-\varsigma_i\right)^q\right),\\ \sqrt{\left(1-\left(1-\kappa_i^2\right)^p\left(1-\kappa_j^2\right)^q\right)\left(1-\left(1-\kappa_j^2\right)^p\left(1-\kappa_i^2\right)^q\right)}\end{array}\right).$$

Thus, we conclude the *Proposition 2* proof.

Proposition 3. Let $x_i=\left(\psi_i(x),\varsigma_i(x),\kappa_i(x)\right)$ where $(i=1,2,\ldots,n)$ be a set of PNS and $p,q\geq 0$, then for any i, j and $i\neq j$ and $1\leq f<n$, we have:

$$\overset{f}{\underset{i=1}{\oplus}}\left(x_i^{p}\otimes x_{f+1}^{q}\right)= \left(\sqrt{1-\prod_{i=1}^{f}\left(1-\psi_i^{2p}\psi_{f+1}^{2q}\right)},\prod_{i=1}^{f}\left(1-\left(1-\varsigma_i\right)^p\left(1-\varsigma_{f+1}\right)^q\right),\sqrt{\prod_{i=1}^{f}\left(1-\left(1-\kappa_i^2\right)^p\left(1-\kappa_{f+1}^2\right)^q\right)}\right). \tag{16}$$

Proof: Based on *Eq. (14)* in *Proposition 1*, if we let f = 2, we have:

$$x_i^{p}\otimes x_{2+1}^{q}=\left(\psi_i^{p}\psi_{2+1}^{q},1-\left(1-\varsigma_i\right)^p\left(1-\varsigma_{2+1}\right)^q,\sqrt{1-\left(1-\kappa_i^2\right)^p\left(1-\kappa_{2+1}^2\right)^q}\right).$$

Then,

$$\overset{2}{\underset{i=1}{\oplus}}\left(x_i^{p}\otimes x_{3}^{q}\right)=\left(x_1^{p}\otimes x_{3}^{q}\right)\oplus\left(x_2^{p}\otimes x_{3}^{q}\right)$$

$$=\left(\begin{array}{l}\sqrt{1-\left(1-\psi_1^{2p}\psi_3^{2q}\right)\left(1-\psi_2^{2p}\psi_3^{2q}\right)},\\ \left(1-\left(1-\varsigma_1\right)^p\left(1-\varsigma_3\right)^q\right)\left(1-\left(1-\varsigma_2\right)^p\left(1-\varsigma_3\right)^q\right),\\ \sqrt{\left(1-\left(1-\kappa_1^2\right)^p\left(1-\kappa_3^2\right)^q\right)\left(1-\left(1-\kappa_2^2\right)^p\left(1-\kappa_3^2\right)^q\right)}\end{array}\right).$$

Next if we let f = fo,

$$\overset{f_o}{\underset{i=1}{\oplus}}\left(x_i^{p}\otimes x_{f_o+1}^{q}\right)= \left(\sqrt{1-\prod_{i=1}^{f_o}\left(1-\psi_i^{2p}\psi_{f_o+1}^{2q}\right)},\prod_{i=1}^{f_o}\left(1-\left(1-\varsigma_i\right)^p\left(1-\varsigma_{f_o+1}\right)^q\right),\sqrt{\prod_{i}^{f_o}\left(1-\left(1-\kappa_i^2\right)^p\left(1-\kappa_{f_o+1}^2\right)^q\right)}\right).$$

Thus *Eq. (16)* from *Proposition 3* holds for f = fo .

When f = fo +1, we have:

$$\overset{f_o+1}{\underset{i=1}{\oplus}}\left(x_i^{p}\otimes x_{f_o+1+1}^{q}\right)=\overset{f_o+1}{\underset{i=1}{\oplus}}\left(x_i^{p}\otimes x_{f_o+2}^{q}\right)=\overset{f_o}{\underset{i=1}{\oplus}}\left(x_i^{p}\otimes x_{f_o+2}^{q}\right)\oplus\left(x_{f_o+1}^{p}\otimes x_{f_o+2}^{q}\right)$$

$$=\left(\sqrt{1-\prod_{i=1}^{f_o}\left(1-\psi_i^{2p}\psi_{f_o+1}^{2q}\right)},\prod_{i=1}^{f_o}\left(1-\left(1-\varsigma_i\right)^p\left(1-\varsigma_{f_o+1}\right)^q\right),\sqrt{\prod_{i=1}^{f_o}\left(1-\left(1-\kappa_i^2\right)^p\left(1-\kappa_{f_o+1}^2\right)^q\right)}\right)\oplus$$
$$\left(\psi_{f_o+1}^{p}\psi_{f_o+2}^{q},\ 1-\left(1-\varsigma_{f_o+1}\right)^p\left(1-\varsigma_{f_o+2}\right)^q,\ \sqrt{1-\left(1-\kappa_{f_o+1}^2\right)^p\left(1-\kappa_{f_o+2}^2\right)^q}\right)$$

$$=\left(\sqrt{1-\prod_{i=1}^{f_o+1}\left(1-\psi_i^{2p}\psi_{f_o+2}^{2q}\right)},\prod_{i=1}^{f_o+1}\left(1-\left(1-\varsigma_i\right)^p\left(1-\varsigma_{f_o+2}\right)^q\right),\sqrt{\prod_{i=1}^{f_o+1}\left(1-\left(1-\kappa_i^2\right)^p\left(1-\kappa_{f_o+2}^2\right)^q\right)}\right).$$

Thus also holds for f = fo +1. Therefore, *Proposition 3* holds. Hence, we can directly deduce the following *Proposition 4* based on *Proposition 3* above.

Proposition 4. Let $x_i=\left(\psi_i\left(x\right),\varsigma_i\left(x\right),\kappa_i\left(x\right)\right)$ where $(i=1,2,\ldots,n)$ be a set of PNS and $p,q\geq 0$, then for any i, j and $i\neq j$ and $1\leq f<n$, we have:

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$$\underset{j=1}{\overset{f}{\oplus}}\left(x_{f+1}{}^{p}\otimes x_{j}{}^{q}\right)=\left(\sqrt{1-\prod_{j=1}^{f}\left(1-\psi_{f+1}{}^{2p}\psi_{j}{}^{2q}\right)},\ \prod_{j=1}^{f}\left(1-\left(1-\varsigma_{f+1}\right)^{p}\left(1-\varsigma_{j}\right)^{q}\right),\sqrt{\prod_{j=1}^{f}\left(1-\left(1-\kappa_{f+1}{}^{2}\right)^{p}\left(1-\kappa_{j}{}^{2}\right)^{q}\right)}\right).$$

Proposition 5. Let $x_i=\left(\psi_i\left(x\right),\varsigma_i\left(x\right),\kappa_i\left(x\right)\right)$ where $(i=1,2,\ldots,n)$ be a set of PNS and $p,q\geq 0$, then for any i, j and $i\neq j$,

$$\underset{i,j=1;i\neq j}{\overset{n}{\oplus}}\left(x_{i}{}^{p}\otimes x_{j}{}^{q}\right)=\left(\begin{array}{l}\sqrt{1-\prod_{i,j=1;i\neq j}^{n}\left(1-\psi_{i}^{2p}\psi_{j}^{2q}\right)},\\ \prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-\left(1-\varsigma_{i}\right)^{p}\left(1-\varsigma_{j}\right)^{q}\right),\\ \sqrt{\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-\left(1-\kappa_{i}^{2}\right)^{p}\left(1-\kappa_{j}^{2}\right)^{q}\right)}\end{array}\right). \tag{18}$$

Proof: By combining the result from *Propositions 2* to *4*, we can prove *Proposition 5* as follows:

Let n = 2, we use *Eq. (15)* from *Proposition 2* to get;

$$\underset{\substack{i,j=1\\ i\neq j}}{\overset{2}{\oplus}}\left(x_{i}{}^{p}\otimes x_{j}{}^{q}\right)=\left(x_{1}{}^{p}\otimes x_{2}{}^{q}\right)\oplus\left(x_{2}{}^{p}\otimes x_{1}{}^{q}\right)=$$

$$\left(\begin{array}{l}\sqrt{1-\left(1-\psi_{1}{}^{2p}\psi_{2}{}^{2q}\right)\left(1-\psi_{2}{}^{2p}\psi_{1}{}^{2q}\right)},\\ \left(1-\left(1-\varsigma_{1}\right)^{p}\left(1-\varsigma_{2}\right)^{q}\right)\left(1-\left(1-\varsigma_{2}\right)^{p}\left(1-\varsigma_{1}\right)^{q}\right),\\ \sqrt{\left(1-\left(1-\kappa_{1}{}^{2}\right)^{p}\left(1-\kappa_{2}{}^{2}\right)^{q}\right)\left(1-\left(1-\kappa_{2}{}^{2}\right)^{p}\left(1-\kappa_{1}{}^{2}\right)^{q}\right)}\end{array}\right),$$

and if n = f, we have:

$$\underset{i,j=1;i\neq j}{\overset{f}{\oplus}}\left(x_{i}{}^{p}\otimes x_{j}{}^{q}\right)=\left(\sqrt{1-\prod_{i,j=1;i\neq j}^{f}\left(1-\psi_{i}{}^{2p}\psi_{j}{}^{2q}\right)},\ \prod_{i,j=1;i\neq j}^{f}\left(1-\left(1-\varsigma_{i}\right)^{p}\left(1-\varsigma_{j}\right)^{q}\right),\sqrt{\prod_{i,j=1;i\neq j}^{f}\left(1-\left(1-\kappa_{i}{}^{2}\right)^{p}\left(1-\kappa_{j}{}^{2}\right)^{q}\right)}\right).$$

Thus when n = f +1, we have:

$$\underset{i,j=1;i\neq j}{\overset{f+1}{\oplus}}\left(x_{i}{}^{p}\otimes x_{j}{}^{q}\right)=\underset{i,j=1;i\neq j}{\overset{f}{\oplus}}\left(x_{i}{}^{p}\otimes x_{j}{}^{q}\right)\underset{i=1}{\overset{f}{\oplus}}\left(x_{i}{}^{p}\otimes x_{f+1}{}^{q}\right)\underset{j=1}{\overset{f}{\oplus}}\left(x_{f+1}{}^{p}\otimes x_{j}{}^{q}\right),$$

where $\underset{i=1}{\overset{f}{\oplus}}\left(x_i{}^{p}\otimes x_{f+1}{}^{q}\right)$ from *Proposition 3*, is

$$\left(\sqrt{1-\prod_{i=1}^{f}\left(1-\psi_{i}{}^{2p}\psi_{f+1}{}^{2q}\right)},\prod_{i=1}^{f}\left(1-\left(1-\varsigma_{i}\right)^{p}\left(1-\varsigma_{f+1}\right)^{q}\right),\sqrt{\prod_{i=1}^{f}\left(1-\left(1-\kappa_{i}{}^{2}\right)^{p}\left(1-\kappa_{f+1}{}^{2}\right)^{q}\right)}\right).$$

and $\overset{f}{\underset{j=1}{\oplus}}\left(x_{f+1}{}^{p}\otimes x_{j}{}^{q}\right)$ from *Proposition 4*, is

$$\left(\sqrt{1-\prod_{j=1}^{f}\left(1-\psi_{f+1}{}^{2p}\psi_{j}{}^{2q}\right)},\prod_{j=1}^{f}\left(1-\left(1-\varsigma_{f+1}\right)^{p}\left(1-\varsigma_{j}\right)^{q}\right),\sqrt{\prod_{j=1}^{f}\left(1-\left(1-\kappa_{f+1}{}^{2}\right)^{p}\left(1-\kappa_{j}{}^{2}\right)^{q}\right)}\right).$$

Hence, we calculate $\overset{f+1}{\underset{i,j=1;i\neq j}{\oplus}}\left(x_{i}{}^{p}\otimes x_{j}{}^{q}\right)$ as follows:

$$\overset{f+1}{\underset{\substack{i,j=1\\i\neq j}}{\oplus}}\left(x_{i}{}^{p}\otimes x_{j}{}^{q}\right)=$$

$$\overset{f+1}{\underset{\substack{i,j=1\\i\neq j}}{\oplus}}\left(x_{i}{}^{p}\otimes x_{j}{}^{q}\right)=\left(\sqrt{1-\prod_{\substack{i,j=1\\i\neq j}}^{f+1}\left(1-\psi_{i}{}^{2p}\psi_{j}{}^{2q}\right)},\prod_{\substack{i,j=1\\i\neq j}}^{f+1}\left(1-\left(1-\varsigma_{i}\right)^{p}\left(1-\varsigma_{j}\right)^{q}\right),\sqrt{\prod_{\substack{i,j=1\\i\neq j}}^{f+1}\left(1-\left(1-\kappa_{i}{}^{2}\right)^{p}\left(1-\kappa_{j}{}^{2}\right)^{q}\right)}\right).$$

Since *Eq. (18)* also holds for n = f +1, thus the statement for *Proposition 5* holds.

Proposition 6. Let $x_{i}=\left(\psi_{i}\left(x\right),\varsigma_{i}\left(x\right),\kappa_{i}\left(x\right)\right)$ where $(i=1,2,\ldots,n)$ be a set of PNS and $p,q\geq 0$, then for any i, j and $i\neq j$,

$$\frac{1}{n\left(n-1\right)}\left(\overset{n}{\underset{i,j=1;i\neq j}{\oplus}}\left(x_{i}{}^{p}\otimes x_{j}{}^{q}\right)\right)=\left(\begin{array}{c}\sqrt{1-\prod\limits_{i,j=1;i\neq j}^{n}\left(1-\psi_{i}{}^{2p}\psi_{j}{}^{2q}\right)^{\frac{1}{n(n-1)}}},\\ \prod\limits_{i,j=1;i\neq j}^{n}\left(1-\left(1-\varsigma_{i}\right)^{p}\left(1-\varsigma_{j}\right)^{q}\right)^{\frac{1}{n(n-1)}},\\ \sqrt{\prod\limits_{i,j=1;i\neq j}^{n}\left(1-\left(1-\kappa_{i}{}^{2}\right)^{p}\left(1-\kappa_{j}{}^{2}\right)^{q}\right)^{\frac{1}{n(n-1)}}}\end{array}\right). \quad (19)$$

Proof: Based on *Proposition 5*, we have:

$$\overset{n}{\underset{i,j=1;i\neq j}{\oplus}}\left(x_{i}{}^{p}\otimes x_{j}{}^{q}\right)=$$
$$\left(\sqrt{1-\prod_{i,j=1;i\neq j}^{n}\left(1-\psi_{i}{}^{2p}\psi_{j}{}^{2q}\right)},\prod_{i,j=1;i\neq j}^{n}\left(1-\left(1-\varsigma_{i}\right)^{p}\left(1-\varsigma_{j}\right)^{q}\right),\sqrt{\prod_{i,j=1;i\neq j}^{n}\left(1-\left(1-\kappa_{i}{}^{2}\right)^{p}\left(1-\kappa_{j}{}^{2}\right)^{q}\right)}\right).$$

Next by using *Operational Law (11)* from *Definition 7*, we compute $\frac{1}{n\left(n-1\right)}\left(\overset{n}{\underset{i,j=1;i\neq j}{\oplus}}\left(x_{i}{}^{p}\otimes x_{j}{}^{q}\right)\right)$ as follows:

$$\frac{1}{n\left(n-1\right)}\left(\overset{n}{\underset{i,j=1;i\neq j}{\oplus}}\left(x_{i}{}^{p}\otimes x_{j}{}^{q}\right)\right)=\left(\begin{array}{c}\sqrt{1-\prod\limits_{i,j=1;i\neq j}^{n}\left(1-\psi_{i}{}^{2p}\psi_{j}{}^{2q}\right)^{\frac{1}{n(n-1)}}},\\ \prod\limits_{i,j=1;i\neq j}^{n}\left(1-\left(1-\varsigma_{i}\right)^{p}\left(1-\varsigma_{j}\right)^{q}\right)^{\frac{1}{n(n-1)}},\\ \sqrt{\prod\limits_{i,j=1;i\neq j}^{n}\left(1-\left(1-\kappa_{i}{}^{2}\right)^{p}\left(1-\kappa_{j}{}^{2}\right)^{q}\right)^{\frac{1}{n(n-1)}}}\end{array}\right).$$

Thus, *Proposition* 6 holds. Next, by utilizing PN-BM in *Definition 8* we can further deduce the PN-NWBM operator as follows.

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Definition 9. Let $x_i = \left(\psi_i\left(x\right), \varsigma_i\left(x\right), \kappa_i\left(x\right)\right)$ where $(i = 1, 2, \ldots, n)$ be a set of PNS and $p, q \geq 0$, then for any i, j and $i \neq j$ PN-NWBM is defined as:

$$\text{PN}-\text{NWBM}(x_1, x_2, x_3, \ldots\ldots, x_n)^{p,q} = \left(\underset{\substack{i,j=1\\ i\neq j}}{\overset{n}{\oplus}} \frac{\omega_i\omega_j}{1-\omega_i}\left(x_i^{\,p} \otimes x_j^{\,q}\right)\right)^{\frac{1}{p+q}}, \tag{20}$$

where $\omega = \left(\omega_1, \omega_2, \ldots, \omega_n\right)$ is the weight vector represents the importance degree of x_i, with $\omega_i > 0$ and $\sum_{i=1}^{n} \omega_i = 1$.

Theorem 1. Given $p, q \geq 0,$ and $x_i = \left(\psi_i\left(x\right), \varsigma_i\left(x\right), \kappa_i\left(x\right)\right)$ where $(i = 1, 2, 3\ldots\ldots, n)$ be a set of PNS, then the aggregated value by PN-NWBM operator in *Eq. (20)* is a Pythagorean neutrosophic number with

$$\psi_i\left(x\right) = \sqrt{\left(1 - \prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1 - \psi_{ij}^{\,2p}\psi_{ij}^{\,2q}\right)^{\frac{\omega_i\omega_j}{1-\omega_i}}\right)^{\frac{1}{p+q}}},$$

$$\varsigma_i\left(x\right) = 1 - \left(1 - \prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1 - \left(1-\varsigma_{ij}\right)^{p}\left(1-\varsigma_{ij}\right)^{q}\right)^{\frac{\omega_i\omega_j}{1-\omega_i}}\right)^{\frac{1}{p+q}}$$

$$\text{and } \kappa_i\left(x\right) = \sqrt{1 - \left(1 - \prod_{i,j=1;i\neq j}^{n}\left(1 - \left(1-\kappa_i^{\,2}\right)^{p}\left(1-\kappa_j^{\,2}\right)^{q}\right)^{\frac{\omega_i\omega_j}{1-\omega i}}\right)^{\frac{1}{p+q}}}.$$

Proof: Based on the derived equations in *Propositions 1* to *6*, we can prove *Theorem* 1 as follows:

We have $x_i^{\,p} \otimes x_j^{\,q} = \left(\psi_i^{\,p}\psi_j^{\,q}, 1-\left(1-\varsigma_i\right)^{p}\left(1-\varsigma_j\right)^{q}, \sqrt{1-\left(1-\kappa_i^{\,2}\right)^{p}\left(1-\kappa_j^{\,2}\right)^{q}}\right)$, then

$$\frac{\omega_i\omega_j}{1-\omega_i}\left(x_i^{\,p} \otimes x_j^{\,q}\right) = \left(\sqrt{1-\left(1-\psi_i^{\,p}\psi_j^{\,q}\right)^{\frac{\omega_i\omega_j}{1-\omega_i}}}, \left[1-\left(1-\varsigma_i\right)^{p}\left(1-\varsigma_j\right)^{q}\right]^{\frac{\omega_i\omega_j}{1-\omega_i}}, \left[\sqrt{\left(1-\left(1-\kappa_i\right)^{p}\left(1-\kappa_j\right)^{q}\right)^{\frac{\omega_i\omega_j}{1-\omega_i}}}\right]\right)$$

$$\Rightarrow \underset{\substack{i,j=1\\ i\neq j}}{\overset{n}{\oplus}} \frac{\omega_i\omega_j}{1-\omega_i}\left(x_i^{\,p} \otimes x_j^{\,q}\right)$$

$$= \left(\sqrt{1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-\psi_i^{\,2p}\psi_j^{\,2q}\right)^{\frac{\omega_i\omega_j}{1-\omega_i}}}, \prod_{\substack{i,j=1\\ i\neq j}}^{n}\left[1-\left(1-\varsigma_i\right)^{p}\left(1-\varsigma_j\right)^{q}\right]^{\frac{\omega_i\omega_j}{1-\omega_i}}, \sqrt{\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-\left(1-\kappa_i^{\,2}\right)^{p}\left(1-\kappa_j^{\,2}\right)^{q}\right)^{\frac{\omega_i\omega_j}{1-\omega_i}}}\right).$$

Therefore, $PN-NWBM\left(x_1,x_2,........x_n\right)^{p,q}=\left(\underset{\substack{i,j=1\\i\neq j}}{\overset{n}{\oplus}}\frac{\omega_i\omega_j}{1-\omega_i}\left(x_i^{\,p}\otimes x_j^{\,q}\right)\right)^{\frac{1}{p+q}}=$

$$\left(\begin{array}{l}\sqrt{\left(1-\prod_{\substack{i,j=1\\i\neq j}}^{n}\left(1-\left(\psi_i\right)^{2p}\left(\psi_j\right)^{2q}\right)^{\frac{\omega_i\omega_j}{1-\omega_i}}\right)^{\frac{1}{p+q}}},\\ 1-\left(1-\prod_{\substack{i,j=1\\i\neq j}}^{n}\left(1-\left(1-\varsigma_i\right)^{p}\left(1-\varsigma_j\right)^{q}\right)^{\frac{\omega_i\omega_j}{1-\omega_i}}\right)^{\frac{1}{p+q}},\\ \sqrt{1-\left(1-\prod_{\substack{i,j=1\\i\neq j}}^{n}\left(1-\left(1-\kappa_i^{\,2}\right)^{p}\left(1-\kappa_j^{\,2}\right)^{q}\right)^{\frac{\omega_i\omega_j}{1-\omega_i}}\right)^{\frac{1}{p+q}}}\end{array}\right), \tag{21}$$

where

$$0\leq\sqrt{\left(1-\prod_{\substack{i,j=1\\i\neq j}}^{n}\left(1-\left(\psi_i\right)^{2p}\left(\psi_j\right)^{2q}\right)^{\frac{\omega_i\omega_j}{1-\omega_i}}\right)^{\frac{1}{p+q}}}\leq 1,$$

$$0\leq 1-\left(1-\prod_{\substack{i,j=1\\i\neq j}}^{n}\left(1-\left(1-\varsigma_{ij}\right)^{p}\left(1-\varsigma_{ij}\right)^{q}\right)^{\frac{\omega_i\omega_j}{1-\omega_i}}\right)^{\frac{1}{p+q}}\leq 1.$$

And $0\leq\sqrt{1-\left(1-\prod_{i,j=1;i\neq j}^{n}\left(1-\left(1-\kappa_i^{\,2}\right)^{p}\left(1-\kappa_j^{\,2}\right)^{q}\right)^{\frac{\omega_i\omega_j}{1-\omega i}}\right)^{\frac{1}{p+q}}}\leq 1$ satisfying *Eq. (7)* and *Eq. (8)* from *Definition 6* which proves *Theorem 1*.

4 | Computational Procedure for PN-NWBM within DEMATEL

In this section, the computational procedure is outlined to show the applicability of the proposed method. The PN-NWBM operator is utilized to aggregate all individual experts' decisions into a single evaluation before initiating the DEMATEL procedure.

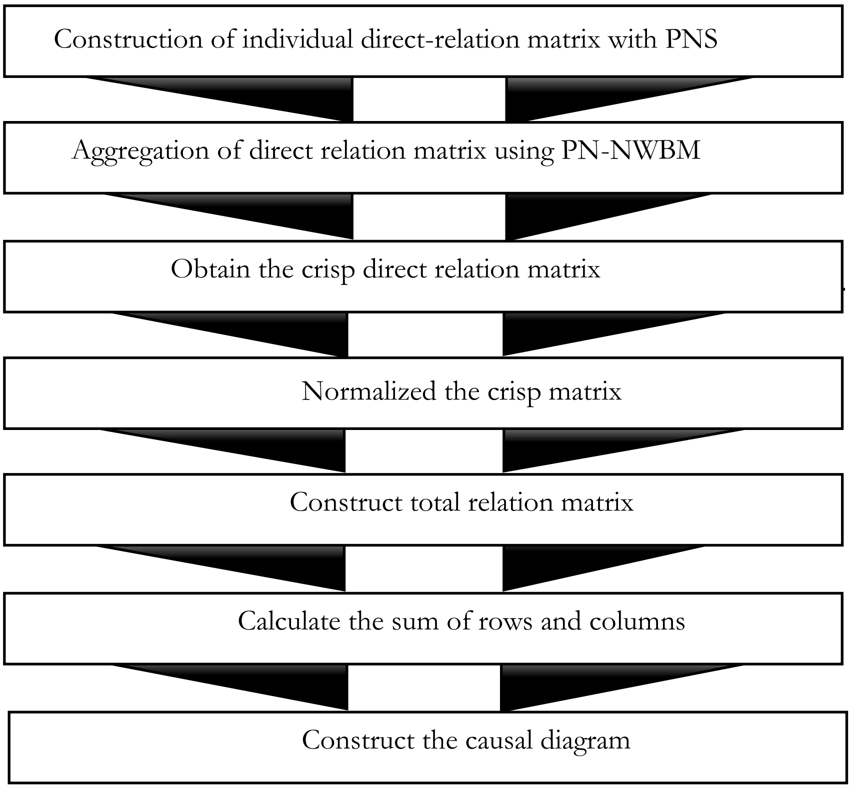

Fig. 1. PN-NWBM DEMATEL computational procedure.

The seven steps algorithm for the computation of the numerical example is explained in detailed below:

Step 1. Construct the initial direct-relation matrix. A direct-relation matrix with Pythagorean neutrosophic number were constructed based on experts' preference. The score is determined using seven linguistic scales that ranged from no influence to very high influence based on Pythagorean neutrosophic linguistic variable as in *Table 1* below.

Table 1. The Pythagorean neutrosophic linguistic variable.

Score	Linguistic Variable	Pythagorean Neutrosophic Number
1	No impact	$< 0.10, 0.80, 0.90 >$
2	Low impact	$< 0.20, 0.70, 0.80 >$
3	Medium low impact	$< 0.35, 0.60, 0.60 >$
4	Medium impact	$< 0.50, 0.40, 0.45 >$
5	Medium high impact	$< 0.65, 0.30, 0.25 >$
6	High impact	$< 0.80, 0.20, 0.15 >$
7	Very high impact	$< 0.90, 0.10, 0.10 >$

Step 2. Obtain the aggregate direct-relation matrix A. Aggregate all the individual direct relation-matrices, $X^k = \{X^1, X^2, \ldots\ldots, X^m\}$ into one collective decision matrix $A = \left[a_{ij}\right]_{m\times n}$ using PN-NWBM in *Eq. (21)* where $a_{ij} = \left(\psi_{ij}, \varsigma_{ij}, \kappa_{ij}\right)$ and $\omega = \left(\omega_1, \omega_2, \ldots\ldots, \omega_m\right)^n$ is the weight vector of x_i *for* $(i = 1, 2, \ldots.., n)$, satisfying $\omega_j \in [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1$.

Step 3. To deneutrosophicate the Pythagorean neutrosophic number $\left(\psi_A(x), \varsigma_A(x), \kappa_A(x)\right)$ to a crisp value, the deneutrosophication formula is used as follows:

$$B = \frac{\psi_A(x) + \varsigma_A(x) + \kappa_A(x)}{3}. \tag{22}$$

Step 4. Normalizing the matrix. Using,

$$Z = \frac{B}{s}, \tag{23}$$

where $s = \max_{1 \le i \le n} \sum_{j=1}^{n} b_{ij}$. All elements in the matrix Z are complying with $0 \le z_{ij} < 1$.

Step 5. Construct the total-influence matrix Tij using

$$T = Z(I - Z)^{-1}, \tag{24}$$

where I is the identity matrix.

Step 6. Calculating the sums of the rows and columns.

In this step, the vectors R and C representing the sum of the rows and the sum of columns from the total-influence matrix T are defined by the following formulas:

$$R = \left[r_i\right]_{n\times 1} = \left[\sum_{j=1}^{n} t_{ij}\right]_{n\times 1}. \tag{25}$$

$$C = \left[c_i\right]_{1\times n} = \left[\sum_{i=1}^{n} t_{ij}\right]_{1\times n}^{T}. \tag{26}$$

The $R+C$ and $R-C$ values are calculated in which these values reflect the importance and relation values, respectively.

Step 7. Construct Network Relationship Map (NRM). The graph is plotted using (R + C, R – C) data set. The R + C is labelled on the horizontal axis and R – C is on vertical axis. The NRMs advantages can reflect the MCDM flow. Each graph node represents the object examined, while the arc between two nodes shows the direction and strength of the influence relationship.

5 | Numerical Example

In this section, a numerical example based on MCDM problem under Pythagorean neutrosophic environment adapted from [17] that used seven term linguistic variable with PNS number. The study used arithmetic mean to aggregate the experts' evaluations, thus in this numerical example PN-NWBM operator is utilized to show the applicability of the proposed method. *Table 2* shows each expert's evaluation in PNS number that will then aggregated to a single direct relation matrix as in *Table 3*.

Table 2. Experts' evaluation in PNS number.

E1	(0,0,0)	(0.1,0.8,0.9)	(0.2,0.7,0.8)	(0.9,0.1,0.1)	(0.5,0.4,0.45)	(0.8,0.2,0.15)	(0.8,0.2,0.15)
	(0.1,0.8,0.9)	(0,0,0)	(0.5,0.4,0.45)	(0.1,0.8,0.9)	(0.2,0.7,0.8)	(0.1,0.8,0.9)	(0.35,0.6,0.6)
	(0.8,0.2,0.15)	(0.2,0.7,0.8)	(0,0,0)	(0.5,0.4,0.45)	(0.1,0.8,0.9)	(0.8,0.2,0.15)	(0.1,0.8,0.9)
	(0.5,0.4,0.45)	(0.65,0.3,0.25)	(0.2,0.7,0.8)	(0,0,0)	(0.65,0.3,0.25)	(0.65,0.3,0.25)	(0.5,0.4,0.45)
	(0.1,0.8,0.9)	(0.8,0.2,0.15)	(0.1,0.8,0.9)	(0.5,0.4,0.45)	(0,0,0)	(0.2,0.7,0.8)	(0.5,0.4,0.45)
	(0.2,0.7,0.8)	(0.5,0.4,0.45)	(0.65,0.3,0.25)	(0.2,0.7,0.8)	(0.1,0.8,0.9)	(0,0,0)	(0.2,0.7,0.8)
	(0.1,0.8,0.9)	(0.1,0.8,0.9)	(0.1,0.8,0.9)	(0.5,0.4,0.45)	(0.35,0.6,0.6)	(0.35,0.6,0.6)	(0,0,0)

Table 2. Continued.

E2	(0,0,0)	(0.2,0.7,0.8)	(0.35,0.6,0.6)	(0.9,0.1,0.1)	(0.35,0.6,0.6)	(0.65,0.3,0.25)	(0.8,0.2,0.15)
	(0.35,0.6,0.6)	(0,0,0)	(0.35,0.6,0.6)	(0.1,0.8,0.9)	(0.2,0.7,0.8)	(0.1,0.8,0.9)	(0.35,0.6,0.6)
	(0.9,0.1,0.1)	(0.1,0.8,0.9)	(0,0,0)	(0.5,0.4,0.45)	(0.2,0.7,0.8)	(0.9,0.1,0.1)	(0.1,0.8,0.9)
	(0.65,0.3,0.25)	(0.65,0.3,0.25)	(0.2,0.7,0.8)	(0,0,0)	(0.5,0.4,0.45)	(0.5,0.4,0.45)	(0.65,0.3,0.25)
	(0.35,0.6,0.6)	(0.65,0.3,0.25)	(0.1,0.8,0.9)	(0.65,0.3,0.25)	(0,0,0)	(0.1,0.8,0.9)	(0.35,0.6,0.6)
	(0.35,0.6,0.6)	(0.5,0.4,0.45)	(0.5,0.4,0.45)	(0.35,0.6,0.6)	(0.35,0.6,0.6)	(0,0,0)	(0.1,0.8,0.9)
	(0.2,0.7,0.8)	(0.35,0.6,0.6)	(0.2,0.7,0.8)	(0.65,0.3,0.25)	(0.1,0.8,0.9)	(0.2,0.7,0.8)	(0,0,0)
E3	(0,0,0)	(0.35,0.6,0.6)	(0.1,0.8,0.9)	(0.8,0.2,0.15)	(0.5,0.4,0.45)	(0.9,0.1,0.1)	(0.9,0.1,0.1)
	(0.2,0.7,0.8)	(0,0,0)	(0.5,0.4,0.45)	(0.35,0.6,0.6)	(0.1,0.8,0.9)	(0.35,0.6,0.6)	(0.1,0.8,0.9)
	(0.8,0.2,0.15)	(0.2,0.7,0.8)	(0,0,0)	(0.35,0.6,0.6)	(0.1,0.8,0.9)	(0.65,0.3,0.25)	(0.2,0.7,0.8)
	(0.5,0.4,0.45)	(0.5,0.4,0.45)	(0.1,0.8,0.9)	(0,0,0)	(0.65,0.3,0.25)	(0.65,0.3,0.25)	(0.5,0.4,0.45)
	(0.1,0.8,0.9)	(0.9,0.1,0.1)	(0.35,0.6,0.6)	(0.5,0.4,0.45)	(0,0,0)	(0.2,0.7,0.8)	(0.5,0.4,0.45)
	(0.1,0.8,0.9)	(0.35,0.6,0.6)	(0.65,0.3,0.25)	(0.1,0.8,0.9)	(0.1,0.8,0.9)	(0,0,0)	(0.2,0.7,0.8)
	(0.35,0.6,0.6)	(0.1,0.8,0.9)	(0.35,0.6,0.6)	(0.35,0.6,0.6)	(0.35,0.6,0.6)	(0.1,0.8,0.9)	(0,0,0)
E4	(0,0,0)	(0.1,0.8,0.9)	(0.1,0.8,0.9)	(0.8,0.2,0.15)	(0.35,0.6,0.6)	(0.65,0.3,0.25)	(0.65,0.3,0.25)
	(0.2,0.7,0.8)	(0,0,0)	(0.35,0.6,0.6)	(0.2,0.7,0.8)	(0.35,0.6,0.6)	(0.1,0.8,0.9)	(0.2,0.7,0.8)
	(0.9,0.1,0.1)	(0.1,0.8,0.9)	(0,0,0)	(0.5,0.4,0.45)	(0.2,0.7,0.8)	(0.9,0.1,0.1)	(0.35,0.6,0.6)
	(0.5,0.4,0.45)	(0.5,0.4,0.45)	(0.1,0.8,0.9)	(0,0,0)	(0.8,0.2,0.15)	(0.5,0.4,0.45)	(0.65,0.3,0.25)
	(0.35,0.6,0.6)	(0.65,0.3,0.25)	(0.2,0.7,0.8)	(0.65,0.3,0.25)	(0,0,0)	(0.35,0.6,0.6)	(0.35,0.6,0.6)
	(0.35,0.6,0.6)	(0.35,0.6,0.6)	(0.35,0.6,0.6)	(0.2,0.7,0.8)	(0.35,0.6,0.6)	(0,0,0)	(0.35,0.6,0.6)
	(0.1,0.8,0.9)	(0.35,0.6,0.6)	(0.1,0.8,0.9)	(0.35,0.6,0.6)	(0.2,0.7,0.8)	(0.2,0.7,0.8)	(0,0,0)
E5	(0,0,0)	(0.2,0.7,0.8)	(0.35,0.6,0.6)	(0.8,0.2,0.15)	(0.5,0.4,0.45)	(0.8,0.2,0.15)	(0.9,0.1,0.1)
	(0.1,0.8,0.9)	(0,0,0)	(0.65,0.3,0.25)	(0.2,0.7,0.8)	(0.1,0.8,0.9)	(0.2,0.7,0.8)	(0.35,0.6,0.6)
	(0.9,0.1,0.1)	(0.1,0.8,0.9)	(0,0,0)	(0.65,0.3,0.25)	(0.35,0.6,0.6)	(0.8,0.2,0.15)	(0.2,0.7,0.8)
	(0.65,0.3,0.25)	(0.5,0.4,0.45)	(0.35,0.6,0.6)	(0,0,0)	(0.8,0.2,0.15)	(0.65,0.3,0.25)	(0.35,0.6,0.6)
	(0.2,0.7,0.8)	(0.8,0.2,0.15)	(0.1,0.8,0.9)	(0.35,0.6,0.6)	(0,0,0)	(0.35,0.6,0.6)	(0.35,0.6,0.6)
	(0.2,0.7,0.8)	(0.65,0.3,0.25)	(0.35,0.6,0.6)	(0.35,0.6,0.6)	(0.2,0.7,0.8)	(0,0,0)	(0.2,0.7,0.8)
	(0.2,0.7,0.8)	(0.2,0.7,0.8)	(0.1,0.8,0.9)	(0.35,0.6,0.6)	(0.2,0.7,0.8)	(0.2,0.7,0.8)	(0,0,0)

Table 3. Aggregated direct relation matrix.

(0,0,0)	(0,0,0)	(0.24,0.7,0.77)	(0.84,0.16,0.13)	(0.44,0.48,0.52)	(0.77,0.22,0.18)	(0.82,0.18,0.15)
(0.21,0.71,0.8)	(0,0,0)	(0.47,0.47,0.49)	(0.2,0.72,0.81)	(0.2,0.72,0.81)	(0.18,0.74,0.83)	(0.28,0.66,0.71)
(0.86,0.14,0.12)	(0.14,0.76,0.86)	(0,0,0)	(0.5,0.42,0.45)	(0.2,0.72,0.81)	(0.82,0.17,0.15)	(0.2,0.72,0.81)
(0.56,0.36,0.37)	(0.57,0.36,0.37)	(0.19,0.73,0.82)	(0,0,0)	(0.68,0.28,0.26)	(0.59,0.34,0.34)	(0.55,0.39,0.4)
(0.24,0.7,0.77)	(0.77,0.22,0.18)	(0.18,0.74,0.83)	(0.55,0.39,0.4)	(0,0,0)	(0.24,0.69,0.76)	(0.41,0.52,0.54)
(0.25,0.68,0.75)	(0.47,0.46,0.49)	(0.52,0.43,0.43)	(0.25,0.68,0.75)	(0.24,0.7,0.77)	(0,0,0)	(0.21,0.71,0.79)
(0.2,0.72,0.81)	(0.24,0.7,0.77)	(0.19,0.74,0.83)	(0.45,0.49,0.51)	(0.25,0.69,0.76)	(0.21,0.7,0.79)	(0,0,0)

Table 4. Crisp matrix.

0.0000	0.5756	0.5709	0.3768	0.4804	0.3893	0.3817
0.5743	0.0000	0.4748	0.5773	0.5772	0.5845	0.5528
0.3737	0.5893	0.0000	0.4594	0.5782	0.3818	0.5782
0.4323	0.4318	0.5786	0.0000	0.4090	0.4235	0.4445
0.5697	0.3893	0.5839	0.4445	0.0000	0.5641	0.4922
0.5608	0.4736	0.4606	0.5615	0.5697	0.0000	0.5705
0.5756	0.5697	0.5825	0.4846	0.5627	0.5701	0.0000

Table 5. Normalized matrix.

0.0000	0.1721	0.1706	0.1126	0.1436	0.1164	0.1141
0.1717	0.0000	0.1419	0.1726	0.1725	0.1747	0.1652
0.1117	0.1762	0.0000	0.1373	0.1728	0.1141	0.1728
0.1292	0.1291	0.1729	0.0000	0.1223	0.1266	0.1329
0.1703	0.1164	0.1745	0.1329	0.0000	0.1686	0.1471
0.1676	0.1416	0.1377	0.1679	0.1703	0.0000	0.1705
0.1721	0.1703	0.1741	0.1449	0.1682	0.1704	0.0000

Table 6. R+C and R-C values.

Factor	R	C	R+C	R-C	Ranking	Category
F1	9.661940497	10.57232732	20.23426781	-0.910386821	6	Effect
F2	11.33145586	10.4298902	21.76134606	0.901565654	1	Cause
F3	10.27204608	11.08162812	21.35367419	-0.809582038	3	Effect
F4	9.466186835	10.01967411	19.48586094	-0.553487274	7	Effect
F5	10.43465839	10.86974451	21.3044029	-0.435086118	4	Effect
F6	10.89789035	10.06108136	20.95897171	0.836808987	5	Cause
F7	11.3647677	10.39460009	21.75936778	0.97016761	2	Cause

Table 4 above shows the crisp matrix and *Table 5* show the normalized direct relation matrix. *Table 6* presents the resulting R+C and R-C values together with its ranking order and cause effect group. *Fig. 2* below illustrates the causal diagram based on the R+C and R-C values from the PN-NWBM DEMATEL calculation. Based on the ranking order, F2 has become the most influential factor and F4 as the least influential factor. It can also be seen that F2, F6 and F7 lies in the cause group while F1, F3, F4 and F5 lies in the effect group.

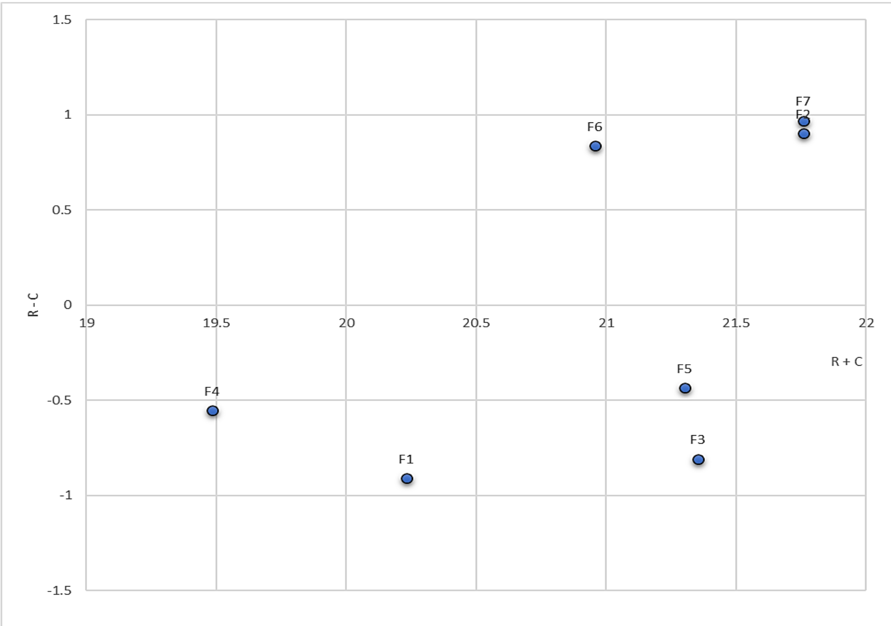

Fig. 2. Causal diagram.

6 | Results and Findings

In this section, we present the sensitivity analysis to explore how variations in parameters p and q influnce the PN-NWBM calculation and also comporative analysis between the PN-NWBM DEMATEL, SVNNWBM DEMATEL and PN-DEMATEL with arithmetic mean aggregation.

6.1 | Sensitivity Analysis

Sensitivity analysis was done by varying the parameters p and q in the calculation using PN-NWBM operators. *Table 7* illustrates how these parameter changes affect the DEMATEL results, including ranking order and Cause-Effect (CE) group category.

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Table 7. Affect the DEMATEL results.

Parameter	F	R+C	R-C	Rank	Difference	Category	Difference
p = 1 , q = 2	F1	20.21064797	-0.907857972	6	No changes in ranking	Effect	No changes in group category
	F2	21.74054272	0.898203904	1		Cause	
	F3	21.33820088	-0.8096818	3		Effect	
	F4	19.47079406	-0.548911372	7		Effect	
	F5	21.28184782	-0.434557005	4		Effect	
	F6	20.93809075	0.829696155	5		Cause	
	F7	21.73527706	0.973108091	2		Cause	
Parameter	**F**	**R+C**	**R-C**	**Rank**	**Difference**	**Category**	**Difference**
p = 2 , q = 1	F1	20.26576603	-0.907394918	6	No changes in ranking	Effect	No changes in group category
	F2	21.80252519	0.902110079	1		Cause	
	F3	21.40046336	-0.809242054	3		Effect	
	F4	19.52622841	-0.548970418	7		Effect	
	F5	21.3451787	-0.437725326	4		Effect	
	F6	21.0007347	0.828744556	5		Cause	
	F7	21.79650599	0.972478081	2		Cause	
Parameter	**F**	**R+C**	**R-C**	**Rank**	**Difference**	**Category**	**Difference**
p = 1 , q = 5	F1	21.25590337	-0.91501019	6	No changes in ranking	Effect	No changes in group category
	F2	22.84333222	0.880278442	1		Cause	
	F3	22.41796735	-0.767736461	3		Effect	
	F4	20.57476121	-0.528378528	7		Effect	
	F5	22.34485545	-0.44844022	4		Effect	
	F6	22.01696838	0.814296201	5		Cause	
	F7	22.81857971	0.964990756	2		Cause	
Parameter	**F**	**R+C**	**R-C**	**Rank**	**Difference**	**Category**	**Difference**
p = 10 , q = 1	F1	22.66661775	-0.900719607	6	Slight changes where F2 interchange rank with F7 and F3 interchange rank with F5	Effect	No changes in group category
	F2	24.05814678	1.036901298	2		Cause	
	F3	23.58671103	-0.898867095	4		Effect	
	F4	21.99559391	-0.474352529	7		Effect	
	F5	23.74793719	-0.462792626	3		Effect	
	F6	23.43020006	0.784888108	5		Cause	
	F7	24.18478804	0.914942452	1		Cause	
Parameter	**F**	**R+C**	**R-C**	**Rank**	**Difference**	**Category**	**Difference**
p = 1 , q = 20	F1	170.131852	-0.736038426	7	Major changes in F2, F3 and F7 ranking while slight changes in F1, F5 and F6	Effect	F2 now become Effect group while the others remain the same
	F2	177.3393278	-1.366489905	1		Effect	
	F3	176.2492714	-0.420580016	2		Effect	
	F4	173.7068337	-0.32371644	5		Effect	
	F5	175.0774596	-0.275871019	3		Effect	
	F6	172.3896808	2.504506013	6		Cause	
	F7	174.8814991	0.618189793	4		Cause	
Parameter	**F**	**R+C**	**R-C**	**Rank**	**Difference**	**Category**	**Difference**
p = 15 , q = 20	F1	19.06379124	-1.005490149	7	Similar with the above changes	Effect	Similar with above where F2 now lies in effect group while the others remain the same
	F2	20.45839382	0.586502985	1		Effect	
	F3	20.40349107	0.088017955	2		Effect	
	F4	18.39145137	1.215849306	5		Effect	
	F5	19.07312988	-0.472231209	4		Effect	
	F6	19.18246055	-0.802675137	6		Cause	
	F7	19.86860682	0.390026249	3		Cause	

The findings on sensitivity analysis are relatively stable across different values of parameters suggesting less fragility and provides greater degree of assurance. This stability instills confidence in the reliability of theresults, suggesting that the outcomes are not heavily contingent on specific assumptions or conditions by maintaining their validity across a spectrum of realistic situations. However bigger variations in the parameter somehow showed some significant effect in the result suggesting influence of bigger parameter involved in the calculation.

6.2 | Comparative Analysis

Next, we conducted a comparative analysis between PN-NWBM aggregation operator with Single Value Neutrosophic Set Normalized Weighted Bonferroni Mean (SVNNWBM), as well as the Pythagorean neutrosophic arithmetic mean aggregation in DEMATEL as shown in *Table 8* below:

Table 8. Comparative analysis.

	PN-NWBM Aggregation				SVN-NWBM Aggregation				Arithmetic Mean Aggregation			
	R+C	Rank	R-C	CE	R+C	Rank	R-C	CE	R+C	Rank	R-C	CE
F1	20.234	6	-0.91	E	20.69	6	-0.91	E	20.5504	6	-0.92	E
F2	21.761	1	0.9	C	22.22	1	0.9	C	22.0567	2	0.88	C
F3	21.354	3	-0.81	E	21.81	3	-0.77	E	21.6428	3	-0.78	E
F4	19.486	7	-0.55	E	19.91	7	-0.56	E	19.7982	7	-0.55	E
F5	21.304	4	-0.44	E	21.75	4	-0.44	E	21.5768	4	-0.41	E
F6	20.959	5	0.84	C	21.39	5	0.81	C	21.2255	5	0.83	C
F7	21.759	2	0.97	C	22.21	2	0.96	C	22.0570	1	0.96	C

The findings of this comparative analysis indicate a striking consistency in the ranking of factors and CE group between PN-NWBM DEMATEL and SVNNWBM-DEMATEL. It can also be seen that conventional DEMATEL method using arithmetic mean as its aggregation operator gives a slight different ranking for the F2 and F7. However the difference is very small difference which is 0.0003. This validates the utilization PN-NWBM aggregation operator within DEMATEL and PNS framework.

7 | Conclusion

In conclusion, this research focused on the application of the BM aggregation operator within a Pythagorean neutrosophic framework. The main objective was to introduce and validate a new normalized weight BM, known as PN-NWBM, for PNS. BMs was introduced in 1950 and have been studied by many authors. While additional extensions have been developed in recent decades, their primary objective remains consistent which to capture the explicit interconnection among the criteria. Throughout the study the normalized weight BM has been proposed as the aggregation operator for combining all the decision makers' decision into a comprehensive evaluation. By incorporating the BM operator into the Pythagorean neutrosophic environment, we aimed to improve the precision and reliability of the decision-making process using DEMATEL method. The proposed PN-NWBM method demonstrated promising results and was rigorously validated through various experiments and comparisons.

Furthermore, this research contributes to the existing literature by expanding the applications of the BM operator and addressing its effectiveness in Pythagorean neutrosophic environments. The findings emphasize the potential of PN-NWBM as a practical means of aggregating information in decision-making problems involving imprecise and uncertain data. It is worth noting that further research can explore additional aspects of the BM operator and Pythagorean neutrosophic environments. These may include investigating its performance in different domains, analyzing its computational complexity, and exploring its potential combination with other aggregation operators. Overall, this research paves the way for utilizing the BM aggregation operator in Pythagorean neutrosophic decision-making, and the proposed PN-NWBM contributes valuable insights and methodologies to this field. To further expand the scope of this research, future investigations should explore aggregation operators within the proposed model and their application in solving complex decision-making problems, see [26]-[30].

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