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Applications of Intuitionistic Fuzzy Sets to Assessment and Multi-Criteria Decision Making

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Abstract

The Intuitionistic Fuzzy Sets (IFSs) are generalizations of Zadeh's fuzzy sets, in which the elements of the universe have apart from Zadeh's membership and the degree of non-membership in [0, 1]. This paper studies applications of intuitionistic Fuzzy Sets (FS) to assessment and multi-criteria decision making, which are very useful when uncertainty characterizes the grades or parameters respectively assigned to the elements of the universal set. Applications to everyday life situations are also presented illustrating our results.

Keywords: Fuzzy sets, Intuitionistic FS, Soft set, Fuzzy assessment, Fuzzy decision making.

1 | Introduction

Assessment is a very important process connected to all human and machine activities, because it helps to improve the performance by avoiding mistakes or weak behaviors of the past. In many cases the assessment is performed using qualitative grades (linguistic expressions) instead of numerical scores. This happens either because the existing data are not exact, or for reasons of elasticity (e.g. from teacher to students). Obviously, in such cases the mean performance of a group cannot be assessed by calculating the mean value of the individual scores of its members. For tackling this problem, we have used in earlier works either triangular fuzzy numbers or grey numbers and we have shown that these two methods are equivalent [1].

Decision Making (DM), on the other hand, which is a fundamental activity of human cognition, can be defined as the process of choosing a solution among two or more alternatives, with the help of suitable criteria, aiming to obtain the best possible outcome for a given problem. The recent technological progress and the rapid changes happening to human society increased the complexity of the DM problems. Thus from the 1950's a systematic approach has been started to develop for DM, known as statistical decision theory, which is based on a synthesis of principles from several scientific fields, including probability theory, statistics, economics, psychology, etc. [2].

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Frequently in everyday life situations, however, the DM process takes place under conditions of uncertainty, which makes the application of the traditional DM methods for the solution of the corresponding problems difficult or even impossible. Several DM methodologies have been proposed for such cases, based on principles of the theory of Fuzzy Sets (FSs) [3] and of their extensions; e.g. see [4]–[7], etc.

It is well known that apart from assessment and DM, fuzzy FSs sets and their extensions have been used successfully in a great variety of other everyday life situations. For some recent interesting applications of FSs the reader may look at [8]–[13].

Atanassov's theory of Intuitionistic Fuzzy Sets (IFSs), introduced in 1986 [14], is a generalization of Zadeh's FSs. In fact, in an IFS all the elements of the universal set of the discourse are characterized, apart from Zadeh's membership degree, by the degree of non-membership too, both of which are taking values in [0, 1]. IFSs have found important applications to everyday life problems in which the use of FSs has not been proved to be too effective for obtaining the required solutions [15].

The main objective of this work is to use IFSs as tools for assessment and for multiple-criteria parametric DM. In fact, assessment cases frequently appear in everyday life, in which the evaluator is not sure for the accuracy of the qualitative grades assigned to the members of a group for characterizing their individual performance with respect to a certain activity (for example the new coach of a football team, the new teacher of a class, etc.). In such cases, using Intuitionistic Fuzzy Pairs (IFPs) instead of the uncertain qualitative grades, we assess the overall performance of the group with respect to the corresponding activity by calculating the mean value of these pairs. Our method is simple and straightforward providing, in case of uncertain individual grades, more trust-worthy results than other fuzzy assessment methods, used earlier in the past, did. In multiple-criteria parametric DM, on the other hand, frequently some of the parameters used happen to be of fuzzy texture (e.g. young people, expensive cars, etc.). In such cases, the characterization of the parameters with intuitionistic f IFPs uzzy pairs helps to make better decisions. A representative example is presented here, concerning the choice of a new employee by a company, illustrating the usefulness of our method in practice. Notice that IFSs have been used as tools by other researchers as well for multiple-criteria DM [16]–[18]. In any case, however, our method, which improves an earlier DM algorithm of Maji et al. [19] utilizing Soft Sets (SSs) as tools, is very easy in use resulting in worthy credit decisions.

The rest of the paper is formulated as follows: Section 2 includes the information about FSs, IFSs and Molodtsov's SSs [20], which is necessary for the understanding of the paper. The application of IFSs to assessment processes is developed in Section 3, whereas Section 4 introduces their application to DM. The last Section 5 includes the general conclusions and a discussion about perspectives of further research on the subject.

2 | Mathematical Background

2.1 | Fuzzy Sets

Zadeh [3] introduced in 1965 the notion of FS for tackling mathematically the partial truths and the definitions having no clear boundaries in the following way:

Definition 1 ([3]). A FS A in the universe U is of the form

$$A = \{(x, m(x)): x \in U\}. \tag{1}$$

In *Eq. (1)*, m: U→ [0, 1] is the membership function of A, and the number m(x) is called the membership degree of x in A. The closer m(x) to 1, the better x satisfies the property of A. An ordinary set is obviously a FS with membership function its characteristic function.

The definition of the membership function is not unique, depending on each observer's subjective criteria. Defining, for example, the FS of the good students of a class, one may consider all students with grades greater than 16 (in a scale from 0 to 20) as good and another one all those with grades greater than 17. As a result, the first observer will attach membership degree 1 to all students with grades between 16 and 17, whereas the second one will attach to them membership degrees <1. The only restriction for the membership function is to follow common sense, so that the corresponding FS is compatible with reality. This does not happen, for instance, if in the previous example students with grades <10 have membership degrees ≥0.5.

Later, when membership degrees were reinterpreted as possibility distributions, FSs were also used for tackling the existing in everyday life uncertainty, which is caused by the shortage of knowledge about the corresponding situation. Until then, the unique tool for dealing mathematically with uncertainty was probability theory. Probability, however, has been proved to be effective only for tackling the uncertainty which is due to randomness, like it happens, for example, with the games of chance. Possibility is an alternative mathematical theory for dealing with uncertainty. Zadeh [21] determined the difference between possibility and probability stating that whatever is probable it must be primarily possible. In particular, FSs are suitable for tackling the uncertainty due to vagueness, where one has difficulty distinguishing between two properties, like a mediocre and a good student. As a general reference for FSs and the related to them uncertainty we propose [22].

2.2 | Intuitionistic Fuzzy Sets

Following the introduction of FSs, a series of extensions and related theories have been proposed for tackling more effectively the existing in each particular case uncertainty [13]. Atanassov [14] in 1986 extended the concept of FS to that of IFS in the following way:

Definition 2 ([14]). An IFS A in the universal set U is of the form

$$A = \{(x, m(x), n(x)): x \in U, m(x), n(x) \in [0,1], \ 0 \leq m(x) + n(x) \leq 1\}. \quad (2)$$

In *Eq. (2)*, m: U → [0, 1] is the membership function and n: U → [0, 1] is the non-membership function of A and m(x), n(x) are the degrees of membership and non-membership respectively for each x in A. Further, h(x) = 1 - m(x) - n(x), is said to be the degree of *hesitation* of x in A. If h(x) = 0, then A is a FS.

The term IFS is due to Atanassov's collaborator Gargov in analogy to the idea of intuitionism introduced by Brouwer at the beginning of the last century. It is recalled that intuitionism rejects Aristotle's law of the excluded middle by stating that a proposition is either true, or not true, or we do not know if it is true or not true. The first part of this statement corresponds to Zadeh's membership degree, the second to Atanassov's non-membership and the third to the degree of hesitation.

Example 1. Let U be the set of all the players of a football team, let A be the IFS of the good players of the team, and let (x, 0.6, 0.3, 0.1) be in A. This means that there is a 60% belief that x is a good player, but simultaneously a 30% belief that he is not a good player and a 10% hesitation to characterize him as a good player or not.

A FS with membership function y = m(x) is obviously an IFS with non-membership function n(x) = 1-m[x) and hesitation degree h(x) = 0, for all x in U. IFSs can be used everywhere the ordinary FSs can be applied, but this is not always necessary. Suppose, for example, that one is interested in the degree of support that the governments of several countries have in their parliaments. For this, he calculates the quotient of the number of the ruling party's members of the parliament to the total number of its members. Here, therefore, the use of IFSs is not necessary. In the case of an election, on the contrary, a candidate can have voted for (membership), voted against (non-membership) and undecided (hesitancy) by the electoral vote. This is an example where the use of IFSs is necessary.

Atanassov [14] proposed the geometrical representation of an IFS shown in *Fig. 1*, where E denotes the universe, $\mu_A(x)$ the membership and $\nu_A(x)$ the non-membership degree of the element x of E.

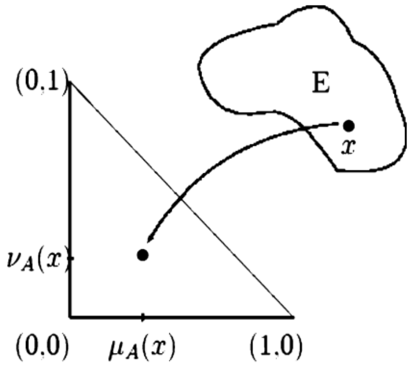

Fig. 1. Geometrical representation of an IFS.

The IFSs are suitable for tackling the uncertainty due to imprecision, which appears frequently in human reasoning and they have found various applications to many sectors of the human activity including career determination [23], weather broadcasting [24], artificial intelligence [14], DM [25], [26] medicine [27]–[30], logic[31], [32], choice of discipline of studies [33], etc. An interesting special issue on IFs and their applications consisting of 14 in total papers has been published in Volumes 8 (2020) and 9(2021) of the journal.

Here, for simplicity, we denote an IFS A by A = <m, n> and the elements of A in the form of IFPs (m, n), with m, n in [0, 1] and $0 \leq m + n \leq 1$. Considering the elements of an IFS A as ordered pairs only we define here addition and scalar multiplication in the usual way for the ordered pairs, i.e. as follows:

Definition 3. Let (m1, n1), (m2, n2) be in the IFS A and let r be a positive number. Then we define:

$$\text{sum } (m1, n1) + (m2, n2) = (m1 + m2,\ n1 + n2). \tag{3}$$
$$\text{scalar product } r(m_1, n_1 = (rm_1, r\,n_1). \tag{4}$$

The sum and the scalar product of the elements of an IFS A with respect to *Definition 3* need not be elements of A, since it could happen that (m1+ n2) + (m1+ n2)>1 or rm1 + rn1 >1. The mean value of a finite number of elements of A, however, defined below, is always in A.

Definition 4. Let A be an IFS and let (m1, n1), (m2, n2) …. , (mk, nk) be a finite number of elements of A. Then the mean value of these elements is defined to be the element of A

$$(m,\ n) = \frac{1}{k}[(m1, n1) + (m2, n2) + \ldots . + (mk, nk)]. \tag{5}$$

In fact, we always have that $\frac{1}{k}$ $[(m_1, n_1) + (m_2, n_2) + \ldots . + (m_k, n_k)] \leq 1$, which means (m, n) is always in A.

For an application of *Eq. (5)* see *Example 3*.

Remark 1: Smarandache, inspired by the various neutralities of the everyday life, like <win – draw – defeat> , <friend – neutral – enemy>, etc., extended further in 1995 the concept of IFS to that of Neutrosophic Set (NS) by introducing the degree of indeterminacy or neutrality in addition to those of membership and non-membership. The information that we have for an element x of the universe U is said to be complete, if m(x) + i(x) + n(x) = 1, incomplete, if m(x) + i(x) + n(x) < 1, and inconsistent, if m(x) + i(x) + n(x) > 1, where m(x), i(x), n(x) denote membership, indeterminacy and non-membership respectively. A NS can contain simultaneously elements leaving room to all the previous forms of information. In an IFS the indeterminacy coincides with the hesitation, whereas in a FS is n(x) = 1 – m(x) and i(x) = 0. For general facts on IFSs we refer to [15].

2.3 | Soft Sets and Parametric Decision Making

The difficulty of defining the membership function of a FS, which was described in Section 2.1, holds also for all their extensions involving membership functions, like IFSs, NSs, etc. Several solutions have been proposed for tackling this problem like:

I. The interval-valued FSs [34], in which the membership degrees are replaced by subintervals of [0, 1].
II. The rough sets [35], in which instead of a membership function one introduces a pair of sets as the lower and upper bound of the original set.
III. The grey numbers (closed intervals of real numbers) [36], where the definition of a membership function is not necessary.
IV. The SSs [20], tackling the uncertainty in a parametric manner, etc.

Molodtsov [20] introduced in 1999 the notion of SS in the following way:

Definition 5 ([20]). Let E be a set of parameters and let f be a map from E into the power set P(U) of the universal set U. Then the SS (f, E) in U is defined as a parameterized family of subsets of U by

$$(f, E) = \{(e, f(e)) \colon e \in A\}. \tag{6}$$

The term "soft" was introduced because the form of (f, E) depends on the parameters of E. A FS in U with membership function y = m(x) is a SS in U of the form (f, [0, 1]), where f(a) = {x ∈: m(x) > a} is the a-cut of the FS, ∀ a ∈ [0, 1].

Maji et al. [37] introduced the tabular representation of a SS for storing it easily in a computer's memory and they used it for parametric DM [19]. The following example illustrates their DM methodology.

Example 2. A company wants to employ a person among six candidates, say A_1, A_2, A_3, A_4, A_5 and A_6. The ideal qualifications for the new employee is to have satisfactory previous experience, to hold a university degree, to have a driving license and to be young. Assume that A_1, A_2, A_6 are the candidates with satisfactory previous experience, A_2, A_3, A_5, A_6 are the holders of a university degree, A_3, A_5 are the holders of a driving license and A_4 is the unique young candidate. Find the best decision for the company.

Solution: Set U = {A_1, A_2, A_3, A_4, A_5, A_6} and let F = {p_1, p_2, p_3, p_4} be the set of the parameters p_1=well experienced, p_2=holder of a university degree, p_3=holder of a driving license and p_4=young. Consider the map f: F → P(U) defined by f(p_1) = { A_1, A_2, A_6}, f(p_2) = { A_2, A_3, A_5, A_6}, f(p_3) = {A_3, A_5}, f(p_4) = { C_4}. Then the SS defined with respect to F and f is equal to

$$(f, F) = \{(p1, \{A1, A2, A6\}), (p2, \{A2, A3, A5, A6\}), (p3, \{A3, A5\}), (p4, \{A4\})\}. \tag{7}$$

The tabular representation of the SS (f, F), shown in *Table 1*, is formed by assigning to each candidate the binary element 1, if he/she satisfies the qualification addressed by the corresponding parameter, or the binary element 0 otherwise.

Table 1. Tabular matrix of the SS (f, F) of Example 2.

	P_1	P_2	P_3	P_4
A_1	1	0	0	0
A_2	1	1	0	0
A_3	0	1	1	0
A_4	0	0	0	1
A_5	0	1	1	0
A_6	1	1	0	0

Then, the choice value of each candidate is determined by adding the entries of the row of the tabular representation of (f, F) where it belongs. Thus, the candidates A_1 and A_4 have choice value 1 and all the others 2. The company, therefore must employ one of the candidates A_2, A_3, A_5 or A_6.

3 | Intuitionistic Fuzzy Assessment

When the assessment is performed with qualitative grades, cases appear in which one is not sure about the accuracy of the qualitative grades assigned to the objects under assessment. In such cases, the use of IFSs as tools provides a potential way for evaluating a group's overall performance. This new methodology is applied in the next example.

Example 3. The new coach of a football team is not sure yet about the quality of his players. He characterized, therefore, the good players as follows: p_1 and p_2 by (1, 0), p_3, p_4 and p_5 by (0.8, 0.1), p_6 and p_7 by (0.7, 0.2), p_8, p_9 and p_{10} by (0.6, 0.2), p_{11} by (0.5, 0.1), p_{12} and p_{13} by (0.5, 0.2), p_{14} by (0.3, 0.6), p_{15} and p_{16} by (0.2, 0.6), p_{17} by (0.1, 0.7), and the remaining three players by (0, 1). In other words, the coach believes that p_1 and p_2 are good players, he is 80% sure that p_3, p_4 and p_5 are good players too, but simultaneously he has a 10% belief that they may not be good players and a 10% hesitation to decide about it, and so on. For the last three players the coach is absolutely sure that they are not good players. Estimate the overall quality of the team according to the opinion of the coach about the individual quality of his players.

Solution: The overall quality of the football team can be estimated by calculating the mean value, say M = (m, n), of the elements of the IFS of the good players of the team. For this, according to the given data, *Eq. (5)* gives that

$$M = (m,n) = \frac{1}{20}[2(1,0) + 3(0.8,0.1) + 2(0.7,0.2) + 3(0.6,0.2) + (0.5,0.1) + 2(0.5,0.2) + (0.3,0.6) + 2(0.2,0.6) + (0.1,0.7) + 3(0,1)]. \tag{8}$$

Therefore, *Eq. (3)* and *Eq. (4)* give that

$$(m,n) = \frac{1}{20}(9.9.,7.3) = (0.495,0.365). \tag{9}$$

Consequently, a random player of the team is a good player by 49.5%, but simultaneously there is a 36.5% chance to be not a good player and a 14% hesitation for deciding whether he is a good player or not. These outcomes give a quite good idea about the overall quality of the football team.

Remark 2:

I. As we have already mentioned in Section 2.1, there is not any objective criterion for defining the membership function of a FS, its definition depending on the subjective criteria of each observer. The same problem obviously holds for all the extensions of FSs involving membership functions and in particular for the membership and non-membership function of the IFSs. As a result, the characterization of the individual performance of the players' by the coach using IFPs in the previous example was purely based on his personal criteria.

II. The assessment method applied in *Example 3* for estimating the overall quality of the football team is of general character. This means that it can be applied to all the cases of evaluating a group's overall performance when the qualitative grades assigned to its members are uncertain, provided that the evaluator is in position to assess the individual performance of the group's members using IFSs (see Remark 2(I)). These cases may include, for example, students' or teachers' performance, mean level of athletes' assessment, chess or bridge players' skills and even computer programs' effectiveness, industrial machines' or artificial intelligence's devices proficiency, race cars' competitions, etc.

III. In an earlier work [38], we introduced a similar assessment method by using NSs as tools instead of IFSs. The use of IFSs, however, involves less calculations and provides equally reliable results. The advantage of using NSs, on the other hand, is that they enable one to handle data providing incomplete or inconsistent assessment information; for example 40% belief that a player is good, 30% doubt about it, and at the same time 20% (40% respectively) belief that he is not a good player.

4 | Intuitionistic Fuzzy Decision Making

The DM method of [19], did not help very much the company of *Example 2* to choose the new employee, since it excluded only two among the six candidates for employment. This is due to the fact that in the tabular matrix of the corresponding SS the characterizations of the candidates by the corresponding parameters are replaced with the binary elements (truth values) 0, 1. In other words, the method of [19] although it starts from a fuzzy basis using SSs, then it uses bivalent logic for making the required decision, e.g. young or not young in *Example 2*. This could lead to inadequate decisions, if some of the parameters have a fuzzy texture; like the parameters "well-experienced" and "young" of *Example 2*.

For tackling this problem, in earlier works we have used grey numbers instead of the binary elements 0, 1in the tabular representation of the corresponding SS (e.g. see [38]). DM cases appear frequently in everyday life, however, in which the decision maker is not sure even for the accuracy of the grey numbers assigned to the parameters in the tabular DM matrix. In such cases we have used NSs as tools for DM instead of grey numbers [7]. The use of IFSs instead of NSs, however, provides equally good results too, requiring less calculations. This is illustrated by *Example 4*.

Example 4. Revisiting *Example 2*, assume that the company has doubts about the accuracy of the binary characterizations 0, 1 assigned to each of the six candidates with respect to the parameters p_1 and p_4. It was decided, therefore, to replace the binary elements of the tabular representation of the corresponding SS by IFPs. In this case the tabular representation of the DM procedure takes the form shown in *Table 2*. Which will be the optimal decision for the company now?

Table 2. Tabular representation of the DM process.

	P_1	P_2	P_3	P_4
A_1	(1, 0)	(0,1)	(0,1)	(0.6, 0.1)
A_2	(1, 0)	(1,0)	(0,1)	(0.2, 0.6)
A_3	(0.5,0.1)	(1,0)	(1,0)	(0.6, 0.2)
A_4	(0.5,0.3)	(0,1)	(0,1)	(1, 0)
A_5	(0.5,0.4)	(1,0)	(1,0)	(0.6, 0.1)
A_6	(1, 0)	(1,0)	(0,1)	(0.4, 0.2)

Solution: The four last columns of *Table 2* contain the elements of the IFSs of the candidates who are well experienced, holders of a university degree, holders of a driving license and young respectively.

By *Eq. (3)* to *Eq. (5)* the choice value of A_1 is equal to

$$\frac{1}{4}\,[(1,0)\ +\ 2(0,1)\ +\ (0.6,0.1)]\ =\ \frac{1}{4}(1.6,2.1)\ =\ (0.4,0.525). \tag{10}$$

In the same way one finds that the choice values of A_2, A_3, A_4, A_5 and A_6 are equal to (0.55, 0.4), (0.775, 0.075), (0.375, 0.575), (0.775, 0.125) and (0.6, 0.3) respectively. The company, therefore, may choose either the candidate with the greatest membership degree, or the candidate with the lower non-membership degree, i.e. either one of the candidates A_3 and A_5, or the candidate A_3. A combination of the two criteria leads to the final choice of the candidate A_3. Observe, however, that, since the hesitation degree of A_3 is 0.15 and of A_5 is 0.1, the choice of A_3 is connected with a slightly greater risk. In final analysis, therefore, all the intuitionistic degrees assigned to each candidate give useful information about his suitability.

Remark 3:

I. Our DM algorithm applied in *Example 4* provides better decisions than the DM method of Maji et al. [19] in cases where the decision maker is not sure for the accuracy of the DM parameters. The characterization of the parameters by IFPs, however, is based completely on the personal criteria of the decision maker, since, as we have already explained, there is not any objective criterion available for defining the membership and non-membership functions of an IFS.

II. This DM algorithm, due to its generality, can be also used in all cases of parametric multi-criteria DM involving fuzzy parameters. Examples that have been already presented in earlier author's works (using GNs and NSs instead of IFSs) are related with decisions for buying a house [38], or a car and for choosing a new player for a football team [7].

III. Weighted DM: In multiple-valued DM processes the decision-maker's goals are not always equally important. In such cases, weight coefficients are assigned to each parameter, whose sum is equal to 1. Assume, for instance, that the weight coefficients 0.4, 0.3, 0.2 and 0.1 have been assigned to the parameters p1, p2, p3 and p4 respectively of *Example 4*. Then the weighted choice value of the candidate A1 is equal to

$$\frac{1}{4}[0.4(1,0) + 0.3(0,1) + 0.2(0,1) + 0.1(0.6,0.1)] = \frac{1}{4}\ (0.46,0.51) = (0.115.\,0.1275). \tag{11}$$

In the same way one finds that the choice values of the candidates A_2, A_3, A_4, A_5 and A_6 are (0.18, 0.065), (0.19, 0.015), (0.075, 0.115), (0.19, 0.0425) and (0.185, 0.055) respectively. The combination of the two criteria, therefore, shows again that the best decision for the company is to employ the candidate A_3.

5 | Discussion and Hints for Future Research

Probability, FSs and the related to them theories [23] are not able to tackle effectively, each one alone, all the types of the existing in everyday life and science uncertainty. Each of them is suitable for tackling some types of uncertainty only, e.g. probability for randomness, FSs for vagueness, IFSs for imprecision in human thinking, NSs for ambiguity and inconsistency, etc. All these theories together, however, provide an adequate framework for dealing with the uncertainty. In particular, suitable combinations of two or more of these theories provide more effective ways for treating the uncertainty. This is, therefore, an interesting area for further research.

For example, Zadeh [39] in 1975 extended FS, which is now referred to as a type-1 *FS*, to the concept of type-2 *FS* for handling more uncertainty connected to the membership function. The membership function of a type-2 FS is three - dimensional, its third dimension being the value of the membership function at each point of its two – dimensional domain, which is called the Footprint of Uncertainty (FOU). The FOU is determined by its two bounding functions, a lower membership function and an upper membership function, both of which are type-1 FSs. When no uncertainty exists about the membership function, then a type-2 FS is reduced to a type-1 FS. Zadeh [39] in the same paper generalized the type-2 FS to the type-n FS, n = 1, 2, 3, … . In the late 1990s Prof. Jerry Mendel and his group cultivated further the theory of type-2 FS [9]. Since then, more and more papers are written about type-2 FSs, like [40], a recent very interesting work about Fourier-based, type -2 fuzzy neural networks effective for the solution of high dimensional problems. Also some remarkable papers have been written about type-3 FSs with important applications to renewable energy modelling prediction [41], to fractional learning algorithms [38], etc. The type-2 and type-3 FSs have been combined with IFSs and IVFSs to produce the concepts of type-2 [43] and type-3 IFS and IVFS [44], [45] respectively. Such kind of combinations frequently enable the successful modeling of complex real life situations leading to the solution of difficult related problems. This is therefore another important area for future research.

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6 | Final Conclusions

In this paper, IFSs were used as tools for assessment and for parametric, multiple-criteria DM processes. This is a very useful approach when one is not sure about the accuracy of the qualitative grades or parameters respectively assigned to the elements of the universal set.

Our DM method in particular, was obtained by adapting a parametric DM method of Maji et al. [19] which uses SSs as tools. In our method the binary elements 0, 1 of the tabular representation of the corresponding SS are replaced by IFPs.

Similar assessment and DM methodologies have been introduced by the present author using NSs instead of IFSs [38], [44]. The use of NSs has the advantage of covering cases with incomplete or inconsistent information about the assessment or the DM data. It involves, however, more calculations than those needed when using IFSs.

In general, the advantage of using IFSs or NSs as tools for assessment is that they enable one to estimate the overall performance of a group with respect to a certain activity when the qualitative grades assigned to its members are uncertain. On the contrary, a limitation of these methods is that they don't provide an exact measurement of the group's mean performance, as the TFNs or the GNs do when there is no uncertainty about the qualitative grades assigned to its members [1] (Sections 5 and 6).

In parametric DM, on the other hand, when some of the parameters involved are of fuzzy texture, the advantage of using elements of IFPs or neutrosophic triplets in the tabular matrix of the decision problem is that they help the decision maker to make a better decision, than that made by using the binary elements 0, 1 instead. This was fully illustrated in this work with the help of *Example 2* and *Example 4*

A limitation of both our assessment and DM methods using IFSs or NSs as tools is that, due to the lack of an objective criterion for defining the membership, non-membership and indeterminacy functions, the characterizations of the individual performance of the members of the group under assessment and of the decision parameters respectively are based on the personal criteria of the evaluator or the decision maker.

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