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On Solving Linear Fractional Programming Transportation Problems with Fuzzy Numbers

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Abstract

In this paper, we present a novel method for solving Fractional Transportation Problems (FTPs) with fuzzy numbers using a ranking function. The proposed method introduces a transformation technique that converts an FTP with fuzzy numbers into an FTP with crisp numbers by employing the robust ranking technique. Subsequently, we formulate two transportation problems, one for maximization and another for minimization, based on the given FTP. The optimal solution for the original FTP is then derived by leveraging the solutions obtained for the formulated transportation problems. By optimizing these single-objective transport problems, our method provides decision-makers with the ability to make satisfactory managerial decisions and evaluate economic operations when confronted with logistic problems involving fractional transportation. To demonstrate the effectiveness of the proposed approach, we present several illustrative examples that showcase its practical application and efficiency.

Keywords: Optimal solution, Fractional transportation problems, Denominator objective restriction method, Linear fractional programming problems.

1 | Introduction

One of the most crucial factors in economic and social activities is the transportation problem, which plays a fundamental role in management science and operations research. Transportation models offer a robust framework for effectively supplying goods to customers. Precisely, the transportation problem involves the shipment of homogeneous goods from multiple sources (Plants) to distinct destinations (warehouses). The main objective of this problem is to identify an optimal shipping schedule that minimizes the overall transportation cost (or maximizes the total transportation profit), all while ensuring compliance with the supply and demand constraints. Linear Fractional Programming (LFP) problems, which belong to the realm of Non-Linear Programming (NLP), constitute a specialized category within this domain, where the objective function and constraints are expressed as ratios of linear functions. The objective of LFP problems is to ascertain the best allocation of available resources, considering specific requirements. These problems often involve combining resources such as land, machinery, labor, and materials to produce different products. The objective of LFP is to determine the allocation of resources that optimizes specific performance

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indicators, including minimizing the unit cost of production, maximizing the profit-to-cost ratio, or optimizing the ratio between the total delivery speed and waste along the shipping route. Linear fractional models find applications in real-life scenarios such as hospital planning, healthcare, construction planning, commercial planning, and economics. Extensive research has been conducted on solving LFP problems. Charnes and Cooper [1] introduced the LFP model and developed a duality theory-based algorithm to solve LFP problems. Subsequently, various algorithms, including the Charnes-Cooper transformation [2], parametric approach [3], column generation approach [4] and penalty function methods [5], have been proposed to address LFP problems. In many practical optimization problems, some or all the problem parameters cannot be precisely determined due to incomplete information or a lack of knowledge. Fuzzy set theory, introduced by Zadeh [6], provides a mathematical framework for modeling uncertainty and vagueness. Fuzzy linear programming, which incorporates fuzzy parameters in the constraints and objective function of linear programming problems, has gained significant attention. Bellman and Zadeh [7] were the first to propose a fuzzy decision-making process based on fuzzy goals and fuzzy constraints. Tanaka et al. [8] developed an interactive fuzzy linear programming method by introducing the concept of a fuzzy feasible solution region. Maleki et al. [23] proposed a method for solving multi-objective fuzzy linear programming problems using comparison functions. Jiménez and Verdegay [9] described models and methods for solving fuzzy linear programming problems where the objective function coefficients and constraint right-hand sides are represented by fuzzy numbers. In recent years, several researchers have focused on solving fuzzy LFP problems. Chang and Hung [10] proposed a method based on the Charnes-Cooper transformation to find the fuzzy optimal solution and corresponding fuzzy dual solution of a fuzzy linear fractional program. Their study introduced a novel framework that integrating fuzzy logic and random variables into a multi-objective LFP model. The proposed approach aimed to optimize inventory management by considering multiple conflicting objectives. The results demonstrated the effectiveness of the fuzzy random-based approach in improving the overall performance of the inventory system. In another study, Khalifa, H.A., and Pavan Kumar [11] proposed a goal programming approach for solving multi-objective LFP problems with LR possibilistic variables. Their research focused on developing a mathematical framework to handle uncertainties and possible information in the context of LFP. The goal programming approach allowed decision-makers to set priorities and achieve satisfactory solutions considering multiple objectives. The study highlighted the importance of incorporating possibilistic variables in LFP models and illustrated the applicability of the proposed approach through numerical examples. Interval-type fuzzy LFP problem in the neuromorphic environment was explored by Khalifa and Kumar. [12]. They presented a fuzzy mathematical programming approach to solve this problem. By incorporating neuromorphic sets and fuzzy logic, the study aimed to address uncertainties and vagueness in the problem parameters. The proposed approach provided a flexible framework for decision-making under uncertain conditions, enhancing the robustness and effectiveness of solving interval-type fuzzy LFP problems. Also, the authors in [13], present a comprehensive study on solving linear interval Fractional Transportation Problems (FTPs) with interval objective functions. The objective of the study is to handle uncertainty in the coefficients of the objective function by proposing a novel approach that combines interval analysis and optimization techniques. The proposed method involves variable transformation, which simplifies the solution process and improves the accuracy of the results. In summary, while LFP and fuzzy programming problems have been studied separately, relatively little research has addressed fuzzy LFP problems, especially within the transportation problem structure. This paper aims to fill this gap by developing an effective algorithm to solve LFP transportation problems where some parameters are represented by triangular fuzzy numbers. For more details on the theory and algorithms for MOPs, interested readers can refer to [14]. Fractional calculus is a mathematical framework that extends the traditional concepts of differentiation and integration to non-integer orders. This powerful tool has found widespread applications in various fields, including physics, engineering, medicine, and finance [15]–[19]. Several notable papers have contributed to the understanding and utilization of fractional calculus [15], [16], [20]–[24]. Fractional calculus provides the mathematical basis for understanding and analyzing the behavior of systems involving fractional derivatives or integrals, while fractional programming applies these concepts to optimization problems. Fractional order operators in the objective function or constraints of Fractional Programming Problems

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(FPPs) capture the underlying fractional dynamics or characteristics of the problem at hand. By leveraging fractional calculus, FPPs can model and optimize systems with non-local or memory-dependent behavior more accurately. Fractional order operators in the objective function or constraints reflect the specific dynamics or properties of the system being modeled, leading to more robust and effective optimization solutions. FPPs appear in numerous practical areas, such as decision problems in management, stock cutting, portfolio selection, and game theory. Various approaches have been applied to FPPs with significant details. For example, Bitran and Novaes [5], Charnes and Cooper [1], Craven [14], Schaible [25], Stancu-Minasian [26], and Schaible and Ibaraki [27]. Charnes and Cooper [1] developed a method indicating that the LFP problem can be optimized by solving two linear programming problems. In a standard scenario of this kind, a trucking company might seek to identify the most economically efficient approach (minimizing cost or maximizing benefit per unit of transported product in LFP) for transporting significant volumes of goods from various storage facilities to multiple stores. The zero-point method was introduced by Pandian and Natarajan [28] as an alternative approach to determining the optimal solution for a traditional transportation problem, eliminating the need for conventional optimality-checking methods [28]. Transportation problems in fuzzy environments have gained considerable attention in recent years. This section provides a literature review of relevant articles addressing fuzzy transportation problems. Kumar [29] proposed the PSK method for solving type-1 and type-3 fuzzy transportation problems. The study presented a novel approach that effectively handled uncertainty and imprecision in transportation models. The PSK method demonstrated promising results in terms of finding optimal solutions, and it was discussed in the context of fuzzy systems applications. In a similar vein, Kumar [30], introduced a simple method for solving type-2 and type-4 fuzzy transportation problems. The study focused on extending the PSK method to handle higher degrees of fuzziness. The proposed approach exhibited efficiency and effectiveness in finding solutions for fuzzy transportation problems, contributing to the advancing of fuzzy logic and intelligent systems. Kumar [31] investigated the application of the PSK method for solving vague traffic problems. The study emphasized the importance of addressing uncertainty and vagueness in traffic-related issues. By employing the PSK method, the research provided insights into finding optimal solutions to traffic problems under uncertain conditions, contributing to the field of artificial life research and development. Mohideen and Kumar [32] conducted a comparative study on transportation problems in fuzzy environments. The research compared different approaches and techniques for solving fuzzy transportation problems and discussed their strengths and limitations. The study aimed to provide a comprehensive understanding of the existing methods in the field and their applicability in various scenarios. These articles collectively contribute to the body of knowledge on fuzzy transportation problems. The PSK method proposed by Kumar [33] and its extensions [32], [34]–[41], offer effective approaches to handling uncertainty and imprecision in transportation models. Additionally, Senthil and Kumar [42] provide valuable insights into the comparative analysis of different methods for solving fuzzy transportation problems. In the real world, the objective coefficients may not be in the form of crisp numbers, so in such problems, the numbers are considering as fuzzy. The proposed method helps us solve FTPs with fewer calculations than the modified distribution method for FTPs in [43]. The modified distribution method, commonly referred to as the MODI method, is employed to obtain a minimum-cost solution for the transportation problem. This method involves initially obtaining an initial solution and subsequently optimizing it through a series of calculations to attain the optimal solution. However, the proposed method achieves the optimal solution of the transportation problem with the least number of calculations. The following objectives collectively aim to provide an efficient and effective approach for solving FTPs while minimizing computational complexity and leveraging existing problem-solving techniques.

- *Propose a method for solving FTPs.*
- *Obtain a feasible solution for FTPs.*
- *Optimize the feasible solution using a modified distribution method.*
- *Address the complexity and computational demands associated with solving FTPs.*
- *Overcome the challenges of automation in solving FTPs.*
- *Handle deficit transportation issues involving multiple origins and destinations.*
- *Optimize single-objective transport problems instead of directly solving fractional transport problems.*

– *Reduce the number of calculations required for solving FTPs.*
– *Enable the utilization of linear programming problem-solving software with fewer linear calculations.*

The organization of the paper is as follows: Section 2 presents necessary the definitions and preliminaries related to the Fractional Transportation Simplex Method (FTSM). Section 3 describes the proposed method. To demonstrate the efficiency of our approach, Section 4 includes solved numerical examples. Finally, Section 5 provides a summary of the paper.

2 | Preliminaries

Linear Fractional Transportation Problems (LFTPs) is a special case of LFP problems. Hence, like a typical LFP problem, solving a LFTP Problem involves two main phases [44]:

I. Obtaining an initial Basic Feasible Solution (BFS).
II. Iteratively refining the current BFS to meet the optimality criterion.

Since the process of finding the initial BFS for LFTP follows the same approach as in the case of LFP, the emphasis will be placed on the second phase.

To provide a general overview, an LFTP typically consists of the following components:

I. A set of m supply stores responsible for shipping goods.
II. A set of n demand points (or shops) that receive the shipped goods.
III. Profit coefficients p_{ij}, which determine the profit p_{ij} obtained by the transportation company when transporting one unit of goods from a supply store to a specific demand point j.
IV. Cost coefficients d_{ij}, which represent the cost d_{ij} associated with transporting one unit of goods from a supply store to a particular demand point j.

Consider the below LFTP:

$$\text{LFTP)} \quad \text{Max } Q\ x) = \frac{P\ x)}{D\ x)} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}x_{ij} + p_0}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}x_{ij} + d_0}.$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \text{ for } i = 1,2,\dots,m. \tag{1}$$

$$\sum_{i=1}^{n} x_{ij} = b_j \text{ for } j = 1,2,\dots,n. \tag{2}$$

$$x_{ij} \geq 0,\ i = 1,2,\dots,m,\ j = 1,2,\dots,n. \tag{3}$$

From this point forward, it is assumed that D(x) is greater than zero for all x belonging to the set S, which is defined by *Eq. (1)* to *Eq. (3)* and represents a feasible solution space. Additionally, it is assumed that a_i and b_j are greater than zero for all i ranging from 1 to m and j ranging from 1 to n, respectively. Finally, it is assumed that the total demand is equal to the total supply. i.e. $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. The objective coefficients in a linear transportation problem aim to minimize the total cost of transportation (or maximize the total profit for transportation). Nevertheless, in LFTPs, the objective coefficients are formulated to either maximize the ratio of the total delivery speed to the total waste along the shipping route or maximize the ratio of the total profit to the total cost [24].

Definition 1 ([43]). A set $X^0 = \{x_{ij},\ i = 1,2,\dots,m;\ j = 1,2,\dots,n\}$ is said to be feasible for LFTP if X^0satisfies *Eqs. (2)* and *(3)*.

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Definition 2 ([44]). Consider the set R of real numbers. A fuzzy number, denoted by $\tilde{a}$, can be described as a mapping that satisfies the following conditions

I. μ_a is continuous.
II. μ_a on $[a_1, a_2]$ is ascending and continuous.
III. μ_a on $[a_3, a_4]$ is descending and continuous.

Where a_1, a_2, a_3 and a_4 are real numbers and fuzzy number is shown as $a = [a_1, a_2, a_3, a_4]$ and it is called a trapezoidal fuzzy number.

Definition 3 ([44]). If $\tilde{a}$ is a trapezoid fuzzy number, the membership function is as follows:

$$\mu_a\ x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2, \\ 1, & a_2 \le x \le a_3, \\ \dfrac{x - a_4}{a_3 - a_4}, & a_3 \le x \le a_4. \end{cases}$$

Definition 4 ([4]). The α-cut set of a fuzzy number a, denoted by A_α, can be defined as follows:

$$A_\alpha = \left\{x \middle| \mu_a\ x) \ge \alpha\right\} = \left[a_\alpha^l, a_\alpha^u\right].$$

Definition 5 ([4]). If $\tilde{a}$ is a fuzzy number, then the robust ranking technique is defined as follows:

$$R\ a) = \frac{1}{2}\int_0^1 (a_\alpha^l + a_\alpha^u) d\alpha,$$

where $[a_\alpha^l, a_\alpha^u]$ represents the α-cut of the fuzzy number a.

Definition 6 ([4]). Consider two fuzzy numbers, a and b. The following statements hold true:

I. $a \ge b$ if and only if $R\ a) \ge R(b)$.
II. $a = b$ if and only if $R\ a) = R(b)$.
III. $a \le b$ if and only if $R\ a) \le R(b)$.

Definition 7 ([4]). Consider $a = (a_1, a_2, a_3, a_4)$ and $b = (b_1, b_2, b_3, b_4)$ as two trapezoidal fuzzy numbers then:

I. $a \oplus b = \ a_1,a_2,a_3,a_4) \oplus\ b_1,b_2,b_3,b_4) = (a_1+b_1,\ a_2+b_2, a_3+b_3, a_4 + b_4)$.
II. $a \ominus b = \ a_1,a_2,a_3,a_4) \ominus\ b_1,b_2,b_3,b_4) = (a_1-b_1,\ a_2-b_2, a_3-b_3, a_4-b_4)$.
III. $ka = k\ a_1,a_2,a_3,a_4) = \begin{cases} ka_1,ka_2,ka_3,ka_4), \text{ for } k \ge 0, \\ ka_4,ka_3,ka_2,ka_1), \text{ for } k<0. \end{cases}$
IV. $a \otimes b = \ a_1,a_2,a_3,a_4) \otimes\ b_1,b_2,b_3,b_4) = \ t_1,t_2,t_3,t_4)$.

Where

$$t_1 = \text{minimum}\{a_1b_1, a_1b_4, a_4b_1, a_4b_4),$$
$$t_2 = \text{minimum}\{a_2b_2, a_2b_3, a_3b_2, a_3b_3),$$
$$t_3 = \text{maximum}\{a_2b_2, a_2b_3, a_3b_2, a_3b_3),$$

And

$$t_4 = \text{maximum}\{a_1b_1, a_1b_4, a_4b_1, a_4b_4).$$

V. $a/b = a_1,a_2,a_3,a_4)/\ b_1,b_2,b_3,b_4) = (a_1/b_4, a_2/b_3, a_3/b_2, a_4/b_1)$.

3 | The Proposed Method

In the real world, objective coefficients in transportation problems may not always be precise or crisp numbers. In such cases, these coefficients are considered as fuzzy numbers. The proposed method offers a solution for FTPs that requires fewer calculations compared to the modified distribution method presented in [10]. The objective of the modified distribution method, commonly referred to as the MODI method, is to obtain a solution to the transportation problem that minimizes the overall cost. It involves obtaining an initial solution and then optimizing it through numerous calculations to reach the optimal solution. In contrast, the proposed method achieves the optimal solution of the transportation problem with significantly fewer calculations. Next, we formulate two separate linear programming problems based on the given LFTP, each focusing on a single objective. The construction of these problems is as follows:

(N) Maximize $P\ X) = \sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}x_{ij} + p_0$, *Eq. (1)* to *Eq. (3),* and (D) Minimize $D\ X) = \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}x_{ij} + d_0$, *Eq. (1)* to *Eq. (3).*

Furthermore, it is assumed that $a_i > 0$ and $b_j > 0$, for i=1, 2, ..., m; j = 1, 2, ..., n, indicating positive values for the supply stores and demand points. Additionally, the total demand is equal to the total supply, i.e. $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ for problem (N) and problem (D).

Inspired by the work in [28], the two below theorems that connect the solutions of LFTP, the problem (D), and the problem (N) are proved to be used in the proposed method.

Theorem 1 ([28]). Consider X_0 as an optimal solution of (N). Suppose that $\{X_n\}$ be a BFSs sequence of (D) in which $Q\ X_k) \leq Q\ X_{k+1})\ for\ all\ k = 0,1,2,\dots,n,$ and $Q(X_n) \geq Q(X_{n+1})$ by applying the simplex method, the solution X_0 is regarded as the initial feasible solution, and subsequently, X_n represents the optimal solution for the LFTP problem.

Proof: the proof is in [28].

Theorem 2 ([28]). Consider X_0 as an optimal solution of (N). Suppose that $\{X_n\}$ be a BFSs sequence of (D) in which $Q(X_k) \leq Q(X_{k+1})$ for all k=0, 1, 2, ..., n. Let X_{n+1} be an optimal solution for problem (D) obtained by utilizing the simplex method, with X_0 considered as the initial feasible solution. Consequently, X_{n+1} is regarded as the optimal solution for the LFTP problem.

Proof: the proof is in [28].

2.1 | Proposed Method

In this context, a novel approach is presented for obtaining an optimal solution to the LFTP problem. The methodology of the proposed method is outlined as follows:

The Proposed Method Consists of the Following Steps

Step 1. Utilize the robust ranking technique to transform the FTP with fuzzy numbers into the FTP with classic or crisp numbers.

Step 2. Formulate the problem as two single objective linear programming problems, denoted as (N) and (D).

Step 3. Solve problem (N) using the simplex method to obtain the optimal solution, denoted as X_0, and the corresponding maximum value of the objective function Q, denoted as Q_0.

Step 4. Use X_0 as the initial feasible solution for problem (D) and solve it using the simplex method to obtain an improved sequence of BFSs $\{X_n\}$ and the corresponding Q values at each improved BFS.

Step 5.

I. If $Q(X_k) \leq Q(X_{k+1})$ holds true for all $k = 0, 1, 2, \ldots, n$, and $Q(X_n) \geq Q(X_{n+1})$ for a given value of n, terminate the computation procedure. According to *Theorem 1*, it can be concluded that $X_n$is an optimal solution to the LFTP problem, and the maximum value of $Q\ X)$ is achieved at $Q(X_n)$.

II. If $Q(X_k) \leq Q(X_{k+1})$ holds true for all $k = 0, 1, 2, \ldots, n$, and X_{n+1} represents an optimal solution to problem (D) for a specific value of n, halt the computation procedure. As per *Theorem 2*, it can be inferred that X_{n+1} is an optimal solution to the LFTP problem, and the maximum value of $Q\ X)$ is attained at $Q\ X_{n+1})$.

3 | Numerical Examples

In this section, we give some illustrative numerical examples to solve LFTP problems.

Example 1. Suppose we are considering a single objective, which is to maximize the ratio of the total delivery speed to the total waste along the shipping route. In this case, we are using fuzzy numbers to represent the values. The fuzzy number representation of the ratio of the total delivery speed to the total waste along the shipping route is shown in *Table 1* [44].

Table 1. Fractional transportation table with fuzzy numbers.

Destination→ Source↓	1	2	3	4	Supply
1	(9, 10, 10, 11)	(13, 13, 14, 16)	(6, 7, 9, 10)	(10, 10, 13, 15)	150
	(12, 14, 17, 17)	(11, 11, 13, 13)	(14, 14, 17, 19)	(5, 7, 9, 11)	
2	(7, 8, 8, 9)	(9,11,13,15)	(10, 13, 16, 17)	(6,6,9,11)	250
	(8, 9, 11, 12)	(5,5,7,7)	(10, 11, 14, 17)	(9,11,13,15)	
3	(7, 8, 10, 11)	(4, 5, 6, 9)	(12, 15, 16, 17)	(7, 9, 10, 10)	200
	(10, 13, 14, 15)	(13, 14, 16, 17)	(9, 11, 12, 16)	(7, 8, 11, 14)	
Demand	140	180	120	16	

Through the utilization of the robust ranking technique, it becomes feasible to convert bi-objective transportation problems, which encompass fuzzy numbers, into bi-objective transportation problems that employ classic or crisp numbers. The resultant problem will exhibit the following characteristics, as depicted in *Table 2*:

Table 2. Fractional transportation table with crisp numbers [42].

Destination→ Source↓	1	2	3	4	Supply
1	10	14	8	12	150
	15	12	16	8	
2	8	12	14	8	250
	10	6	13	12	
3	9	6	15	9	200
	13	15	12	10	
Demand	140	180	120	160	

From the problem, we formulate two linear programming problems, denoted as (N) and (D), as illustrated in *Tables 3* and *4*, respectively.

Table 3. Problem (N).

Destination→ Source ↓	**1**	**2**	**3**	**4**	**Supply**
1	10	14	8	12	150
2	8	12	14	8	250
3	9	6	15	9	200
Demand	140	180	120	160	

Table 4. Problem (D).

Destination→ Source ↓	**1**	**2**	**3**	**4**	**Supply**
1	15	12	16	8	150
2	10	6	13	12	250
3	13	15	12	10	200
Demand	140	180	120	160	

The optimal solution of (N) is as follows:

($x_{14} = 150$, $x_{21}=60$, $x_{22}=180$, $x_{24}=10$, $x_{31}=80$, $x_{33}=120$) and the value of $Q_0\ x) = \frac{176}{137}$. Subsequently, the optimal solution obtained from problem (N) can be employed as the initial simplex solution for problem (D), as depicted in *Table 5*.

Table 5. The optimal solution of (N) as an initial simplex solution to (D).

Destination→ Source ↓	**1**	**2**	**3**	**4**	**Supply**
1	15	12	16	8	150
				150	
2	10	6	13	12	250
	60	**180**		**10**	
3	13	15	12	10	200
	80		**120**		
Demand	140	180	120	160	

The value of Q for each improved BFS is determined by employing the modified distribution method. Now, the variables x_{34} and x_{24} enter and leave from the basis, respectively. A rectangular loop (3,4)-(2,4)-(2,1)-(3,1)-(3,4) is constructed. Therefore, we have the reduced table as follows (*Table 6*):

Table 6. Improved table.

Destination→ Source ↓	**1**	**2**	**3**	**4**	**Supply**
1	15	12	16	8	150
				150	
2	10	6	13	12	250
	70	**180**			
3	13	15	12	10	200
	70		**120**	**10**	
Demand	140	180	120	160	

The optimal solution of (D) is ($x_{14} = 150$, $x_{21}=70$, $x_{22}=180$, $x_{31}=70$, $x_{33}=120$, $x_{34} = 10$) and the value of $Q_1\ x) = \frac{704}{543}$. Since $Q_1 > Q_0$, based on Step 4 of the proposed method, the optimal solution for the aforementioned LFP problem is determined to be ($x_{14} = 150$, $x_{21}=70$, $x_{22}=180$, $x_{31}=70$, $x_{33}=120$, $x_{34} = 10$) and Max $Q\ x) = \frac{704}{543}$.

Example 2. Suppose that there are single objectives being considered: the second objective function aims to maximize the ratio of the total profit to the total cost. The values associated with this objective function are represented as fuzzy numbers and are presented in *Table 7* as follows:

Table 7. Fractional transportation table with fuzzy numbers.

Destination→ Source ↓	1	2	3	4	Supply
1	(2, 3, 5, 6)	(3, 5, 8, 10)	(2, 3, 6, 9)	(0, 1, 3, 4)	180
	(0, 1, 1, 2)	(1, 2, 3, 6)	(1, 2, 5, 8)	(5, 5, 7, 7)	
2	(1, 1, 2, 4)	(3, 5, 6, 6)	(0, 0, 2, 2)	(2, 3, 5, 6)	220
	(2, 4, 5, 5)	(1, 2, 3, 6)	(4, 4, 7, 8)	(0, 1, 3, 4)	
3	(1, 1, 2, 4)	(0, 0, 1, 3)	(2, 2, 5, 7)	(2, 2, 4, 4)	200
	(3, 4, 6, 7)	(1, 3, 4, 4)	(2, 2, 3, 5)	(1, 2, 2, 3)	
Demand	130	190	120	160	

Through the utilization of the robust ranking technique, bi-objective transportation problems that incorporate fuzzy numbers can be transformed into bi-objective transportation problems that utilize crisp or classic numbers. This conversion process leads to a resulting problem that possesses the following characteristics, as illustrated in *Table 8*.

Table 8. Fractional transportation table with crisp numbers [42].

Destination→ Source ↓	1	2	3	4	Supply
1	4	6	5	2	180
	1	3	4	6	
2	2	5	1	4	220
	4	3	6	2	
3	2	1	4	3	200
	5	3	3	2	
Demand	130	190	120	160	

We can find the two LP problems (N) and (D) from the above problem as follows (*Tables 9* and *10*):

Table 9. Problem (N).

Destination→ Source ↓	1	2	3	4	Supply
1	4	6	5	2	180
2	2	5	1	4	220
3	2	1	4	3	200
Demand	130	190	120	160	

And problem (D):

Table 10. Problem (D).

Destination→ Source ↓	1	2	3	4	Supply
1	1	3	4	6	180
2	4	3	6	2	220
3	5	3	3	2	200
Demand	130	190	120	160	

The optimal solution of (N) is ($x_{11} = 50, x_{12}=130, x_{22}=60, x_{24}=160, x_{31}=80, x_{33}=120$) and the value of $Q_0\ x) = \frac{128}{85}$. Now, we use the optimal solution of (N) as an initial simplex solution to (D) as follows (*Table 11*):

Table 11. The optimal solution of (N) as an initial simplex solution to (D).

Destination→ Source ↓	1	2	3	4	Supply
1	15	12	16	8	180
	50	**130**			
2	10	6	13	12	220
		60		**160**	
3	13	15	12	10	200
	80		**120**		
Demand	130	190	120	160	

The value of Q at any improved BFS is found using a modified distribution method. Now, the variables x_{32} and x_{24} enter and leave from the basis, respectively. We construct a rectangular loop (3,2)-(1,2)-(1,1)-(3,1)-(3,2). Therefore, we have the below-reduced table (*Table 12*):

Table 12. Improved table.

Destination→ Source ↓	1	2	3	4	Supply
1	15	12	16	8	180
	130	**50**			
2	10	6	13	12	250
		60		**160**	
3	13	15	12	10	200
	80		**120**		
Demand	140	180	120	160	

The optimal solution of (D) is as follows:

($x_{11} = 130, x_{12}=50, x_{22}=60, x_{24}=160, x_{31}=80, x_{33} = 120$) and the value of $Q_1\ x) = \frac{29}{17}$. Since $Q_1 > Q_0$ and based on *Step 4* of the proposed method, the optimal solution for the given LFP problem is as follows:

$$(x_{11} = 130, x_{12}=50, x_{22}=60, x_{24}=160, x_{31}=80, x_{33} = 120) \text{ and Max Q } x) = \frac{29}{17}.$$

Remark 1: The proposed method assumes a certain level of availability and compatibility with linear programming problem-solving software. It may limit its applicability in cases where such software is not accessible or where alternative problem-solving approaches are preferred. Also, the proposed method focuses on optimizing single-objective transport problems rather than directly solving fractional transport problems. While this approach reduces computational complexity, it may not capture the full complexity of certain fractional transport problems that require multi-objective optimization.

5 | Conclusion

In conclusion, this paper presented an approach for solving FTPs by first obtaining a feasible solution and then optimizing it using a modified distribution method. However, it is worth noting that this method exhibits high complexity and computational demands, posing challenges for automation. Moreover, the presence of deficit transportation issues, involving multiple origins and destinations, further complicates the problem-solving process. To address these challenges, our proposed method adopts a different approach. Instead of directly solving FTPs, we utilize the classic single-objective transport problem-solving method. By optimizing single-objective transport problems, we minimize the computational burden associated with solving FTPs. This approach also allows for using linear programming problem-solving software, which reduces the number of linear calculations required. It is important to acknowledge that FTPs remain an active area of research, and future studies may develop new methods to tackle multi-objective FTPs. These advancements will contribute to further improvements in solving complex transportation problems. In summary, our proposed method offers

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a promising solution for FTPs by leveraging the strengths of the single-objective transport problem-solving method. By mitigating computational complexity and utilizing existing software tools, we aim to facilitate more efficient and effective resolution of FTPs. We look forward to future research endeavors that will continue to advance the field of FTPs. In the future, new methods may be developed to solve multi-objective FTPs. Furthermore, there is potential for the discovery of novel methods to address fractional multi-objective FTPs with fuzzy coefficients.

Author Contribution

Conceptualization, A.S. and M.J.E.; methodology, A.S.; software, A.S.; validation, A.S.; formal analysis, A.S.; investigation, A.S.; resources, A.S.; data maintenance, A.S.; writing-creating the initial design, A.S. and M.J.E.; writing-reviewing and editing, A.S. and M.J.E.; visualization, A.S.; monitoring, A.S.; project management, M.J.E.; funding procurement, M.J.E. All authors have read and agreed to the published version of the manuscript.

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Data Availability

We have not used any data during the study.

Conflicts of Interest

The authors declare no conflict of interest.

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