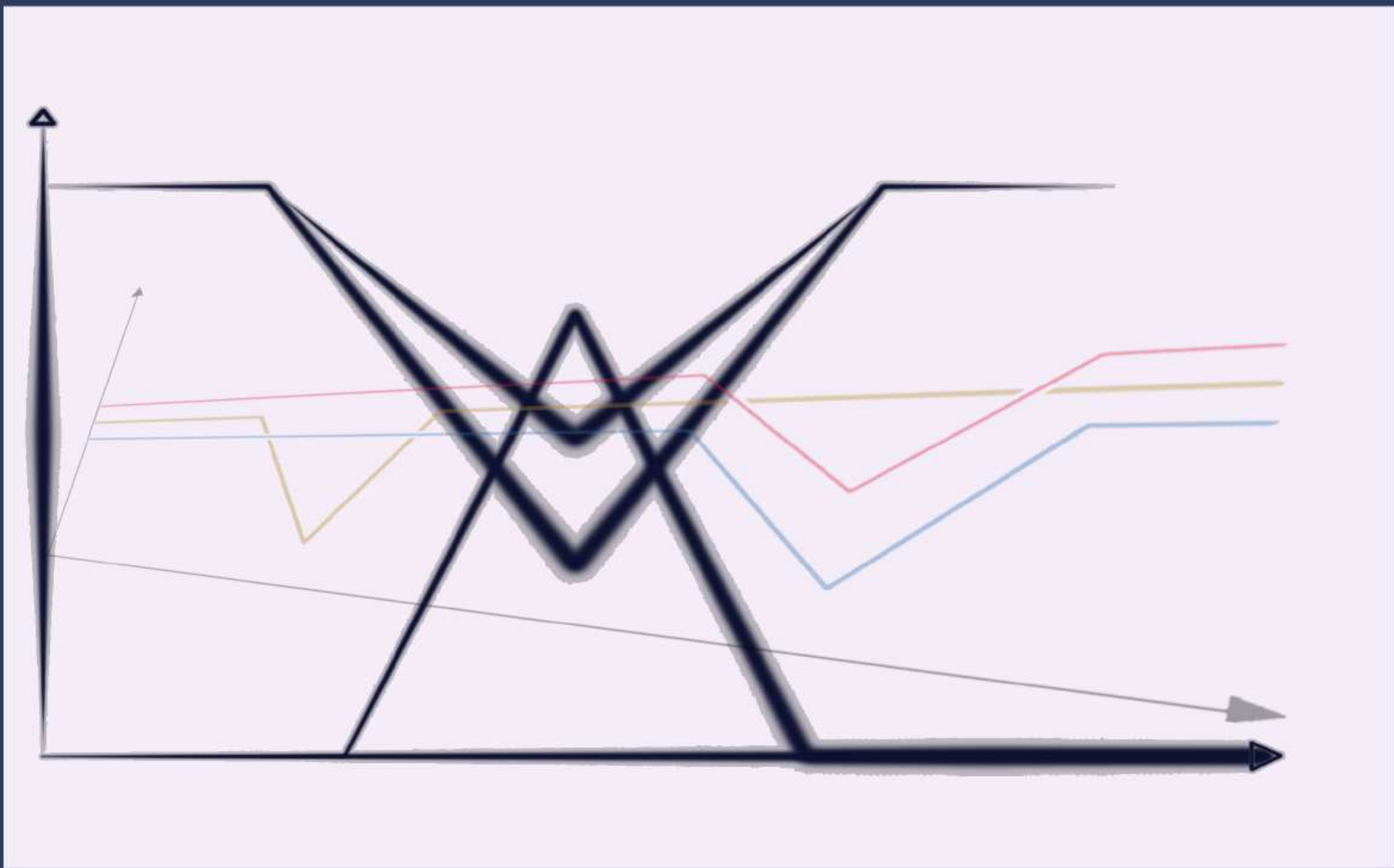
Journal of Fuzzy Extension & Applications
www.journal-fea.com



Journal of Fuzzy Extension & Applications

Print ISSN: 2783-1442
Online ISSN: 2717-3453

J. Fuzzy. Ext. Appl

J. Fuzzy. Ext. Appl

Journal of Fuzzy Extension and Applications

www.journal-fea.com

J. Fuzzy. Ext. Appl. Vol. 2, No. 1 (2021) 1–105.

TABLE OF CONTENTS

E-ISSN: 2717-3453 | P-ISSN: 2783-1442

Journal of Fuzzy Extension and Applications

www.journal-fea.com

J. Fuzzy. Ext. Appl. Vol. 2, No. 1 (2021) 1–15.

Paper Type: Research Paper

New Plithogenic Sub Cognitive Maps Approach with Mediating Effects of Factors in COVID-19 Diagnostic Model

Nivetha Martin[1,*] R. Priya[2], Florentin Smarandache[3]

[1] Department of Mathematics, Arul Anandar College (Autonomous), Karumathur, India; nivetha.martin710@gmail.com.
[2] Department of Mathematics, PKN Arts College, Madurai, India; iampriyaravi@gmail.com.
[3] Department of Mathematics, University of New Mexico, Gallup, NM 87301, USA; fsmarandache@gmail.com.

Citation:

Martin, N., Priya, R., & Smarandache, F. (2021). New Plithogenic sub cognitive maps approach with mediating effects of factors in COVID-19 diagnostic model. *Journal of fuzzy extension and application*, 2 (1), 1-15.

Received: 12/10/2020 Reviewed: 23/11/2020 Revised: 25/12/2020 Accept: 07/01/2021

Abstract

The escalation of COVID-19 curves is high and the researchers worldwide are working on diagnostic models, in the way this article proposes COVID-19 diagnostic model using Plithogenic cognitive maps. This paper introduces the new concept of Plithogenic sub cognitive maps including the mediating effects of the factors. The thirteen study factors are categorized as grouping factors, parametric factors, risks factors and output factor. The effect of one factor over another is measured directly based on neutrosophic triangular representation of expert's opinion and indirectly by computing the mediating factor's effects. This new approach is more realistic in nature as it takes the mediating effects into consideration together with contradiction degree of the factors. The possibility of children, adult and old age with risk factors and parametric factors being infected by corona virus is determined by this diagnostic model.

Keywords: Plithogenic cognitive maps, Sub cognitive maps, Diagnostic model, COVID-19.

1 | Introduction

Robert Axelrod [1] developed cognitive maps, a graphical representation of decision maker's perception towards the problem. The factors of the problem are taken as the nodes and the influence of one factor over the other is represented by directed edges. The edge weights assume either of the values -1, 0, 1. The positive influence of one factor is indicated by 1, no influence by 0 and negative influence by -1. The edge weights in Cognitive maps are crisp in nature and it does not include the partial influence of the factors. Suppose if factor A has some influence on factor B, the edge weight is assumed to be 0 as it lacks the completeness. This limitation of cognitive maps is handled by extending cognitive maps to Fuzzy Cognitive Maps (FCM) by Kosko [2].

Corresponding Author: nivetha.martin710@gmail.com
http://dx.doi.org/10.22105/jfea.2020.250164.1015

In FCM the edge weights take values between the range of values from [-1, 1]. Fuzzy cognitive maps consider both partial and complete influence of the factors. The applications of Fuzzy cognitive maps are extensively investigated by many researchers. Vasantha Kandasamy [3] introduced the method of Combined Disjoint Block Fuzzy Cognitive Maps to study the influence of the factors. In this method the factors are grouped into disjoint blocks and the influence of the factors is determined based on expert's opinion. Combined overlap block fuzzy cognitive maps were also introduced by Vasantha Kandasamy [3] in which the overlapping factors are subjected to investigation. These combined overlap and block FCM are extensively used to design optimal solutions to the decision – making problems on finding the effect of smart phone on children [4], role of happiness in family [5], analysis of environmental education for the next generation [6], reasons of failure of engineering students [7].

The concept of fuzzy introduced by Zadeh [8] was extended to neutrosophic sets by Smarandache [9]. Neutrosophic sets consists of truth, indeterminacy and falsity membership functions and it is significant in handling the notion of indeterminacy in decision – making environment. Neutrosophic representation of expert's opinion in problem solving scenario resolves the mishaps of indeterminacy [10]. Neutrosophic sets are widely used in making feasible decisions. Sudha et al. [11] assessed MCDM problems by the method of TODIM using neutrosophic aggregate weights.

Singh et al. [12] and [13] has used pentagonal neutrosophic numbers to investigate social media linked MCGDM skill. Paul et al. [14] developed a generalized neutrosophic solid transportation model to handle deficit supply. Das and Tripathy [15] developed multi-polar neutrosophic causal modelling for examining the state of institutional culture. Kandasamy and Smarandache introduced Neutrosophic Cognitive Maps (NCM) [16]. In NCM the edge weights are represented as neutrosophic sets. Neutrosophic cognitive maps are used in finding the risk factors of breast cancer [17], situational analysis [18], medical diagnosis [19], selection approach of leaf diagnosis [20]. The different methods of Fuzzy cognitive maps are also discussed in neutrosophic environment. Farahani et al. [21] compared combined block and overlap block NCM and applied the methods to find the hidden patterns and indeterminacies in a case study of Attention-Deficit/Hyperactivity Disorder (ADHD).

Smarandache introduced the concept of plithogeny [22]. Plithogenic sets are the extension of crisp set, fuzzy set, intutionistic and neutrosophic sets and it can be termed as higher order sets. The significance of Plithogenic sets is the incorporation of degree of appurtenance and contradiction degree with respect to the attributes of the decision-making components. Plithogenic sets are widely used in making optimal decisions. Abdel-Basset developed Plithogenic decision making model to solve supply chain problem; constructed Plithogenic MCDM approach for evaluating the financial performance of the manufacturing industries [23].

Concentric Plithogenic hypergraphs, Plithogenic n– super hypergraphs developed by Martin and Smarandache [24] and [25] finds novel applications in decion-making. Plithogenic hypersoft sets introduced by Rana et al. [26] was extended to Plithogenic fuzzy whole hypersoft set by Smarandache [27] and to combined Plithogenic hypersoft sets by Martin and Smarandache [28]. Plithogenic cognitive maps was introduced by Martin and Smarandache [29]. PCM involves contradiction degree in addition to the influence of the factors. Based on this approach of PCM the contradiction degree of the experts was also discussed.

The extension of cognitive models to FCM, NCM and PCM measures only the direct influence of one factor over another, but not the mediating effects of the factors. The factors are grouped as combined or overlapping blocks to study the effects, but the factors are not classified into various groups of factors to study the individual impacts or the combined effects of one factor of a group over the factors of other groups. The connection matrix representing the association between the factors taken for study is fully considered for determining the influence of the factors in ON position over the factors in OFF position.

In this paper the PCM introduced by Nivetha and Florentin is examined profoundly and the concept of Plithogenic sub cognitive maps are introduced to determine the mediating effects of the factors. Also the factors 13 factors taken for study are classified as grouping factors, parametric factors, risks factors and output factor. The proposed approach is modeled to diagnose Covid-19. Several Covid-19 predictive models are framed by researchers.

Ibrahim Yasser et al. [30] has used neutrosophic classifier in developing a framework COVID -19 confrontation. Khalifia [31] used the concept of neutrosophic and deep learning approach to diagnose COVID-19 chext X ray. These models are the forecasting models of COVID -19 are based on neutrosophic sets. A PCM Covid 19 diagnosis model with contradiction degree of expert's opinion is proposed by Broumi. Covid -19 predictive models based on Plithogenic sets are limited and this motivated the authors to explore more in this area of decision-making.

In all these COVID-19 neutrosophic and Plithogenic prediction models the study factors are not categorized and the mediating effects are not studied. But in this paper the mediating effects are considered to determine the cumulative effect of one factor over another together with the respective contradiction degree of the factors with respect to the dominating factor. The proposed approach of PSCM with linguistic representation of impacts based on expert's opinion will pave way for determining the true impacts of the factors. The paper is organized as follows: Section 2 presents the methodology of PSCM of determining mediating effects. Section 3 comprises of the application of the proposed approach to COVID 19 diagnosis model. Section 4 discusses the results and the last section concludes the work.

2| Methodology

2.1| Plithogenic Sub Cognitive Maps with Mediating Effects of the Factors

Plithogenic sub cognitive maps are the subgraphs of Plithogenic cognitive maps that comprises of one of the grouping factors, all the parametric factors, one of the risk factors and the output factor. In general, the factors are categorized based on the nature of the problem, but generally the factors are essentially classified as grouping factors, measuring factors, risk factors and output factors. Measuring factors are the factors that take certain parameters into consideration and sometime they are the describing factors of the grouping factors. The grouping factors are the factors based on which the Plithogenic sub cognitive maps are constructed. The vertex representing the grouping factors in the Plithogenic cognitive maps is the prime vertex in the PSCM.

Let C1, C2, C3, C4, C5, C6, C7 be the factors taken for study to explore the decision-making problem. C1 and C2 are taken as the grouping factors, C3, C4 are taken as the parametric factors, C5, C6 are taken as the risk factors and C7 is the output factor. The intra association between the factors of each kind are not considered, if considered the Plithogenic sub cognitive maps are known as Plithogenic super sub cognitive maps.

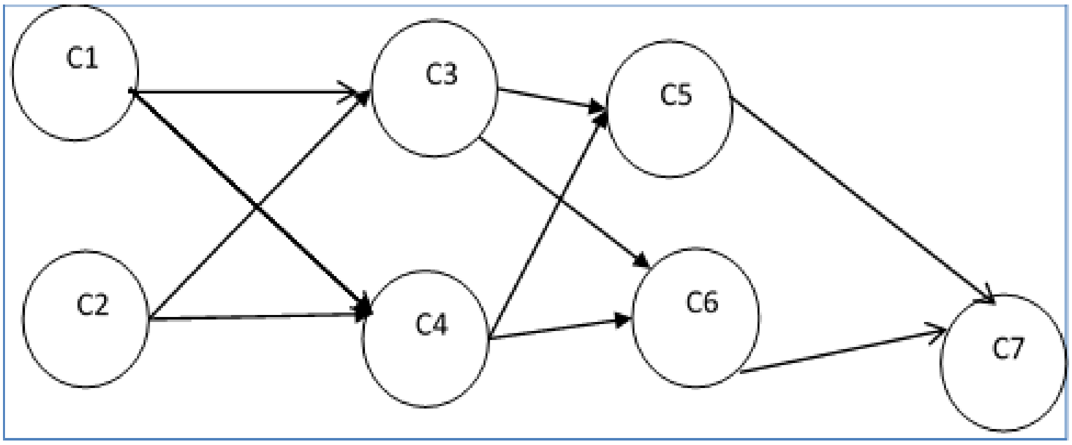


Fig. 1. The Plithogenic cognitive maps.

The Plithogenic cognitive maps represented in the *Fig. 1* presents all the possible inter relationships between the factors, to study the inter association between the grouping factors and the output factor through the mediating effects of the parametric factors and or on the risk factors, the Plithogenic sub cognitive maps are derived from Plithogenic cognitive maps.

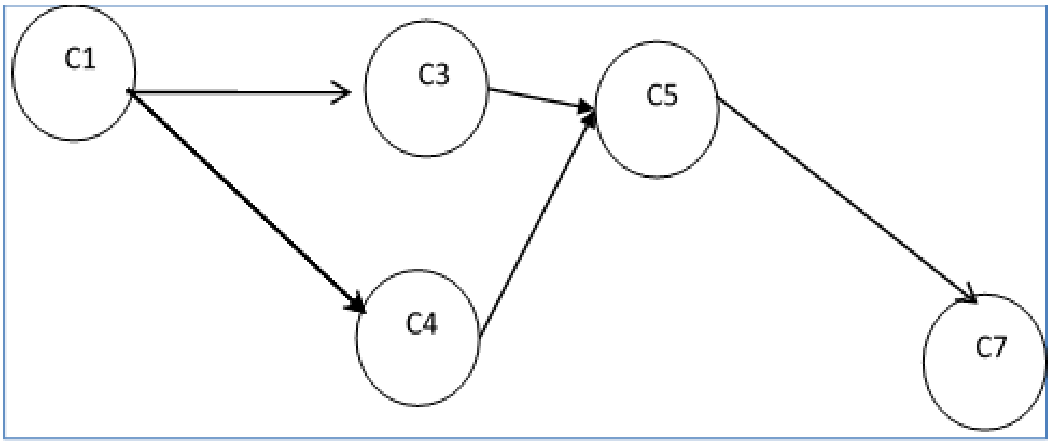


Fig. 2. Prime vertex with inter-association.

In *Fig. 2* the grouping factor C1 is taken as the prime vertex and the inter-association between the parametric factors and C5, one of the risk factors is represented to find the ultimate association between the grouping factor C1 and the output factor C7. The other Plithogenic sub cognitive maps to examine the inter associations between other set of factors can also constructed as in *Fig. 3*.

Martin et al.|J. Fuzzy. Ext. Appl. 2(1) (2021) 1-15

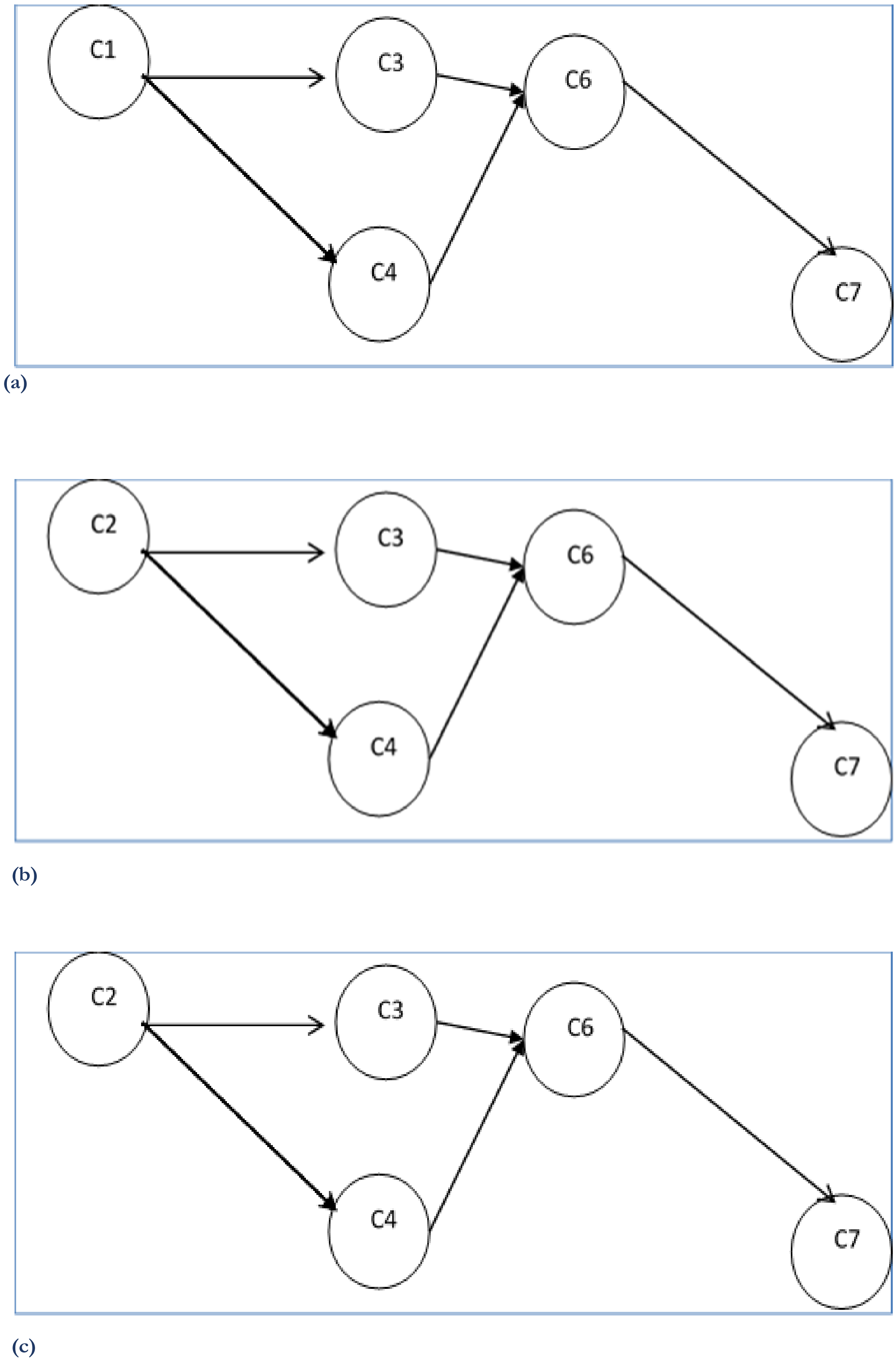


Fig 3. Plithogenic sub cognitive maps.

In each of the Plithogenic sub cognitive maps, the effects of the mediating factors representing the association between the grouping factor and the output factor are considered.

The steps involved in determining the mediating effects of the factors are presented as follows:

Step 1. The factors of the decision-making problem are categorized as grouping factors, parametric factors, risk factors and output factor.

Step 2. The Plithogenic cognitive maps representing the association between all the factors is represented graphically and the association or the connection matrix is constructed. The connection matrix M comprises of the direct effects and the indirect unknown effects between the factors in terms of linguistic variables. The association between the parametric and the risk factors are known. The levels of association between the grouping factors and the parametric factors are unknown as of certain time, until the real data is collected. The association between the grouping factor and the output factor is determined by finding the cumulative effect i.e. known and the unknown levels of impact are taken to find it. These linguistic representations are quantified using triangular neutrosophic numbers in this method, but it can be quantified by using other kinds of linguistic representations.

Step 3. The Plithogenic sub cognitive maps are constructed to determine the individual association of the grouping factor and that of the output factor. The association matrix M1 presenting the relationships between the vertices of the PSCM is derived from M. The matrix M1 also comprises of the levels of associations and the impacts.

Step 4. The factors presented under each categorization are graded as dominant when it is kept in ON position as in Plithogenic cognitive maps. The contradiction degree of the factors that are considered in PSCM are also taken into account in determining the impact of the factors.

Step 5. The factors in the PSCM are kept in ON position to determine the impact of the ON factors over the output factor and the procedure of finding the fixed point of Plithogenic cognitive maps is adopted.

3| COVID 19 Diagnostic Model

This section comprises of the formulation of COVID 19 diagnostic model. The thirteen factors taken in developing diagnostic model are categorized as grouping factors, parametric factors, risks factors and output factor. The factors C1, C2, C3 are the grouping factors based on the age group vulnerable to Covid-19, C4, C5, C6 are the parametric factors, C7, C8, C9, C10, C11, C12 are the risk factors and C13 is the output factor. The factors are

- *C1 Children,*
- *C2 Adult,*
- *C3 Old age,*
- *C4 Blood sugar level,*
- *C5 Blood pressure level,*
- *C6 State of immune system,*
- *C7 Risk of heart diseases,*
- *C8 Risk of Cancer,*
- *C9 Risk of Chronic kidney disease,*
- *C10 Asthma effect,*
- *C11 Risk of Liver disease,*

Martin et al.|J. Fuzzy. Ext. Appl. 2(1) (2021) 1-15

- *C12 Risk of Sickle cell disease,*
- *C13 Possibility of being Infected by Corona Virus.*

This model aims in determining the association between grouping factors and the output factor with the intervention of the mediating effects of the parametric and the risk factors. The comprehensive outlook of the association between the factors are presented graphically in *Fig. 4,* and the respective connection matrix M.

The general connection matrix with possible direct effects D (D = H or M or L) and unknown indirect cumulative effects Y & Z. The matrix also comprises of the direct effects of the parametric factors over the risk factors. The effects between the grouping factors and parametric factors are represented as D and it indicates the low or medium or high impacts between the factors. The association between the parametric factors and the risk factors are represented using linguistic variables. The cumulative effects of the grouping factors on the risk factors through the mediating effects of the parametric factors are represented as Y and on the output factor is represented as Z. The connection matrix in the Plithogenic cognitive maps presents the direct associations, but in the constructed connection matrix both the direct and the indirect associations are included to make optimal decisions and this is the distinguishing aspect of this model from the earlier models.

Based on the above description of the model with the direct and indirect effects between the factors of various kinds, the individual examination of one of the grouping factors in relation to the parametric factors and one of the risk factors subjected to the output factor is modelled as follows using the notion of Plithogenic sub cognitive maps. Let us now determine the association between the grouping factor C2 and the output factor C13 with the consideration of the mediating effects. The association refers to the relational impacts between adults and the possibility of being infected by corona. To determine the associational impacts, firstly the relational impacts of the adult [chosen for study] with the parametric factors are initially assigned as linguistic variables on investigation. The levels of the relational impacts are either high or medium or low or sometimes in combination.

Let us consider the factor C2 to be dominant amidst the grouping factors, C4 to be dominant amidst parametric factors and C7 to be the dominant among the risk factors. The associational impact between the adult [possessing the factors C4, C5 and C6 at various levels and the risk factor C7] and the possibility of being infected by Corona Virus is determined as follows. The contradiction degree of the factors is presented.

Table1. Contradiction degree of the factors.

C1	C2	C3
1/3	0	2/3

C4	C5	C6
0	1/3	2/3

C7	C8	C9	C10	C11	C12
0	1/6	2/6	3/6	4/6	5/6

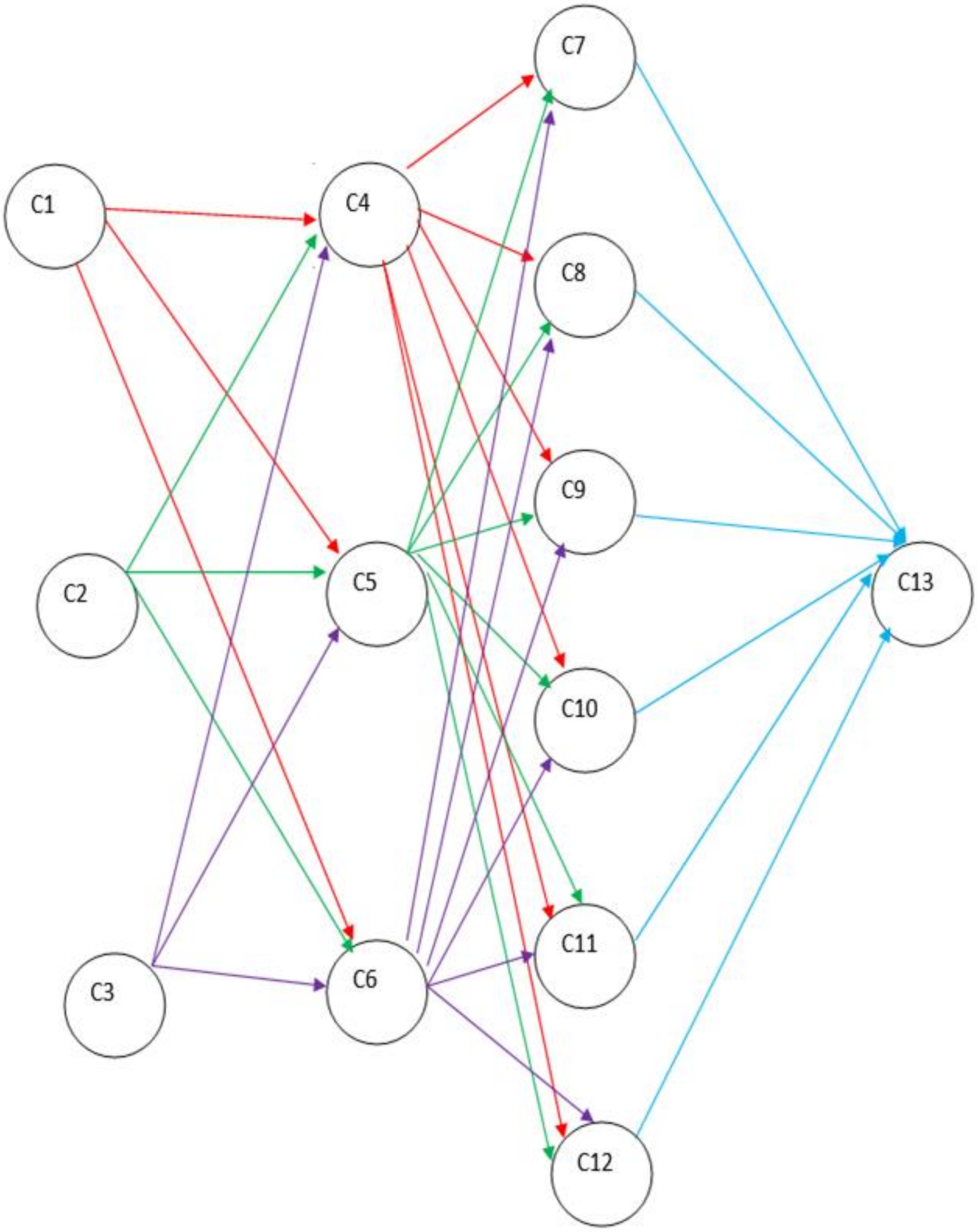


Fig. 4. Association between the factors.

Here D = H/M/L, let us first analyse the case where the relational impacts are high (D=H) and the connection matrix obtained is

$$
\begin{array}{c}
\\ C1 \\ C4 \\ C5 \\ C6 \\ C7 \\ C13
\end{array}
\begin{pmatrix}
C1 & C4 & C5 & C6 & C7 & C13 \\
0 & H & H & H & 2H & 3H \\
0 & 0 & 0 & 0 & H & 2H \\
0 & 0 & 0 & 0 & H & 2H \\
0 & 0 & 0 & 0 & H & 2H \\
0 & 0 & 0 & 0 & 0 & H \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

$$
\begin{array}{c}
\\ C1 \\ C4 \\ C5 \\ C6 \\ C7 \\ C13
\end{array}
\begin{pmatrix}
C1 & C4 & C5 & C6 & C7 & C13 \\
0 & D & D & D & Y & Z1 \\
0 & 0 & 0 & 0 & H & Z2 \\
0 & 0 & 0 & 0 & H & Z3 \\
0 & 0 & 0 & 0 & H & Z4 \\
0 & 0 & 0 & 0 & 0 & H \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

Fig. 5 represents the respective Plithogenic sub cognitive map and the corresponding connection matrix with known and unknown effects is M1.

The impact Y between C1 and C7 is determined as follows, the relational impact between C1 and C4 is H, the relational impact between C4 and C7 is H, as represented in the general connection matrix and so the cumulative impact between C1 and C7 is 2H, thus the indirect effects Y and Z are determined by taking the cumulative effects between the factors.

Table 2. Quantification of linguistic variable.

Linguistic Variable	Triangular Neutrosophic Representation	Crisp Value
High	(O.1,0.25,0.35)(0.6,0.2,0.3)	0.18
Moderate	(0.35,0.55,0.65)(0.8,0.1,0.2)	.48
Low	(0.65,0.85,0.95)(0.9,0.10.1)	.83

The modified Plithogenic fuzzy connection matrix P(E) is

$$
\begin{array}{c}
\\ C1 \\ C4 \\ C5 \\ C6 \\ C7 \\ C13
\end{array}
\begin{pmatrix}
C1 & C4 & C5 & C6 & C7 & C13 \\
0 & 0.83 & 0.83 & 0.83 & 1.66 & 2.49 \\
0 & 0 & 0 & 0 & .83 & 1.66 \\
0 & 0 & 0 & 0 & .83 & 1.66 \\
0 & 0 & 0 & 0 & .83 & 1.66 \\
0 & 0 & 0 & 0 & 0 & .83 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

Martin et al. | J. Fuzzy. Ext. Appl. 2(1) (2021) 1-15

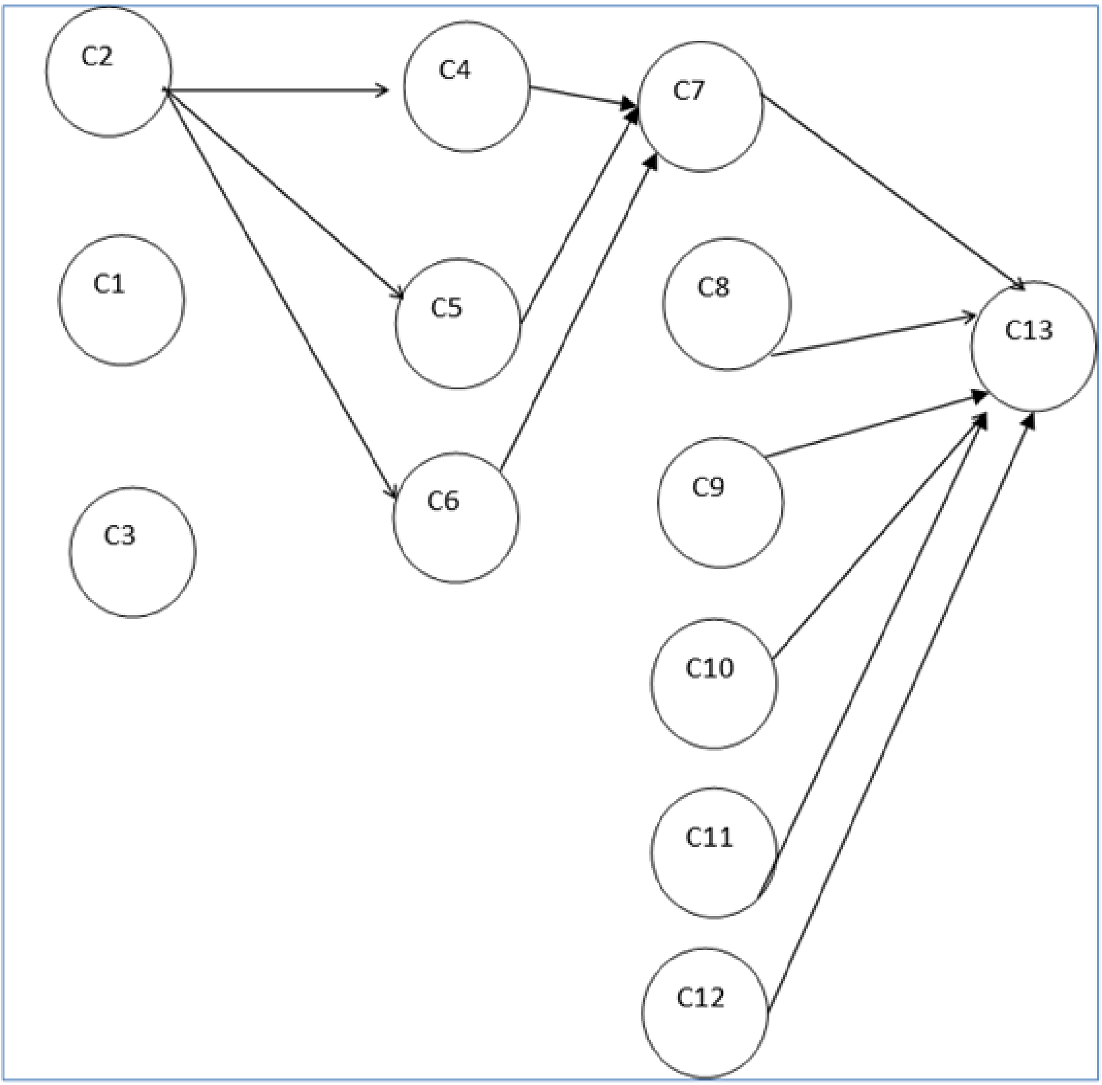


Fig. 5. Respective plithogenic sub cognitive map.

Let us consider the instantaneous state vector as X= (1 1 0 0 1 0)

$X *_p$ P (E) = F where F= (a b c d e f);

a= Max $[1_{\wedge p}\, 0,\ 1_{\wedge p}\, 0,\ 0_{\wedge p}\, 0,\ ,0_{\wedge p}\, 0,\ 1_{\wedge p}\, 0,\ 0_{\wedge p}\, 0,\ ]$;

b=max $[1_{\wedge p}\, 0.83,\ 1_{\wedge p}\, 0,\ 0_{\wedge p}\, 0,\ ,0_{\wedge p}\, 0,\ 1_{\wedge p}\, 0,\ 0_{\wedge p}\, 0,\ ]$;

c=max $[1_{\wedge p}\, 0.83,\ 1_{\wedge p}\, 0,\ 0_{\wedge p}\, 0,\ ,0_{\wedge p}\, 0,\ 1_{\wedge p}\, 0,\ 0_{\wedge p}\, 0,\ ]$;

d=max $[1_{\wedge p}\, 0.83,\ 1_{\wedge p}\, 0,\ 0_{\wedge p}\, 0,\ ,0_{\wedge p}\, 0,\ 1_{\wedge p}\, 0,\ 0_{\wedge p}\, 0,\ ]$;

e=max $[1_{\wedge p}\, 1.66,\ 1_{\wedge p}\, 0.83,\ 0_{\wedge p}\, 0.83,\ ,0_{\wedge p}\, 0.83,\ 1_{\wedge p}\, 0,\ 0_{\wedge p}\, 0,\ ]$;

f=max $[1_{\wedge p}\, 2.49,\ 1_{\wedge p}\, 1.66,\ 0_{\wedge p}\, 1.66,\ ,0_{\wedge p}\, 1.66,\ 1_{\wedge p}\, .83,\ 0_{\wedge p}\, 0,\ ]$;

$X *_p$ P (E)= (0 .83 .8 .94 1.66 2.49)→(1 1 .8 .94 1 1)=X1;

$X1 *_P$ P(E)= ((0 0.83 .89 .94 1.66 2.49) → (1 1 .89 .94 1 1)=X2;

$X2 *_P$ P(E)= (0 0.83 .89 .94 1.66 2.49) → (1 1 .89 .94 1 1) = X3;

X2 = X3.

The fixed point is:

$$(1\ \ 1\ \ .89\ \ .94\ \ 1\ \ 1), \tag{1}$$

thus obtained represents the influence of the factors in ON position over the other factors.

Let us consider D, the relational impacts to be moderate (i.e.) D= M. The respective connection matrix is as follows:

$$\begin{array}{cc} & \begin{array}{cccccc} C1 & C4 & C5 & C6 & C7 & C13 \end{array} \\ \begin{array}{c} C1 \\ C4 \\ C5 \\ C6 \\ C7 \\ C13 \end{array} & \begin{pmatrix} 0 & 0.48 & 0.48 & 0.48 & 0.96 & 1.44 \\ 0 & 0 & 0 & 0 & 0.48 & 0.96 \\ 0 & 0 & 0 & 0 & 0.48 & 0.96 \\ 0 & 0 & 0 & 0 & 0.48 & 0.96 \\ 0 & 0 & 0 & 0 & 0 & 0.48 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Let us now consider the instantaneous state vector as X= (1 1 0 0 1 0)

$X *_P$ P(E)=(0 0 .48 .65 .83 0.96 1.44)→(1 1 .65 .83 1 1) = X1;

$X1 *_P$ P(E)=((0 0.48 .65 .83 .96 1.44) → (1 1 .65 .83 1 1) = X2;

X1 = X2.

The fixed point is:

$$(1\ \ 1\ \ .65\ \ .83\ \ 1\ \ 1), \tag{2}$$

Let us consider D, the relational impacts to be moderate (i.e.) D= L. The respective connection matrix is as follows

$$\begin{array}{cc} & \begin{array}{cccccc} C1 & C4 & C5 & C6 & C7 & C13 \end{array} \\ \begin{array}{c} C1 \\ C4 \\ C5 \\ C6 \\ C7 \\ C13 \end{array} & \begin{pmatrix} 0 & 0.18 & 0.18 & 0.18 & 0.36 & 0.54 \\ 0 & 0 & 0 & 0 & 0.18 & 0.36 \\ 0 & 0 & 0 & 0 & 0.18 & 0.36 \\ 0 & 0 & 0 & 0 & 0.18 & 0.36 \\ 0 & 0 & 0 & 0 & 0 & 0.18 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

$X1 *_P$ $X *_P$ Let us consider the instantaneous state vector as *X=(1 1 0 0 1 0)*

P(E)=(0 0 .18 .45 .73 0.36 .54)→(1 1 .45 .73 1 .54) = X1;

P(E)=((0 .18 .45 .73 .36 .44) → (1 1 .45 .73 1 0.54) = X2;

X1 = X2.

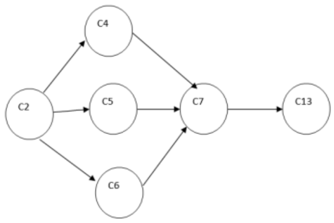
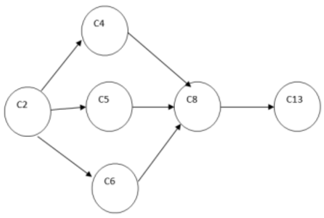
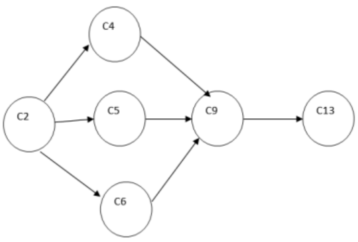
The fixed point is

$$(1\ \ 1\ \ .45\ \ .73\ \ 1\ \ 0.54), \tag{3}$$

Eqs. (1)- (3) represents the association of the adult (grouping factor) with the possibility of corona virus (output factor) together with the intervention of the various levels of the mediating effects of the parametric and the risk factor of heart disease.

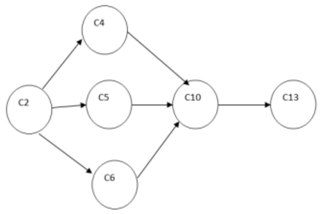
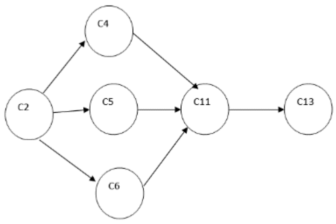
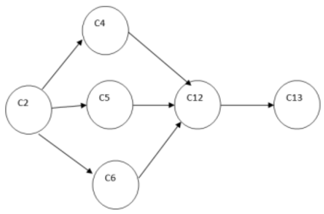
The same procedure is repeated to analyse the association of the adult (grouping factor) with the possibility of corona virus (output factor) together with the intervention of the various levels of the mediating effects of the parametric and other risk factors. The fixed points thus obtained by considering the grouping factor C2, the output factor C13 , the diverse levels of parametric factors C4, C5, C6 with varying risk factors are tabulated as follows.

Table 3. Diverse levels of parametric factors.

Levels/Risk Factors	**High**	**Moderate**	**Low**
 C7	(1 1 .89 .94 1 1)	(1 1 .65 .83 1 1)	(1 1 .45 .73 1 0.54)
 C8	(1 1 .83 .94 1 1)	(1 1 .65 .83 1 1)	(1 1 .45 .73 1 0.54)
 C9	(1 1 .89 .94 1 1)	(1 1 .65 .83 1 1)	(1 1 .45 .73 1 0.54)

Martin et al. | J. Fuzzy. Ext. Appl. 2(1) (2021) 1-15

Table 3. (Continued).

Levels/Risk Factors	High	Moderate	Low
 C10	(1 1 .89 .94 1 1)	(1 1 .65 .83 1 1)	(1 1 .45 .73 1 0.54)
 C11	(1 1 .83 .94 1 1)	(1 1 .65 .83 1 1)	(1 1 .45 .73 1 0.54)
 C12	(1 1 .89 .94 1 1)	(1 1 .65 .83 1 1)	(1 1 .45 .73 1 0.54)

It is quite evident that the mediating effects of the risk factors in determining the association of the grouping factor with the output factor by considering various levels of parametric factors yields the same fixed point and thus the association between the grouping factor and the output factor exists and it is invariable with respect to the risk factors, also all the risk factors strongly indicates the association. The same procedure can be applied to determine the association between other grouping factors and the output factor together with the mediating effects of the factors by varying the contradiction degree with respect to the dominant factors

4| Discussion

The proposed diagnostic model provides the platform to take decisions on the possibility of corona virus to the persons of various age group with the various levels of common parametric values and the risk factors. In the first case of taking the relational impacts to be high, the blood sugar level was considered to be dominant as diabetic patients are highly vulnerable to COVID-19 and the other parametric factors are also considered. In addition to it the risk of heart diseases is taken to be dominant. The first case investigates the possibility of corona virus to the adult with high blood sugar level, and with the risk of heart diseases. The same fashion of investigation is done with various levels of the relational impacts and also with the consideration of other risk factors.

5| Conclusion

The paper introduces the concept of Plithogenic sub cognitive maps to design COVID-19 diagnostic model with the mediating effects of the factors. This model can be extended to Plithogenic super sub cognitive maps with the consideration of the intra relationship between the factors. This model assists in making exclusive examination of the possibility of corona virus infection to various age group of people with several mediating factors. This decision-making model will surely disclose novel vistas of constructing many more diagnostic models of this kind. The model can be extended by classifying the factors based on the needs of decision-making problem.

References

[1] Axelrod, R. (Ed.). (2015). *Structure of decision: The cognitive maps of political elites*. Princeton university press.

[2] Kosko, B. (1986). Fuzzy cognitive maps. *International journal of man-machine studies, 24*(1), 65-75.

[3] Vasantha Kandasamy, W. B., & Smarandache, F. (2004). Analysis of social aspects of migrant labourers living with hiv/aids using fuzzy theory and neutrosophic cognitive maps: With special reference to rural tamil nadu in india. arXiv:math/0406304

[4] Gogineni, A., & Srinivasarao, N. (2017). Investigation of effect of smart phone on children using combined disjoint block fuzzy cognitive maps (CDBFCM). Retrieved from https://www.semanticscholar.org/paper/Investigation-of-effect-of-smart-phone-on-children-Gogineni-Srinivasarao/7c50c9661eba98921e576f5edd913ca8a0521ae0

[5] Devadoss, A. V., Anand, M. C. J., Rozario, A. A., & Bavia, M. S. (2013). The role of happiness in the family using combined disjoint block fuzzy cognitive maps (CDBFCMs). *International journal of business intelligents, 2*, 229-235.

[6] Devadoss, A. V., Anand, M. C. J., Felix, A., & Alexandar, S. (2015). A analysis of environmental education for the next generation using combined disjoint block fuzzy cognitive maps (CDBFCMS). *International journal of business intelligents, 4*(2), 102-106

[7] Anusha, G. (2019). Reasons for the failure of B.Tech students in mathematics using combined disjoint blocked fuzzy cognitive maps(CDBFCM). *International journal of engineering and advanced technology (IJEAT), 9*(2), 2249-8958.

[8] Zadeh, L. A. (1965). Fuzzy set theory. *Information and control, 8*(3), 338-353.

[9] Smarandache, F. (2005). Neutrosophic set-a generalization of the intuitionistic fuzzy set. *International journal of pure and applied mathematics, 24*(3), 287.

[10] Smarandache, F. (2005). Neutrosophic set-a generalization of the intuitionistic fuzzy set. *International journal of pure and applied mathematics, 24*(3), 287.

[11] Sudha, A. S., Gomes, L. F. A. M., & Vijayalakshmi, K. R. (2020). Assessment of MCDM problems by TODIM using aggregated weights. *Neutrosophic sets and systems, 35*, 78-98.

[12] Singh, N., Chakraborty, A., Biswas, S. B., & Majumdar, M. (2020). A study of social media linked MCGDM skill under pentagonal neutrosophic environment in the banking industry. *Neutrosophic sets and systems, 35*, 119-141.

[13] Singh, N., Chakraborty, A., Biswas, S. B., & Majumdar, M. (2020). Impact of social media in banking sector under triangular neutrosophic arena using MCGDM technique. *Neutrosophic sets and systems, 35*, 153-176.

[14] Paul, N., Sarma, D., & Bera, A. S. U. K. (2020). A generalized neutrosophic solid transportation model with insufficient supply. *Neutrosophic sets and systems, 35*, 177-187.

[15] Das, R., & Tripathy, B. C. (2020). Neutrosophic multiset topological space. *Neutrosophic sets and systems, 35*, 142-152.

[16] Kandasamy, W. V., & Smarandache, F. (2003). *Fuzzy cognitive maps and neutrosophic cognitive maps.* Infinite study.

[17] William, M. A., Devadoss, A. V., & Sheeba, J. J. (2013). A study on neutrosophic cognitive maps (NCMs) by analyzing the risk factors of breast cancer. *International journal of scientific and engineering research, 4*(2), 1-4.

[18] Zafar, A., & Anas, M. (2019). Neutrosophic cognitive maps for situation analysis. *Neutrosophic sets and systems, 29*(1), 7.

[19] Kumar, M., Bhutani, K., & Aggarwal, S. (2015, August). Hybrid model for medical diagnosis using Neutrosophic Cognitive Maps with Genetic Algorithms. *2015 IEEE international conference on fuzzy systems (FUZZ-IEEE)* (pp. 1-7). IEEE.

[20] Shadrach, F.D., Kandasamy, G. (2020). Neutrosophic cognitive maps (NCM) based feature selection approach for early leaf disease diagnosis. *Journal of ambient intelligence and humanized computing*. https://doi.org/10.1007/s12652-020-02070-3.

[21] Farahani, H., Smarandache, F., & Wang, L. L. (2015). *A comparison of combined overlap block fuzzy cognitive maps (COBFCM) and combined overlap block neutrosophic cognitive map (COBNCM) in finding the hidden patterns and indeterminacies in psychological causal models: case study of ADHD*. Infinite Study.

[22] Smarandache, F. (2017). *Plithogeny, plithogenic set, logic, probability, and statistics*. Pons, Brussels.

[23] Abdel-Basset, M., Mohamed, R., Zaied, A. E. N. H., & Smarandache, F. (2019). A hybrid plithogenic decision-making approach with quality function deployment for selecting supply chain sustainability metrics. *Symmetry, 11*(7), 903.

[24] Martin, N., & Smarandache, F. (2020). Concentric Plithogenic hypergraph based on Plithogenic hypersoft sets–a novel outlook. *Neutrosophic sets and systems, 33*(1), 5.

[25] Smarandache, F., & Martin, N. (2020). *Plithogenic n-super hypergraph in novel multi-attribute decision making*. Infinite Study.

[26] Rana, S., Qayyum, M., Saeed, M., Smarandache, F., & Khan, B. A. (2019). *Plithogenic fuzzy whole hypersoft set, construction of operators and their application in frequency matrix multi attribute decision making technique.* Infinite Study.

[27] Smarandache, F. (2018). extension of soft set to hypersoft set, and then to Plithogenic hypersoft set. *Neutrosophic sets and systems, 22*, 168-170.

[28] Martin, N., & Smarandache, F. (2020). *Introduction to combined Plithogenic hypersoft sets*. Infinite Study.

[29] Martin, N., & Smarandache, F. (2020). *Plithogenic cognitive maps in decision making*. Infinite Study.

E-ISSN: 2717-3453 | P-ISSN: 2783-1442

Journal of Fuzzy Extension and Applications

www.journal-fea.com

J. Fuzzy. Ext. Appl. Vol. 2, No. 1 (2021) 16–22.

Paper Type: Research Paper

A Study of Maximal and Minimal Ideals of n-Refined Neutrosophic Rings

Mohammad Abobala*

Department of Mathematics, Faculty of Science, Tishreen University, Lattakia, Syria; mohammadabobala777@gmail.com.

Citation:

Abobala, M. (2021). A study of maximal and minimal ideals of n-refined neutrosophic rings. *Journal of fuzzy extension and application*, 2 (1), 16-22.

Received: 01/11/2020 Reviewed: 19/12/2020 Revised: 24/12/2020 Accept: 08/02/2021

Abstract

If R is a ring, then $R_n(I)$ is called a refined neutrosophic ring. Every AH-subset of $R_n(I)$ has the form $P = \sum_{i=0}^{n} P_i I_i = \{a_0 + a_1 I + \cdots + a_n I_n : a_i \in P_i\}$, where P_i are subsets of the classical ring R. The objective of this paper is to determine the necessary and sufficient conditions on P_i which make P be an ideal of $R_n(I)$. Also, this work introduces a full description of the algebraic structure and form for AH-maximal and minimal ideals in $R_n(I)$.

Keywords: n-Refined neutrosophic ring, n-refined AH-ideal, Maximal ideal, Minimal ideal.

1 | Introduction

Robert Neutrosophy is a new kind of generalized logic proposed by Smarandache [12]. It becomes a useful tool in many areas of science such as number theory [16] and [20], solving equations [18], [21], and medical studies [11] and [15]. Also, there are many applications of neutrosophic structures in statistics [14], optimization [8], and decision making [7]. On the other hand, neutrosophic algebra began in [4], Smarandache and Kandasamy defined concepts such as neutrosophic groups and neutrosophic rings. These notions were handled widely by Agboola et al. in [6], [10], where homomorphisms and AH-substructures were studied [3], [13], [17].

Corresponding Author: mohammadabobala777@gmail.com

http://dx.doi.org/10.22105/jfea.2020.250164.1015

Recently, there is an arising interest by the generalizations of neutrosophic algebraic structures. Authors proposed n-refined neutrosophic groups [9], rings [1], modules [2] and [22], and spaces [5] and [19].

If R is a classical ring, then the corresponding refined neutrosophic ring is defined as follows:

$R_n(I)=\{a_0 + a_1 I + \cdots + a_n I_n\ ;\ a_i \in R\}$.

Addition and multiplication on $R_n(I)$ are defined as:

$$\sum_{i=0}^{n} x_i I_i + \sum_{i=0}^{n} y_i I_i = \sum_{i=0}^{n} (x_i + y_i) I_i\ ,\ \sum_{i=0}^{n} x_i I_i \times \sum_{i=0}^{n} y_i I_i = \sum_{i,j=0}^{n} (x_i \times y_j) I_i I_j.$$

Where $\times$ is the multiplication defined on the ring R and $I_i I_j = I_{\min(i,j)}$.

Every AH-subset of $R_n(I)$ has the form $P = \sum_{i=0}^{n} P_i I_i = \{a_0 + a_1 I + \cdots + a_n I_n : a_i \in P_i\}$. There is an important question arises here. This question can be asked as follows:

What are the necessary and sufficient conditions on the subsets P_i which make P be an ideal of $R_n(I)$? On the other hand, can we determine the structure of all AH-maximal and minimal ideals in the n-refined neutrosophic ring $R_n(I)$?

Through this paper, we try to answer the previous questions in the case of n-refined neutrosophic rings with unity. All rings through this paper are considered with unity.

2| Preliminaries

Definition 1. [1]. Let $(R,+,\times)$ be a ring and $I_k; 1 \le k \le n$ be n indeterminacies. We define $R_n(I)=\{a_0 + a_1 I + \cdots + a_n I_n\ ;\ a_i \in R\}$ to be n-refined neutrosophic ring. If $n=2$ we get a ring which is isomorphic to 2-refined neutrosophic ring $R(I_1, I_2)$.

Addition and multiplication on $R_n(I)$ are defined as:

$$\sum_{i=0}^{n} x_i I_i + \sum_{i=0}^{n} y_i I_i = \sum_{i=0}^{n} (x_i + y_i) I_i\ ,\ \sum_{i=0}^{n} x_i I_i \times \sum_{i=0}^{n} y_i I_i = \sum_{i,j=0}^{n} (x_i \times y_j) I_i I_j.$$

Where $\times$ is the multiplication defined on the ring R.

It is easy to see that $R_n(I)$ is a ring in the classical concept and contains a proper ring R.

Definition 2. [1]. Let $R_n(I)$ be an n-refined neutrosophic ring, it is said to be commutative if $xy = yx$ for each $x, y \in R_n(I)$, if there is $1 \in R_n(I)$ such $1.x = x.1 = x$, then it is called an n-refined neutrosophic ring with unity.

Theorem 1. [1]. Let $R_n(I)$ be an n-refined neutrosophic ring. Then (a) R is commutative if and only if $R_n(I)$ is commutative, (b) R has unity if and only if $R_n(I)$ has unity, and (c) $R_n(I) = \sum_{i=0}^{n} R I_i = \{\sum_{i=0}^{n} x_i I_i : x_i \in R\}$.

Definition 3. [1]. (a) Let $R_n(I)$ be an n-refined neutrosophic ring and $P = \sum_{i=0}^{n} P_i I_i = \{a_0 + a_1 I + \cdots + a_n I_n : a_i \in P_i\}$ where P_i is a subset of R, we define P to be an AH-subring if P_i is a subring of R for all i, AHS-subring is defined by the condition $P_i = P_j$ for all i, j. (b) P is an AH-ideal if P_i is an two sides ideal of R for all i, the AHS-ideal is defined by the condition $P_i = P_j$ for all i, j. (c) The AH-ideal P is said to be null if $P_i = R$ or $P_i = \{0\}$ for all i.

Definition 4. [1]. Let $R_n(I)$ be an n-refined neutrosophic ring and $P = \sum_{i=0}^{n} P_i I_i$ be an AH-ideal, we define AH-factor $R(I)/P = \sum_{i=0}^{n} (R/P_i) I_i = \sum_{i=0}^{n} (x_i + P_i) I_i \,; x_i \in R$.

Theorem 2. [1]. Let $R_n(I)$ be an n-refined neutrosophic ring and $P = \sum_{i=0}^{n} P_i I_i$ be an AH-ideal: $R_n(I)/P$ is a ring with the following two binary operations:

$$\sum_{i=0}^{n}(x_i + P_i)I_i + \sum_{i=0}^{n}(y_i + P_i)I_i = \sum_{i=0}^{n}(x_i + y_i + P_i)I_i,$$

$$\sum_{i=0}^{n}(x_i + P_i)I_i \times \sum_{i=0}^{n}(y_i + P_i)I_i = \sum_{i=0}^{n}(x_i \times y_i + P_i)I_i.$$

Definition 5. [1]. (a) Let $R_n(I), T_n(I)$ be two n-refined neutrosophic rings respectively, and $f_R : R \to T$ be a ring homomorphism. We define n-refined neutrosophic AHS-homomorphism as $f : R_n(I) \to T_n(I); f(\sum_{i=0}^{n} x_i I_i) = \sum_{i=0}^{n} f_R(x_i) I_i$, (b) f is an n-refined neutrosophic AHS-isomorphism if it is a bijective n-refined neutrosophic AHS-homomorphism, and (c) AH-Ker $f = \sum_{i=0}^{n} Ker(f_R) I_i = \{\sum_{i=0}^{n} x_i I_i \,;\, x_i \in Ker\, f_R\}$.

3| Main Discussion

Theorem 3. Let $R_n(I) = \{a_0 + a_1 I + \cdots + a_n I_n \,;\, a_i \in R\}$ be any n-refined neutrosophic ring with unity 1. Let $P = \sum_{i=0}^{n} P_i I_i = \{a_0 + a_1 I + \cdots + a_n I_n : a_i \in P_i\}$ be any AH-subset of $R_n(I)$, where P_i are subsets of R. Then P is an ideal of $R_n(I)$ if and only if (a) P_i are classical ideals of R for all I and (b) $P_0 \le P_k \le P_{k-1}$. For all $0 < k \le n$.

Proof. First of all, we assume that (a), (b) are true. We should prove that P is an ideal. Since P_i are classical ideals of R, then they are subgroups of $(R,+)$, hence P is a subgroup of $(R_n(I),+)$. Let $r = r_0 + r_1 I_1 + \cdots + r_n I_n$ be any element of $R_n(I)$, $x = x_0 + x_1 I_1 + \cdots + x_n I_n$ be an arbitrary element of P, where $x_i \in P_i$. We have For $n = 0$, the statement $r.x \in P$ *is* true clearly. We assume that it is true for $n = k$, we must prove it for $k + 1$.

$$r.x = (r_0 + r_1 I_1 + \cdots + r_k I_k + r_{k+1} I_{k+1})(x_0 + x_1 I_1 + \cdots + x_k I_k + x_{k+1} I_{k+1}) =$$

$$(r_0 + r_1 I_1 + \cdots + r_k I_k)(x_0 + x_1 I_1 + \cdots x_k I_k) + r_{k+1} I_{k+1}(x_0 + \cdots + x_{k+1} I_{k+1}) + (r_0 + \cdots r_k I_k) x_{k+1} I_{k+1}.$$

We remark

$(r_0 + r_1 I_1 + \cdots + r_k I_k)(x_0 + x_1 I_1 + \cdots x_k I_k) \in P_0 + P_1 I_1 + \cdots + P_k I_k$ (by induction hypothesis).

On the other hand, we have

$$r_{k+1} I_{k+1}(x_0 + \cdots + x_{k+1} I_{k+1}) = (r_{k+1} x_0 + r_{k+1} x_{k+1}) I_{k+1} + r_{k+1} x_1 I_1 + \cdots + r_{k+1} x_k I_k.$$

Since all P_i are ideals and $P_0 \leq P_{k+1}$, we have $r_{k+1}x_i \in P_i$ *and* $r_{k+1}x_0 + r_{k+1}x_{k+1} \in P_{k+1}$, hence $r_{k+1}I_{k+1}(x_0 + \cdots + x_{k+1}I_{k+1}) \in P$. Also, $(r_0 + \cdots r_kI_k)x_{k+1}I_{k+1} = r_0x_{k+1}I_{k+1} + r_1x_{k+1}I_1 + \cdots + r_kx_{k+1}I_k$. Under the assumption of theorem, we have $r_0x_{k+1} \in P_{k+1}$ and $r_ix_{k+1} \in P_{k+1} \leq P_i$.

For all $1 \leq i \leq k$. Thus P is an ideal.

For the converse, we assume that P is an ideal of $R_n(I)$. We should prove (a) and (b).

It is easy to check that if $P = P_0 + \cdots + P_nI_n$ is a subgroup of $(R_n(I), +)$, then every P_i is a subgroup of *(R,+)*. Now we show that (b) is true.

For every $1 \leq i \leq n$, we have an element I_i, that is because R is a ring with unity, hence. Let x_0 be any element of p_0 , we have $x_0 \in P$, and $x_0I_i \in P$.

Thus $x_0 \in P_i$, which means that $P_0 \leq P_i$ for all $1 \leq i \leq n$.

Also, for every $x_i \in P_i$, we have $x_iI_i \in P$, thus $x_iI_iI_{i-1} = x_iI_{i-1} \in P$, so that $x_i \in P_{i-1}$, which means that $P_i \leq P_{i-1}$ and (b) holds.

Example 1. Let Z be the ring of integers, $Z_3(I) = \{a + bI_1 + cI_2 + dI_3; a, b, c, d \in Z\}$ be the corresponding 3-refined neutrosophic ring, we have:

$\mathrm{P} =< 16 > +< 2 > \mathrm{I}_1 +< 4 > \mathrm{I}_2 +< 8 > \mathrm{I}_3 = \{16\mathrm{x} + 2\mathrm{yI}_1 + 4\mathrm{zI}_2 + 8\mathrm{tI}_3; \mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{t} \in \mathrm{Z}\}$
is an ideal of $\mathrm{Z}_3(\mathrm{I})$, that is because, $< 16 >\leq< 8 >\leq< 4 >\leq< 2 >$.

Now, we are able to describe all AH-maximal and minimal ideals in $R_n(I)$.

Theorem 4. Let $R_n(I)=\{a_0 + a_1I + \cdots + a_nI_n\ ;\ a_i \in R\}$ be any n-refined neutrosophic ring with unity 1.

Let $P = \sum_{i=0}^{n} P_iI_i = \{a_0 + a_1I + \cdots + a_nI_n\colon a_i \in P_i\}$ be any ideal of $R_n(I)$. Then (a) non trivial AH-maximal ideals in $R_n(I)$ have the form $P_0 + RI_1 + \cdots + RI_n$, where P_0 is maximal in R and (b) non trivial AH-minimal ideals in $R_n(I)$ have the form P_1I_1, where P_1 is minimal in R.

Proof. (a) assume that P is an AH-maximal ideal on the refined neutrosophic ring $R_n(I)$, hence for every ideal $M=(M_0 + M_1I_1 + \cdots + M_nI_n)$ with property $P \leq M \leq R_n(I)$, we have $M=P$ or $M=R_n(I)$. This implies that $M_i = R$ or $M_i = P_i$, which means that P_0 is maximal in R. On the other hand, we have $P_0 \leq P_k \leq P_{k-1}$. For all $0 < k \leq n$, thus $P_i \in \{P_0, R\}$ *for* all $1 \leq i \leq n$. Now suppose that there is at least j such that $P_j = P_0$, we get that $P_0 + \cdots + P_jI_j + \cdots RI_n \leq P_0 + RI_1 + \cdots + RI_j + .. + RI_n$, hence P is not maximal. This means that $P_0 + RI_1 + \cdots + RI_n$, where P_0 is maximal in R is the unique form of AH-maximal ideals.

For the converse, we suppose that P_0 is maximal in R and $P_i = R$. For all $1 \leq i \leq n$. Consider $M=(M_0 + M_1I_1 + \cdots + M_nI_n)$ as an arbitrary ideal of $R_n(I)$ with AH-structure. If $P \leq M \leq R_n(I)$, then $P_i \leq M_i \leq R$ and, this means that $P_0 = M_0$ *or* $M_0 = R$, that is because P_0 is maximal.

According to *Theorem 3*, we have $M_0 \leq M_i \leq M_{i-1}$. Now if $M_0 = R$, we get $M_i = R$, thus $M=R_n(I)$.

If $M_0 = P_0$, we get $M = P$. This implies that P is maximal.

(b) It is clear that if P_1 is minimal in R, then $P_1 I_1$ is minimal in $R_n(I)$. For the converse, we assume that $P = P_0 + P_1 I_1 + \cdots + P_n I_n$ is minimal in $R_n(I)$, consider an arbitrary ideal with AH-structure $M = (M_0 + M_1 I_1 + \cdots + M_n I_n)$ of $R_n(I)$ with the property $M \leq P$, we have: $M = \{0\}$ *or* $M = P$ which means that $M_1 = P_1$ *or* $M_1 = \{0\}$. Hence P_1 is minimal.

According to *Theorem 3*, we have $M_0 \leq M_k \leq M_{k-1}$ for all k. Now, suppose that there is at least $j \neq 1$ such that $P_j \neq \{0\}$, we get $P_j I_j \leq P_0 + P_1 I_1 + \cdots + P_n I_n$. Thus P is not minimal, which is a contradiction with respect to assumption. Hence any non trivial minimal ideal has the form $P_1 I_1$, where P_1 is minimal in R.

Example 2. Let $R=Z$ be the ring of integers, $Z_n(I) = \{a_0 + a_1 I_1 + \cdots + a_n I_n;\ a_i \in Z\}$ be the corresponding n-refined neutrosophic ring, we have

(a) the ideal $P =< 2 > +ZI_1 + \cdots + ZI_n$ is AH-maximal, that is because <2> is maximal in R and (b) there is no AH-minimal ideals in $Z_n(I)$, that is because R has no minimal ideals.

Example 3. Let $R = Z_{12}$ be the ring of integers modulo 12, $Z_{12_n}(I)$ be the corresponding n-refined neutrosophic ring, we have

(a) the ideal $P =< 6 > I_1 = \{0, 6I_1\}$ is AH-minimal, that is because <6> is minimal in R.

(b) the ideal $Q =< 2 > +Z_{12} I_1 + \cdots + Z_{12} I_n$ is maximal, that is because <2> is maximal in R.

Now, we show that *Theorem 4* is not available if the ring R has no unity, we construct the following example.

Example 4. Consider $2Z_2(I) = \{(2a + 2bI_1 + 2cI_2); a, b, c \in Z\}$ the 2-refined neutrosophic ring of even integers, let $P = (2Z + 4ZI_1 + 4ZI_2) = \{(2a + 4bI_1 + 4cI_2); a, b, c \in Z\}$ be an AH-subset of it. First of all, we show that P is an ideal of $2Z_2(I)$. It is easy to see that *(P,+)* is a subgroup. Let $x = (2m + 4nI_1 + 4tI_2)$ be any element of P, $r = (2a + 2bI_1 + 2cI_2)$ be any element of $2Z_2(I)$, we have $rx = (4am, +[8an + 4bm + 8bn + 8bt + 8cn] + I_2[8at + 8ct + 4cm]) \in P$. Thus P is an ideal and the inclusion's condition is not available, that is because *2Z* is not contained in *4Z*.

4| Conclusion

In this article, we have found a necessary and sufficient condition for any subset to be an ideal of any n-refined neutrosophic ring with unity. On the other hand, we have characterized the form of maximal and minimal ideals in this class of neutrosophic rings. As a future research direction, we aim to study Köthe's Conjecture on n-refined neutrosophic rings about the structure of nil ideals and the maximality/minimality conditions if R has no unity.

Abobala |J. Fuzzy. Ext. Appl. 2(1) (2021) 16-22

4.1 | Open Problems

According to our work, we find two interesting open problems.

Describe the algebraic structure of the group of units of any n-refined neutrosophic ring.

What are the conditions of AH-maximal and minimal ideals if R has no unity?.

References

[1] Smarandache, F., & Abobala, M. (2020). n-Refined neutrosophic rings. *International journal of neutrosophic science*, *5*, 83-90.

[2] Sankari, H., & Abobala, M. (2020). n-refined neutrosophic modules. *Neutrosophic sets and systems, 36*, 1-11.

[3] Sankari, H., & Abobala, M. (2020). AH-Homomorphisms in neutrosophic rings and refined neutrosophic rings. *Neutrosophic sets and systems*, 38.

[4] Kandasamy, W. V., & Smarandache, F. (2006). *Some neutrosophic algebraic structures and neutrosophic n-algebraic structures*. Infinite Study.

[5] Smarandache F., and Abobala, M. (2020). n-refined neutrosophic vector spaces. *International journal of neutrosophic science, 7*(1), 47-54.

[6] Abobala, M., Hatip, A., & Alhamido, R. (2019). A contribution to neutrosophic groups. *International journal of neutrosophic science, 0*(2), 67-76.

[7] Abdel-Basset, M., Gamal, A., Son, L. H., & Smarandache, F. (2020). A bipolar neutrosophic multi criteria decision making framework for professional selection. *Applied sciences*, *10*(4), 1202. https://doi.org/10.3390/app10041202

[8] Abdel-Basset, M., Mohamed, R., Zaied, A. E. N. H., Gamal, A., & Smarandache, F. (2020). Solving the supply chain problem using the best-worst method based on a novel Plithogenic model. In *Optimization theory based on neutrosophic and plithogenic sets* (pp. 1-19). Academic Press.

[9] Abobala, M. (2019). n-refined neutrosophic groups I. *International journal of neutrosophic science*, *0*(1), 27-34.

[10] Agboola, A. A. A., Akwu, A. D., & Oyebo, Y. T. (2012). Neutrosophic groups and subgroups. *International .J .Math. Combin*, *3*, 1-9. http://mathcombin.com/upload/file/20150127/1422320633982016018.pdf#page=6 http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.641.3352&rep=rep1&type=pdf

[11] Abdel-Basset, M., Manogaran, G., Gamal, A., & Chang, V. (2019). A novel intelligent medical decision support model based on soft computing and IoT. *IEEE internet of things journal*, *7*(5), 4160-4170.

[12] Smarandache, F. (2013). n-Valued refined neutrosophic logic and its applications to physics. *Progress in physics*, *4*, 143-146.

[13] Abobala, M., & Lattakia, S. (2020). Classical homomorphisms between n-refined neutrosophic rings. *International journal of neutrosophic science*, *7*, 74-78.

[14] Alhabib, R., & Salama, A. A. (2020). the neutrosophic time series-study its models (linear-logarithmic) and test the coefficients significance of its linear model. *Neutrosophic sets and systems*, *33*, 105-115.

[15] Abdel-Basset, M., Mohamed, M., Elhoseny, M., Chiclana, F., & Zaied, A. E. N. H. (2019). Cosine similarity measures of bipolar neutrosophic set for diagnosis of bipolar disorder diseases. *Artificial intelligence in medicine*, *101*, 101735. https://doi.org/10.1016/j.artmed.2019.101735

[16] Sankari, H., & Abobala, M. (2020). *Neutrosophic linear diophantine equations with two variables* (Vol. 38). Infinite Study.

[17] Abobala, M. (2020). Ah-subspaces in neutrosophic vector spaces. *International journal of neutrosophic science*, *6*, 80-86.

[18] Edalatpanah, S. A. (2020). Systems of neutrosophic linear equations. *Neutrosophic sets and systems*, *33*(1), 92-104.

[19] Abobala, M. (2020). A study of ah-substructures in n-refined neutrosophic vector spaces. *International journal of neutrosophic science*, *9*, 74-85.

[20] Abobala, M. (2021). Foundations of neutrosophic number theory. *Neutrosophic sets and systems*, *39*(1), 10.

[21] Abobala, M. (2020). On some neutrosophic algebraic equations. *Journal of new theory*, (33), 26-32.

[22] Abobala, M. (2021). Semi homomorphisms and algebraic relations between strong refined neutrosophic modules and strong neutrosophic modules. *Neutrosophic sets and systems*, *39*(1), 9. https://digitalrepository.unm.edu/cgi/viewcontent.cgi?article=1748&context=nss_journal

Abobala |J. Fuzzy. Ext. Appl. 2(1) (2021) 16-22

E-ISSN: 2717-3453 | P-ISSN: 2783-1442

Journal of Fuzzy Extension and Applications

www.journal-fea.com

J. Fuzzy. Ext. Appl. Vol. 2, No. 1 (2021) 23–32.

Paper Type: Research Paper

Neutrosophic Soft Matrices and Its Application in Medical Diagnosis

Mamoni Dhar *

Department of Mathematics, Science College, Kokrajhar-783370, Assam, India; mamonidhar@gmail.com.

Citation:

Dhar, M. (2021). Neutrosophic soft matrices and its application in medical diagnosis. *Journal of fuzzy extension and application, 2* (1), 23-32.

Received: 01/09/2020 Reviewed: 19/10/2020 Revised: 08/01/2021 Accept: 01/02/2021

Abstract

In real life situations, there are many issues in which we face uncertainties, vagueness, complexities, and unpredictability. Neutrosophic sets are a mathematical tool to address some issues which cannot be met using the existing methods. Neutrosophic soft matrices play a crucial role in handling indeterminant and inconsistent information during decision making process. The main focus of this article is to discuss the concept of neutrosophic sets, neutrosophic soft sets, and neutrosophic soft matrices theory which are very useful and applicable in various situations involving uncertainties and imprecisions. Thereafter our intention is to find a new method for constructing a decision matrix using neutrosophic soft matrices as an application of the theory. A neutrosophic soft matrix based algorithm is considered to solve some problems in the diagnosis of a disease from the occurrence of various symptoms in patients. This article deals with patient-symptoms and symptoms-disease neutrosophic soft matrices. To come to a decision, a score matrix is defined where multiplication based on max-min operation and complementation of neutrosophic soft matrices are taken into considerations.

Keywords: Fuzzy sets, Soft sets, Soft matrix, Neutrosophic soft sets, Neutrosophic soft matrix.

1 | Introduction

The theory of fuzzy sets introduced by Zadeh [1] showed many applications in many areas of research. The idea of fuzzy sets was needed because it deals with those uncertainties and vagueness which cannot be handled using crisp sets. Fuzzy sets has membership function which assigns to each element of the Universe of discourse, a number from the unit interval [0,1], to indicate the degree of belongingness of the set under consideration. In fuzzy set theory, the membership of an element to a fuzzy set is a single value between zero and one. But in reality it may not always happen that the degree of non membership of an element in a fuzzy set is equal to one minus the membership degree because there may exist some hesitation degree as well.

Corresponding Author: mamonidhar@gmail.com

http://dx.doi.org/10.22105/jfea.2020.250164.1015

Dhar |J. Fuzzy. Ext. Appl. 2(1) (2021) 23-32

Therefore, a generalization of fuzzy set was realized by Atanassov [2] who introduced a new set and named it intuitionistic fuzzy set. Intuitionistic fuzzy set incorporated the degree of hesitation known as hesitation margin.

Intuitionistic fuzzy sets which is generalization of fuzzy sets is useful in some situations when the problem involving linguistic variable are considered. For example, in decision making problems particularly in the cases of medical diagnosis, sales analysis, new product marketing, financial services, etc. there is a fair chance of a non null hesitation part in each moment of evaluation of an unknown project.

In real life situations, most of the problems in economics, social sciences, environment, etc. have various uncertainties. However, most of the existing mathematical tools for formal modeling, reasoning and computation are crisp deterministic and precise in character. There are theories namely theory of probability, evidence, fuzzy set, intuitionistic fuzzy set, rough set, etc. for dealing with uncertainties. These theories have their own difficulties as pointed out by Molodsov [3], and as such the novel concept of soft set theory was initiated. Soft set theory has rich potential for application in solving practical problems in economics, social science, medical sciences, etc. Maji et al. ([4] and [5]) have studied the theory of fuzzy soft set. In work [6], it can be seen that the theory of fuzzy soft sets have been extended to intuitionistic fuzzy soft sets. Smarandhache [7] generalized soft set to Hypersoft sets and further used it in decision making processes. Vellapandi and Gunasekaran [9], studied a new decision making approach using multi soft set logic.

Intuitionistic fuzzy sets can only handle the incomplete information considering both the truth membership and falsity membership values. It does not handle the interminant and inconsistent information that exists in belief system. Smarandache [9], introduced the concept of neutrosophic sets as a mathematical tool to deal with some situations which involves impreciseness, inconsistencies and interminancy. It is expected that neutrosophic sets will produce more accurate results than those obtained using fuzzy sets or intuitionistic fuzzy sets. Many researchers as can be found in ([10]-[14]) who have worked on the applications of neutrosophic sets in decision making processes. Applications of neutrosophic soft sets now catching momentum.

2| Definitions and Preliminaries

Some basic definitions which are useful in the subsequent sections of the article are discussed in this section.

Definition 1. Soft set. Let U be the initial Universe of discourse and E is the set of parameter. Let $P(U)$ denote the power set of U. A pair (E,F) is called a soft set over U where F is a mapping given by $F: E \to P(U)$. Clearly soft set is a mapping from parameters to $P(U)$.

Example 1. Let $U=\{u_1,u_2,u_3,u_4\}$ be a set of four types of ornaments and $E=\{costly(e_1), medium(e_2), cheap(e_3)\}$ be the set of parameters. If $A=\{e_1,e_3\}\subseteq E$. Let $F(e_1)=\{u_1,u_4\}$ *and* $F(e_3)=\{u_2,u_3\}$. Then the soft set can be described as $(F,E)=\{(e_1, \{u_1,u_4\}),(e_3, \{u_2,u_3\})\}$ over U which describe the "quality of furnitures" which Mr. X is going to buy. This soft set can be represented in the following form.

U	Costly(e_1)	Medium(e_2)	Cheap(e_3)
U_1	1	0	0
U_2	0	0	1
U_3	0	0	1
U_4	1	0	0

Definition 2. Fuzzy soft set. Let U be the Universe of discourse and E is the set of parameters. Let $P(U)$ denotes the collections of all fuzzy subsets of U. Let $A \subseteq E$. A pair (F_A, E) is called fuzzy soft set over U where F_A is a mapping given by $F_A : E \rightarrow P(U)$.

Example 2. Let $U = \{u_1, u_2, u_3, u_4\}$ be a set of four types of ornaments and E={*costly*(e_1), *medium*(e_2), *cheap*(e_3), *very cheap*(e_4)} be the set of parameters. Let us consider the following case

$F_A(e_1)$={(u_1,0.7),(u_2, 0.8), (u_3, 0.0), (u_4, 0.5)},

$F_A(e_3)$={(u_1,0.3),(u_2, 0.4), (u_3, 0.6), (u_4, 0.5)}.

U	Costly(e_1)	Medium(e_2)	Cheap(e_3)
U_1	0.7	0.0	0.3
U_2	0.8	0.0	0.4
U_3	0.0	0.0	0.6
U_4	0.5	0.0	0.5

Definition 3. Intuitionistic fuzzy sets. Let U be the Universe of discourse. Then the intuitionistic fuzzy set A is an object having of the form $A = \{< x, \mu_A(x), \nu_A(x) >, x \in U\}$, where the function $\mu_A(x), \nu_A(x) : U \rightarrow [0,1]$ define the degree of membership and degree of non membership of the element $x \in X$ to the set A with the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1.$

Definition 4. Intuitionistic fuzzy soft sets. Let U be the Universe of discourse and E is the set of parameters. Let $P(U)$ denotes the collections of all intuitionistic fuzzy subsets of U. Let $A \subseteq E$. A pair (F_A, E) is called an intuitionistic fuzzy soft set over U where F_A is a mapping given by $F_A : E \rightarrow P(U).$

Definition 5. Neutrosophic sets. Let U be the Universe of discourse. The neutrosophic set A on the Universe of discourse U is defined as $A = \{< T_A(x), I_A(x), F_A(x) >: x \in U\}$, where the characteristic functions $T, I, F : U \rightarrow [0,1]$ and $^-0 \leq T + I + F \leq 3^+$; T, I, F are neutrosophic components which defines the degree of membership, the degree of interminancy and the degree of non membership, respectively.

Definition 6. Neutrosophic soft sets. Let U be the Universe of discourse and E is the set of parameters. Let $P(U)$ denotes the collections of all neutrosophic subsets of U. Let $A \subseteq E$. A pair (F_A, E) is called a neutrosophic soft set over U where F_A is a mapping given by $F_A : E \rightarrow P(U)$. The following example will illustrate the concept.

Let U be the set of houses under consideration and E be the set of parameters where each parameter includes neutrosophic words.

Example 3. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ be a set of six types of ornaments and E={*costly*(e_1), *medium*(e_2), *cheap*(e_3), *very cheap*(e_4)} be the set of parameters. Let us consider the following case

$F_A(e_1)=\{(u_1,0.3, 0.4, 0.2),(u_2, 0.5, 0.4, 0.1), (u_3, 0.4, 0.4, 0.2), (u_4, 0.5, 0.2, 0.1), (u_5, 0.6, 0.2, 0.2), (u_6, 0.5, 0.2, 0.2)\}$,

$F_A(e_2)=\{(u_1,0.4, 0.5, 0.1),(u_2, 0.4, 0.2, 0.3), (u_3, 0.1, 0.6, 0.2), (u_4, 0.6, 0.2, 0.1), (u_5, 0.3, 0.4, 0.2), (u_6, 0.5, 0.3, 0.1)\}$,

$F_A(e_4)=\{(u_1,0.5, 0.2, 0.2),(u_2, 0.4, 0.5, 0.1), (u_3, 0.5, 0.4, 0.2), (u_4, 0.5, 0.2, 0.3), (u_5, 0.4, 0.4, 0.2), (u_6, 0.5, 0.3, 0.1)\}$.

The tabular representation of the NSS is

U	Costly(e_1)	Medium(e_2)	Cheap(e_3)
U_1	(0.3,0.4,0.2)	(0.4,0.5,0.1)	(0.5, 0.2,0.2)
U_2	0.(0.5,0.4,0.1)8	0(0.4,0.2,0.3).0	(0.4,0.5,0.1)
U_3	(0.4,0.4,0.2)	(0.1,0.6,0.2)	(0.5,0.2,0.2)
U_4	(0.5,0.2,0.1)	(0.6,0.2,0.1)	(0.4,0.2,0.3)
U_5	(0.6,0.2,0.2)	((0.3,0.4,0.2)	(0.4,0.4,0.2)
U_6	(0.5,0.2,0.2)	(0.5,0.3,0.1)	(0.5,0.3,0.1)

Definition 7. Neutrosophic soft matrix. Let (F_A,E) is a neutrosophic soft set over U where F_A is a mapping given by $F_A : E \to P(U)$ and $P(U)$ is the collection of all neutrosophic subsets of U. Then the subsets of UXE is uniquely defined by $R_A = \{(u,e) : e \in A, u \in F_A(e)\}$ and this is called a relation form of (F_A,E). Now the relation R_A characterized by truth membership function $T_A : U \times E \to [0,1]$, interminancy membership function $I_A : U \times E \to [0,1]$ and falsity membership function $F_A : U \times E \to [0,1]$ where $T_A(u,e)$ is the truth membership value, $I_A(u,e)$ is the interminancy membership value $F_A(u,e)$ is the falsity membership value of the object u associated with the parameter e.

Let $U = \{u_1, u_2, u_3, ...u_m\}$ be the Universe set and $E = \{x_1, x_2, x_3, ...x_n\}$ be the set of parameters. Then R_A can be represented by tabular form as follows.

R_N	e_1	e_2	……	e_n
U_1	$(T_{A_{11}}, I_{A_{11}}, F_{A_{11}})$	$(T_{A_{12}}, I_{A_{12}}, F_{A_{12}})$	……	$(T_{A_{1n}}, I_{A_{1n}}, F_{A_{1n}})$
U_2	$(T_{A_{21}}, I_{A_{21}}, F_{A_{21}})$	$(T_{A_{22}}, I_{A_{22}}, F_{A_{22}})$	……	$(T_{A_{2n}}, I_{A_{2n}}, F_{A_{2n}})$
…..	….	….	…..	….
u_m	$(T_{A_{m1}}, I_{A_{m1}}, F_{A_{m1}})$	$(T_{A_{m2}}, I_{A_{m2}}, F_{A_{m2}})$	…….	$(T_{A_{mn}}, I_{A_{mn}}, F_{A_{mn}})$

where $(T_{A_{mn}}, I_{A_{mn}}, F_{A_{mn}}) = (T_A(u_m, e_n), I_A(u_m, e_n), F_A(u_m, e_n))$, If $a_{ij} = (T_A(u_i, e_j), I_A(u_i, e_j), F_A(u_i, e_j))$, a matrix can be defined as

$$a_{ij} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}.$$

This is called neutrosophic soft matrix corresponding to the neutrosophic soft set (F_A,E) over U.

Example 4. Let $U=\{u_1,u_2,u_3,u_4,u_5,u_6\}$ be the Universal set and $E=\{e_1, e_2, e_3,,e_4)$, be the set of parameters, $A=\{e_1, e_2, e_3,)$.

Let us consider the following case

$F_A(e_1)$={(u_1,0.3, 0.4, 0.2),(u_2, 0.5, 0.4, 031), (u_3, 0.4, 0.5, 0.4), (u_4, 0.6, 0.3, 0.2), (u_5, 0.8, 0.1, 0.3), (u_6, 0.7, 0.2, 0.1)},

$F_A(e_2)$={(u_1,0.4, 0.5, 0.2),(u_2, 0.6, 0.2, 0.3), (u_3, 1, 0, 0.4), (u_4, 0.6, 0.2, 0.5), (u_5, 0.3, 0.4, 0.3), (u_6, 0.5, 0.4, 0.4)},

$F_A(e_3)$={(u_1,0.6, 0.2, 0.3),(u_2, 0.4, 0.3, 0.3), (u_3, 0.5, 0.1, 0.4), (u_4, 0.4, 0.2, 0.3), (u_5, 0.6, 0.4, 0.2), (u_6, 0.7, 0.3, 0.2)}.

Then the $NSS(F_A,E)$ is a parameterized family $\{(F_A(e_1), F_A(e_2), F_A(e_3)\}$ of all NSS over U and gives an approximate description of the object. Hence neutrosophic soft matrix can be represented by

$$A=\begin{bmatrix} (0.3,0.4,0.2) & (0.4,0.5,0.2) & (0.6,0.2,0.3) \\ (0.5,0.4,0.3) & (0.6,0.2,0.3) & (0.4,0.3,0.3) \\ (0.4,0.5,0.4) & (0.1,0.0,0.4) & (0.5,0.1,0.4) \\ (0.6,0.3,0.2) & (0.6,0.2,0.5) & (0.4,0.2,0.3) \\ (0.8,0.1,0.3) & (0.3,0.4,0.3) & (0.6,0.4,0.2) \\ (0.7,0.2,0.1) & (0.5,0.4,0.4) & (0.7,0.3,0.2) \end{bmatrix}.$$

Definition 8. Complement of neutrosophic soft matrices. Let $A=[(T_{ij}^A,I_{ij}^A,F_{ij}^A)]\in NSM_{m\times n}$ then the complement of the neutrosophic soft matrix A is denoted by A^C and is defined as $A^c=[(F_{ij}^A,I_{ij}^A,T_{ij}^A)]\in NSM_{m\times n}$ for all i and j. If the above mentioned neutrosophic fuzzy matrix is considered then the complement of the said matrix will be

$$A^c=\begin{bmatrix} (0.2,0.4,0.3) & (0.2,0.5,0.4) & (0.3,0.2,0.6) \\ (0.3,0.4,0.5) & (0.3,0.2,0.6) & (0.3,0.3,0.4) \\ (0.4,0.5,0.4) & (0.4,0.0,0.1) & (0.4,0.1,0.5) \\ (0.2,0.3,0.6) & (0.5,0.2,0.6) & (0.3,0.2,0.4) \\ (0.3,0.1,0.8) & (0.3,0.4,0.3) & (0.2,0.4,0.6) \\ (0.1,0.2,0.7) & (0.4,0.4,0.5) & (0.2,0.3,0.7) \end{bmatrix}.$$

Definition 9. Max-min product of neutrosophic soft matrices. Let $A=[(T_{ij}^A,I_{ij}^A,F_{ij}^A)]$ and $B=[(T_{ij}^B,I_{ij}^B,F_{ij}^B)]$ be two neutrosophic soft matrices. Then the max-min product of the two neutrosophic soft matrices A and B is denoted as $A*B$ is defined as

$A*B=[max\,min(T_{ij}^A,T_{ij}^B), min\,max(I_{ij}^A,I_{ij}^B), min\,max(F_{ij}^A,F_{ij}^B)]$ for all I and j.

Definition 10. Score matrix. The score matrix A and b is defined as $S/(A,B)=[V-W]$, where $V=\lfloor v_{ij}\rfloor, v_{ij}=T_{ij}^A+I_{ij}^A-F_{ij}^A$ and $W=\lfloor w_{ij}\rfloor, w_{ij}=T_{ij}^B+I_{ij}^B-F_{ij}^B$ are called membership value matrices.

In this article, it is intended to find $S\{(A*B),(A*B^c)\}$. Suppose $S\{(A*B),(A*B^c)\}=V'-W'$. Then $V'=\left[v'_{ij}\right], v'_{ij}=T_{ij}^{AB}+I_{ij}^{AB}-F_{ij}^{AB}$ and $W'=\left[w'_{ij}\right], w'_{ij}=T_{ij}^{AB^c}+I_{ij}^{AB^c}-F_{ij}^{AB^c}$.

In this article, with the help of the above mentioned definitions, it is intended to get an approximate conception of the diseases revealing from some symptoms which the patients may convey to the doctors. The following sections deals with the algorithm followed by a case study to make the concept clear.

3| Algorithm

Step 1. Input neutrosophic soft sets *(F,E)* and *(G,E)* and obtain neutrosophic soft matrices *A* and *B*.

Step 2. Write the neutrosophic soft complement set $(G,E)^C$ and obtain neutrosophic soft complement matrix B^C.

Step 3. Compute patient symptom disease matrix *A*B*.

Step 4. Compute patient symptom non disease matrix $A*B^C$.

Step 5. Compute V', W'.

Step 6. Compute score matrix $S(A*B, A*B^C)$.

Step 7. Identify maximum score for the patient P_i and conclude that the patient P_i is suffering from the disease D_i.

4| Case Studies

Suppose the test results of four patients $P=\{P_1,P_2,P_3,P_4\}$ as the Universal set where P_1,P_2,P_3 and P_4 represents patients Ram, Shyam, Jadu, and Madhu with systems $S=\{s_1,s_2,s_3,s_4,s_5\}$ where s_1,s_2,s_3,s_4 and s_5 represents symptoms temperature, headaches, coughs, stomach pain, and body pain, respectively. Let the possible diseases relating to the above symptoms $D=\{D_1,D_2,D_3\}$ be viral fever, typhoid, and malaria.

Again let the set $S=\{s_1,s_2,s_3,s_4,s_5\}$ be a Universal set where s_1,s_2,s_3,s_4 and s_5 represents symptoms temperature, headaches, coughs, stomach pain, and body pain, respectively. Let the possible diseases relating to the above symptoms $D=\{D_1,D_2,D_3\}$ be viral fever, typhoid, and malaria. Suppose that *NSS(F,S)* over *P*, where *F* is a mapping $F:S\to F^P$ gives a collection of an approximate description of patient symptoms in the hospital. Let

(F,S)={F(s_1)={(p_1,0.7, 0.1, 0.2),(p_2, 0.1, 0.8, 0.1), (p_3, 0.6, 0.1, 0.4), (p_4, 0.5, 0.2, 0.4)},

{F(s_2)={(p_1,0.6, 0.1, 0.3),(p_2, 0.4, 0.4, 0.5), (p_3, 0.8, 0.1, 0.2), (p_4, 0.5, 0.4, 0.1) },

F(s_3)={(p_1,0.2, 0.8, 0.2),(p_2, 0.6, 0.1, 0.3), (p_3, 0.0, 0.6, 0.4), (p_4, 0.3, 0.4, 0.5)}, {F(s_4)={(p_1,0.1, 0.6, 0.2),(p_2, 0.1, 0.7, 0.2), (p_3, 0.5, 0.3, 0.3), (p_4, 0.7, 0.2, 0.1)},

{F(s_5)={(p_1,0.1, 0.6, 0.3),(p_2, 0.1, 0.8, 0.2), (p_3, 0.6, 0.5, 0.1), (p_4, 0.3, 0.4, 0.4)}.

The neutrosophic soft set is represented by the following neutrosophic soft matrix to describe the patient symptoms relationship.

$$A=\begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array}\begin{array}{c} \begin{array}{ccccc} s_1 & s_2 & s_3 & s_4 & s_5 \end{array} \\ \begin{bmatrix} (0.7,0.1,0.2) & (0.6,0.1,0.3) & (0.2,0.8,0.2) & (0.6,0.1,0.2) & (0.1,0.6,0.3) \\ (0.1,0.8,0.1) & (0.4,0.4,0.5) & (0.6,0.1,0.3) & (0.1,0.7,0.2) & (0.1,0.8,0.2) \\ (0.6,0.1,0.4) & (0.8,0.1,0.2) & (0.6,0.0) & (0.5,0.3,0.3) & (0.6,0.5,0.1) \\ (0.5,0.2,0.4) & (0.5,0.4,0.1) & (0.3,0.4,0.5) & (0.7,0.2,0.1) & (0.3,0.4,0.4) \end{bmatrix} \end{array}.$$

Again let the set $S=\{s_1,s_2,s_3,s_4,s_5\}$ be a Universal set where s_1,s_2,s_3,s_4 and s_5 represents symptoms temperature, headaches, coughs, stomach pain, and body pain, respectively. Let the possible diseases relating to the above symptoms $D=\{D_1,D_2,D_3\}$ be viral fever, typhoid, and malaria. Suppose that $NSS(G,D)$ over S, where G is a mapping $G:D\to F^S$ gives a collection of an approximate description of medical knowledge of the three diseases and their symptoms. Let

(G, D)={G(D_1)={(s_1,0.6, 0.2, 0.3),(s_2, 0.3, 0.5, 0.4), (s_3, 0.1, 0.8, 0.1), (s_4, 0.4, 0.5, 0.3), (s_5, 0.7, 0.4, 0.2)},

{G(D_2)={(s_1,0.6, 0.2, 0.2),(s_2, 0.2, 0.6, 0.3), (s_3, 0.5, 0.4, 0.3), (s_4, 0.7, 0.2, 0.1), (s_5, 0.1, 0.8, 0.2) },

G(D_3)={(s_1,0.3, 0.4, 0.3),(s_2, 0.7, 0.2, 0.4), (s_3, 0.7, 0.2, 0.3), (s_4, 0.3, 0.4, 0.4), (s_5, 0.2, 0.7, 0.3)}.

Neutrosophic soft set can be represented by the following neutrosophic soft matrix

$$B=\begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{array}\begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \begin{bmatrix} (0.6,0.2,0.3) & (0.6,0.2,0.2) & (0.3,0.4,0.3) \\ (0.3,0.5,0.4) & (0.2,0.6,0.3) & (0.7,0.2,0.4) \\ (0.1,0.8,0.1) & (0.5,0.4,0.3) & (0.7,0.2,0.3) \\ (0.4,0.5,0.3) & (0.7,0.2,0.1) & (0.3,0.4,0.4) \\ (0.7,0.4,0.2) & (0.1,0.8,0.2) & (0.2,0.7,0.3) \end{bmatrix} \end{array}.$$

Neutrosophic soft complement matrix

$$B^c=\begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{array}\begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \begin{bmatrix} (0.3,0.2,0.6) & (0.2,0.2,0.6) & (0.3,0.4,0.3) \\ (0.4,0.5,0.3) & (0.3,0.6,0.2) & (0.4,0.2,0.7) \\ (0.1,0.8,0.1) & (0.3,0.4,0.5) & (0.3,0.2,0.7) \\ (0.3,0.5,0.4) & (0.1,0.2,0.7) & (0.4,0.4,0.3) \\ (0.2,0.4,0.7) & (0.2,0.8,0.1) & (0.3,0.7,0.2) \end{bmatrix} \end{array}.$$

Max-min compositions of two neutrosophic soft matrices will produce the following results:

Let us suppose $A*B=[C_{ij}]_{m\times n}$ where

C11=[max(0.6,0.3,0.1,0.4,0.1),min(0.2,0.5,0.8,0.5,0.6), min(0.2, 0.4, 0.2,0.3,0.3)]=(0.6, 0.2, 0.2),

C12=[max(0.6,0.2,0.2,0.6,0.1),min(0.2,0.6,0.8,0.2,0.8), min(0.2, 0.3, 0.3,0.2,0.3)]=(0.6, 0.2, 0.2),

C13=[max(0.3,0.6,0.2,0.3,0.1),min(0.4,0.2,0.8,0.4,0.7), min(0.3, 0.4, 0.3,0.4,0.3)] =(0.6, 0.2, 0.3),

C21=[max(0.6,0.3,0.1,0.1,0.1),min(0.8,0.5,0.8,0.7,0.8), min(0.3, 0.5, 0.3,0.3,0.2)]=(0.6, 0.5,0.2),

C22=[max(0.1,0.2,0.5,0.1,0.1),min(0.8,0.6,0.4,0.7,0.8), min(0.2, 0.5, 0.3,0.2,0.2)]=(0.5, 0.4,0.2),

C23=[max(0.1,0.4,0.6,0.1,0.1),min(0.8,0.4,0.2,0.7,0.8), min(0.3, 0.5, 0.3,0.4,0.2)]=(0.6, 0.2,0.3),

C31=[max(0.6,0.3,0.0,0.4,0.6),min(0.2,0.5,0.8,0.5,0.5), min(042, 0.4, 0.4,0.3,0.2)]=(0.6, 0.2,0.2),

C32=[max(0.6,0.2,0.0,0.5,0.1),min(0.2,0.6,0.6,0.3,0.8), min(0.4, 0.3, 0.4,0.3,0.2)]=(0.7, 0.2,0.2),

C33=[max(0.3,0.7,0.0,0.3,0.2),min(0.4,0.2,0.6,0.4,0.7), min(0.4, 0.4, 0.4,0.4,0.3)]=(0.7, 0.2, 0.3),

C41=[max(0.5,0.3,0.1,0.4,0.3),min(0.2,0.5,0.8,0.5,0.4), min(0.4, 0.4, 0.5,0.3,0.4)]=(0.5, 0.2,0.3),

C42=[max(0.5,0.2,0.3,0.7,0.1),min(0.2,0.6,0.4,0.3,0.8), min(0.4, 0.3, 0.5,0.1,0.4)]=(0.7, 0.2,0.1),

C43=[max(0.3,0.5,0.3,0.3,0.2),min(0.4,0.4,0.4,0.4,0.7), min(0.4, 0.4, 0.5,0.4,0.4)]=(0.5, 0.4,0.4).

Then

$$A*B=\begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array}\begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \begin{bmatrix} (0.6,0.2,0.2) & (0.6,0.2,0.2) & (0.6,0.2,0.3) \\ (0.6,0.5,0.2) & (0.5,0.4,0.2) & (0.6,0.2,0.3) \\ (0.6,0.2,0.2) & (0.6,0.2,0.2) & (0.7,0.2,0.3) \\ (0.5,0.2,0.3) & (0.7,0.2,0.1) & (0.5,0.4,0.4) \end{bmatrix} \end{array}.$$

Hence

$$V = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \begin{bmatrix} 0.6 & 0.6 & 0.5 \\ 0.9 & 0.7 & 0.5 \\ 0.6 & 0.6 & 0.6 \\ 0.4 & 0.8 & 0.5 \end{bmatrix} \end{array}.$$

Similarly if $A \times B^C$, is calculated then it is obtained that

$$A \times B^c = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \begin{bmatrix} (0.4,0.2,0.2) & (0.3,0.2,0.3) & (0.4,0.2,0.3) \\ (0.4,0.2,0.3) & (0.3,0.4,0.2) & (0.4,0.2,0.2) \\ (0.4,0.2,0.3) & (0.6,0.2,0.1) & (0.4,0.2,0.2) \\ (0.4,0.2,0.3) & (0.3,0.2,0.2) & (0.4,0.4,0.3) \end{bmatrix} \end{array}.$$

Hence

$$W = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \begin{bmatrix} 0.4 & 0.2 & 0.3 \\ 0.3 & 0.5 & 0.4 \\ 0.3 & 0.7 & 0.4 \\ 0.3 & 0.3 & 0.5 \end{bmatrix} \end{array},$$

and finally it is observed that

$$V - W = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \begin{bmatrix} 0.2 & 0.4 & 0.2 \\ 0.6 & 0.2 & 0.1 \\ 0.3 & -0.1 & 0.2 \\ 0.1 & 0.5 & 0.0 \end{bmatrix} \end{array}.$$

It is clear from the above matrix that the patients $\{p_1, p_4\}$ suffering from disease $\{D_2\}$ and patients $\{p_2, p_3\}$ are suffering from disease $\{D_1\}$.

5 | Conclusions

It is seen that this method of finding the patients suffering from various diseases from studying the occurrence of systems with the help of one simple application. In future our effort will be to apply this method to various other situations involving some of the uncertain parameters.

References

[1] Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, 8, 338-353.

[2] Atanassov, K. (2016). Intuitionistic fuzzy sets. *International journal bioautomation*, 20(1), 87-96.

[3] Molodtsov, D. (1999). Soft set theory—first results. *Computers & mathematics with applications*, *37*(4-5), 19-31.

[4] Maji, P. K. Biswas, R., & Roy, A. R. (2001). Fuzzy soft sets. *Journal of fuzzy mathematics*, *9*(3), 589-602.

[5] Maji, P. K., Biswas, R., & Roy, A. R. (2003). Soft set theory. *Computers & mathematics with applications*, *45*(4-5), 555-562.

[6] Maji, P. K. (2009, December). More on intuitionistic fuzzy soft sets. *International workshop on rough sets, fuzzy sets, data mining, and granular-soft computing* (pp. 231-240). Berlin, Heidelberg: Springer.

[7] Smarandache, F. (2018). Extension of soft set to hypersoft set, and then to plithogenic hypersoft set. *Neutrosophic sets syst*, *22*, 168-170.

[8] Smarandache, F. (2019). Neutrosophic set is a generalization of intuitionistic fuzzy set, inconsistent intuitionistic fuzzy set (picture fuzzy set, ternary fuzzy set), pythagorean fuzzy set, spherical fuzzy set, and q-rung orthopair fuzzy set, while neutrosophication is a generalization of regret theory, grey system theory, and three-ways decision (revisited). *Journal of new theory*, (29), 1-31.

[9] Vellapandi, R., & Gunasekaran, S. (2020). A new decision making approach for winning strategy based on muti soft set logic. *Journal of fuzzy extension and applications*, *1*(2), 119-129.

[10] Maji, P. K. (2013). Neutrosophic soft sets. *Annals of fuzzy mathematics and informatics*, *5*(1), 157-168.

[11] Maji, P. K. (2012). A neutrosophic soft set approach to a decision making problem. *Annals of fuzzy mathematics and informatics*, *3*(2), 313-319.

[12] Deli, I., & Broumi, S. (2015). Neutrosophic soft matrices and NSM-decision making. *Journal of intelligent & fuzzy systems*, *28*(5), 2233-2241.

[13] Das, S., Kumar, S., Kar, S., & Pal, T. (2019). Group decision making using neutrosophic soft matrix: An algorithmic approach. *Journal of king saud University-computer and information sciences*, *31*(4), 459-468.

[14] Kumar Das, S. (2020). Application of transportation problem under pentagonal neutrosophic environment. *Journal of fuzzy extension and applications*, *1*(1), 27-41.

E-ISSN: 2717-3453 | P-ISSN: 2783-1442

Journal of Fuzzy Extension and Applications

J. Fuzzy. Ext. Appl

www.journal-fea.com

J. Fuzzy. Ext. Appl. Vol. 2, No. 1 (2021) 33–40.

Paper Type: Short Communication

Neutrosophication of Statistical Data in a Study to Assess the Knowledge, Attitude and Symptoms on Reproductive Tract Infection among Women

Somen Debnath*

Department of Mathematics, Umakanta Academy, Agartala-799001, Tripura, India; somen008@rediffmail.com.

Citation:

Debnath , S. (2021). Neutrosophication of statistical data in a study to assess the knowledge, attitude and symptoms on reproductive tract infection among women. *Journal of fuzzy extension and application*, *2* (1), 33-40.

Received: 08/10/2020 Reviewed: 22/11/2020 Revised: 16/01/2021 Accept: 02/02/2021

Abstract

Statistics mainly concerned with data that may be qualitative or quantitative. Earlier we have used the notion of statistics in the classical sense where we assign values that are crisp. But in reality, we find some areas where the crisp concept is not sufficient to solve the problem. So, it seems difficult to assign a definite value for each parameter. For this, fuzzy sets and logic have been introduced to give the flexibility to analyze and classify statistical data. Moreover, we may come across such parameters that are indeterminate, uncertain, imprecise, incomplete, unknown, unsure, approximate, and even completely unknown. Intuitionistic fuzzy set explain uncertainty at some extent. But it is not sufficient to study all sorts of uncertainty present in real-life. It means that there exists data which are neutrosophic in nature. So, neutrosophic data plays a significant role to study the concept of indeterminacy present in a data without any restriction. The main objective of preparing this article is to highlighting the importance of neutrosophication of statistical data in a study to assess the symptoms related to Reproductive Tract Infections (RTIs) or Sexually Transmitted Infections (STIs) among women by sampling estimation.

Keywords: Neutrosophic statistics, Neutrosophic data, Sample size, Sexually transmitted disease.

1 | Introduction

Nowadays the concept of vagueness or ambiguity or uncertainty is very common in solving different real- life problems. Zadeh is the pioneer of introducing fuzzy sets in his work given in [18], which is recognized by the Mathematical fraternities and researchers from all over the world, as the most efficient tool to handle the vague concept in a systematic manner. So many theories and results have been introduced since then and it has been successfully implemented in various fields. To realize the importance of non-membership value it has been extended by introducing the intuitionistic fuzzy set initiated by Atanassov given in [1].

Corresponding Author: somen008@rediffmail.com

http://dx.doi.org/10.22105/jfea.2021.272508.1073

Still, all these theories are not sufficient to express the indeterminacy involves in solving problems, which led to the introduction of the neutrosophic set proposed by Smarandache given in [13]. The Neutrosophic set enables us to handle incomplete, inconsistent, uncertain, and even unknown data. Since the introduction of the neutrosophic set theory, a lot of theories have been developed, some of them are given in [4], [5], [7], [10], [14], [15], [17].

Statistics mainly dealt with numerical data and by embedding this idea it has been used successfully in various applications of mathematics, economics, biology, psychology, probability, etc. By statistics, we simply mean classical statistics. In real life environment, due to indeterminacy involved in data, we cannot always get numerical data in exact form but sometimes we need to approximate them, which laid the foundation of neutrosophic statistics, introduced by Smarandache [16], as an extension of classical statistics. If any data is denoted by the symbol d then the neutrosophication of the same data is denoted by N_d and its set-theoretic representation is in the form $<T_d, I_d, F_d>$, where T_d, I_d and F_d are used to represent the true value, indeterminate value and the false value respectively of the given data. Under the neutrosophic environment, the statistical data may be partially known, partially unknown, and may be completely unknown and it is due to the incomplete knowledge or information as far as the problem is concerned. Unlike classical statistics where data are known (crisp values), in neutrosophic statistics there exists some indeterminacy (approximate the crisp values) in the data. If there is no indeterminacy then we say that neutrosophic statistics coincides with classical statistics. In neutrosophic statistics, a sample size cannot be denoted by a single value but it can be represented by an interval. For example, in neutrosophic statistics, we consider an approximate sample size as $n= [40, 80]$ i.e. between 40 and 80.

As per the World Health Organization (WHO), STIs have a profound and great impact on sexual and reproductive health worldwide. STIs like herpes and syphilis can increase the risk of HIV acquisition. Mother to child transmission of STIs can result in low-birth-weight, neonatal death, prematurity, and congenital deformities. Low and middle-income countries rely on identifying consistent, easily recognizable signs and symptoms to guide treatment, without the use of laboratory tests. This is known as syndrome management. This approach allows health workers in rural areas to diagnose a specific infection based on observed syndrome (i.e. vaginal discharge, urethral discharge, abdominal pain, genital ulcers,etc). Rizwan et al. [9] conducted a study on assessed knowledge, attitude and, practices on STIs among married women aged 15-45 years in rural Haryana. Geetha [6] conducted a study on the prevalence of RTIs among rural married women in Tamil Nadu, India. Shingade et al. [12] conducted a study on treatment-seeking behavior for RTI/STIs among married women in urban slums of Mumbai, India. Sangeetha [11] study reproductive tract infections among pregnant women in the reproductive age group in Hubli, Karnataka, India. In 2019, Durai et al. [2] studied reproductive tract infection in rural India. Kafle et al. [3] discussed prevalence and factors associated with reproductive tract infection. Naderi et al. [8] conducted a study based on reproductive tract infection among women suffering from rheumatoid arthritis in India.

Reproductive tract infection (both sexually and non-sexually) of the reproductive tract is a colossal public health problem that is exceedingly prevalent worldwide. RTIs are one of the most prevalent health morbidities among women throughout the world especially, in developing countries. These are due to the infection of the reproductive system and the group of communicable diseases that are transmitted predominantly by sexual contact and caused by a wide range of bacterial, viral, protozoan, and fungal agents. Sexual diseases, whose mode of transmission is known, are largely preventable, but due to lack of knowledge about the sexual disease, it spreads rapidly in India as it has a large population. In Southeast Asia, young people do not typically have access to sex education and are poorly informed about how to protect themselves from unwanted pregnancies, STDs, and HIV/AIDS. Women suffer more adverse consequences from STDs than men because generally it is easier for STDs to be transferred from men to women.

Debnath | J. Fuzzy. Ext. Appl. 2(1) (2021) 33-40

About one-third of women from rural India had reported anyone new symptoms of RTI/STIs. Women especially in rural India have a lot of misconceptions regarding RTIs. They are submissive to men socially and economically and they have less control over their sexuality. Low-level of literacy and ignorance among rural women result in misconceptions about the illness. RTIs are often asymptomatic in women and may often go, undiagnosed, and untreated until complications like infertility supervene. Some of the common complications of RTI's include highly destructing, Pelvic Infertility Disease (PID), ectopic pregnancy, infertility, adverse outcomes of pregnancy, neonatal morbidity, and death. SIT's facilitate HIV transmission.

In India, steps are taken to reduce RTI/STI's but still, it is major problem affecting the reproductive age group of women. Despite all the efforts, inadequate knowledge and attitude on RTI/STI's and limited adherence to treatment are the major factors contributing to the success of RTI/STI's. Knowledge, attitude, and awareness about symptoms are essential to prevent and treat RTI/STI's during reproductive age group women.

Earlier crisp theory was introduced to assess data. But human theories do not always obey the crisp concept. That is two valued logic is no more an appropriate tool to emulate human thinking. Some theories are vague, imprecise, uncertain or fuzzy in nature. However, fuzzy data cannot define all sorts of uncertainty present in various fields. This leads to the introduction of neutrosophic data. With an aid of neutrosophic data we can explain uncertainty involved in a data. So, there is a need of neutrosophic data in a present study. The purpose of the study is to assess the knowledge, attitude, and self-reported symptoms on RTI/STI's among rural women where all the information related to RTI/STI's are indeterminate or neutrosophic in nature. Also, we approximate the sample size to make our study simple and concrete.

2| Neutrosophic Data

It refers to the kind of statistical data that contains some/complete indeterminacy and it is represented in the form of an interval. It is an extension of classical statistical data in which the data represented by a single definite numerical value. We have the following definitions:

Definition 1. [16]. A neutrosophic data is said to be discrete neutrosophic data if the values are isolated. For example $8+\Im_1$, where $\Im_1 \in [1, 3]$, $5+7\Im_2$, where $\Im_2 \in [2, 5]$, 17, etc.

If the values form one or more intervals then it is called the continuous neutrosophic data. For example: $[0.1, 0.6]$ or $[0.2, 0.8]$ (not sure exactly).

Definition 2. [16]. A data is said to be quantitative neutrosophic data if it is numerical. For example: any value in the interval $[3, 8]$; $23, 45, 67$ or 34 (don't know exactly).

A categorical data is called qualitative neutrosophic data. For example a pen or pencil, black or white.

Definition 3. [16]. A data is said to be a univariate neutrosophic data if it consists of observations on a neutrosophic single attribute. In the case of two or more attributes, it is called multivariate neutrosophic data.

Definition 4. [16]. A table that contains categories, frequencies and, relative frequencies with some indeterminacies is called a neutrosophic frequency distribution table.

For example, number of failed subjects (when we do not know exactly) by students can be represented in the following neutrosophic frequency distribution *Table 1*.

Table 1. Neutrosophic frequency distribution.

Number of Failed Subjects	Neutrosophic Frequency	Neutrosophic Cumulative Frequency
0	60	[0.214, 0.316]
1	[40, 80]	[0.167, 0.348]
2	[60, 90]	[0.24, 0.41]
3	[30, 50]	[0.115,0.238]
Total 0-3	[190, 280]	

3| Methodology (Sampling Criteria)

The study was mainly designed to assess the knowledge, attitude, and self-reported symptoms on RTI/STIs among women at selected villages in South Tripura district, India. In this work, quantitative research is used where we have the indeterminacy in sample size. Sample individuals are asked questions and they reply voluntarily but sometimes they have a biased response which may influence the results and there may be some malicious people who might answer the opposite of the questions. The study is conducted in the 9 villages of Rural Health and Training Center (RHTC) at South Tripura district which covers 32 villages with a population of 13,992.

The population of these villages is 702,284,443,123,837,271,501,327 and 211, respectively. The number of outpatients attending RHTC is 120-150 patients daily. These villages are situated at a distance of 2-3 km from RHTC and the majority of the population is Hindu. By occupation, most of the village people are farmers. The target population for the study included married women in the age group of 15-49 years. They are willing to participate and able to speak and understand Bengali (non-Tribe) or Kokborok (Tribe) or English (official) languages and they live with their husband. The sample size is determined by the assumption that only 35.4% to 44.6% of the women are knowledgeable on RTI/STIs with the marginal error of 5% to 10% and 90% to 95% CI. Based on this assumption, the approximate sample size for the study is determined using the formula given by:

$_{approx} = \frac{^{2}pq}{min\,(d^2)}$, where = value corresponding to 90% to 95% level of significance ([1.64, 1.96]).

p =expected proportion of knowledge on maternal care among women ([0.354, 0.446]).

q =1- p = [1-0.446, 1-0.354] = [0.554, 0.646].

d =absolute precision= [0.05, 0.1].

Then, $_{approx} = \frac{[0.518, 1.123]}{min[0.0025, 0.01]}$ = [207,449].

The size of the sample size should be ranging from 207 to 449 based on our survey. So it is called a neutrosophic sample size. If both the values converge then it reduces to classical sample size.

Debnath |J. Fuzzy. Ext. Appl. 2(1) (2021) 33-40

The data are collected from the sample size of [207,449] (as there exists indeterminacy) to find out their knowledge, attitude, and self-reported symptoms on RTI/STIs. The study findings are arranged and presented in the following sections:

Section A: Distribution of women according to their background variables.

Section B: The knowledge, attitude and self-reported symptoms and score of women on RTI/STIs.

Section C: The correlation between the knowledge and attitude of women.

Section D: The association between the knowledge, attitude, self-reported symptoms, and background variables of women.

Distribution of women according to their Demographic variables shown in *Table 2*.

Table 2. Distribution of women according to their demographic variables.

Demographic Variable	N	%
Age (in years)	**[80,240]**	
18-26	110	[0.277,O.653]%
27-35	[17,99]	[0.245,0.531]%
36-45		[0.047,0.342]%
Educational Status		
No formal education	72	[0.16,0.348]%
Primary	[70,150]	[0.19,0.526]%
Upper primary	[30,90]	[0.077,0.337]%
High school	[20,82]	[0.051,0.304]%
Higher secondary	[10,50]	[0.024,0.202]%
Graduate	5	[0.011,0.024]
Occupational Status		
Unemployed/house wives	[120,250]	[0.376,0.742]%
Unskilled worker	25	[0.056,0.121]%
Skilled worker	20	[0.044,0.097]%
Clerical/shop owner/farmer	[37,149]	[0.11,0.72]%
Professional	5	[0.011,0.024]%
Type of Family		
Nuclear family	120	[0.267,0.58]%
Joint family	[50,280]	[0.229,0.64]%
Extended family	[37,49]	[0.085,0.224]%
Information about RTI/STIs		
Friends	[20,120]	[0.057,0.39]%
Family	45	[0.1,0.217]%
School/college	[20,40]	[0.047,0.176]%
Television	[35,80]	[0.087,0.317]%
Magazine	[20,70]	[0.05,0.272]%
Internet	[24,37]	[0.055,0.168]%
Hospital/clinic	35	[0.077,0.169]%
Others	[8,22]	[0.017,0.106]%

All the above findings can also be represented in the form of diagrams to visualize the information at a glance (the knowledge, attitude, and self-reported symptoms score of women's on RTI/STIs).

Distribution of level of knowledge on RTI/STIs $(approx = [207, 449])$ are given in *Table 3*.

Table 3. Distribution of level of knowledge on RTI/STIs.

Level of Knowledge on RTI/STIs	N	%
Adequate (76-100%)	0	0
Moderately Adequate (51-75%)	[120,180]	[0.308,0.674]%
Inadequate (0-50%)	[87,269]	[0.326,0.691]%

Distribution of level of women's attitude on RTI/STI $(_{approx} = [207, 449])$ are given in *Table 4*.

Table 4. Distribution of level of women's attitude on RTI/STIs.

Level of Attitude on RTI/STIs	N	%
Highly favorable (76-100%)	0	0
Favorable (51-75%)	[150,332]	[0.562,0.853]%
Unfavorable (<50%)	[57,117]	[0.146,0.438]%

Distribution of level of self-reported symptoms on RTI/STIs among women $(_{approx} = [207, 449])$ given in *Table 5*. Mean (M), Standard Deviation (S.D), and Coefficient of Variation (C.V) of knowledge and attitude on RTI/STIs $(_{approx} = [207, 449])$ are given in *Table 6*.

Table 5. Distribution of level of self reported symptoms on RTI/STIs among women.

Level of Self Reported Symptoms	n	%
No symptoms	60	[0.134,0.29]%
Mild symptoms (1-5)	[120,330]	[0.5,0.79]%
Severe symptoms (6-10)	[27,59]	[0.065,0.247]

Table 6. Knowledge and attitude on RTI/STIs $(_{approx} = [207, 449])$.

Knowledge And Attitude on RTI/STIs	Maximum Score	M	S.D	$C.V = \frac{S.D}{M} \times 100$	Range of C.V
Total knowledge	48	[14.3, 16.5]	[1.5,2.6]	[10.5,15.7]	5.2
General knowledge	6	[1.4, 2.6]	[0.45,0.88]	[32.1,33.8]	1.7
Signs and symptoms	15	[2.5, 3.5]	[1.1, 1.9]	[44, 54.2]	10.2
Prevention	10	[2.6,4.8]	[0.9,1.7]	[34.6, 35.4]	0.8
Complication	17	[3.4,5.6]	[1.4,2.6]	[41.1,46.4]	5.3
Total attitude	45	[28.6, 30.8]	[2.3, 3.6]	[8.04,11.68]	3.64

Table 6 shows that the signs and symptoms aspect has the highest range of C.V score and prevention has the lowest range of C.V score and the C.V score of total knowledge is [10.5, 15.7]. On the other hand, the C.V score of total attitude is [8.04, 11.68]. So it is well justified fact that less range of precaution leads to a wide range of signs and symptoms. Therefore, this preliminary study will help the health workers to take necessary steps to check the signs and symptoms by spreading awareness among the people so that they will be able to take preventive measures as necessary.

4| Limitation

Our study is limited to married women with age limit 15-49 years and they live with their husband. The assessment of attitude is limited for only one time and information on self-reported symptoms is obtained from the verbal response of the women and more importantly, here we use inclusion criteria for sampling estimation.

Debnath |J. Fuzzy. Ext. Appl. 2(1) (2021) 33-40

5| Conclusions

The study is conducted under a neutrosophic environment in which there exists some or complete indeterminacy to assess the knowledge, attitude, and self-reported symptoms among women on RTI/STIs in selected villages of South Tripura District in India by assuming an approximate sample size and it is predicted as per our survey report is concerned. We also find the C.V score of knowledge and attitude. Current findings underscore the urgent need for culturally relevant and effective reproductive health education for rural women. Specifically, community-based interventions targeting these women in improvised environments should be implemented to reduce the extreme health disparity gap in the lives of women in rural areas. In the future, the same study can be conducted as a compared study between urban and rural women. A similar study can be conducted on female health workers to improve preventive and primitive health services and enhance the literacy of women which helps to improve reproductive health.

Conflicts of Interest

Declare no conflict of interest.

Acknowledgements

Author is thankful to the reviewer for his/her valuable comments.

References

[1] Atanassov, K. (1986). Intuitionistic fuzzy sets. *Fuzzy sets and systems, 20*, 87-96.

[2] Durai, V., Varadharajan, S., & Muthuthandavan, A. R. (2019). Reprductive tract infections in rural India-a population based study. *Journal of family medicine and primary care, 8, 3578-3583.*

[3] Kafle, P., & Bhattarai, S. S. (2016). Prevalence and factors associated with reproductive tract infections in Gongolia village, Rupandehi district, Nepal. *Advances in Public Health.* https://doi.org/10.1155/2016/8063843

[4] Maji, P. K. (2013). Neutrosophic soft set. *Annals of fuzzy mathematics and informatics, 5, 157-168*.

[5] Maji, P. K. (2012). A neutrosophic soft set approach to a decision making problem. *Annals of fuzzy mathematics and informatics, 3*, 313-319.

[6] Mani, G., Annadurai, K., & Danasekaran, R. (2013). Healthcare seeking behaviour of symptoms of reproductive tract infections among rural married women in Tamil Nadu-A community based study. *Online journal of health and allied sciences, 12, 1-4.*

[7] Mitra Basu, T., & Mondal, S. K. (2015). Neutrosophic soft matrix and its application in solving group decision making problems from medical science. *Computer communication and collaboration, 3(1), 1-31.*

[8] Naderi, J., Chopra, A., & Relwani, R. (2020). Reproductive tract infection among women suffering from rheumatoid arthritis in India: a clinical-based, cross-sectional study. *Jundishapur journal of microbiology, 13.* DOI:10.5812/jjm.97176

[9] Rizwan, S. A., Rath, R. S., & Vivek, G. (2015). KAP study on sexually transmitted infections/Reproductive tract infections (STIs/RTIs) among married women in rural Haryana. *Indian dermatology online journal*, *6*(1), 9-12.

[10] Sahin, M., Alkhazaleh, S., & Ulucay, V. (2015). Neutrosophic soft expert sets. *Applied mathematics, 6(1)*, 116-127.

[11] Sangeetha, S. (2012). A study of reproductive tract infections among pregnant women in the reproductive age group, in Urban Field Practice Area in Hubli, Karnataka, India. *Annals of tropical medicine and public health*, *5*(3), 209-*213.*

[12] Shingade, P. P., Kazi, Y., & Madhavi, L. H. (2015). Treatment seeking behavior for sexually transmitted infections/reproductive tract infections among married women in urban slums of Mumbai, India. *South east asia journal of public health*, *5*(2), 65-70.

[13] Smarandache, F. (2005). Neutrosophic set-a generalization of the intuitionistic fuzzy sets. *International journal of pure and applied mathematics*, *24*(3), 287–297.

[14] Smarandache, F. (1998). *Neutrosophy, neutrosophic probability, set and logic*. American Research Press.

[15] Smarandache, F. (2001). Preface: an introduction to neutrosophy, neutrosophic logic, neutrosophic net, and neutrosophic probability and statistics. *Proceeding of the first international conference on neutrosophy, neutrosophic logic, set, probability and statistics.* https://dl.acm.org/doi/proceedings/10.5555/779586

[16] Smarandache, F. (2014). Introduction to neutrosophic statistics. Zendo. DOI: 10.5281/zenodo.8840

[17] Tas, F., Topal, S., & Smarandache, F. (2018). Clustering neutrosophic data sets and neutrosophic valued metric spaces. Symmetry, *10*(10). https://doi.org/10.3390/sym10100430

[18] Zadeh, L. A. *(1965).* Fuzzy set. *Information and control*, *8*, *338-353.*

Debnath |J. Fuzzy. Ext. Appl. 2(1) (2021) 33-40

E-ISSN: 2717-3453 | P-ISSN: 2783-1442

Journal of Fuzzy Extension and Applications

www.journal-fea.com

J. Fuzzy. Ext. Appl. Vol. 2, No. 1 (2021) 41–68.

Paper Type: Research Paper

Interval Type-2 Fuzzy Logic System for Remote Vital Signs Monitoring and Shock Level Prediction

Uduak Umoh[1,*], Samuel Udoh[1], Abdultaofeek Abayomi[2], Alimot Abdulazeez[1]

[1] Department of Computer Science, University of Uyo, Uyo, PMB 1017, Uyo, Akwa Ibom State, Nigeria; uduakumoh@uniuyo.edu.ng;samueludoh@uniuyo.edu.ng; alimotolaide4real@gmail.com.

[2] Department of Information and Communication Technology, Mangosuthu University of Technology, P.O. Box 12363 Jacobs, 4026 Durban, South Africa; taofeekab@yahoo.com.

Citation:

Umoh, U., Udoh, S., Abayomi, A., & Abdulazeez, A. (2021). Interval type-2 fuzzy logic system for remote vital signs monitoring and shock level prediction. *Journal of fuzzy extension and application, 2* (1), 41-68.

Received: 08/11/2020 Reviewed: 24/12/2020 Revised: 06/01/2021 Accept: 17/02/2021

Abstract

Interval Type-2 Fuzzy Logic Systems (IT2 FLSs) have shown popularity, superiority, and more accuracy in performance in a number of applications in the last decade. This is due to its ability to cope with uncertainty and precisions adequately when compared with its type-1 counterpart. In this paper, an Interval Type-2 Fuzzy Logic System (IT2FLS) is employed for remote vital signs monitoring and predicting of shock level in cardiac patients. Also, the conventional, Type-1 Fuzzy Logic System (T1FLS) is applied to the prediction problems for comparison purpose. The cardiac patients' health datasets were used to perform empirical comparison on the developed system. The result of study indicated that IT2FLS could coped with more information and handled more uncertainties in health data than T1FLS. The statistical evaluation using performance metrices indicated a minimal error with IT2FLS compared to its counterpart, T1FLS. It was generally observed that the shock level prediction experiment for cardiac patients showed the superiority of IT2FLS paradigm over T1FLS.

Keywords: Uncertainty, Mamdani fuzzy inference, Cardiac patient, Health data, Deffuzification.

1 | Introduction

Non-linear problems, especially problems that are more relevant to everyday life activities cannot be solved with conventional models as they are not well suited. The increase in popularity of the fuzzy logic systems in problem solving can be attributed to its ability to incorporate human reasoning in its algorithm. The notion of Fuzzy Sets (FSs) was introduced in [1] as a method of representing uncertainty and vagueness in a way that elements are not limited to binary Membership Functions (MFs) of 0 or 1 but rather is a continuum in 0 and 1. A Type-1 Fuzzy System (T1FLS) is a rule based system which can be viewed as a process that maps crisp inputs to outputs by using the theory of fuzzy sets [2].

Corresponding Author: uduakumoh@uniuyo.edu.ng

http://dx.doi.org/10.22105/jfea.2021.255064.1028

T1FLSs have the ability to process information using linguistic variables and make decision with imprecise, vague, ambiguous and uncertain data. The beauty of T1FLSs is its' ability to represent nonlinear input/output relationships by using IF/THEN statements, called rules [3]. T1FLS consists of four components: fuzzifier, fuzzy rules, inference engine, and defuzzifier [4]. T1FLSs have achieved great success in handling many different real-world problems such as classification, regression, control, decision making, prediction, and so on [5-11]. However, due to the complexity and uncertainty in many real-world problems, T1FLs cannot adequately cope with or minimize the effects of the uncertainties posed by the complex nature of many real-world problems. These uncertainties can be as a result of uncertainty in inputs, uncertainty in outputs, uncertainty that is related to the linguistic differences, uncertainty caused by the change of conditions in the operation and uncertainty associated with the noisy data when training the fuzzy logic system [12].

In order to address this problem, [13] has recognized this potential limitation and introduced a higher type of fuzzy sets which is the concept of Type-2 Fuzzy Logic System (T2FLS) from Type-2 Fuzzy Set (T2FS). T2FLS is an extension of T1FLS with additional design degrees of freedom where the MFs are themselves fuzzy with the actual membership grade of an element assumed to lie within a closed interval of 0 and 1. However, T2FLS suffers computational complexity and to resolve this problem, Interval Type-2 Fuzzy Logic System (IT2FLS) is used which is a simplified version of T2FLS with reduced computational intensity making it quite practicable. Recently, T2FLSs and IT2FLSs have been successfully applied to handle a great deal of uncertainties in prediction problems and the results are very encouraging [14-15]. Consequently, IT2FLSs have dominated the research field and T2FLS are widely applied in various are due to their simpler structure and reduced computational cost [16]-[23].

In this study, remote vital signs monitoring and prediction of shock level in cardiac patients using an IT2FLS based on Mamdani fuzzy inference is presented. Our motivation is to apply IT2FS to reduce prediction error in the task of modeling uncertainties in health data. Mamdani Fuzzy Inference System (FIS) is employed as the background algorithm. Also, the study investigates the prediction of problems using conventional T1FLS for comparison purpose. The rest of the paper is structured as follows: Section 2 involves the preliminaries and reviews of the concepts on TIFS and IT2IFS. In Section 3, the design of IT2FLS for remote vital signs monitoring and prediction of shock level in cardiac patients is carried out. Our model results and discussion are presented in Section 4 and Section 5 involves conclusion of study.

2| An Overview of Type-1 and Interval Type-2 Fuzzy Sets

In this section, basic concepts of TIFS and IT2IFS are reviewed to deliver the fundamental knowledge required for comprehension of the work.

2.1| Type-1 Fuzzy Sets

According to [24], a classical fuzzy set otherwise called a T1FS, denoted by A, is characterized by a Type-1 Membership Function(T1MF), $\mu_A(x)$ where $x \in X$, and X is the domain of definition of the variable as seen in *Eq. (1)*.

$$A = \{\{(x, \mu_A(x))\} | \forall x \in X\}. \tag{1}$$

Umoh et al. |J. Fuzzy. Ext. Appl. 2(1) (2021) 41-68

Where μ_A is called aType-1 membership function of T1FS, A and maps each element of A to a membership value called membership grade between 0 and 1. *Eqs. (2)* and *(3)* represent the T1FS, A in its discrete and continuous forms respectively.

$$A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \cdots + \mu_A(x_n)/x_n, \quad (2)$$

$$A = \int_X \mu A(x)/x. \quad (3)$$

Where x_1, x_2, x_n are members of fuzzy set A and $\mu_A(x_1)$, $\mu_A(x_2)$, $\mu_A(x_n)$ are their respective membership grades.

2.2 | Type-1 Fuzzy Logic Systems (T1FLS)

The T1FLS also known as T1FIS is both, intuitive and numerical. The inference engine combines rules and gives a mapping from input type-1 fuzzy sets to output type-1 fuzzy sets. A fuzzy rule is a conditional statement in the form of IF-THEN where it contains two parts, the IF part called the antecedent part and the THEN part called the consequent part. Every T1FIS is associated with a set of rules obtained either from numerical data, or from problem domain experts with meaningful linguistic interpretations. In literature, we have two rules methods: Mamdani fuzzy rules [25] and [26] and Tagaki Sugeno Kang (TSK) [27] rules respectively. In Mamdani rule, the rule consequents are fuzzy sets while in TSK, the rule consequents are crisp functions of the inputs and can be illustrated in the form as follows:

$$\text{Mamdani: } R_k \text{IF } x_i \text{ is } A^k_i \text{and,., and } x_n \text{ is } A^k_n \text{ then } y_k \text{ is } B^k_i. \quad (4)$$

$$\text{TSK: } R_k \text{IF } x_i \text{ is } A^k_i \text{ and,., and } x_n \text{ is } A^k_n \text{ then } y_k = a * x_1 + b * x_2 + c. \quad (5)$$

Where, y is a T1FS, and a, b, c are crisp coefficients.

The membership grades in the fuzzy input sets are combined with those in the fuzzy output sets using the composition method and the outcome is combined with rules in an antecedent/consequent format, and then aggregated according to approximate reasoning theory. The structure of a T1FIS is shown in *Fig. 1* and consists of four basic components namely: the fuzzifier, the fuzzy rule-base, the inference engine, and the defuzzifier.

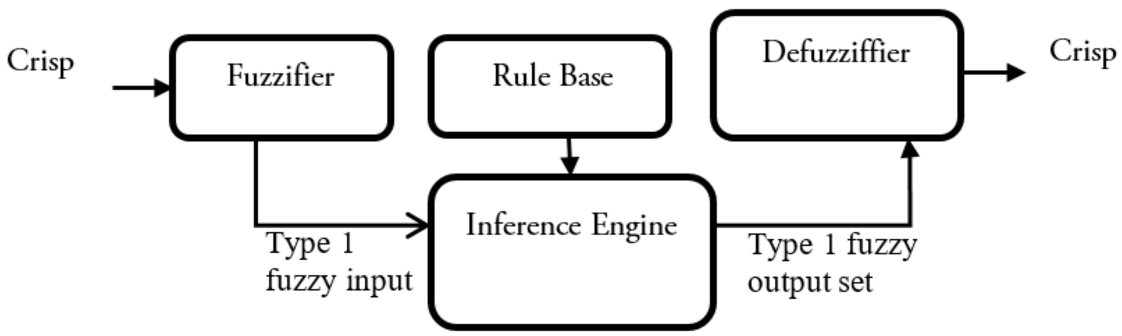


Fig. 1. Structure of a type 1 fuzzy logic system [18].

The fuzzifier maps the crisp input into a T1FS sets by evaluating the crisp inputs based on the antecedent part of the rules and assigns each crisp input a degree of membership in its input fuzzy set. The fuzzy rule-base is a collection of IF–THEN statements called rules in the form represented in *Eqs. (4)* and *(5)*. The inference engine combines the type-1 fuzzy input set and the rules to produce a type-1 fuzzy output and the defuzzifier maps the type-1 fuzzy output set, T1FSs that is produced by the inference engine into crisp values. The centroid of area defuzzification method based on [24] is applied to compute the crisp number as seen in *Eq. (6)*.

$$\text{Crisp Output, } Z = \sum_{i}^{n} \acute{Y}_k Z_k = \frac{\sum_{i=1}^{n} \acute{Y}_k Z_k}{\sum_{i=1}^{n} \acute{Y}_k} = \frac{\sum_{i=1}^{n} Z_k}{\sum_{i=1}^{n} Y_k}. \tag{6}$$

Where, $\sum_{i=1}^{n} \acute{Y}_k Z_k$ is the membership value in the membership function and $\acute{Y}_k$is the centre of membership function which is the running point in a discrete universe. The expression can be interpreted as the weighted average of the elements in the support set [28].

2.3 | Interval Type-2 Fuzzy Set (IT2FS)

An IT2FS is a simplified version of T2FS (1) introduced in [13] as an extension of T1FSand is characterized by a Type-2 Membership Function (T2MF), $\mu_{\tilde{A}}(x,u)$ and $0 \le \mu_{\tilde{A}}(x,u) \le 1$ as represented in *Eq. (7)* [12]. For an IT2FS, for which X and U are discrete, the domain of $\tilde{A}$ is equal to the union of all of its embedded T1 FSs as seen in *Eq. (8)*.

$$\tilde{A} = \left\{\left((x,u), \mu_{\tilde{A}}(x,u)\right) \middle| \forall\ x\ \in X, \forall\ u\ \in Jx \subseteq\ [0,1], \tag{7}$$

$$\tilde{A} = \sum_{i=1}^{n} \left[\sum_{u \in Jx} [1/u]\right] / x_i\ . \tag{8}$$

Where $x\ \in X$ and $u\ \in J \subseteq\ [0,1x]$; x is the primary variable with a domain X and $u \in U$ is the secondary variable with domain J_x at each $x \in X$. Jx is the primary membership of x and the secondary grades of all equal 1 [15, 18]. Hence, a type-2 membership grade can be any subset in [0, 1] [12]. Uncertainty about Ã is conveyed by the union of all the primary memberships, which is called the Footprint Of Uncertainty (FOU) of A as shown in *Eq. (9)* and *Fig. 2,* respectively.

$$\mu_{\tilde{A}}(x,u) = 1, \text{FOU}\left(\tilde{A}\right) = \bigcup_{\forall x \in X} J_x = \{(x,u) \colon u\ \in Jx\ \subseteq\ [0,1]\}. \tag{9}$$

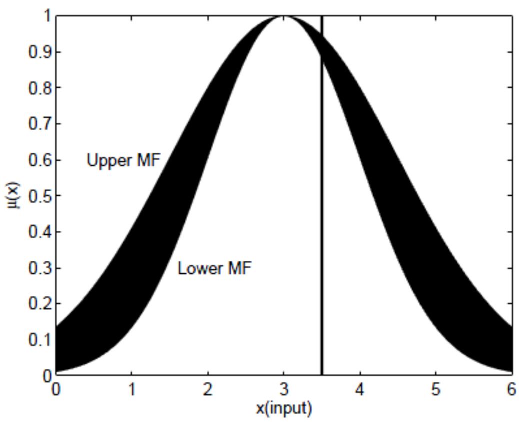


Fig. 2. Interval type-2 fuzzy set [29].

The Upper Membership Function (UMF) and Lower Membership Function (LMF) of $\tilde{A}$comprises two type-1 MFs that bound the FOU. The UMF is associated with the upper bound of *FOU* $(\tilde{A})$ and is denoted by $\overline{\mu}_{\tilde{A}}(x), \forall x \in X$, and the LMF is associated with the lower bound of *FOU* $(\tilde{A})$ and is denoted $\underline{\mu}_{\tilde{A}}(x), \forall x \in X$, that is:

$$\overline{\mu}_{\tilde{A}}(x) \equiv \overline{\mathrm{FOU}\left(\tilde{A}\right)} \quad \forall x \in X, \tag{10}$$

$$\underline{\mu}_{\tilde{A}}(x) \equiv \underline{\mathrm{FOU}\left(\tilde{A}\right)} \quad \forall x \in X, \tag{11}$$

$$J_x = \{(x, u): u \in [\underline{\mu}_{\tilde{A}}(x), \overline{\mu}_{\tilde{A}}(x)]\}. \tag{12}$$

From *Eq. (3)*,

$$\tilde{A} = \mathrm{FOU}\left(\tilde{A}\right) = \bigcup_{\forall x \in X} [\underline{\mu}_{\tilde{A}}(x), \overline{\mu}_{\tilde{A}}(x)]. \tag{13}$$

The LMF of $\tilde{A}$ are twice T1MFs bound the FOU in *Eqs.(9)* and *(10)* and J_x is an interval set. The set theory operations of union, intersection and complement are applied to compute IT2FSs. For an IT2FS, for which X and U are discrete, the domain of $\tilde{A}$ is equal to the union of all of its embedded T1 FSs, so that $\tilde{A}$ can be expressed as;

$$\tilde{A} = 1/\mathrm{FOU}\left(\tilde{A}\right) = 1/\bigcup_{x \in X} [\underline{\mu}_{\tilde{A}}(x), \overline{\mu}_{\tilde{A}}(x)]. \tag{14}$$

3 | Interval Type-2 Fuzzy Logic System (IT2FLS)

Fig. 3 shows a structure of an IT2FLS which is a version of T2FL. IT2FLS is a FLS that uses at least one IT2FS in mapping from crisp inputs to crisp outputs. IT2FLS is employed to reduce the computational burden of T2FLS while preserving the advantages of T2FLS. T1FLS cannot adequately handle the effect of uncertainties posed by the complex nature of real-world data because it uses T1FS which is characterized by membership functions that are certain. To address this problem, IT2FL is introduced as an extension of T1FL to provide additional design degrees of freedom with membership functions that are themselves fuzzy. IT2FL can be very useful when used in situations where lots of uncertainties are presented and have the potential to provide better performance than T1FLS. An IT2FIS is characterized by five components: a fuzzifier, a rule-base, an inference-engine, type-reducer and defuzzifier - that are inter-connected. Also, an IT2FIS is characterized by IF–THEN rules, in this case, the antecedent and consequents parts are Type-2 fuzzy sets [12].

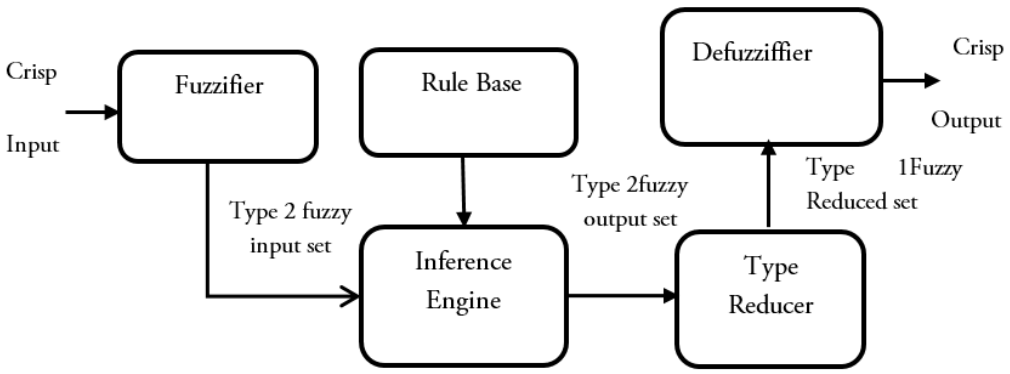


Fig. 3. The structure of a type -2 fuzzy logic system [30].

$$\text{Mamdani: } R_k \text{IF } x_i \text{ is } \widetilde{A}_i^k \text{ and, ... , and } x_n \text{ is } \widetilde{A}_n^k \text{ then } y_k \text{ is } \tilde{B}_i^k. \tag{15}$$

Where x_i, $i = 1, 2\ldots, n$ are the antecedents, y is the consequent of the k*th* rule of IT2FLS. The $\tilde{A}^i$'s are the MFs $\mu_{\tilde{A}_i^k}(x_i)$ of antecedent part of the *i*th input x_i, The B^i is the MFs $\mu_{\tilde{B}_i^k}(y)$ of the consequent part of the output y_j. The firing strength is evaluated from the input and antecedent operations to produce an IT1 set [31].

$$F^i(x') = [\underline{f}'(x'), \overline{f}'(x')] \equiv [\underline{f}', \overline{f}'] \tag{16}$$

Where $F^i(x')$ is the antecedent of rule i and $\mu_{F^i}(x')$ is the degree of membership of x in F. $\overline{\mu}_{\tilde{f}^i}(x)$ and $\underline{\mu}_{\tilde{f}^i}(x)$ are upper and lower MFs of $\mu_{\tilde{f}^i}$, $i = 1$ to m respectively. The inference engine combines the fired rules to produce a mapping from input to output in IT2FSs. The output fuzzy set, $\mu_{\tilde{E}_j^l}(y_j)$, is evaluated by combining the fired output consequent sets through the union of the kth rule fired output consequent sets. An exact iterative method of type-reduction is performed to compute the centroid of an *IT2FS* which is a T1FS. The type-reduced set gives an interval of uncertainty for the output of an IT2FLS: the more uncertainties in IT2FLS, the more uncertainties about its MFs and the larger the type-reduced set, and vice-versa. IT2FS are characterized by their left- and right-end points required to compute the centroid of an IT2FS [12], [32] and a detailed definition oftype-reduction *Eq. (17)* can be found in [15] and [30], [31] respectively. In this paper, we adopt Karnik and Mendel (KM), Algorithms [32] to calculate the exact end-points in *Eqs.(18)* and *(19)*, respectively we obtain the defuzzified crisp output for each output *k* using *Eq. (20).*

$$\begin{aligned} Y_{TR}(x') &= [y_l(x'), y_r(x')] \equiv [y_l, y_r] \\ &= \int_{y^1\in[y_l^1, y_r^1]} \cdots \int_{y^1\in[y_l^N, y_r^N]} \int_{f^1\in[\underline{f}^1, \overline{f}^1]} \cdots \int_{f^N\in[\underline{f}^N, \overline{f}^N]} 1/\frac{\sum_{i=1}^N f^i y^i}{\sum_{i=1}^N f^i}. \end{aligned} \tag{17}$$

$$y_r = \frac{\sum_{i=1}^N f_r^i y_r^i}{\sum_{i=1}^N f_r^i}. \tag{18}$$

$$y_l = \frac{\sum_{i=1}^N f_l^i y_l^i}{\sum_{i=1}^N f_l^i}. \tag{19}$$

$$Y_k(X) = \frac{y_{lk} + y_{rk}}{2}. \tag{20}$$

The performance criteria in *Eqs. (21)* and *(22)* are defined and applied to measure our experimental results: Mean Squared Error (MSE) and Root Mean Squared Error (RMSE).

$$\text{MSE} = \frac{1}{N} \sum_{i=1}^N (y^x - y)^2, \tag{21}$$

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^N (y^x - y)^2}. \tag{22}$$

Umoh et al. |J. Fuzzy. Ext. Appl. 2(1) (2021) 41-68

Where y^x is desired output, y is the computed output and N is the number of data items, respectively.

4 | Model Experiment

The model experiment was carried out in two (2) major stages namely: clinical data collection and design of interval type-2 fuzzy logic model for prediction problems in patients. Data of cardiovascular patients were collected. The data were pre-processed and stored in a database. The interval type-2 fuzzy logic model was designed. Inputs to the model were systolic blood pressure, diastolic blood pressure, temperature, heart rate and respiratory rate while cardiac shock level served as the desired output. The model employed, triangular membership function, Mamdani inference strategy, Karnik-Mendel reduction method and centroid deffuzification process for cardiac shock prediction in patients.

4.1 | Clinical Data Collection

Clinical data were collected for cardiovascular patients at the Federal Medical Centre Yenagoa, Bayelsa, Nigeria and University of Uyo Teaching Hospital, Akwa Ibom State, Nigeria, respectively for the period of 2016-2018. To represent ethical concerns, a written permission was processed and duly approved before data collection commenced. Expert knowledge on cardiovascular disease and disease diagnosis was captured with 50 questionnaire and interview from the head of cardiology unit in the department of medicine in both Federal Medical Centre, Yenagoa, Bayelsa State, Nigeria and University of Uyo Teaching Hospital, Akwa Ibom State, Nigeria, respectively. A total of 1000 patients sample data were obtained from the University of Uyo Teaching Hospital, and in the Federal Medical Centre Yenagoa, Bayelsa, Nigeria. The sample dataset for the first 35 patients extracted from Federal Medical Centre Yenagoa are presented in the *Table 1*.

4.2 | Interval Type-2 Fuzzy Logic Model for Prediction Problems in Patients

The data collected in Section 4.1 were stored in a database and served as input to the interval type-2 fuzzy logic controller. In the fuzzy controller system, the data were fuzzified using fuzzy linguistic variable and membership triangular membership function. Inference was made based on fuzzy rules which produced interval type 2 fuzzy set. Karnik-Mendel method was employed to reduce the type 2 fuzzy set to type 1 fuzzy set. The fuzzy output was obtained through centroid defuziffication process. Notification message module was incorporated in the system. This would enable alerts and vital information to be sent to doctors-on-call, caregivers, ambulance agency and designated family members for necessary action based on detection threshold from the cardiac shock prediction results. Interval Type-2 Fuzzy logic frameworks for prediction problems in cardiac patient is presented in *Fig. 4*.

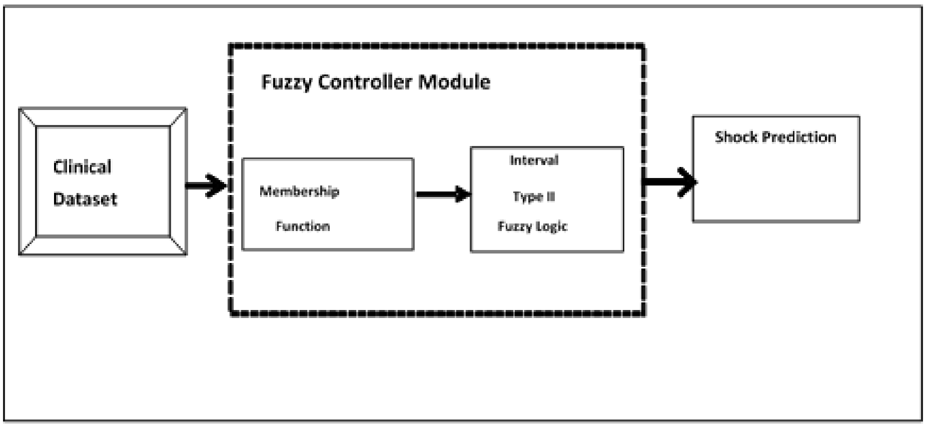


Fig. 4. Interval type-2 fuzzy logic framework for prediction problems in cardiac patient.

Table 1. Sample data extraction from Federal Medical Centre Yenagoa, Bayelsa.

Age	Sex	BPDiastolic	BPSystolic	Temp oC	Weight (kg)	Respiratory Rate(cm)	Pulse Rate(b/m)
70	F	120	90	36.9	82	20	80
53	F	140	100	36.7	90	22	68
60	M	170	100	36.1	77.4	24	120
45	M	100	70	35.2	80	22	70
57	M	105	70	36.9	79	18	122
63	F	170	90	36.2	77.4	20	80
37	M	125	80	36.4	62.5	20	80
41	F	120	70	35.2	55	20	86
56	M	130	90	36.8	92	20	80
60	F	150	110	36.9	77.4	20	86
57	M	98	60	36.6	91	20	76
56	M	120	80	36.2	79	22	73
44	M	140	90	36.9	55	20	80
52	M	120	90	36.7	48	20	90
57	F	150	120	39	50	22	80
54	M	120	80	36.8	80	24	78
48	M	112	60	36.7	50	24	62
49	F	110	80	36.3	41.4	20	60
64	F	140	80	36.7	94.1	16	78
58	F	120	70	36.7	80	20	82
50	F	130	80	36.4	49.8	20	68
59	M	130	100	36.1	53	22	90
66	F	150	90	36.4	93	21	78
43	M	130	80	36.5	50.6	23	64
69	M	160	90	36.7	96.5	20	80
59	M	130	90	36.4	51.2	19	68
44	M	120	80	36.7	68	17	59
42	M	140	80	36.7	71	20	72
61	F	145	100	36.8	58	22	85
40	M	90	70	35.1	62	24	69
71	M	120	70	36.3	50	20	70
59	F	120	80	36.4	70	22	90
51	M	120	80	37.4	60	21	80
65	F	130	80	36.3	68.8	20	68
53	M	190	110	36.1	59.5	24	78

In this paper, derived input parameters are defined as temperature, heart rate, blood pressure and respiratory rate, the membership functions and the associated linguistic variables, the universe of discourse and fuzzy linguistic variable are well defined. *Table 2* presents the input variables and the universe of discourse.

Table 2. The input variables and the universe of discourse.

Input Variables and their Universe of Discourse					
Heart Rate	**Respiratory Rate**	**Body Temperature**	**Blood Pressure**		**Prediction**
			Systolic	**Diastolic**	
[0, 200]	[0, 60]	[0 ,41.5]	[0, 200]	[0, 150]	[0 , 1]

Triangular Membership Function (TMF) method in *Eq. (23)* was used to evaluate each input and output Membership Functions (MFs) as follows:

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \le a \\ \frac{x-a}{b-a} & \text{if } a \le x \le b \\ \frac{c-x}{c-b} & \text{if } b \le x \le c \\ 0 & \text{if } x \ge a \end{cases}. \tag{23}$$

Where, a, b, c represent the x coordinates of the three vertices of $\mu_A(x)$in a fuzzy set A. a , c is the lower boundary and Upper boundary where membership degree is zero respectively, b is the centre where membership degree is 1 . x is the coordinate of three vertices (taking by assumption). The IT2FL UMF and LMF are derived from general model in *Eqs.(24)* and *(25)* and evaluated for all the input variables respectively. The Membership function values for shock prediction are shown in *Table 3*.

$$\begin{pmatrix} \bar{\mu}(x) \\ \text{IT2FL UMF} \end{pmatrix} = \begin{cases} 0, & x \le y_1 \\ \frac{x-y_1}{q_1-y_1}, & \text{if } y_1 \le x \le q_1 \\ 1, & \text{if } q_1 \le x \le r_2 \\ \frac{r_2-x}{r_2-q_2}, & \text{if } q_2 \le x \le r_2 \end{cases}. \tag{24}$$

$$\begin{pmatrix} \underline{\mu}(x) \\ \text{IT2FL LMF} \end{pmatrix} = \begin{cases} 0, & x \le y_2 \\ \frac{x-y_2}{q_2-y_2}, & \text{if } y_2 \le x \le \frac{r_1(q_2-y_2)+y_2(r_1-q_1)}{(q_2-y_2)+(r_1-q_1)} \\ \frac{r_2-x}{r_2-q_2}, & \text{if } x > \frac{r_1(q_2-y_2)+y_2(r_1-q_1)}{(q_2-y_2)+(r_1-q_1)} < x < r_1 \\ 0, & \text{if } x \ge r_2 \end{cases}. \tag{25}$$

Where *y* is left end point of both UMF and LMF and *r* is right end point of both UMF and LMF and *q* is the peak point.

Respiratory rate UMF and LMF (low)

$$\overline{\mu}_{RR}(low)(x) = \begin{cases} 0, & x \le 0.0863 \\ \dfrac{x - 0.0863}{q_1 - 0.0863}, & if\ 0.0863 \le x \le q_1 \\ 1, & if\ q_1 \le x \le q_2 \\ \dfrac{13.4 - x}{13.4 - q_2}, & if\ q_2 \le x \le 13.4 \\ 0, & if\ x > 13.4 \end{cases}$$

$$\underline{\mu}_{RR}(low)(x) = \begin{cases} 0, & x \le 1.69 \\ \dfrac{x - y_2}{q_2 - y_2}, & if\ 1.69 \le x \le \dfrac{12.01(q_2 - 1.69) + 1.69(12.01 - q_1)}{(q_2 - 1.69) + (12.01 - q_1)} \\ \dfrac{13.4 - x}{13.4 - q_2}, & if\ x > \dfrac{12.01(q_2 - 1.69) + 1.69(12.01 - q_1) < x < 12.01}{(q_2 - 1.69) + (12.01 - q_1)} \\ 0, & if\ x \ge 13.4 \end{cases}$$

Respiratory Rate UMF and LMF Normal (NM)

$$\overline{\mu}_{RR}(N)(x) = \begin{cases} 0, & \le 7.61 \\ \dfrac{x - 7.61}{q_1 - 7.61}, & if\ 7.61 \le x \le q_1 \\ 1, & if\ q_1 \le x \le q_2 \\ \dfrac{29.6 - x}{29.6 - q_2}, & if\ q_2 \le x \le 29.6 \\ 0, & if\ x > 29.6 \end{cases}$$

$$\underline{\mu}_{RR}(N)(x) = \begin{cases} 0, & x \le 10.13 \\ \dfrac{x - 10.13}{q_2 - 10.13}, & if\ 10.13 \le x \le \dfrac{26.8(q_2 - 10.13) + 10.13(26.8 - q_1)}{(q_2 - 10.13) + (26.8 - q_1)} \\ \dfrac{29.6 - x}{29.6 - q_2}, & if\ x > \dfrac{26.8(q_2 - 10.13) + 10.13(26.8 - q_1) < x < 26.8}{(q_2 - 10.13) + (26.8 - q_1)} \\ 0, & if\ x \ge 29.6 \end{cases}$$

For Respiratory Rate UMF and LMF High (H)

$$\overline{\mu}_{RR}(H)(x) = \begin{cases} 0, & x \le 22.8 \\ \dfrac{x - 22.8}{q_1 - 22.8}, & if\ 22.8 \le x \le q_1 \\ 1, & if\ q_1 \le x \le q_2 \\ \dfrac{59.8 - x}{59.8 - q_2}, & if\ q_2 \le x \le 59.8 \\ 0, & if\ x > 59.8 \end{cases}$$

$$\underline{\mu}_{RR}(H)(x) = \begin{cases} 0, & x \le 26.1 \\ \dfrac{x - 26.1}{q_2 - 26.1}, & if\ 26.1 \le x \le \dfrac{54.8(q_2 - 26.1) + 26.1(54.8 - q_1)}{(q_2 - 26.1) + (26.8 - q_1)} \\ \dfrac{59.8 - x}{59.8 - q_2}, & if\ x > \dfrac{54.8(q_2 - 26.1) + 26.1(54.8 - q_1) < x < 54.8}{(q_2 - 26.1) + (54.8 - q_1)} \\ 0, & if\ x \ge 59.8 \end{cases}$$

For Heart Rate UMF and LMF low (L)

$$\overline{\mu}_{HR}(L)(x) = \begin{cases} 0, & x \le -0.208 \\ \dfrac{x - (-0.208)}{q_1 - (-0.208)}, & if\ -0.208 \le x \le q_1 \\ 1, & if\ q_1 \le x \le q_2 \\ \dfrac{60 - x}{60 - q_2}, & if\ q_2 \le x \le 60 \\ 0, & if\ x > 60 \end{cases}$$

$$\underline{\mu}_{HR}(L)(x) = \begin{cases} 0, & x \le 4.48 \\ \dfrac{x - 4.48}{q_2 - 4.48}, & if\ 4.48 \le x \le \dfrac{56.2(q_2 - 4.48) + 4.48(56.2 - q_1)}{(q_2 - 4.48) + (56.2 - q_1)} \\ \dfrac{60 - x}{60 - q_2}, & if\ x > \dfrac{56.2(q_2 - 4.48) + 4.48(56.2 - q_1) < x 56.2}{(q_2 - 4.48) + (56.2 - q_1)} \\ 0, & if\ x \ge 60 \end{cases}$$

For Heart Rate UMF and LMF Normal (N)

$$\overline{\mu}_{HR}(N)(x) = \begin{cases} 0, & x \le 49.5 \\ \dfrac{x-49.5}{q_1-49.5}, & if\ 49.5 \le x \le q_1 \\ 1, & if\ q_1 \le x \le q_2 \\ \dfrac{123-x}{123-q_2}, & if\ q_2 \le x \le 123 \\ 0, & if\ x > 123 \end{cases}$$

$$\underline{\mu}_{HR}(N)(x) = \begin{cases} 0, & x \le 53.03 \\ \dfrac{x-53.03}{q_2-53.03}, & if\ 4.48 \le x \le \dfrac{56.2(q_2-53.03)+53.03(56.2-q_1)}{(q_2-53.03)+(56.2-q_1)} \\ \dfrac{123-x}{123-q_2}, & if\ x > \dfrac{119(q_2-53.03)+53.03(119-q_1)<x119}{(q_2-53.03)+(119-q_1)} \\ 0, & if\ x \ge 123 \end{cases}$$

For Heart Rate UMF and LMF High (H)

$$\overline{\mu}_{HR}(H)(x) = \begin{cases} 0, & x \le 107 \\ \dfrac{x-107}{q_1-107}, & if\ 107 \le x \le q_1 \\ 1, & if\ q_1 \le x \le q_2 \\ \dfrac{198-x}{198-q_2}, & if\ q_2 \le x \le 198 \\ 0, & if\ x > 198 \end{cases}$$

$$\underline{\mu}_{HR}(H)(x) = \begin{cases} 0, & x \le 111 \\ \dfrac{x-111}{q_2-111}, & if\ 111 \le x \le \dfrac{191(q_2-111)+111(191-q_1)}{(q_2-111)+(191-q_1)} \\ \dfrac{198-x}{198-q_2}, & if\ x > \dfrac{191(q_2-111)+111(191-q_1)<x191}{(q_2-111)+(191-q_1)} \\ 0, & if\ x \ge 198 \end{cases}$$

For Body Temperature UMF and LMF (low)

$$\overline{\mu}_{Temp}(low)(x) = \begin{cases} 0, & x \le 0.145 \\ \dfrac{x-0.145}{q_1-0.145}, & if\ 0.145 \le x \le q_1 \\ 1, & if\ q_1 \le x \le q_2 \\ \dfrac{22.5-x}{22.5-q_2}, & if\ q_2 \le x \le 22.5 \\ 0, & if\ x > 22.5 \end{cases}$$

$$\underline{\mu}_{Temp}(low)(x) = \begin{cases} 0, & x \le 1.85 \\ \dfrac{x-1.85}{q_2-1.85}, & if\ 1.85 \le x \le \dfrac{20.3(q_2-1.85)+1.85(20.3-q_1)}{(q_2-1.85)+(20.3-q_1)} \\ \dfrac{22.5-x}{22.5-q_2}, & if\ x > \dfrac{20.3(q_2-1.85)+1.85(20.3-q_1)<x20.3}{(q_2-1.85)+(20.3-q_1)} \\ 0, & if\ x \ge 22.5 \end{cases}$$

For Body Temperature UMF and LMF (No

$$\overline{\mu}_{Temp}(normal)(x) = \begin{cases} 0, & x \le 9.59 \\ \dfrac{x-9.59}{q_1-9.59}, & if\ 9.59 \le x \le q_1 \\ 1, & if\ q_1 \le x \le q_2 \\ \dfrac{30.02-x}{30.02-q_2}, & if\ q_2 \le x \le 30.02 \\ 0, & if\ x > 30.02 \end{cases}$$

$$\underline{\mu}_{Temp}(normal)(x) = \begin{cases} 0, & x \le 11.48 \\ \dfrac{x-11.48}{q_2-11.48}, & if\ 11.48 \le x \le \dfrac{28.3(q_2-11.48)+11.48(28.3-q_1)}{(q_2-11.48)+(28.3-q_1)} \\ \dfrac{30.02-x}{30.02-q_2}, & if\ x > \dfrac{28.3(q_2-11.48)+11.48(28.3-q_1)<x28.3}{(q_2-11.48)+(28.3-q_1)} \\ 0, & if\ x \ge 30.02 \end{cases}$$

For Body Temperature UMF and LMF (High)

$$\overline{\mu}_{Temp}(high)(x) = \begin{cases} 0, & x \le 25.1 \\ \dfrac{x-25.1}{q_1-25.1}, & if\ 25.1 \le x \le q_1 \\ 1, & if\ q_1 \le x \le q_2 \\ \dfrac{37.7-x}{37.7-q_2}, & if\ q_2 \le x \le 37.7 \\ 0, & if\ x > 37.7 \end{cases}$$

$$\underline{\mu}_{Temp}(high)(x) = \begin{cases} 0, & x \le 26.0 \\ \dfrac{x-26.0}{q_2-26.0}, & if\ 26.0 \le x \le \dfrac{36.3(q_2-26.0)+26.0(36.3-q_1)}{(q_2-26.0)+(36.3-q_1)} \\ \dfrac{37.7-x}{37.7-q_2}, & if\ x > \dfrac{36.3(q_2-26.0)+26.0(36.3-q_1)<x36.3}{(q_2-26.0)+(36.3-q_1)} \\ 0, & if\ x \ge 37.7 \end{cases}$$

For Body Temperature UMF and LMF very high

$$\overline{\mu}_{Temp}(\text{veryhigh})(x) = \begin{cases} 0, & x \le 30.5 \\ \dfrac{x-30.5}{q_1-30.5}, & \text{if } 30.5 \le x \le q_1 \\ 1, & \text{if } q_1 \le x \le q_2 \\ \dfrac{41.4-x}{41.4-q_2}, & \text{if } q_2 \le x \le 41.4 \\ 0, & \text{if } x > 41.4 \end{cases}$$

$$\underline{\mu}_{Temp}(veryhigh)(x) = \begin{cases} 0, & x \le 31.9 \\ \dfrac{x-31.9}{q_2-31.9}, & if\ 31.9 \le x \le \dfrac{40(q_2-31.9)+31.9(40-q_1)}{(q_2-31.9)+(40-q_1)} \\ \dfrac{41.4-x}{41.4-q_2}, & if\ x > \dfrac{40(q_2-31.9)+31.9(40-q_1)<x<40}{(q_2-31.9)+(40-q_1)} \\ 0, & if\ x \ge 41.4 \end{cases}$$

For Systolic Blood Pressure UMF and LMF for Low

$$\overline{\mu}_{systolicBP}(low)(x) = \begin{cases} 0, & x \le 2.34 \\ \dfrac{x-2.34}{q_1-2.34}, & if\ 2.34 \le x \le q_1 \\ 1, & if\ q_1 \le x \le q_2 \\ \dfrac{90.03-x}{90.03-q_2}, & if\ q_2 \le x \le 90.03 \\ 0, & if\ x > 90.03 \end{cases}$$

$$\underline{\mu}_{systolicBP}(low)(x) = \begin{cases} 0, & x \le 11.2 \\ \dfrac{x-11.2}{q_2-11.2}, & if\ 11.2 \le x \le \dfrac{84(q_2-11.2)+11.2(84-q_1)}{(q_2-11.2)+(84-q_1)} \\ \dfrac{90.03-x}{90.03-q_2}, & if\ x > \dfrac{84(q_2-11.2)+11.2(84-q_1)<x<84}{(q_2-11.2)+(84-q_1)} \\ 0, & if\ x \ge 90.03 \end{cases}$$

For Systolic Blood Pressure UMF and LMF (Normal)

$$\overline{\mu}_{systolicBP}(normal)(x) = \begin{cases} 0, & x \le 74.42 \\ \dfrac{x-74.42}{q_1-74.42}, & if 74.42 \le x \le q_1 \\ 1, & if\ q_1 \le x \le q_2 \\ \dfrac{139-x}{139-q_2}, & if\ q_2 \le x \le 139 \\ 0, & if\ x > 139 \end{cases}$$

$$\underline{\mu}_{systolicBP}(normal)(x) = \begin{cases} 0, & x \le 78.76 \\ \dfrac{x-78.76}{q_2-78.76}, & if\ 78.76 \le x \le \dfrac{135(q_2-78.76)+78.76(135-q_1)}{(q_2-78.76)+(135-q_1)} \\ \dfrac{139-x}{139-q_2}, & if\ x > \dfrac{135(q_2-78.76)+78.76(135-q_1)<x<135}{(q_2-78.76)+(135-q_1)} \\ 0, & if\ x \ge 139 \end{cases}$$

Umoh et al. |J. Fuzzy. Ext. Appl. 2(1) (2021) 41-68

For Systolic Blood Pressure UMF and LMF (High)

$$\overline{\mu}_{systolicBP}(high)(x) = \begin{cases} 0, & x \le 151 \\ \dfrac{x-151}{q_1-151}, & if151 \le x \le q_1 \\ 1, & if\ q_1 \le x \le q_2 \\ \dfrac{159.7-x}{159.7-q_2}, & if\ q_2 \le x \le 159.7 \\ 0, & if\ x > 159.7 \end{cases}$$

$$\underline{\mu}_{systolicBP}(high)(x) = \begin{cases} 0, & x \le 128 \\ \dfrac{x-128}{q_2-128}, & if\ 128 \le x \le \dfrac{154(q_2-128)+128(154-q_1)}{(q_2-128)+(154-q_1)} \\ \dfrac{159.7-x}{159.7-q_2}, & if\ x > \dfrac{154(q_2-128)+128(154-q_1)<x<154}{(q_2-128)+(154-q_1)} \\ 0, & if\ x \ge 159.7 \end{cases}$$

For Systolic Blood Pressure UMF and LMF Very High

$$\overline{\mu}_{systolicBP}(veryhigh)(x) = \begin{cases} 0, & x \le 151 \\ \dfrac{x-151}{q_1-151}, & if\ 151 \le x \le q_1 \\ 1, & if\ q_1 \le x \le q_2 \\ \dfrac{179.9-x}{179.9-q_2}, & if\ q_2 \le x \le 179.9 \\ 0, & if\ x > 179.9 \end{cases}$$

$$\underline{\mu}_{systolicBP}(veryhigh)(x) = \begin{cases} 0, & x \le 155 \\ \dfrac{x-155}{q_2-155}, & if\ 155 \le x \le \dfrac{177(q_2-155)+155(177-q_1)}{(q_2-155)+(177-q_1)} \\ \dfrac{179.9-x}{179.9-q_2}, & if\ x > \dfrac{177(q_2-155)+155(177-q_1)<x<177}{(q_2-155)+(177-q_1)} \\ 0, & if\ x \ge 179.9 \end{cases}$$

For Systolic Blood Pressure UMF and LMF Severe

$$\overline{\mu}_{systolicBP}(severe)(x) = \begin{cases} 0, & x \le 171 \\ \dfrac{x-171}{q_1-171}, & if\ 171 \le x \le q_1 \\ 1, & if\ q_1 \le x \le q_2 \\ \dfrac{198-x}{198-q_2}, & if\ q_2 \le x \le 198 \\ 0, & if\ x > 198 \end{cases}$$

$$\underline{\mu}_{systolicBP}(veryhigh)(x) = \begin{cases} 0, & x \le 174 \\ \dfrac{x-174}{q_2-174}, & if174 \le x \le \dfrac{194(q_2-174)+174(194-q_1)}{(q_2-174)+(194-q_1)} \\ \dfrac{198-x}{198-q_2}, & if\ x > \dfrac{194(q_2-174)+174(194-q_1)<x<194}{(q_2-174)+(194-q_1)} \\ 0, & if\ x \ge 198 \end{cases}$$

For Diastolic Blood Pressure UMF and LMF Low

$$\overline{\mu}_{DiastolicBP}(low)(x) = \begin{cases} 0, & x \le 1.778 \\ \dfrac{x-1.778}{q_1-1.778}, & if\ 1.778 \le x \le q_1 \\ 1, & if\ q_1 \le x \le q_2 \\ \dfrac{67.53-x}{67.53-q_2}, & if\ q_2 \le x \le 67.53 \\ 0, & if\ x > 67.53 \end{cases}$$

$$\underline{\mu}_{DiastolicBP}(low)(x) = \begin{cases} 0, & x \le 6.45 \\ \dfrac{x-6.45}{q_2-6.45}, & if6.45 \le x \le \dfrac{63.84(q_2-6.45)+6.45(63.84-q_1)}{(q_2-6.45)+(63.84-q_1)} \\ \dfrac{67.53-x}{67.53-q_2}, & if\ x > \dfrac{63.84(q_2-6.45)+6.45(63.84-q_1)<x<63.84}{(q_2-6.45)+(63.84-q_1)} \\ 0, & if\ x \ge 67{,}53 \end{cases}$$

For Diastolic Blood Pressure UMF and LMF Normal

$$\overline{\mu}_{DiastolicBP}(Normal)(x) = \begin{cases} 0, & x \le 54.29 \\ \dfrac{x-54.29}{q_1-54.29}, & if\ 54.29 \le x \le q_1 \\ 1, & if\ q_1 \le x \le q_2 \\ \dfrac{102.8-x}{102.8-q_2}, & if\ q_2 \le x \le 102.8 \\ 0, & if\ x > 102.8 \end{cases}$$

$$\underline{\mu}_{DiastolicBP}(Normal)(x) = \begin{cases} 0, & x \le 57.3 \\ \dfrac{x-57.3}{q_2-57.3}, & if 57.3 \le x \le \dfrac{98.5(q_2-57.3)+57.3(98.5-q_1)}{(q_2-57.3)+(98.5-q_1)} \\ \dfrac{102.8-x}{102.8-q_2}, & if\ x > \dfrac{98.5(q_2-57.3)+57.3(98.5-q_1)<x<98.5}{(q_2-57.3)+(98.5-q_1)} \\ 0, & if\ x \ge 102.8 \end{cases}$$

For Diastolic Blood Pressure UMF and LMF High

$$\overline{\mu}_{DiastolicBP}(High)(x) = \begin{cases} 0, & x \le 90.6 \\ \dfrac{x-90.6}{q_1-90.6}, & if\ 90.6 \le x \le q_1 \\ 1, & if\ q_1 \le x \le q_2 \\ \dfrac{117-x}{117-q_2}, & if\ q_2 \le x \le 117 \\ 0, & if\ x > 117 \end{cases}$$

$$\underline{\mu}_{DiastolicBP}(High)(x) = \begin{cases} 0, & x \le 92.3 \\ \dfrac{x-92.3}{q_2-92.3}, & if 92.3 \le x \le \dfrac{115(q_2-92.3)+92.3(115-q_1)}{(q_2-92.3)+(115-q_1)} \\ \dfrac{117-x}{117-q_2}, & if\ x > \dfrac{115(q_2-92.3)+92.3(115-q_1)<x<115}{(q_2-92.3)+(115-q_1)} \\ 0, & if\ x \ge 117 \end{cases}$$

For Diastolic Blood Pressure UMF and LMF Very High

$$\overline{\mu}_{DiastolicBP}(VeryHigh)(x) = \begin{cases} 0, & x \le 111.3 \\ \dfrac{x-111.3}{q_1-111.3}, & if\ 111.3 \le x \le q_1 \\ 1, & if\ q_1 \le x \le q_2 \\ \dfrac{134.9-x}{134.9-q_2}, & if\ q_2 \le x \le 134.9 \\ 0, & if\ x > 134 \end{cases}$$

$$\underline{\mu}_{DiastolicBP}(VeryHigh)(x) = \begin{cases} 0, & x \le 112.4 \\ \dfrac{x-112.4}{q_2-112.4}, & if 112.4 \le x \le \dfrac{134(q_2-112.4)+112.4(134-q_1)}{(q_2-112.4)+(134-q_1)} \\ \dfrac{134.9-x}{134.9-q_2}, & if\ x > \dfrac{134(q_2-112.4)+112.4(134-q_1)<x<134}{(q_2-112.4)+(134-q_1)} \\ 0, & if\ x \ge 134.9 \end{cases}$$

For Diastolic Blood Pressure UMF and LMF Severe

$$\overline{\mu}_{DiastolicBP}Severe(x) = \begin{cases} 0, & x \le 128.2 \\ \dfrac{x-128.2}{q_1-128.2}, & if\ 128.2 \le x \le q_1 \\ 1, & if\ q_1 \le x \le q_2 \\ \dfrac{148.5-x}{148.5-q_2}, & if\ q_2 \le x \le 148.5 \\ 0, & if\ x > 148.5 \end{cases}$$

$$\underline{\mu}_{DiastolicBP}(Severe)(x) = \begin{cases} 0, & x \le 131 \\ \dfrac{x-131}{q_2-131}, & if 131 \le x \le \dfrac{146(q_2-131)+131(146-q_1)}{(q_2-131)+(146-q_1)} \\ \dfrac{148.5-x}{148.5-q_2}, & if\ x > \dfrac{146(q_2-131)+131(146-q_1)<x<146}{(q_2-131)+(146-q_1)} \\ 0, & if\ x \ge 148.5 \end{cases}$$

Table 3. Membership function values for Shock Prediction.

Linguistic Term	Upper Membership Function	Lower Membership Function
Low	0,0.1733,0.3	0.0282,0.173,0.271
Normal	0.2,0.3532,0.5	0.234,0.3558,0.464
High	0.4,0.5833,0.8	0.43,0.584,0.7666
Very High	0.6,0.806,1	0.632,0.806,0.9718

In this paper, 900 fuzzy rules are defined based on (4) and part of the rules are presented in *Fig. 5* while sample rule is given as;

If HeartRate is low and Respiratory Rate is normal and Temp is very high and Blood Pressure is high THEN Prediction is high.

1					
2	Heart Rate	Respiratory Rate	Temp	BP	Patient Risk Level
3	Low	low	low	low	high
4	Low	low	low	normal	low
5	Low	low	low	high	high
6	Low	low	low	veryhigh	high
7	Low	low	normal	severe	veryhigh
8	Low	low	normal	low	high
9	Low	low	normal	normal	low
10	Low	low	normal	high	low
11	Low	low	high	veryhigh	high
12	Low	low	high	severe	veryhigh
13	Low	low	high	low	low
14	Low	low	high	normal	minimal risk
15	Low	normal	veryhigh	high	high
16	Low	normal	veryhigh	veryhigh	high
17	Low	normal	veryhigh	severe	veryhigh HD
18	Low	normal	veryhigh	low	low
19	Low	normal	low	normal	low
20	Low	normal	low	high	low
21	Low	normal	low	veryhigh	minimal risk
22	Low	normal	low	severe	very highrisk
23	Low	normal	normal	low	risk

Fig. 5. Rule base for cardiac patient shock level prediction.

Given the crisp input vector $v = [55, 25, 7, 135]$, their degree of memberships is calculated from the respective triangular membership functions as shown in *Table 3*. *Table 4* presents the firing rules based on the set input values. Rule evaluation of the firing rules 20, 21, 165, 51, 100, 101, 35 and 36 against the fuzzy set in *Table 3* yields the result in *Table 5*. The Karnik Mendel Type reduction model is applied by selecting the leftmost (*L*) and rightmost (*R*) switch points. In our study, the switch point is selected at $L=5$ and $R = 2$ and then we compute for the values of y_l and y_r using *Eqs. (24)* and *(25)*, respectively. The defuzzification is carried out using *Eq. (26)*.

Table 3. Fuzzified value (fuzzy set).

Lingusitic Variable				
HeartRate $[\overline{\mu}^1, \underline{\mu}^1]$ (55)	Respiratory Rate$[\overline{\mu}^2, \underline{\mu}^2]$ (25)	Body Temp. $[\overline{\mu}^3, \underline{\mu}^3]$ (7)	Systolic Blood Pressure $[\overline{\mu}^4, \underline{\mu}^4]$ (135)	Diastolic Blood Pressure $[\overline{\mu}^4, \underline{\mu}^4]$ (102)
U_L[1.644,0.053]	U_N[2.067,0.197]	U_L[0.649,0.569]	U_H[0.360,1.579]	U_H[0.950,0.9065]
U_N[0.154,0.062]	U_H[0.129,0.170]		U_{VH}[0.659,0.455]	

Table 4. The firing rules.

Rule No.	Firing Interval	Consequent
R20	$\overline{f}^1, \underline{f}^1,]$ = [1.644*2.067*0.649*0.360, 0.053*0.197*0.569*1.579] = [0.360, 0.053]	$[\overline{y}^1, \underline{y}^1]$ = L[0.149, 1.525]
R21	$\overline{f}^2, \underline{f}^2,]$ = [1.644*2.067*0.649*0.659, 0.053*0.197*0.589*0.455] = [0.649,0.053]	$[\overline{y}^2, \underline{y}^2]$ =M[0.3497, 0.6598]
R165	$[\overline{f}^3, \underline{f}^3,]$= [0.129*2.067*0.649*0.360, 0.170*0.197*0.569*1.579] = [0.129,0.170]	$[\overline{y}^3, \underline{y}^3]$= L[0.105, 0.631]
R51	$[\overline{f}^4, \underline{f}^4,]$=[0.1544*2.067*0.649*0.659, 0.062*0.197*0.569*0.455]=[0.154,0.062]	$[\overline{y}^4, \underline{y}^4]$=L[0.15, 0.1525]
R100	$[\overline{f}^5, \underline{f}^5]$=[1.644*0.129*0.649*0.360, 0.053*0.170*0.569*1.579]=[0.129,0.053]	$[\overline{y}^5, \underline{y}^5]$=H[0.60, 0.598]
R101	$[\overline{f}^6, \underline{f}^6]$ =[1.644*0.129*0.649*0.659’ 0.053*0.170*0.569*0.455]=[0.129,0.053]	$[\overline{y}^6, \underline{y}^6]$= VH[0.80, 0.798]
R35	$[\overline{f}^7, \underline{f}^7,]$=[2.067*0.129*0.649*0.360, 0.197*0.170*0.569*1.579]=[0.129,0.170]	$[\overline{y}^7, \underline{y}^7]$=L[0.148,0.1525]
R36	$[\overline{f}^8, \underline{f}^8,]$=[2.067*0.129*0.649*0.659, 0.197*0.170*0.569*0.455]=[0.129,0.170]	$[\overline{y}^8, \underline{y}^8]$=H[0.5302, 0.5975]

The defuzzification is carried out using

$$y_l = \min_{L\in[1,N-1]} \frac{\sum_{n=1}^{L} \overline{f}^n \underline{y}^n + \sum_{n=L+1}^{N} \underline{f}^n \underline{y}^n}{\sum_{n=1}^{L} \overline{f}^n + \sum_{n=L+1}^{N} \underline{f}^n}. \tag{26}$$

$$y_r = \max_{R\in[1,N-1]} \frac{\sum_{n=1}^{R} \underline{f}^n \overline{y}^n + \sum_{n=R+1}^{N} \overline{f}^n \overline{y}^n}{\sum_{n=1}^{R} \underline{f}^n + \sum_{n=R+1}^{N} \overline{f}^n}. \tag{27}$$

Umoh et al. |J. Fuzzy. Ext. Appl. 2(1) (2021) 41-68

The switch point for *L=5* and *R = 2*,

$$y_l = \frac{\bar{f}^1\underline{y}^1 + \bar{f}^2\underline{y}^2 + \bar{f}^3\underline{y}^3 + \bar{f}^4\underline{y}^4 + \bar{f}^5\underline{y}^5 + \underline{f}^6\underline{y}^6 + \underline{f}^7\underline{y}^7 + \underline{f}^8\underline{y}^8}{\bar{f}^1 + \bar{f}^2 + \bar{f}^3 + \bar{f}^4 + \bar{f}^5 + \underline{f}^6 + \underline{f}^7 + \underline{f}^8}. \tag{28}$$

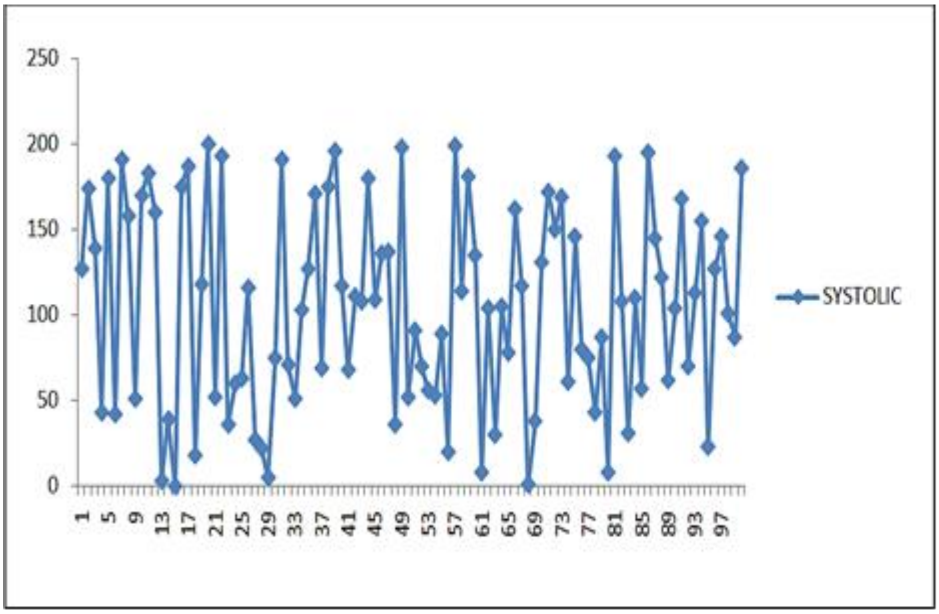
$$y_r = \frac{\underline{f}^1\bar{y}^1 + \underline{f}^2\bar{y}^2 + \bar{f}^3\bar{y}^3 + \bar{f}^4\bar{y}^4 + \bar{f}^5\bar{y}^5 + \bar{f}^6\bar{y}^6 + \bar{f}^7\bar{y}^7 + \bar{f}^8\bar{y}^8}{\underline{f}^1 + \underline{f}^2 + \bar{f}^3 + \bar{f}^4 + \bar{f}^5 + \bar{f}^6 + \bar{f}^7 + \bar{f}^8}. \tag{29}$$

$$y_l = \frac{\begin{array}{c}0.360 * 1.525 + 0.649 * 0.6598 + 0.129 * 0.631 + 0.154 * 0.1525 \\ + 0.129 * 0.598 + 0.053 * 0.798 + 0.170 * 0.1525 + 0.170 * 0.5975\end{array}}{0.360 + 0.649 + 0.129 + 0.154 + 0.129 + 0.053 + 0.170 + 0.170} = 1.9997.$$

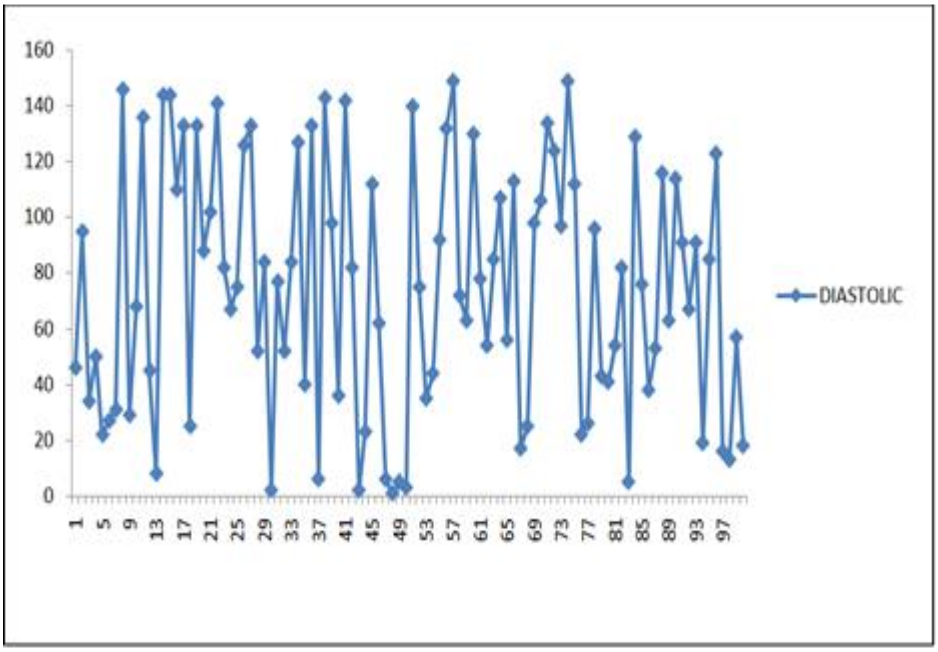
$$y_r = \frac{\begin{array}{c}0.053 * 0.149 + 0.053 * 0.3497 + 0.129 * 0.105 + 0.154 * 0.15 \\ + 0.129 * 0.60 + 0.129 * 0.80 + 0.129 * 0.148 + 0.129 * 0.5302\end{array}}{0.053 + 0.053 + 0.129 + 0.154 + 0.129 + 0.129 + 0.129 + 0.129} = 0.36593.$$

5| Results and Discussion

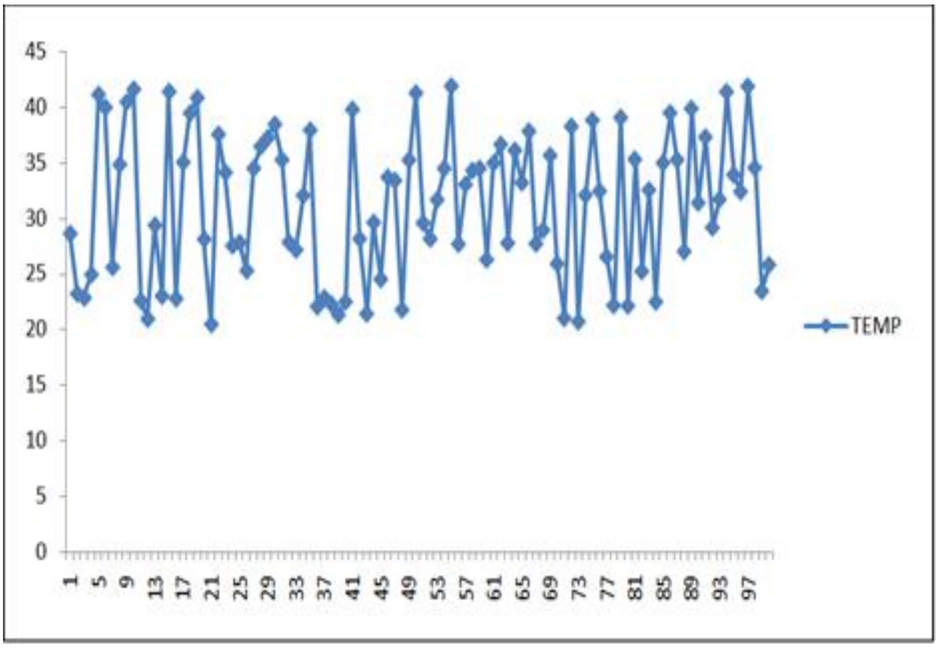
In this paper, experimental results for prediction problem as applied to predicting shock level in cardiac patients using interval type-2 fuzzy logic system have been presented. The effectiveness and generalization capability of IT2FLS have been tested using 1000 datasets obtained from University of Uyo Teaching Hospital and Federal Medical Centre, Yenagoa all in Nigeria. *Table 1* shows part of the details of the cardiac patient's health datasets. Five cardiac health variables namely: systolic blood pressure, diastolic blood pressure, temperature, heart rate and respiratory rate served as the input while cardiac shock level served as the desired output. Triangular membership function was employed for fuzzification of the input. Fuzzy inference was derived using Mamdani inference process. Investigation of the performance of IT2FLS and that of its type-1 counterpart were carried out. Statistical evaluation was carried out using performance metrics of Mean Absolute Difference (MAD), Mean Absolute Percentage Error (MAPE), Mean Square Error (MSE) and Root Mean Squared Error (RMSE), respectively. *Fig. 6* shows the plots of transformed cardiac patients' health dataset for (a) blood pressure diastolic, (b) blood pressure systolic, (c) temperature, (d) respiratory rate, and (e) heart rate, respectively.



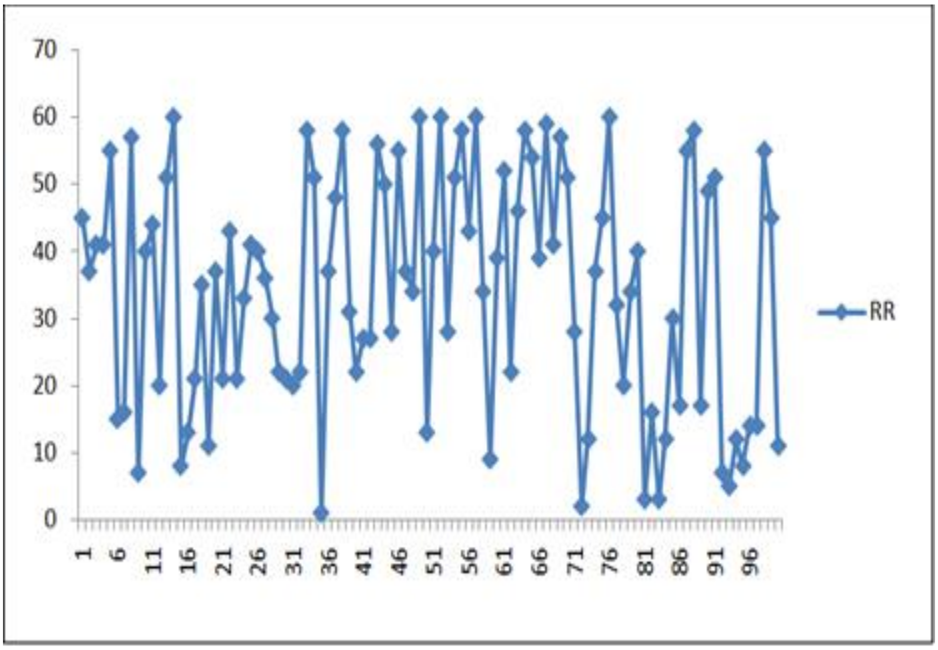
(a)


160
140
120
100
80
60
40
20
0
1
5
9
13
17
21
25
29
33
37
41
45
49
53
57
61
65
69
73
77
81
85
89
93
97
DIASTOLIC

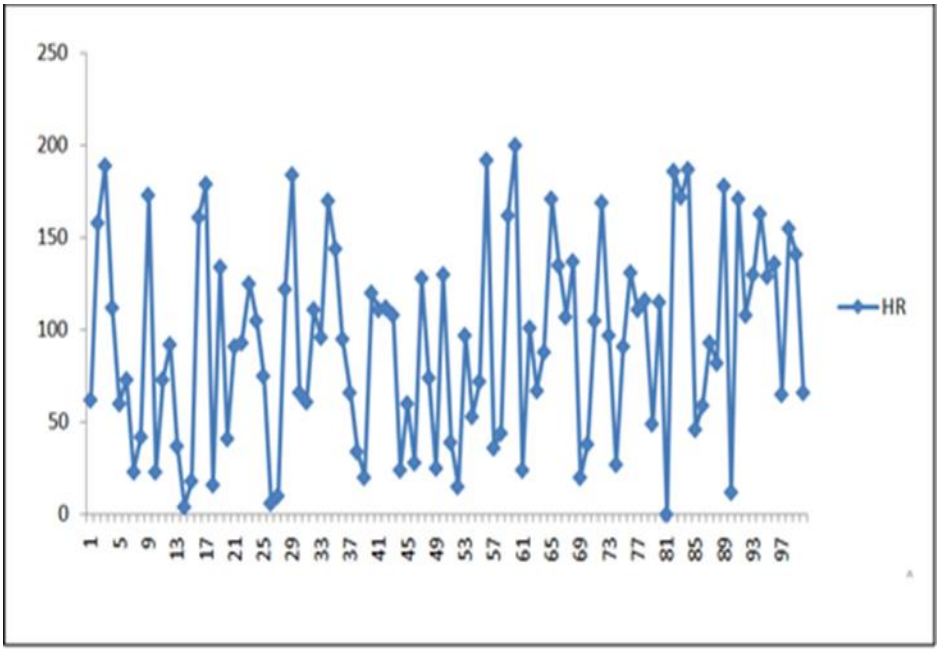
(b)


45
40
35
30
25
20
15
10
5
0
1
5
9
13
17
21
25
29
33
37
41
45
49
53
57
61
65
69
73
77
81
85
89
93
97
TEMP

(c)



(d)

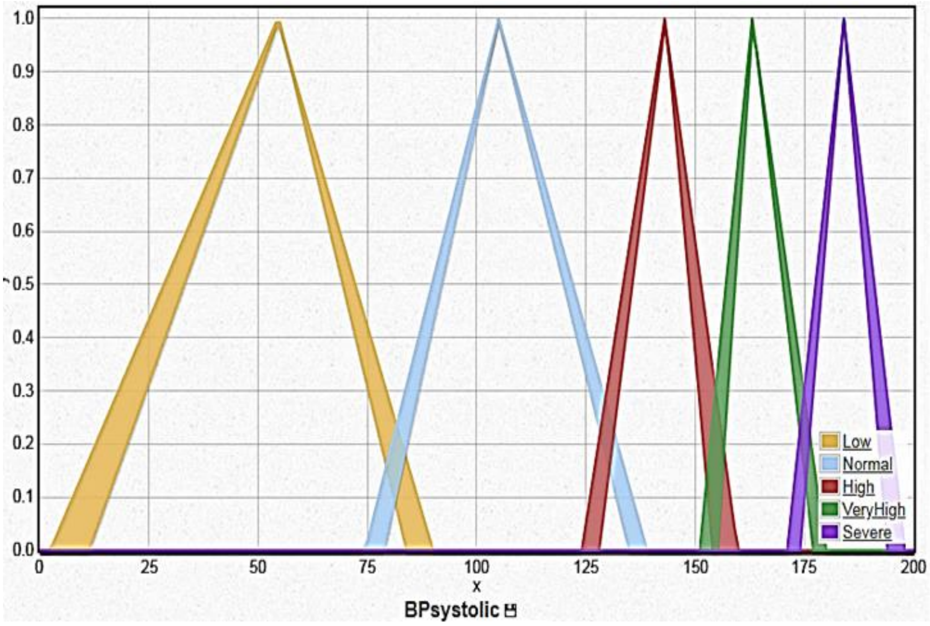


(e)

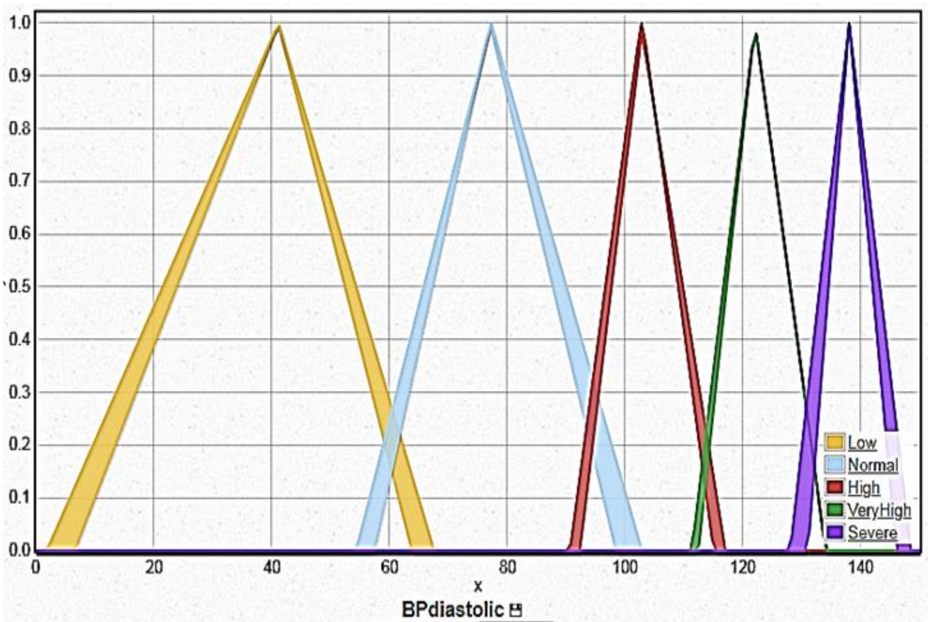
Fig. 6. Plots of the transformed cardiac patients' health dataset for (a) blood pressure diastolic, (b) blood pressure systolic, (c) temperature, (d) respiratory rate, (e) heart rate.

5.1| Interval Type-2 Fuzzy Logic Results for Remote Vital Signs Monitoring and Shocks Level Prediction

With the fuzzy rule-base of Fig. 4 and both the input and output membership functions plots are depicted in *Fig. 7(a-e)*, respectively. Applying the IT2FLC to the proposed problem, we obtain the system's response. *Fig. 8* gives the prediction results for IT2FLS. *Fig. 9* presents the system's response of the IT2FLS for the actual and predicted shock level.

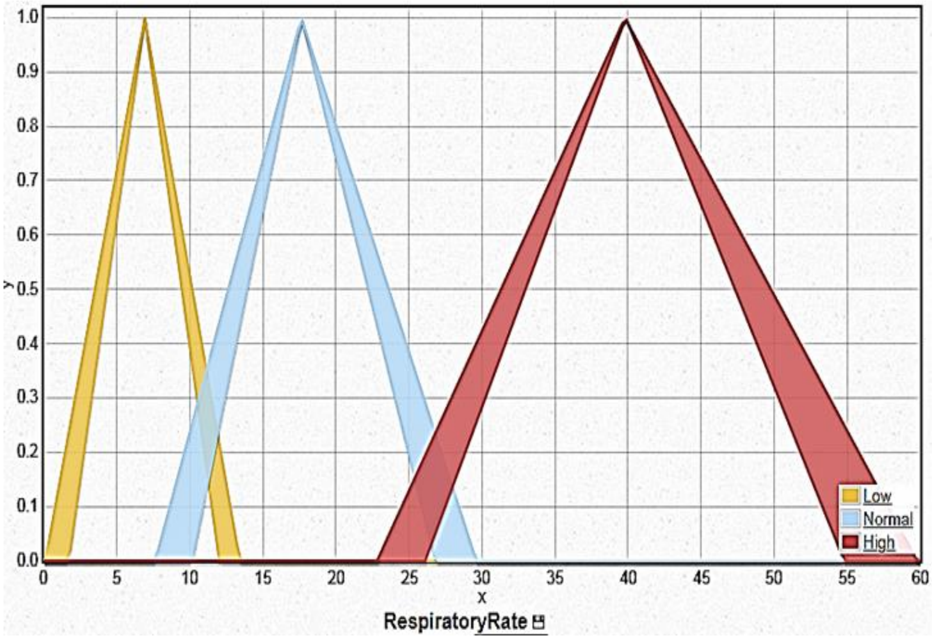


(a)

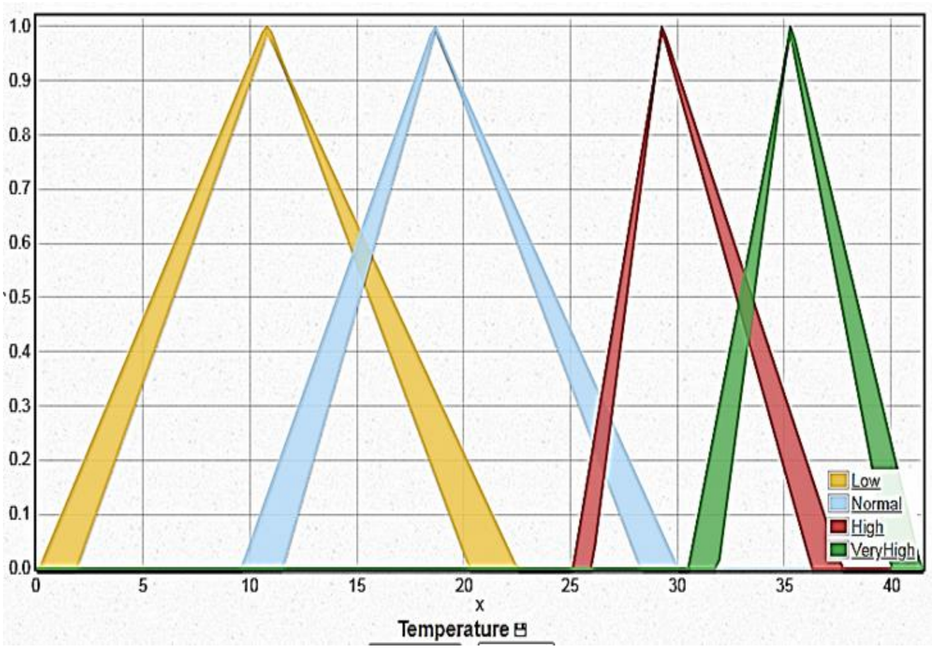


(b)

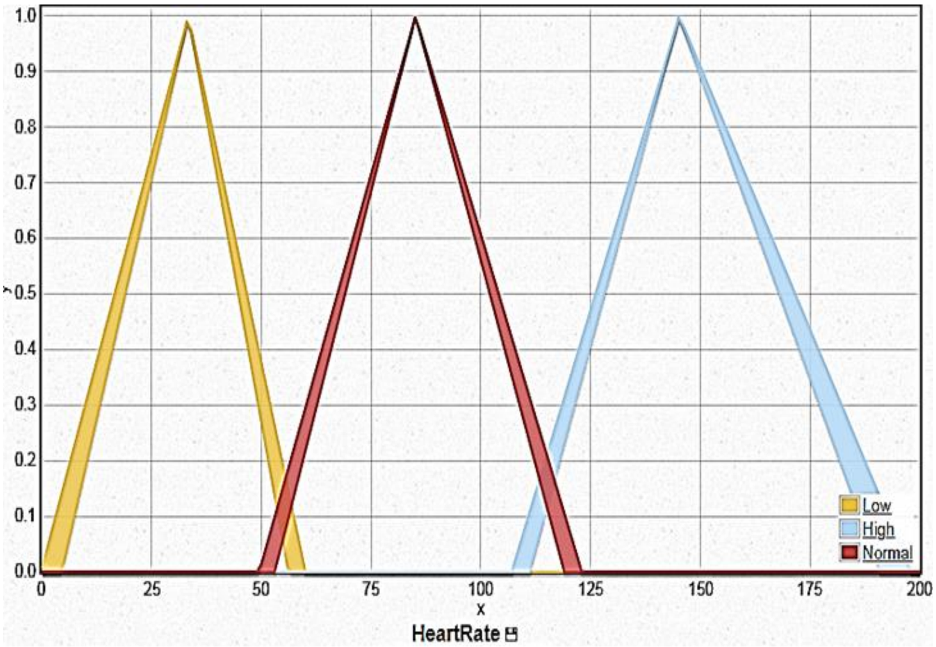
Umoh et al. |J. Fuzzy. Ext. Appl. 2(1) (2021) 41-68


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
y
0
5
10
15
20
25
30
35
40
45
50
55
60
x
RespiratoryRate
Low
Normal
High

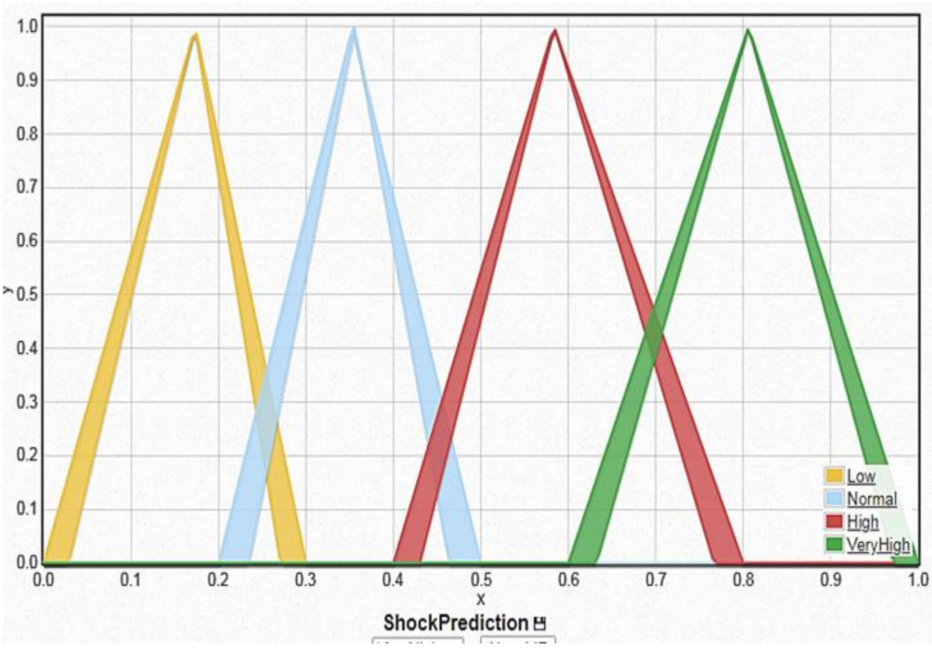
(c)


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
5
10
15
20
25
30
35
40
x
Temperature
Low
Normal
High
VeryHigh

(d)



(e)



(d)

Fig. 7. Inputs and output membership functions plot for (a) bp-diastolic (b) bp-systolic (c) temperature (d) respiratory rate (e) heart rate and (f) shock level prediction.

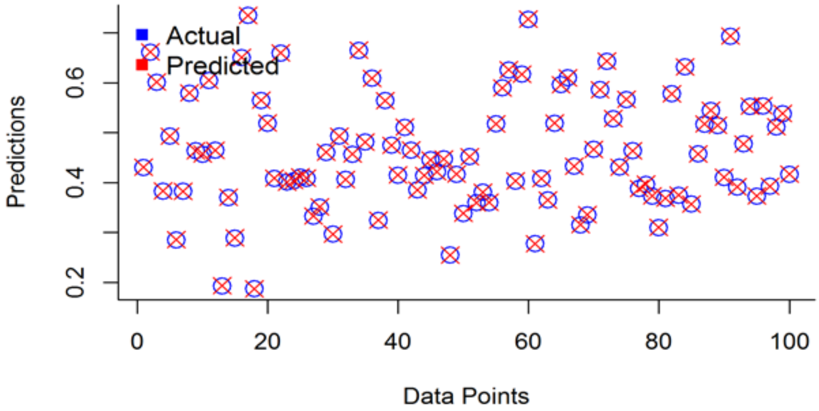


Fig. 8. The prediction results for (a) IT2FLS.

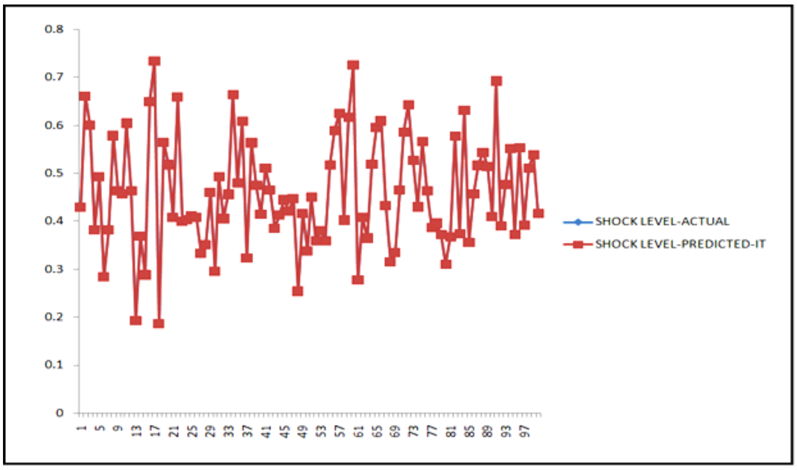


Fig. 9. System's response of the IT2FLS for the actual and predicted shock level.

5.2| Type-1 Fuzzy Logic Results for Remote Vital Signs Monitoring and Shocks Level Prediction

Fig. 10 gives the prediction results for T1FLS while *Fig. 11* presents the system's response of the T1FLS for the actual and predicted shock level. *Fig. 12* shows the plots of the system's response for the actual shock level and predicted shock level for T2FLS and T1FLS.

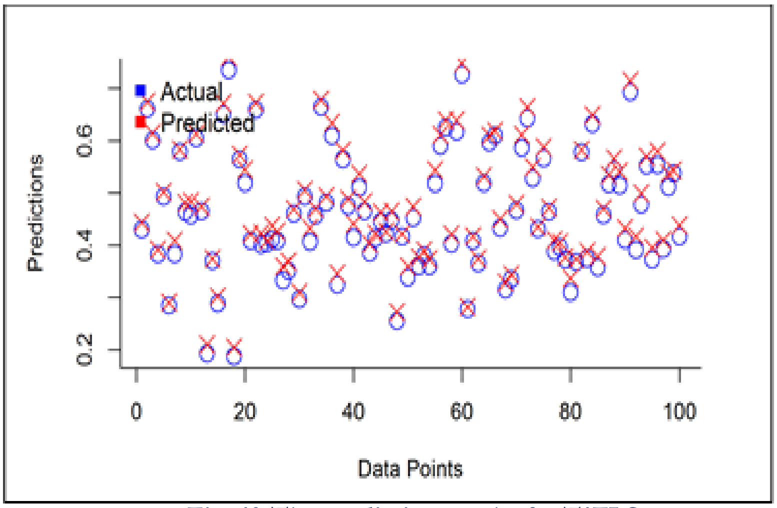


Fig. 10. **The prediction results for T1FLS.**

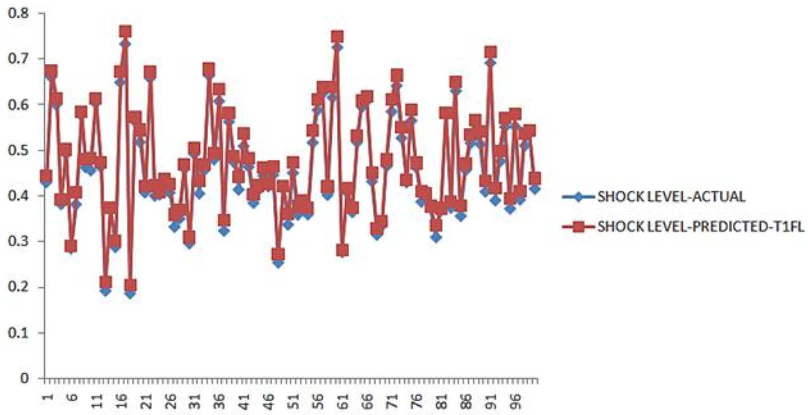


Fig. 11. System's response of T1FLS for the actual and predicted shock level.

5.3| Performance Evaluation

Fig. 12 and *Table 5* give the results of the performance of IT2FLS with that of T1FLS where, shock level-actual is the actual shock level, Shock Level-Predicted-IT2FLS is the predicted shock level by IT2FLS and Shock Level-Predicted-T1FLS is the predicted shock level by T1FLS, respectively. Statistical analysis for comparison is also made between IFLS and IT2FLS. Two experiments are conducted in order to explore these analyses. In each case, the performance metrics are the MSE and the RMSE. *Figs. 13-15* show the plots of the two models using their test MSEs and RMSEs. *Figs. 13 (a)* and *(b)* give the prediction errors for IT2FLS and T1FLS, while the prediction accuracy of IT2FLS and T1FLS are shown in *Figs.14* (a) and (b). *Figs. 15* (a) and (b) gives the error models against error values for IT2FLS and T1FLS, respectively. *Table 6* compares the performance of IT2FLS with that of T1FLS with respect to MSE and RMSE. The results of the statistical comparison of the model performance are presented in *Table 6*.

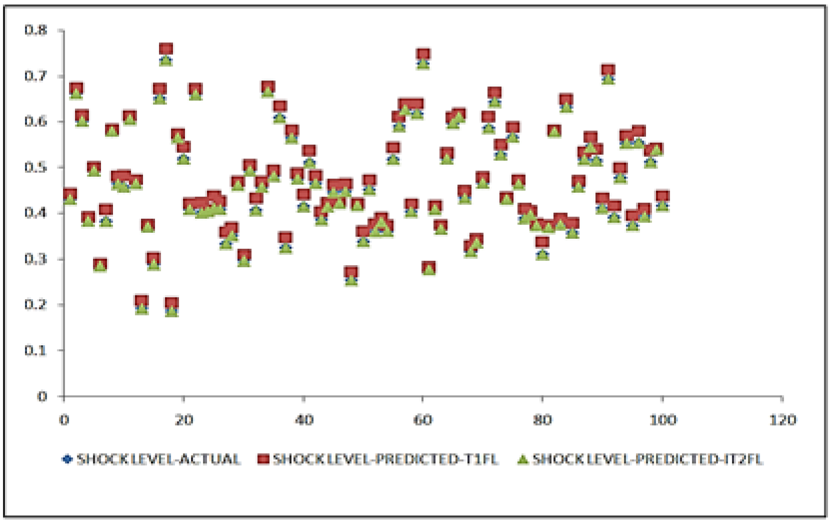


Fig. 12. Plots of the system's response of actual shock level and predicted shock level for IT2FLS and T1FLS.

From *Fig. 12* it is observed that with the use of IT2FLS, the model is able to model uncertainty adequately in predicting shock level of a cardiac patient and in many applications better than the T1FLS. Also, with MFs that are intervals, the IT2FLS is able to model uncertainty in predicting shock level of a cardiac patient in many applications better than T1FLS which MFs are not represented as intervals values.

6| Conclusion

This paper investigated the predictive capability of Interval Type-2 Fuzzy Logic System (IT2FLS) based on Mamdani fuzzy inference and applied to in monitoring and predicting shock level in cardiac patients. Implementation of conventional Type-1 Fuzzy Logic System (T1FLS) was carried out for the purpose of comparison. By the use of IT2FLS, we have been able to predict different shock levels for cardiac patients. Specifically, the following conclusions are made:

- *The IT2FLS copes with more information and handle more uncertainties in health data.*
- *The IT2FLS performs significantly well compared to TIFLS with model errors of 0.0005 against 0.079.*
- *The IT2FLSs with interval MF can reduce the effects of uncertainties in most health applications.*

In the future, we intend to explore Tagaki Sugeno Kang (TSK) fuzzy inference to conduct more experiments using the same data sets. Also, we intend to optimize our system using flower pollination algorithm for performance improvement. More so, we will apply other fuzzy modeling functions such as triangular and trapezoidal functions, respectively.

Table 5. Results of shock level actual and predicted for T2FLS and T1FLS.

S/No.	Shock Level-Actual	Shock Level-Predicted IT2FLS	Shock Level-Predicted T1FLS
1.	0.430209738	0.430534538	0.443508938
2.	0.661076429	0.661401229	0.674375629
3.	0.600932789	0.601582389	0.614556789
4.	0.382681769	0.383006569	0.391656169
5.	0.492674633	0.492999433	0.501649033
6.	0.284818273	0.285467873	0.289792673
7.	0.381977912	0.382627512	0.408576312
8.	0.578692885	0.579342485	0.583667285
9.	0.463544732	0.463869532	0.481168732
10.	0.456734044	0.457383644	0.483332444
11.	0.604593743	0.605243343	0.613892943
12.	0.464519101	0.464843901	0.473493501
13.	0.192299351	0.192948951	0.210248151
14.	0.369657904	0.370307504	0.374632304
15.	0.28822936	0.28887896	0.30185336
16.	0.650264118	0.650588918	0.672212918
17.	0.734445205	0.734770005	0.760718805
18.	0.186542266	0.187191866	0.204491066
19.	0.564212161	0.564861761	0.573511361
20.	0.518631549	0.518956349	0.544905149
21.	0.408237197	0.408561997	0.421536397
22.	0.659320718	0.659645518	0.672619918
23.	0.400810079	0.401134879	0.422758879
24.	0.403371577	0.404021177	0.408345977
25.	0.410957191	0.411606791	0.437555591
26.	0.408079746	0.408729346	0.426028546
27.	0.332620921	0.333270521	0.359219321
28.	0.350599807	0.351249407	0.368548607
29.	0.46014474	0.46079434	0.46944394
30.	0.296303978	0.296628778	0.309603178
31.	0.492720862	0.493045662	0.506020062
32.	0.40598575	0.40631055	0.43225935

6.1| Ethical Issues

Ethical issues came to play in this research as the research involved gathering data from cardiovascular patients' records. However, the research was not involved in the direct collection of the data, but a review of patients' files and medical histories after due permission was granted by the responsible authorities. Hence, we discuss the ethical issues under two areas:

- *Consent form. The consent form through written permission was obtained from the health authority before embarking on the research. A sample of the authorization clearance is used by the ethical committee of Federal Medical Centre, Yenagoa and University Teaching Hospital, Uyo*
- *Data protection. Data protection is ensured by not revealing patients' personal details such as Name, Address, Occupation and many others. Hence, data gathered excluded this information.*

Table 6. Comparison of T1FLS and IT2FLS in shock level prediction.

Models	Performance	MSE	RMSE
T1FLS	Prediction Error	0.004	0.018
	Model Error	0.0003	0.079
	Prediction Accuracy	0.9997	0.9821
IT2FLS	Prediction Error	0.0001	0.0006
	Model Error	2.57E-07	0.0005
	Prediction Accuracy	1	0.9995

References

[1] Zadeh, L. A. (1965). Fuzzy sets. *Inf. Control*, *8*, 338–353.

[2] Negnevitsky, M. (2005). *Artificial intelligence: a guide to intelligent systems*. Pearson education.

[3] Chiu, S. (1997). Extracting fuzzy rules from data for function approximation and pattern classification. In D. Dubois, H. Prade, and R. Yager (Eds.) *Fuzzy information engineering: a guided tour of applications*. John Wiley & Sons.

[4] Mamdani, E. H. (1974, December). Application of fuzzy algorithms for control of simple dynamic plant. In *Proceedings of the institution of electrical engineers* (Vol. 121, No. 12, pp. 1585-1588). IET.

[5] Ramkumar, V., Mihovska, A. D., Prasad, N. R., & Prasad, R. (2010). Fuzzy-logic based call admission control for a heterogeneous radio environment. *The 12th international symposium on wireless personal multimedia communications (WPMC 2009)*. Japan, Sendai, Japan. https://vbn.aau.dk/en/publications/fuzzy-logic-based-call-admission-control-for-a-heterogeneous-radi

[6] Selvi, M. P., & Sendhilnathan, S. (2016). Fuzzy based mobility management in 4G wireless Networks. *Brazilian archives of biology and technology*, *59*(SPE2).

[7] Mali. G. U. (2017). Fuzzy based vertical handoff decision controller for future networks. *International journal of advanced engineering, management and science (IJAEMS)*, 3(1), 111-119.

[8] Abbasi, R., Bidgoli, A., & Abbasi, M. (2012). A new fuzzy algorithm for improving quality of service in real time wireless sensor networks. *International journal of advanced smart sensor network systems (IJASSN)*, *2*(2), 1-14.

[9] Dogman, A., Saatchi, R., & Al-Khayatt, S. (2012). Quality of service evaluation using a combination of fuzzy C-means and regression model. *World academy of science, engineering and technology*, *6*, 562-571.

[10] Umoh, U. A., & Inyang, U. G. (2015). A fuzzfuzzy-neural intelligent trading model for stock price prediction. *International journal of computer science issues (IJCSI)*, *12*(3), 36.

[11] Umoh, U., & Asuquo, D. (2017). Fuzzy logic-based quality of service evaluation for multimedia transmission over wireless ad hoc networks. *International journal of computational intelligence and applications*, *16*(04), 1750023.

[12] Mendel, J. M., & John, R. B. (2002). Type-2 fuzzy sets made simple. *IEEE transactions on fuzzy systems*, *10*(2), 117-127.

[13] Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning—I. *Information sciences*, *8*(3), 199-249.

[14] Hagras, H. (2007). Type-2 FLCs: A new generation of fuzzy controllers. *IEEE computational intelligence magazine*, *2*(1), 30-43.

[15] Tan, W. W., & Chua, T. W. (2007). [Review of Uncertain rule-based fuzzy logic systems: introduction and new directions, by J. M. Mendel]. *IEEE Computational intelligence magazine*, *2*(1), 72-73.

[16] Postlethwaite, B. E. (1993). [Review of Fuzzy control and fuzzy systems, by W. Pedrycz]. *International journal of adaptive control and signal processing*, *5*(1), 87-153. https://doi.org/10.1002/acs.4480050208

[17] Hagras, H. A. (2004). A hierarchical type-2 fuzzy logic control architecture for autonomous mobile robots. *IEEE transactions on fuzzy systems*, *12*(4), 524-539.

[18] Mendel, J. M., John, R. I., & Liu, F. (2006). Interval type-2 fuzzy logic systems made simple. *IEEE transactions on fuzzy systems*, *14*(6), 808-821.

[19] Zimmermann, H. J. (2011). *Fuzzy set theory—and its applications*. Springer Science & Business Media.

[20] Castillo, O., & Melin, P. (2012). *Recent advances in interval type-2 fuzzy systems*. Springer.

[21] Isizoh, A. N., Okide, S. O., Anazia, A. E., & Ogu, C. D. (2012). Temperature control system using fuzzy logic technique. *International journal of advanced research in artificial intelligence*, *1*(3), 27-31.

[22] Roy, N., & Mitra, P. (2016). Electric load forecasting: an interval type-ii fuzzy inference system based approach. *International journal of computer science and information technologies (IJCSIT)*, *7*(6), 2515-2522.

[23] Umoh, U. A**.**, Inyang, U. G., & Nyoho, E. E. (2019). Interval type-2 fuzzy logic for fire outbreak detection. *International journal on soft computing, artificial intelligence and applications (IJSCAI)*, 8(3), 1-20.

[24] Castillo, O., Melin, P., Kacprzyk, J., & Pedrycz, W. (2007, November). Type-2 fuzzy logic: theory and applications. *2007 IEEE international conference on granular computing (GRC 2007)* (pp. 145-145). IEEE.

[25] Mamdani, E. H., & Assilian, S. (1975). An experiment in linguistic synthesis with a fuzzy logic controller. *International journal of man-machine studies*, *7*(1), 1-13.

[26] Mamdani, E. H. (1976). Advances in the linguistic synthesis of fuzzy controllers. *International journal of man-machine studies*, *8*(6), 669-678.

[27] Umoh, U. A., Nwachukwu, E. O., Obot, O. U., & Umoh, A. A. (2011). Fuzzy-neural network model for effective control of profitability in a paper recycling plant. *American journal of scientific and industrial research (AJSIR)*, *2*(4), 552-558. DOI: 10.5251/ajsir.2011.2.4.552.558

[28] Wu, H., & Mendel, J. M. (2002). Uncertainty bounds and their use in the design of interval type-2 fuzzy logic systems. *IEEE Transactions on fuzzy systems*, *10*(5), 622-639.

[29] Wu, D. (2005). *Design and analysis of type-2 fuzzy logic systems* (Master's Thesis, Department of Electrical and Computer Engineering, National University of Singapore). Retrieved from https://core.ac.uk/download/pdf/48633921.pdf

[30] Wu, D., & Tan, W. W. (2005, May). Computationally efficient type-reduction strategies for a type-2 fuzzy logic controller. *The 14th IEEE international conference on fuzzy systems, 2005. FUZZ'05.* (pp. 353-358). IEEE.

[31] Karnik, N. N., & Mendel, J. M. (2001). Centroid of a type-2 fuzzy set. *Information SCiences*, *132*(1-4), 195-220.

E-ISSN: 2717-3453 | P-ISSN: 2783-1442

Journal of Fuzzy Extension and Applications

J. Fuzzy. Ext. Appl

www.journal-fea.com

J. Fuzzy. Ext. Appl. Vol. 2, No. 1 (2021) 69–88.

Paper Type: Research Paper

Evaluation and Selection of Supplier in Supply Chain with Fuzzy Analytical Network Process Approach

Mohammad Ghasempoor Anaraki[1,*], Dmitriy S. Vladislav[2], Mahdi Karbasian[3], Natalja Osintsev[4], Victoria Nozick[5]

[1] Deparment of Industrial Engineering, Najafabad Branch, Islamic Azad University, Esfahan, Iran; me_ghasempoor@yahoo.com.

[2] Department of Production Engineering, South Ural State University, Lenin Prosp. 76, 454080 Chelyabinsk, Russia; dmitriy.s.vladislav@gmail.com

[3] Department of Industrial Engineering, Malek Ashtar University, Esfahan, Iran; mkarbasian@yahoo.com.

[4] Fraunhofer-Institut für Holzforschung Wilhelm-Klauditz Institut WKI, Bienroder Weg 54 E, 38108 Brunswick, Germany; nataljaosintsev@gmail.com.

[5] Operations and Information Management Group, Aston Business School, Aston University, Birmingham B4 7ET, United Kingdom; victorianozick20@yahoo.com.

Citation:

Ghasempoor Anaraki, M., Vladislav, D. S., Karbasian, M., Osintsev, N., & Nozick, V. (2021). Evaluation and selection of supplier in supply chain with fuzzy analytical network process approach. *Journal of fuzzy extension and application, 2* (1), 69-88.

Received: 23/10/2020 Reviewed: 02/12/2020 Revised: 04/01/2021 Accept: 11/02/2021

Abstract

One of the most important issues concerning the designing a supply chain is selecting the supplier. Selecting proper suppliers is one of the most crucial activities of an organization towards the gradual improvement and a promotion in performance. This intricacy is because suppliers fulfil a part of customer's expectancy and selecting among them is multi-criteria decision, which needs a systematic and organized approach without which this decision may lead to failure. The purpose of this research is proposing a new method for assessment and rating the suppliers. We have identified several evaluation criteria and attributes; the selection among them was by the Simple Multi-Attribute Rating Technique (SMART) method, then we have specified the connection and the influence of the criteria on each other by DEMATEL method. After that, suppliers were graded by using the Fuzzy Analytical Network Process (FANP) approach and the most efficient one was selected. The innovation of this research is combining the SMART method, DEMATEL method, and Analytical Network Process in Fuzzy state which lead to more exact and efficient results which is proposed for the first time by the researchers of this study.

Keywords: Supply Chain Management (SCM), Supplier selection, Multi Criteria Decision Making (MCDM), DEMATEL, Fuzzy Analytical Network Process (FANP).

1 | Introduction

The objective of supplier selection is to identify suppliers with the highest potential for meeting an organization's needs undeviatingly and at an acceptable price. Selection is considered a comparison of suppliers using a set of criteria and measures. However, the level of detail used for examining potential suppliers may vary depending on an organization's needs. The ultimate goal of selection is to identify suppliers with the highest potential. A supply chain is composed of all links from suppliers to customers: suppliers, manufacturing plants, warehouses, distribution centers, and retailers.

Corresponding Author: me_ghasempoor@yahoo.com

http://dx.doi.org/10.22105/jfea.2021.274734.1078

Supplier selection and evaluation are recognized as a strategic and crucial component of supply chain strategy. A good coordination between a manufacturer and suppliers is necessary because the failure of coordination contributes to excessive delays, products with poor quality and ultimately results in poor customer services. When companies turn to outside suppliers and manufacturers and become more dependent on outsourcers, the consequences of poor decision-making become more severe. As a result, it is too important for an outsourced-type manufacturer to assess, manage and select its suppliers. The objective of supplier selection process is to identify high potential suppliers for meeting a manufacturer's needs undeviatingly and at an acceptable overall performance. Selecting suppliers among a large number of possible ones with various levels of capabilities and potential is an extremely difficult task and a Supplier Selection. Multi Criteria Decision Making (MCDM) problem. supplier selection decisions are intricate owing to the fact that various criteria must be taken into consideration in the decision-making process. To select the prospective suppliers, the organization judges each supplier's ability to meet consistently and cost-effectively its needs using selection criteria and proper measures.

Criteria and measures are developed to be applicable to all the suppliers being considered and to indicate the organization's needs and its supply and technology strategy. It may be demanding to convert its needs into useful criteria, due to the fact that needs are often expressed as general qualitative concepts while criteria should be specific requirements that can be quantitatively assessed. As more companies become interested in developing and executing strategic partnership with their suppliers, an effective tool is needed to help these organizations in prequalifying their suppliers based on their overall performances, in selecting the best suppliers and in developing and managing the strategic partnership. A number of alternative approaches have been proposed to consider these criteria, called mathematical programming models, multiple attribute decision aid methods, cost-based methods, statistical and probabilistic methods, combined methodologies, and other methods. Consequently, significant amounts of researches have been conducted for supplier assessment and selection problem since the 60s.

Increasing in the number of commercial competition and expanse of global markets cause the organizations to pay more attention to improving the quality of their activities and processes taking into account the measures and competitive criteria, which entails the suppliers selection. Harland et al. [1] argued about the main aim of Supply Chain Management (SCM) is to improve the competitive advantage by focusing on productivity of suppliers' processes, technology, and their abilities. Waters [2] studied the reason of supply chain existence is to meet customers' expectations taking into consideration bringing benefits for the various segments of the chain and ultimate aim of a chain is to maximize the benefits and whole values of segments of the chain. The main reason that organizations focus on their supply chains is due to the short life of products and the variable customers' expectations as a threat, and information technology improvement as an opportunity. Karpak et al. [3] surveyed the main aim of SCM is considered as improving the operational effectiveness, profitability and competitive state of an agency and its supply chain which includes all segments. Pang & Bai [4] argued about evaluation and selection of suppliers are known as a strategic and important part of choosing a long-term approach regarding the supply chain. A thorough cooperation between the factory and the supplier is needed owing to the fact that the failure of corporation leads to excessive delays, poor quality product, and superfluous costs and eventually results in poor servicing; hence, substandard decision-making makes more problems. Therefore, the issue of the best supplier selection and evaluation for a producer is considered a crucial case develops a supplier evaluation approach based on the Analytic Network Process (ANP) and fuzzy synthetic evaluation under a fuzzy environment. The importance weights of various criteria are considered as linguistic variables.

These linguistic ratings can be expressed in triangular fuzzy numbers by using the fuzzy extent analysis. Fuzzy synthetic evaluation is used to select a supplier alternative and the Fuzzy ANP (FANP) method is applied to calculate the importance of the criteria weights. Then an integrated FANP and fuzzy synthetic evaluation methodology is proposed for evaluating and selecting the most suitable suppliers. De Boer et al. [5] stated the most important subjects concerning the purchase management are: supplier selection, the commercial partner, and the issue of determining the optimal amount of order. Weber et al. [6], [7] found the weak point in most of the supplier selection methods is not considering the criteria as variable components and in a span. Although the criteria are quantified using methods such as Analytical Hierarchy Process (AHP), they considered price, delivery time, and quality as criteria to select the suppliers Carrera and Mayorga [8] provided a fuzzy set application in supplier selection for new product development. The model quantifies these four multiple criteria in terms of Taguchi quality loss and then uses an AHP to combine them into one global variable for decision-making.

Kubat and Yuce [9] proposed a hybrid intelligent approach for supply chain management system, which combines AHP, Fuzzy AHP and Genetic Algorithm (GA). Some researchers have applied the FANP based approach to solve complex decision-making problems [10]-[19]; some scholars also proposed decision-making models in generalized fuzzy sets [20]-[29]. Narasimhan et al. [30] proposed an AHP-based methodology for supplier selection and performance evaluation. Jeong and Lee [31] proposed a Multi-Criteria Supplier Selection (MCSS) model to deal with the supplier selection problems in the SCM, where a fuzzy-based methodology is used to assess the ratings for the qualitative factors, such as profitability and quality. Xia and Wu [32] proposed an integrated AHP approach based on rough sets theory with multiple criteria and with supplier's capacity constraints. Among the available multi-attribute decision-making methods, only the ANP can be used to evaluate the most suitable suppliers systematically due to the dependencies and feedbacks caused by the mutual effects of the criteria. Kumar et al. [33] studied a fuzzy multi-objective integer programming problem incorporating three important goals: cost-minimization, quality-maximization and maximization of on-time delivery with the realistic constraints such as meeting the buyers' demand, vendors' capacity, vendors' quota flexibility, etc. In the proposed model, various input parameters have been treated as vague with a linear membership function of fuzzy type. The model acts as a decision tool facilitating the vendor selection and their quota allocation under different degrees of information vagueness in the decision parameters of a supply chain modeling.

Recent information and communication developments caused that global organizations spread out their markets throughout the world. In this environment, local exclusive markets have been replaced with global competitive ones. Therefore, organizations must concentrate on their main operations to survive in such an environment. To do so, managers have intended to cooperate with some financial partners in long-term relations. In this paper, the aim is to develop a FANP model to evaluate the potential suppliers and select the best one(s) with respect to the vendor important factors. Additionally, ANP is developed by fuzzy sets theory to cover the indeterminacy of decisions made in this field. The authors have augmented the model with a non-linear programming model to elicit eigenvectors from fuzzy comparison matrices. Hybridization of these two concepts can model supplier selection problem in all circumstances and reaches the optimal choice.

2| Methodology

2.1| The Steps of Methodology

This study includes the following steps:

Step1: Criteria selection.

Step 2: Choosing and building a prototype.

Step 3: Screening criteria by Simple Multi-Attribute Rating Technique (SMART) method.

Step 4: Identifying the effects of the criteria on each other by DEMATEL method.

Step 5: Forming matrixes of FANP method.

Step 6: Selecting the best supplier.

2.2| SMART Method

SMART was introduced by Winterfeldt and Edwards in 1986 [34], [35], in which a limited number of alternatives are examined based on a limited number of attributes. The present method aimed to rank the alternatives by a combination of quantitative and qualitative attributes. This is a convenient technique because of its ease of use, which is used in many cases such as evaluation of nuclear waste disposal sites and ERP system selection [36]. SMART assigns the center of gravity of weights method to the purposes through the following way. Suppose that w_1 is the weight of the most important (the first) purpose, w_2 is the weight of the second important purpose, w_3 is the weight of the third purpose, etc.

$$W_1= \frac{1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}}{n},$$

$$W_2= \frac{0+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}}{n},$$

$$W_k= \frac{0+0+0+\cdots+\frac{1}{n}}{n}.$$

The sum of all weights, in this way, equals to one. The more the number of purposes, the less errors in estimating the weights.

2.3| DEMATEL Method

The DEcision-MAking Trial and Evaluation Laboratory (DEMATEL) method was introduced by Fonetla and Gabus in 1971 [37]-[40], mainly used to study very complex global issues. The DEMATEL method is applied to construct a network relation design in order to examine the internal relation among the attributes.

The steps used in DEMATEL technique are as the followings:

First Step: Determine the fundamental elements of the system.

Second Step: Assign the given elements to the points of a diagraph and determine the effects between them. Elements are compared in pairs and the judgments are made just for the direct effects of the elements.

Third Step: Ask the experts for the intensity of the final effects of elements on each other. This intensity is in the form of grading (for instance 0-4, 0-10, or 0-100).

Fourth Step: Show the numbers resulted from the previous steps in a $\widehat{M}$ matrix.

Fifth Step: Add the numbers in each row and find the row with the highest sum *(α)* and then divide each number of $\widehat{M}$ matrix by $\alpha.(M=\alpha.\widehat{M})$.

Sixth Step: the result of the following equation is the final direct and indirect effect on each other:

$$S = M + M2 + M3 + \ldots\ldots + Mt = \frac{M(I-M)}{(I-M)}\ ;\ \lim_{t\to\infty} M\,t = 0 = \frac{M}{(I-M)} = M(I-M)\text{-}1\,.$$

We use the FANP method that has been developed in [41].

2.4| The Analytic Network Process

Saaty [42], proposed ANP to decompose a multi-criteria decision making problem into components. Final decision is gained by a logical determination of components' values and aggregation of them [43]-[45]. A comprehensive research was performed by Taslicali and Ercan [46], comparing the analytic hierarchy process and the analytic network process. The authors concluded that the ANP compensates weaknesses of the AHP. They mentioned some of the drawbacks of AHP as followings. AHP can model linear and strictly hierarchical structure, while ANP can be applied to tackle more general structure including interrelationships between different criteria in different clusters or within the same cluster. Moreover, it is indicated that ANP is more accurate in complex situation due to its capability of modeling complex structure and the way in which comparisons are performed. Hence, the ANP can be considered as a more general form of the AHP in which dependencies and feedbacks between elements of a decision can be modeled.

The following sections describe the steps of the ANP. Although ANP is one of the most complete and comprehensive multi-attribute decision-making methods as it encompass the criteria and alternatives in an integrated manner, a great drawback of this method is the pair-wise comparison section. This section consists of deterministic comparisons, while real world has an indeterminate nature. Therefore, fuzzy sets theory is adopted in this research to cope with this drawback.

Criteria Definition. In this step, criteria which affect the decision being made must be defined. In order to define the criteria, a group of managers who make the decision or consultants (e.g., an expert group) can be an appropriate choice. In order to select suppliers, various criteria have been introduced through years during which supplier selection problem have been challenged by academicians and practitioners.

Network Formation. Network formation comprises two steps described as follows:

– *Clustering. Some clusters are formed with respect to the criteria. Then, the criteria are assigned to the clusters to which are mostly related. Finally, alternatives make a separate cluster.*

– *Connecting. In this step, the related clusters are connected with respect to the dependencies between their corresponding criteria. The connections which reflect interrelationships and feedback structure can be either inner (between two criteria within the same cluster) or outer (between two different clusters). An inner connection is like a loop on the corresponding cluster. Connection between two criteria is signed with an arrow from the affecting criterion to the dependent one. As it is shown in Fig. 1, the dotted arrows show connections between criteria within one cluster or two different clusters, while the connection between two clusters are represented by solid arrows as a result of connections between criterion 3 and criterion 4 from two clusters.*

Pair-Wise Comparisons. Pairwise comparisons are performed between each pair of criteria with respect to a control criterion. Control criterion is the criterion to which some other criteria are dependent. In other words, the group of criteria connected to a specific (control) criterion is compared pair-wisely. In addition to the comparisons of criteria, clusters of the network must be compared pair-wisely with respect to the control cluster.

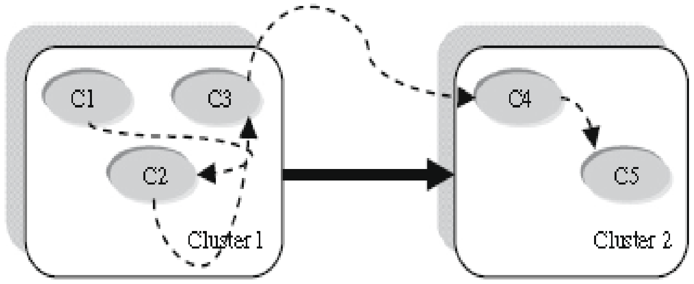


Fig. 1. A network with clusters and connections.

The comparisons are performed using a similar scale to the AHP's (Saaty [42]) (shown in *Table 1*). Similar to the AHP, each comparison results in a matrix with an eigenvector showing the final priorities regarding the control criterion. *Eq. (1)* shows the normalization of the comparison matrices, while aij are the scores assigned to factors i being compared with factors *j*. Calculation of eigenvector w is indicated in *Eq. (2)*, where wi are the relative importance of factors *i*:

$$A=[a_{ij}/\textstyle\sum_j a_{ij}], \tag{1}$$

$$\Rightarrow \begin{bmatrix} a_{11}/\sum_{i=1}^{n} a_{i1} & a_{12}/\sum_{i=1}^{n} a_{i2} & \ldots a_{1n}/\sum_{i=1}^{n} a_{in} \\ a_{21}/\sum_{i=1}^{n} a_{i1} & a_{22}/\sum_{i=1}^{n} a_{i2} & \ldots a_{2n}/\sum_{i=1}^{n} a_{in} \\ \ldots & & \\ a_{n1}/\sum_{i=1}^{n} a_{i1} & a_{n2}/\sum_{i=1}^{n} a_{i2} & \ldots a_{nn}/\sum_{i=1}^{n} a_{in} \end{bmatrix} \Rightarrow \begin{bmatrix} \dfrac{\sum_{j=1}^{n}(a_{1j}/\sum_{j=1}^{n} a_{ij})}{n} \\ \dfrac{\sum_{j=1}^{n}(a_{2j}/\sum_{j=1}^{n} a_{ij})}{n} \\ \ldots \\ \dfrac{\sum_{j=1}^{n}(a_{nj}/\sum_{j=1}^{n} a_{ij})}{n} \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ \ldots \\ w_n \end{bmatrix} \tag{2}$$

Supermatrix Calculation. Un-weighted supermatrix is constructed by putting the eigenvectors together (please refer to the super matrix in Statement).

Table 1. Scoring scales pair-wise comparison.

Score	Definition	
1	Equal importance	Two activities contribute equally to the objective.
2	Weak importance	
3	Moderate importance	Experience and judgment slightly favor one activity over another.
4	Moderate plus	
5	Strong importance	Experience and judgment strongly favor one activity over another.
6	Strong plus	
7	Very strong demonstrated importance	An activity is favored very strongly over another; its dominance demonstrated in practice.
8	Very, very strong	The evidence favoring one activity over another is
9	Extreme importance	of the highest possible order of affirmation.

This super matrix consists of blocks w_{ij} which are the matrices of eigenvectors corresponding to the comparison of elements of cluster i with respect to the control elements of cluster j.

$$W = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1n} \\ w_{21} & w_{22} & \cdots & w_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ w_{n1} & w_{n2} & \cdots & w_{nn} \end{bmatrix} \quad (3)$$

Weighted Supermatrix. The un-weighted supermatrix is not necessarily column stochastic, i.e., all columns do not sum to one. To normalize the columns, each eigenvector in a column is multiplied by the clusters' relative priorities obtained using comparison matrix of the clusters. The resulted column stochastic matrix is the weighted supermatrix.

Limit Supermatrix. In the network model, the direct dependencies are signed by connections and the indirect ones are neglected. For example in *Fig. 1*, *C5* is dependent to *C4*, *C4* is dependent to *C3*, but *C5* is not directly connected to *C3*. These indirect dependencies are not taken into account in weighted super matrix as it is formed regarding only direct dependencies and feedbacks in the model. To cope with this problem, weighted super matrix is powered until it converges, i.e., its rows stabilize to a unique value. The resulted matrix is limit super matrix.

Selection. Final priorities or weights of alternatives are the values in rows corresponding to the alternatives based upon which final decision is made.

3| Proposed Algorithm

To cope with the complexity of the problem as well as cover different dependent and independent aspects and criteria, the analytic network process is applied. The effective application of the ANP is accomplished providing that decision makers know the objectives, decision environment and decision elements. A suitably modeled decision is the result of this knowledge, as the decision makers need it to define the criteria and their dependencies. Additionally, pairwise comparisons must be realistic as much as possible.

As the complete and accurate information is not always available and the decision making process is indeterminate in nature, the proposed ANP is accompanied with fuzzy sets theory. To model the indeterminacy of a comparison, fuzzy comparison is a solution in which a limited and continuous interval of numbers is utilized. With the assumption of triangular fuzzy numbers, paired numbers (l_{ij}, u_{ij}) representing the lower and upper possible values of a_{ij}, indicate the preference of i to j, resulting a matrix like the matrix shown in *Statement (4)*. The conventional way to elicit the weights in ANP is not applicable.

Therefore, a special optimization model is adopted to elicit the best weights. Calculation of the eigenvectors is described in the following sub-section, as it is dissimilar to the conventional ANP.

$$\begin{bmatrix} 1 & (l12, u12) & \ldots & (l1n, u1n) \\ (l21, u21) & 1 & \ldots & (l2n, u2n) \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ (ln1, un1) & (ln2, un2) & \ldots & 1 \end{bmatrix}. \quad (4)$$

As mentioned, the fuzzy comparison matrices are similar to the one demonstrated in *Statement (4)*. It is clear that the reciprocal value of (l_{ij} , u_{ij}) is ($1 / u_{ij}$, $1 / l_{ij}$) (Chang [47]).

If $W = (w1, w2, \ldots, wn)$ is the eigenvector of the matrix shown in *Eq. (4)*, in which w_i are the relative weights of i from the comparison matrix, following conditions are met

$$w_1 + w_2 + \ldots + w_n = 1, \quad (6)$$

$$\forall\ i, j$$

$$\text{s.t. } i \in \{1, 2, 3, \ldots, n-1\};\ j \in \{2, 3, \ldots, n\} \rightarrow lij \leq wi / wj \leq uij, \quad (7)$$

$$\forall\ i \in \{1, 2, \ldots, n\} \quad wi > 0. \quad (8)$$

Therefore, vector W must be determined such that it meets the preceding conditions as well as demonstrates the decision-makers preferences. $\tilde{N}$ is a normalized fuzzy set of triangular fuzzy numbers. This set is defined by three values $a \leq b \leq c$ for any number of which a membership function is defined. This membership function has the following attributes:

It is a continuous function such that $R \rightarrow [0,1]$.

Following function is confirmed.

$$\mu N(x) = \begin{cases} (c-x) / (c-b) & b \leq x \leq c, \\ (x-a) / (b-a) & a \leq x \leq b, \\ 0 & \text{otherwise}. \end{cases} \quad (8)$$

For which

$$\int_{-\infty}^{\infty} \mu N(x) dx = 1. \quad (9)$$

In

this kind of fuzzy numbers, b is the central value with the highest probability, a and c represent the fuzziness. The triangular numbers are selected as the real w_i is a unique number. It is assumed to apply symmetric triangular fuzzy numbers, hence the best value is obtained when w_i / w_j closes to $(l_{ij} + u_{ij}) / 2$ using fuzzy comparison values (l_{ij} , u_{ij}). By means of this concept, weights of criteria are elicited. Supposing *Eq. (10)*, the membership function of the number φ_{ij} is defined as *Eq. (11)* in interval (l_{ij} , u_{ij}).

$$\varphi_{ij} = w_i / w_j , \quad m_{ij} = (l_{ij} + u_{ij}) / 2, \quad (10)$$

$$M(\varphi_{ij}) = \begin{cases} (\varphi_{ij} - m_{ij}) / (u_{ij} - m_{ij}) & ; \quad \varphi_{ij} > m_{ij}, \\ (m_{ij} - \varphi_{ij}) / (m_{ij} - l_{ij}) & ; \quad \varphi_{ij} \leq m_{ij}, \\ 0 & ; \quad \varphi_{ij} > u_{ij} \text{ or } \varphi_{ij} < l_{ij}. \end{cases} \quad (11)$$

It must be noted that the membership function is a triangular function with the base to the top. This function is an index of how distant w_i / w_j is from average m_{ij}. Thus, W is optimum when minimizes *Eq. (12)* and follows *Eqs. (5)–(7)*.

Ghasempoor Anaraki et al. |J. Fuzzy. Ext. Appl. 2(1) (2021) 69-88

$$\sum_{i<j} M(\varphi ij) \quad ; \ i \in \{1,2,3,\ldots,n\text{-}1\}, \ j \in \{2,3,4,\ldots,n\}. \tag{12}$$

So the resulted mathematical model is as follows

$$\begin{aligned} &\text{Min} \sum_{i<j} M(\varphi ij), \\ &\text{s.t.} \quad -w_i + l_{ij} w_j \leq 0 \quad i \in \{1,2,3,\ldots,n\text{-}1\}, \end{aligned} \tag{13}$$

$$j \in \{2,3,4,\ldots,n\}, \ \ w_i - u_{ij} w_j \leq 0 \quad i \in \{1,2,3,\ldots,n\text{-}1\}, \tag{14}$$

$$j \in \{2,3,4,\ldots,n\}, \tag{15}$$

$$w_1 + w_2 + \ldots + w_n = 1, \tag{16}$$

$$w_i > 0 \quad ; \quad \forall \, I \in \{1,2,3,\ldots,n\}. \tag{17}$$

If the mathematical model is infeasible, the judgments are inconsistent and must be revised. As the membership function is multifunctional, optimization of this function is onerous with the conventional methods. To do so, the distance between φ_{ij} and m_{ij} is minimized using the model described by *Eq. (18)*. It must be noted that minimization of square of the distance is the same as the distance itself.

$$\begin{aligned} &\text{Min} \quad \sum_{i<j} \left(\frac{\varphi ij - mij}{(uij - lij)/2}\right)^2, \\ &\text{s.t} \quad -w_i + l_{ij} w_j \leq 0 \quad i \in \{1,2,3,\ldots,n\text{-}1\}, \ j \in \{2,3,4,\ldots,n\}, \\ &w_i - u_{ij} w_j \leq 0 \quad i \in \{1,2,3,\ldots,n\text{-}1\}, \ j \in \{2,3,4,\ldots,n\}, \\ &w_1 + w_2 + \ldots + w_n = 1, \\ &w_i > 0 \ ; \ \forall \ i \in \{1,2,3,\ldots,n\}. \end{aligned} \tag{18}$$

Solving this model using optimizer software packages, the eigenvector of fuzzy comparison matrix is gained which can be utilized in the ANP. It is notable that the model can be solved by the optimizer, because the number of elements being compared pair-wisely is recommended to be less than seven (Saaty, [42]). Pang and bai [4]) recommend the numbers in the following tables for making decision in a fuzzy state.

Table 2. Linguistic expression for fuzzy scale of relative weights of criteria.

Definition of Linguistic Variables for Relative Weights of Criteria	Triangular Fuzzy Scale	Triangular Fuzzy Reciprocal Scale
Just equal	(1,1,1)	(1,1,1)
Equally important	M1=(1,1,3)	(1/3,1,1)
Weakly important	M3=(1,3,5)	(1/5,1/3,1)
Essentially important	M5=(3,5,7)	(1/7,1/5,1/3)
Strongly important	M7=(5,7,9)	(1/9,1/7,1/5)
Absolutely important	M9=(7,9,9)	(1/9,1/9,1/7)
Intermediate values between two adjacent	M2, M4, M6, M8	

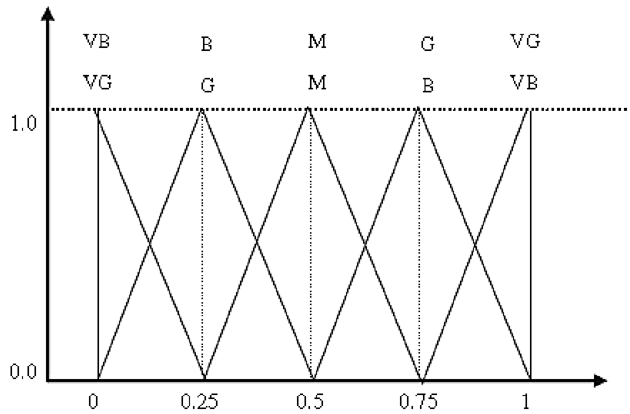


Fig. 2. Membership functions of linguistic values for criteria rating.

Table 3. Linguistic values and mean of fuzzy numbers for alternatives.

Linguistic Values of Negative Criteria for Alternatives	Linguistic Values for Criteria For Alternatives	The Mean of Fuzzy Numbers
Very bad (VB)	Very good (VG)	1
Bad (B)	Good (G)	0.75
Medium (M)	Medium (M)	0.5
Good (G)	Bad (B)	0.25
Very good (VG)	Very bad (VB)	0

5| Numerical Example

Suppose that a firm has three suppliers *(A_1, A_2, A_3)*. Considering the introduced model, and the information about each supplier, the best supplier will be selected.

Step 1. Criteria Selection. The following criteria are selected by the experts from the criteria in ***Table 1***: quality, on time delivery, cost, reliability, services, technical capabilities, location, financial status, partnership, operational control, supplier reputation, packing, background, reciprocity, flexibility, discounts, transport, and risks.

Step 2. Choosing and Building a Prototype. All supplier selection criteria can be classified into two categories as following:

Company status.

Performance criteria.

Step 3. Screening Criteria by SMART. For this purpose, we have considered the following indicators for screening: 1) applicability, 2) being measurable, 3) the frequency of the criteria in other researches, and 4) being perfect. The weight of these indicators:

$$w_1 = 0.521\ ,\ w_2 = 0.271\ ,\ w_3 = 0.145\ ,\ w_4 = 0.63.$$

Screening and ranking the criteria by using these indicators and SMART method. In this stage, the criteria which have the highest points are selected as following.

Table 4. Selected criteria.

No	Name	Selected Criteria	Points
1	C1	Cost	94.56
2	C2	On time delivery	85.79
3	C3	Quality	83.87
4	C4	Flexibility	63.73
5	C5	Services	64.48
6	C6	Financial status	67.12
7	C7	Supplier reputation	69.65
8	C8	Technical capabilities	76.26
9	C9	Background	67.90
10	C10	Location	57.60

Step 4. Identify the Effects of the Criteria on Each Other by DEMATEL. Our experts were asked to rate and grade the effects of criteria on each other from 0 – 4 in some matrixes, the result of which is as the following:

These matrixes are for the *C1* to *C5* criteria:

$$\widehat{M} = \begin{bmatrix} 0 & 3 & 4 & 3 & 2 \\ 3 & 0 & 2 & 2 & 1 \\ 3 & 2 & 0 & 1 & 1 \\ 2 & 1 & 2 & 0 & 1 \\ 2 & 1 & 2 & 1 & 0 \end{bmatrix},$$

$$M = \begin{bmatrix} 0 & 0.25 & 0.33 & 0.25 & 0.166 \\ 0.25 & 0 & 0.166 & 0.166 & 0.083 \\ 0.25 & 0.166 & 0 & 0.083 & 0.083 \\ 0.166 & 0.083 & 0.166 & 0 & 0.083 \\ 0.166 & 0.083 & 0.166 & 0.083 & 0 \end{bmatrix},$$

$$M(I-M)-1 = \begin{bmatrix} 0.814 & 0.827 & 1.000 & 0.737 & 0.586 \\ 0.776 & 0.46 & 0.676 & 0.521 & 0.404 \\ 0.727 & 0.563 & 0.492 & 0.428 & 0.377 \\ 0.583 & 0.43 & 0.55 & 0.291 & 0.327 \\ 0.583 & 0.43 & 0.55 & 0.366 & 0.251 \end{bmatrix}.$$

These matrixes are for the C6 to C10 criteria:

$$\widehat{M}=\begin{bmatrix}0&3&4&2&1\\3&0&1&2&0\\2&3&0&2&0\\1&1&0&0&0\\2&2&1&2&0\end{bmatrix},$$

$$M=\begin{bmatrix}0&0.3&0.4&0.2&0.1\\0.3&0&0.1&0.2&0\\0.2&0.3&0&0.2&0\\0.1&0.1&0&0&0\\0.2&0.2&0.1&0.2&0\end{bmatrix},$$

$$M(I-M)^{-1}=\begin{bmatrix}0.435&0.716&0.66&0.591&0.144\\0.518&0.313&0.344&0.445&0.052\\0.481&0.578&0.255&0.473&0.048\\0.195&0.203&0.1&0.104&0.019\\0.478&0.504&0.346&0.475&0.048\end{bmatrix}.$$

The final diagraph of the pre-mentioned matrixes is as it follows.

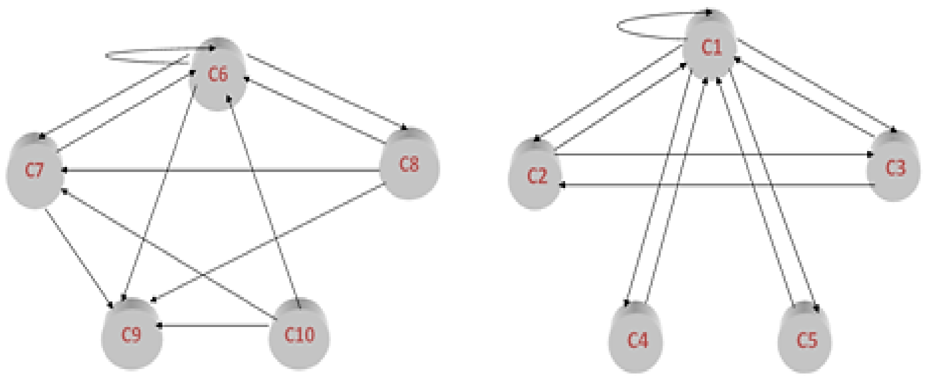


Fig. 3. Effects of criteria on each other.

The final diagraph of the whole criteria is as the following.

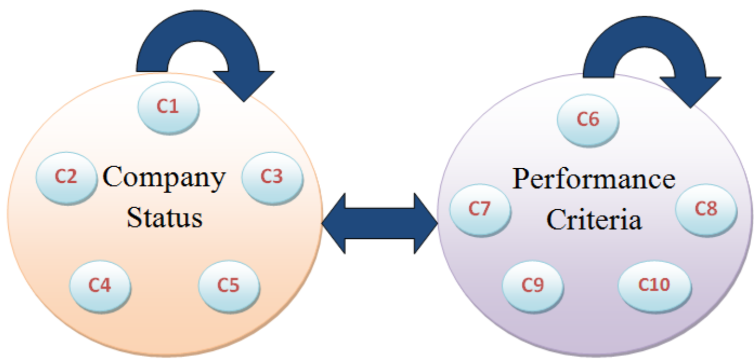


Fig. 4. Final diagraph of effects of criteria on each other.

By adding the suppliers, this diagraph is resulted:

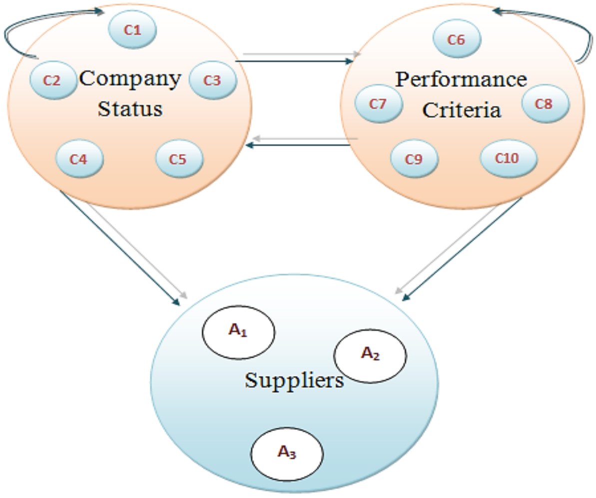


Fig. 5. Final diagraph of effects of criteria and suppliers on each other.

Step 5. Forming matrixes of FANP. The suppliers' information is categorized in the following tables:

Table 5. Suppliers compared with cost.

Cost	A1	A2	A3
A1	1	1.05	1.15
A2	0.95	1	1.1
A3	0.87	0.91	1

The cost calculation in the following tables is the result of the above-mentioned table:

Table 6. Suppliers compared with cost.

Cost	A_1	A_2	A_3
A_1	1	1.05	1.15
A_2	0.95	1	1.1
A_3	0.87	0.91	1
Sum	2.82	2.96	3.25

The normalized form is

Table 7. Suppliers compared with cost.

Cost	A1	A2	A3
A1	0.3546	0.3547	0.3538
A2	0.3369	0.3378	0.3385
A3	0.3085	0.3075	0.3077

To sum up:

$$W_1 = (0.3546 + 0.3547 + 0.3538\ )\ /\ 3 = 0.3544,$$

$W_2 = (0.3369 + 0.3378 + 0.3385) / 3 = 0.3377,$

$W_3 = (0.3085 + 0.3075 + 0.3077) / 3 = 0.3079.$

Other information for the other criteria is provided in a fuzzy state, some of which are mentioned if the following tables:

Table 8. Suppliers compared with on time delivery.

On Time Delivery	A_1	A_2	A_3
A_1	(1,1)	(2/5 , 2/3)	(1/3 , 1/2)
A_2	(3/2 , 5/2)	(1 , 1)	(2/5 , 2/3)
A_3	(2 , 3)	(3/2 , 5/2)	(1 , 1)

Therefore:

$$\text{Min } (900w_1^2/w_2^2 - 960w_1/w_2 + 144w_1^2/w_3^2 - 120w_1/w_3 + 900w_2^2/w_3^2 - 960w_2/w_3 + 537)$$

s.t.

$-w_1 + 2/5\, w_2 \leq 0,$

$w_1 - 2/3\, w_2 \leq 0,$

$-w_1 + 1/3\, w_3 \leq 0,$

$w_1 - 1/2\, w_3 \leq 0,$

$-w_2 + 2/5\, w_3 \leq 0,$

$w_2 - 2/3\, w_3 \leq 0,$

$w_1 + w_2 + w_3 = 1.$

To sum up:

$w_1 = 0.1732,$

$w_2 = 0.3016,$

$w_3 = 0.5251,$

and the information about the flexibility of suppliers are given in the folloing table.

Table 9. Suppliers compared with flexibility.

Flexibility	A_1	A_2	A_3
A_1	(1 , 1)	(1/2 , 3/2)	(1 , 2)
A_2	(2/3 , 2)	(1 , 1)	(1 , 2)
A_3	(1/2 , 1)	(1/2 , 1)	(1 , 1)

Therefore:

$$\text{Min } (4w_1^2/w_2^2 - 8w_1/w_2 + 4w_1^2/w_3^2 - 12w_1/w_3 + 4w_2^2/w_3^2 -12w_2/w_3 + 22)$$

s.t.

$-w_1 +1/2\, w_2 \leq 0,$

$w_1 - 3/2\, w_2 \leq 0,$

$-w_1 + w_3 \leq 0,$

$w_1 - 2w_3 \leq 0,$

$-w_2 + w_3 \leq 0,$

$w_2 - 2w_3 \leq 0,$

$w_1 + w_2 + w_3 = 1.$

To sum up:

$w_1 = 0.3750,$

$w_2 = 0.3750,$

$w_3 = 0.2500.$

Step 6. Selecting the Best Supplier. We assign the resulted weights to un-weighted supermatrix and continue the calculations.

Table 10. Un-weighted supermatrix respecting the change.

	A1	A2	A3	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
A1	0.0000	0.0000	0.0000	0.3544	0.1732	0.1732	0.1732	0.1732	0.3333	0.3333	0.3333	0.3750	0.5062
A2	0.0000	0.0000	0.0000	0.3377	0.3016	0.3016	0.3016	0.3016	0.3333	0.3333	0.3333	0.3750	0.3072
A3	0.0000	0.0000	0.0000	0.3079	0.5251	0.5251	0.5251	0.5251	0.3333	0.3333	0.3333	0.2500	0.1865
C1	0.0000	0.0000	0.0000	0.8140	0.7760	0.7270	0.5830	0.5830	0.5000	0.5000	0.2000	0.3000	0.1000
C2	0.0000	0.0000	0.0000	0.8270	0.0000	0.5630	0.0000	0.0000	0.6000	0.3000	0.7000	0.5000	0.3000
C3	0.0000	0.0000	0.0000	1.0000	0.6760	0.0000	0.0000	0.0000	0.5000	0.3000	0.4000	0.4000	0.2000
C4	0.0000	0.0000	0.0000	0.7370	0.0000	0.0000	0.0000	0.0000	0.5000	0.0000	0.4000	0.4000	0.2000
C5	0.0000	0.0000	0.0000	0.5860	0.0000	0.0000	0.0000	0.0000	0.5000	0.0000	0.5000	0.5000	0.3000
C6	0.0000	0.0000	0.0000	0.8000	0.5000	0.7000	0.1000	0.4000	0.4350	0.5180	0.4810	0.0000	0.4780
C7	0.0000	0.0000	0.0000	0.0000	0.7000	0.7000	0.8000	0.7000	0.7160	0.0000	0.5780	0.0000	0.5040
C8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2000	0.0000	0.6600	0.0000	0.0000	0.0000	0.0000
C9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5910	0.4450	0.4730	0.0000	0.4750
C10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

For calculating the limit supermatrix, weighted supermatrix should be multiplied in itself for several times to reach the stability, as it was mentioned. In this way the limit supermatrix is resulted such as the table above. Taking into consideration the resulted weights, the volunteer suppliers are ranked as the following:

$A_3 > A_2 > A_1$.

Table 11. Weighted supermatrix respecting the change.

	A1	A2	A3	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
A1	0	0	0	0.0614	0.0474	0.0469	0.0697	0.0645	0.0555	0.1088	0.0704	0.1209	0.1423
A2	0	0	0	0.0585	0.0825	0.0817	0.1214	0.1124	0.0555	0.1088	0.0704	0.1209	0.0863
A3	0	0	0	0.0534	0.1437	0.1423	0.2114	0.1957	0.0555	0.1088	0.0704	0.0806	0.0524
C1	0	0	0	0.1412	0.2124	0.1970	0.2348	0.2173	0.0833	0.1632	0.0422	0.0967	0.0281
C2	0	0	0	0.1434	0	0.1525	0	0	0.0999	0.0979	0.1479	0.16129	0.0843
C3	0	0	0	0.1734	0.1851	0	0	0	0.0833	0.0979	0.0845	0.1290	0.0562
C4	0	0	0	0.1278	0	0	0	0	0.0833	0	0.0845	0.1290	0.0562
C5	0	0	0	0.1016	0	0	0	0	0.0833	0	0.1056	0.1612	0.08434
C6	0	0	0	0.138793	0.13691	0.18970	0.04027	0.14909	0.07247	0.16912	0.1016	0	0.13438
C7	0	0	0	0	0.191681	0.189707	0.281928	0.260912	0.119296	0	0.12215	0	0.141696
C8	0	0	0	0	0	0	0.040275	0	0.109965	0	0	0	0
C9	0	0	0	0	0	0	0	0	0.098469	0.145287	0.09996	0	0.133543
C10	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 12. Limit supermatrix respecting the change.

	A1	A2	A3	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
A1	0	0	0	0.0284	0.0284	0.0284	0.0284	0.0284	0.0284	0.0284	0.0284	0.0284	0.0284
A2	0	0	0	0.0344	0.0344	0.0344	0.0344	0.0344	0.0344	0.0344	0.0344	0.0344	0.0344
A3	0	0	0	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447
C1	0	0	0	0.06541	0.06542	0.06583	0.06545	0.06589	0.06533	0.06533	0.06526	0.06534	0.06557
C2	0	0	0	0.03897	0.03885	0.03830	0.0387	0.03859	0.03810	0.03885	0.03888	0.03871	0.03846
C3	0	0	0	0.03914	0.03964	0.03980	0.03917	0.03929	0.03946	0.03926	0.03940	0.03957	0.03986
C4	0	0	0	0.02360	0.02306	0.02318	0.02356	0.02360	0.02361	0.02352	0.02312	0.02394	0.02396
C5	0	0	0	0.02149	0.02171	0.02176	0.02127	0.02128	0.02119	0.02199	0.02195	0.02124	0.02174
C6	0	0	0	0.05132	0.05189	0.05199	0.05178	0.05131	0.05163	0.05177	0.05149	0.05187	0.05167
C7	0	0	0	0.04535	0.04542	0.04542	0.04575	0.04510	0.04588	0.04577	0.04571	0.04518	0.04560
C8	0	0	0	0.00832	0.00831	0.00832	0.00832	0.00835	0.00841	0.00801	0.00800	0.00812	0.00801
C9	0	0	0	0.01706	0.01705	0.0170	0.01700	0.01708	0.01705	0.01708	0.01705	0.01700	0.017076
C10	0	0	0	0	0	0	0	0	0	0	0	0	0

6| Conclusion

The supplier selection process is a MCDM issue. However, the exact and certain information is not always available for making decision. As it was noted earlier, most of the information is qualitative, therefore, in this study linguistic and fuzzy variables and a form of FANP are used to comprise the uncertain and ambiguous states; taking into account the combination of the pre-mentioned method with Smarter and Dematel methods, a precise process has been designed which can select, evaluate, grade, and determine the connections and effects of elements in the best form, and as a result they lead to selecting the best supplier. This model is able to cover all levels of decision making for supplier selection, suppliers rates, and proportional and final weights of each criterion. These weights signify the importance of each criterion in relation to the purpose, which is supplier selection. Taking into consideration the characteristics of this model, the mutual effects of the decision making elements can be applied to calculation and the decisions can be made in the best form.

References

[1] selection under interval-valued intuitionistic uncertain linguistic environment. *Information sciences, 486*, 254-270.

[2] Schramm, V. B., Cabral, L. P. B., & Schramm, F. (2020). Approaches for supporting sustainable supplier selection-A literature review. *Journal of cleaner production*, 123089. https://doi.org/10.1016/j.jclepro.2020.123089

[3] Narasimhan , R ., Talluri, S., & Mendez, D. (2001). Supplier evaluation and rationalization via data envelopment analysis: an empirical examination. *Journal of supply chain management, 37*(3),28-37.

[4] Jeong, C. S., & Lee, Y. H. (2002). A multi-criteria supplier selection (MCSS) model for supply chain management. *VISION: the journal of business perspective*, 51–60. https://www.researchgate.net/profile/Young-Hae-Lee/publication/264235100_Multi-Criteria_Supplier_SelectionMCSS_Model_for_Supply_Chain_Management/links/550123d90cf2aee14b58ecd7/Multi-Criteria-Supplier-SelectionMCSS-Model-for-Supply-Chain-Management.pdf

[5] Xia, W., & Wu, Z. (2007). Supplier selection with multiple criteria in volume discount environments. *Omega, 35*(5), 494-504.

[6] Kumar, M., Vrat, P., & Shankar, R. (2006). A fuzzy programming approach for vendor selection problem in a supply chain. *International journal of production economics*, *101*(2), 273-285.

[7] Edwards, W., & Barron, F. H. (1994). SMARTS and SMARTER: Improved simple methods for multi-attribute utility measurement. *Organizational behavior and human decision processes, 60*(3), 306–325.

[8] Lootsma, F. A. (1996). A model for the relative importance of the criteria in the multiplicative AHP and SMART. *European journal of operational research, 94*(3), 467-476.

[9] Haddara, M. (2014). ERP selection: the SMART way. *Procedia technology, 16*, 394-403.

[10] Cheng, C. C., Chen, C. T., Hsu, F. S., & Hu, H. Y. (2012). Enhancing service quality improvement strategies of fine-dining restaurants: New insights from integrating a two-phase decision-making model of IPGA and DEMATEL analysis. *International journal of hospitality management*, *31*(4), 1155-1166.

[11] Sumrit, D., & Anuntavoranich, P. (2013). Using DEMATEL method to analyze the causal relations on technological innovation capability evaluation factors in Thai technology-based firms. *International transaction journal of engineering, management, & applied sciences & technologies*, *4*(2), 81-103.

[12] Wu, H. H., & Tsai, Y. N. (2011). A DEMATEL method to evaluate the causal relations among the criteria in auto spare parts industry. *Applied mathematics and computation, 218*(5), 2334-2342.

[13] Yamazaki, M., Ishibe, K., Yamashita, S., Miyamoto, I., Kurihara, M., & Shindo, H. (1997). An analysis of obstructive factors to welfare service using DEMATEL method. *Reports of the faculty of engineering, 48*, 25-30.

[14] Razmi, J., Rafiei, H., & Hashemi, M. (2009). Designing a decision support system to evaluate and select suppliers using fuzzy analytic network process. *Computers & industrial engineering*, *57*(4), 1282-1290.

[15] Saaty, T. L. (1980). *The analytic hierarchy process, planning, priority setting, resource allocation*. McGraw-Hill.
[16] Saaty, T. L. (1996). *Decision making with dependence and feedback: the analytic network process*. RWS Publications.
[17] Saaty, T. L. (1999). Fundamentals of the analytical network process. *Proceeding of ISHP* (pp. 48–63). Kobe.
[18] Saaty, T. L., & Vergas, L. G. (2006). *Decision making with analytic network process*. New York: Springer Sciences.
[19] Taslicali, A. K., & Ercan, S. (2006). The analytic hierarchy and the analytic network process in multicriteria decision making: a comparative study. *Journal of aeronautics and space technologies, 2*(4), 55–65.
[20] Chang, D. Y. (1996). Application of the extent analysis method on fuzzy AHP. *European journal of operational research, 95*, 649–655.
[21] Das, S. K., Edalatpanah, S. A., & Mandal, T. (2018). A proposed model for solving fuzzy linear fractional programming problem: numerical point of view. *Journal of computational science, 25*, 367-375.
[22] Edalatpanah, S. A., & Smarandache, F. (2019). Data envelopment analysis for simplified neutrosophic sets. *Neutrosophic sets and systems, 29*, 215-226.
[23] Edalatpanah, S. A. (2020). Data envelopment analysis based on triangular neutrosophic numbers. *CAAI transactions on intelligence technology, 5*(2), 94-98.
[24] Edalatpanah, S. A. (2018). Neutrosophic perspective on DEA. *Journal of applied research on industrial engineering, 5*(4), 339-345.
[25] Edalatpanah, S. A. (2020). Neutrosophic structured element. *Expert systems, 37*(5), e12542. https://doi.org/10.1111/exsy.12542
[26] Behera, D., Peters, K., Edalatpanah, S. A., & Qiu, D. (2020). New methods for solving imprecisely defined linear programming problem under trapezoidal fuzzy uncertainty. *Journal of information and optimization sciences*, 1-27. https://doi.org/10.1080/02522667.2020.1758369
[27] Shafi Salimi, P., & Edalatpanah, S. A. (2020). Supplier selection using fuzzy AHP method and D-Numbers. *Journal of fuzzy extension and applications, 1*(1), 1-14.
[28] Karaşan, A., & Kahraman, C. (2019). A novel intuitionistic fuzzy DEMATEL–ANP–TOPSIS integrated methodology for freight village location selection. *Journal of intelligent & fuzzy systems, 36*(2), 1335-1352.
[29] Liu, H. C., Quan, M. Y., Li, Z., & Wang, Z. L. (2019). A new integrated MCDM model for sustainable supplier selection under interval-valued intuitionistic uncertain linguistic environment. *Information sciences, 486*, 254-270.
[30] Schramm, V. B., Cabral, L. P. B., & Schramm, F. (2020). Approaches for supporting sustainable supplier selection-A literature review. *Journal of cleaner production*, 123089. https://doi.org/10.1016/j.jclepro.2020.123089
[31] Narasimhan , R ., Talluri, S., & Mendez, D. (2001). Supplier evaluation and rationalization via data envelopment analysis: an empirical examination. *Journal of supply chain management, 37*(3),28-37.
[32] Jeong, C. S., & Lee, Y. H. (2002). A multi-criteria supplier selection (MCSS) model for supply chain management. *VISION: the journal of business perspective*, 51–60. https://www.researchgate.net/profile/Young-Hae-Lee/publication/264235100_Multi-Criteria_Supplier_SelectionMCSS_Model_for_Supply_Chain_Management/links/550123d90cf2aee14b58ecd7/Multi-Criteria-Supplier-SelectionMCSS-Model-for-Supply-Chain-Management.pdf
[33] Xia, W., & Wu, Z. (2007). Supplier selection with multiple criteria in volume discount environments. *Omega, 35*(5), 494-504.
[34] Kumar, M., Vrat, P., & Shankar, R. (2006). A fuzzy programming approach for vendor selection problem in a supply chain. *International journal of production economics, 101*(2), 273-285.
[35] Edwards, W., & Barron, F. H. (1994). SMARTS and SMARTER: Improved simple methods for multi-attribute utility measurement. *Organizational behavior and human decision processes, 60*(3), 306–325.
[36] Lootsma, F. A. (1996). A model for the relative importance of the criteria in the multiplicative AHP and SMART. *European journal of operational research, 94*(3), 467-476.
[37] Haddara, M. (2014). ERP selection: the SMART way. *Procedia technology, 16*, 394-403.
[38] Cheng, C. C., Chen, C. T., Hsu, F. S., & Hu, H. Y. (2012). Enhancing service quality improvement strategies of fine-dining restaurants: New insights from integrating a two-phase decision-making model of IPGA and DEMATEL analysis. *International journal of hospitality management, 31*(4), 1155-1166.

[39] Sumrit, D., & Anuntavoranich, P. (2013). Using DEMATEL method to analyze the causal relations on technological innovation capability evaluation factors in Thai technology-based firms. *International transaction journal of engineering, management, & applied sciences & technologies*, *4*(2), 81-103.

[40] Wu, H. H., & Tsai, Y. N. (2011). A DEMATEL method to evaluate the causal relations among the criteria in auto spare parts industry. *Applied mathematics and computation*, *218*(5), 2334-2342.

[41] Yamazaki, M., Ishibe, K., Yamashita, S., Miyamoto, I., Kurihara, M., & Shindo, H. (1997). An analysis of obstructive factors to welfare service using DEMATEL method. *Reports of the faculty of engineering*, *48*, 25-30.

[42] Razmi, J., Rafiei, H., & Hashemi, M. (2009). Designing a decision support system to evaluate and select suppliers using fuzzy analytic network process. *Computers & industrial engineering*, *57*(4), 1282-1290.

[43] Saaty, T. L. (1980). *The analytic hierarchy process, planning, priority setting, resource allocation*. McGraw-Hill.

[44] Saaty, T. L. (1996). *Decision making with dependence and feedback: the analytic network process*. RWS Publications.

[45] Saaty, T. L. (1999). Fundamentals of the analytical network process. *Proceeding of ISHP* (pp. 48–63). Kobe.

[46] Saaty, T. L., & Vergas, L. G. (2006). *Decision making with analytic network process*. New York: Springer Sciences.

[47] Taslicali, A. K., & Ercan, S. (2006). The analytic hierarchy and the analytic network process in multicriteria decision making: a comparative study. *Journal of aeronautics and space technologies*, *2*(4), 55–65.

[48] Chang, D. Y. (1996). Application of the extent analysis method on fuzzy AHP. *European journal of operational research*, *95*, 649–655.

Ghasempoor Anaraki et al. |J. Fuzzy. Ext. Appl. 2(1) (2021) 69-88

E-ISSN: 2717-3453 | P-ISSN: 2783-1442

Journal of Fuzzy Extension and Applications

www.journal-fea.com

J. Fuzzy. Ext. Appl. Vol. 2, No. 1 (2021) 89–105.

Paper Type: Research Paper

A Simplified Method for Solving Transportation Problem with Triangular Fuzzy Numbers under Fuzzy Circumstances

Ladji Kané[1,*], Hamala Sidibé[1], Souleymane Kané[1], Hawa Bado[1], Moussa Konaté[1], Daouda Diawara[1], Lassina Diabat[1]

[1] Faculty of Economics and Management (FSEG), University of Social Sciences and Management of Bamako (USSGB), Quartier du Fleuve Rue 310, Porte 238, Mali; fsegmath@gmail.com; garalo2018@gmail.com; badohawa@yahoo.fr; konatefseg@gmail.com.

Citation:

Kané, L., Sidibé, H., Kané, S., Bado, H., Konaté, M., Diawara, D., & Diabat, L. (2021). A simplified method for solving transportation problem with triangular fuzzy numbers under fuzzy circumstances. *Journal of fuzzy extension and application, 2* (1), 89-105.

Received: 14/12/2020 Reviewed: 19/02/2021 Revised: 08/03/2021 Accept: 17/03/2021

Abstract

Transportation Problem (TP) is an important network structured linear programming problem that arises in several contexts and has deservedly received a great deal of attention in the literature. The central concept in this problem is to find the least total transportation cost of a commodity in order to satisfy demands at destinations using available supplies at origins in a crisp environment. In real life situations, the decision maker may not be sure about the precise values of the coefficients belonging to the transportation problem. The aim of this paper is to introduce a formulation of TP involving Triangular fuzzy numbers for the transportation costs and values of supplies and demands. We propose a two-step method for solving fuzzy transportation problem where all of the parameters are represented by non-negative triangular fuzzy numbers i.e., an Interval Transportation Problems (TPIn) and a Classical Transport Problem (TP). Since the proposed approach is based on classical approach it is very easy to understand and to apply on real life transportation problems for the decision makers. To illustrate the proposed approach two application examples are solved. The results show that the proposed method is simpler and computationally more efficient than existing methods in the literature.

Keywords: Fuzzy linear programming, Transportation problem, Triangular fuzzy numbers.

1 | Introduction

Transportation problem is an important network structured linear programming problem that arises in several contexts and has deservedly received a great deal of attention in the literature. The central concept in this problem is to find the least total transportation cost of a commodity in order to satisfy demands at destinations using available supplies at origins. Transportation problem can be used for a wide variety of situations such as scheduling, production, investment, plant location, inventory control, employment scheduling and many others.

Corresponding Author: fsegmath@gmail.com

http://dx.doi.org/10.22105/jfea.2021.275280.1084a

In general, transportation problems are solved with the assumptions that the transportation costs and values of supplies and demands are specified in a precise way i.e., in crisp environment. However, in many cases the decision maker has no crisp information about the coefficients belonging to the transportation problem. If the nature of the information is vague, that is, if it has some lack of precision, the corresponding coefficients or elements defining the problem can be formulated by means of fuzzy sets, and thus fuzzy transportation problems arise. Several researchers have carried out investigations on fuzzy transportation problem. Zimmermann [4] developed Zimmermann's fuzzy linear programming into several fuzzy optimization methods for solving the transportation problems. ÓhÉigeartaigh [5] proposed an algorithm for solving transportation problems where the supplies and demands are fuzzy sets with linear or triangular membership functions. Chanas et al. [6] investigated the transportation problem with fuzzy supplies and demands and solved them via the parametric programming technique. Their method provided solution which simultaneously satisfies the constraints and the goal to a maximal degree.

In addition, Chanas et al. [7] formulated the classical, interval and fuzzy transportation problem and discussed the methods for solution for the fuzzy transportation problem. Chanas and Kuchta [8] discussed the type of transportation problems with fuzzy cost coefficients and converted the problem into a bicriterial transportation problem with crisp objective function. Their method only gives crisp solutions based on efficient solutions of the converted problems. Jimenez and Verdegay [9] and [10] investigated the fuzzy solid transportation problem in which supplies, demands and conveyance capacities are represented by trapezoidal fuzzy numbers and applied a parametric approach for finding the fuzzy solution. Liu and Kao [11] developed a procedure, based on extension principle to derive the fuzzy objective value of fuzzy transportation problem, in that the cost coefficients and the supply and demand quantities are fuzzy numbers.

Gani and Razak [12] presented a two-stage cost minimizing fuzzy transportation problem in which supplies and demands are as trapezoidal fuzzy numbers and used a parametric approach for finding a fuzzy solution with the aim of minimizing the sum of the transportation costs in the two stages. Li et al. [13] proposed a new method based on goal programming for solving fuzzy transportation problem with fuzzy costs. Lin [14] used genetic algorithm for solving transportation problems with fuzzy coefficients. Dinagar and Palanivel [15] investigated fuzzy transportation problem, with the help of trapezoidal fuzzy numbers and applied fuzzy modified distribution method to obtain the optimal solution in terms of fuzzy numbers. Pandian and Natarajan [16] introduced a new algorithm namely, fuzzy zero-point method for finding fuzzy optimal solution for such fuzzy transportation problem in which the transportation cost, supply and demand are represented by trapezoidal fuzzy numbers. Kumar and Kaur [17] proposed a new method based on fuzzy linear programming problem for finding the optimal solution of fuzzy transportation problem.

Gupta et al. [18] proposed a new method named as Mehar's method, to find the exact fuzzy optimal solution of fully fuzzy multi-objective transportation problems. Ebrahimnejad [19] applied a fuzzy bounded dual algorithm for solving bounded transportation problems with fuzzy supplies and demands. Shanmugasundari and Ganesan [20] developed the fuzzy version of Vogel's and MODI methods for obtaining the fuzzy initial basic feasible solution and fuzzy optimal feasible solution, respectively, without converting them into classical transportation problem. Also, Chandran and Kandaswamy [21] proposed an algorithm to find an optimal solution of a fuzzy transportation problem, where supply, demand and cost coefficients all are fuzzy numbers. Their algorithm provides decision maker with an effective solution in comparison to existing methods. Ebrahimnejad [22] using an example showed that their method will not always lead to a fuzzy optimal solution. Moreover, Kumar and Kaur [23] pointed out the limitations and shortcomings of the existing methods for solving fuzzy solid transportation problem and to overcome these limitations and shortcomings proposed a new method to find the fuzzy optimal solution of unbalanced fuzzy solid transportation problems.

In addition, Ebrahimnejad [24] proposed a two-step method for solving fuzzy transportation problem where all of the parameters are represented by non-negative triangular fuzzy numbers. Some researchers applied generalized fuzzy numbers for solving transportation problems. Kumar and Kaur [25] proposed a new method based on ranking function for solving fuzzy transportation problem by assuming that transportation cost, supply and demand of the commodity are represented by generalized trapezoidal fuzzy numbers. After that, Kaur and Kumar [26] introduced a similar algorithm for solving a special type of fuzzy transportation problem by assuming that a decision maker is uncertain about the precise values of transportation cost only but there is no uncertainty about the supply and demand of the product. Ebrahimnejad [27] demonstrated that once the ranking function is chosen, the fuzzy transportation problem introduced by Kaur and Kumar [26] is converted into crisp one, which is easily solved by the standard transportation algorithms.

The contributions of the present study are summarized as follows: (1) in the TPTri under consideration, all of the parameters, such as the transportation costs, supplies and demands are considered as fuzzy numbers, (2) according to the proposed approach, the TPTri is converted into an TPIn and a TP. The integration of the optimal solution of the four sub-problems provides the optimal solution of the TPTri, (3) in contrast to most existing approaches, which provide a precise solution, the proposed method provides a fuzzy optimal solution, (4) In contrast to existing methods that include negative parts in the obtained fuzzy optimal solution and fuzzy optimal cost, the proposed method provides a fuzzy optimal solution and optimal cost, (5) similarly, to the competing methods in the literature, the proposed method is applicable for all types of triangular fuzzy numbers. and (6) the complexity of computation is greatly reduced compared with commonly used existing methods in the literature.

The rest of this paper is organized as follows. In Section 2, we recall the definitions of interval number linear programming, interval numbers and the existing method for solving linear programming problem involving interval numbers. In Section 3, a new method is proposed for obtaining the fuzzy optimal solution of the TPTri. The advantages of the proposed method are discussed in Section 4. Two application examples are provided to illustrate the effectiveness of the proposed method in Section 5. Finally, concluding remarks are presented in Section 6.

2| Materials and Methods

In this section, some basic definitions, arithmetic operations for closed Intervals numbers and of linear programming problems involving interval numbers are presented [28].

2.1| A New Interval Arithmetic

In this section, some arithmetic operations for two intervals are presented [28].

Let $\Re = \{\bar{a} = [a^1, a^3]: a^1 \leq a^3 \; with \; a^1, a^3 \in \mathbb{R}\}$ be the set of all proper intervals and $\bar{\Re} = \{\bar{a} = [a^1, a^3]: a^1 > a^3 \text{ with } a^1, a^3 \in \mathbb{R}\}$ be the set of all improper intervals on the real line $\mathbb{R}$. We shall use the terms "interval" and "interval number" interchangeably. The mid-point and width (or half-width) of an interval number are defined as the midpoint and width (or half-width) of an interval number $\bar{a} = [a^1, a^3]$ are defined as $m(\bar{a}) = \left(\frac{a^3+a^1}{2}\right)$ and $w(\bar{a}) = \left(\frac{a^3-a^1}{2}\right)$. The interval number $\bar{a}$ can also be expressed in terms of its midpoint and width as $\bar{a} = [a^1, a^3] = \langle m(\bar{a}), w(\bar{a})\rangle = \langle \frac{a^3+a^1}{2}, \frac{a^3-a^1}{2} \rangle$.

For any two intervals $\bar{a} = [a^1, a^3] = \langle m(\bar{a}), w(\bar{a}) \rangle$ and $\bar{b} = [b^1, b^3] = \langle m(\bar{b}), w(\bar{b}) \rangle$, the arithmetic operations on $\bar{a}$ and $\bar{b}$ are defined as:

Addition: $\bar{a} + \bar{b} = \langle m(\bar{a}) + m(\bar{b}), w(\bar{a}) + w(\bar{b}) \rangle$;

Soustraction : $\bar{a} - \bar{b} = \langle m(\bar{a}) - m(\bar{b}), w(\bar{a}) + w(\bar{b}) \rangle$, $\alpha\bar{a} = \begin{cases} \langle \alpha m(\bar{a}), \alpha w(\bar{a}) \rangle \text{ if } \alpha \geq 0 \\ \langle \alpha m(\bar{a}), -\alpha w(\bar{a}) \rangle \text{ if } \alpha < 0 \end{cases}$;

$$\text{Multiplication : } \bar{a} \times \bar{b} = \begin{cases} \langle m(\bar{a})m(\bar{b}) + w(\bar{a})w(\bar{b}), m(\bar{a})w(\bar{b}) + m(\bar{b})w(\bar{a}) \rangle \; if \; a^1 \geq 0, b^1 \geq 0 \\ \langle m(\bar{a})m(\bar{b}) + m(\bar{a})w(\bar{b}), m(\bar{b})w(\bar{a}) + w(\bar{b})w(\bar{a}) \rangle \; if \; a^1 < 0, b^1 \geq 0 \\ \langle m(\bar{a})m(\bar{b}) - w(\bar{a})w(\bar{b}), m(\bar{b})w(\bar{a}) - m(\bar{a})w(\bar{b}) \rangle \; if a^3 < 0, b^1 \geq 0 \end{cases}.$$

2.2 | Formulation of a Linear Programming Problem Involving Interval Numbers (LPIn)

We consider the Linear Programming Problems involving Interval numbers (LPIn) as follows [28],

$$\begin{cases} \text{Max } \bar{Z}(\bar{x}) \approx \sum_{j=1}^{n} \bar{c}_j \bar{x}_j \\ \text{Subject to the constraints.} \\ \sum_{j=1}^{n} \bar{a}_{ij} \bar{x}_j \preccurlyeq \bar{b}_i \end{cases}$$

For all the rest of this paper, we will consider the following notations:

$$\bar{x} = [\bar{x}_j]_{n\times1} = \left[[x_j^1, x_j^3]\right]_{n\times1} = \left[\langle m(\bar{x}_j), w(\bar{x}_j) \rangle\right]_{n\times1},$$

$$\bar{c} = [\bar{c}_j]_{1\times n} = \left[[c_j^1, c_j^3]\right]_{1\times n} = \left[\langle m(\bar{c}_j), w(\bar{c}_j) \rangle\right]_{1\times n},$$

$$\bar{b} = [\bar{b}_i]_{m\times1} = \left[[b_i^1, b_i^3]\right]_{m\times1} = \left[\langle m(\bar{b}_i), w(\bar{b}_i) \rangle\right]_{m\times1} \text{ and}$$

$$\bar{A} = [\bar{a}_{ij}]_{m\times n} = \left[[a_{ij}^1, a_{ij}^3]\right]_{m\times n} = \left[\langle m(\bar{a}_{ij}), w(\bar{a}_{ij}) \rangle\right]_{m\times n}.$$

For all the rest of this paper, we will consider the following LPIn [28],

$$\begin{cases} \text{Min/Max } \bar{Z}(\bar{x}_1, \dots, \bar{x}_n) \approx \sum_{j=1}^{n} [c_j^1, c_j^3][x_j^1, x_j^3] \\ \text{Subject to the constraints} \\ \sum_{j=1}^{n} [a_{ij}^1, a_{ij}^3]\,[x_j^1, x_j^3] \left(\substack{\preccurlyeq \\ \approx \\ \succcurlyeq}\right) [b_i^1, b_i^3] \\ 1 \leq j \leq n \text{ and } 1 \leq i \leq m \end{cases},$$

LPIn is equivalent to

$$\begin{cases} \text{Min/Max } \bar{Z}(\bar{x}_1,\dots,\bar{x}_n) \approx \sum_{j=1}^{n} \langle m(\bar{c}_j), w(\bar{c}_j)\rangle\langle m(\bar{x}_j), w(\bar{x}_j)\rangle \\ \qquad\qquad \text{Subject to the constraints} \\ \sum_{j=1}^{n} \langle m(\bar{a}_{ij}), w(\bar{a}_{ij})\rangle \ \langle m(\bar{x}_j), w(\bar{x}_j)\rangle \left(\lesseqgtr\right) \langle m(\bar{b}_i), w(\bar{b}_i)\rangle \\ \qquad\qquad 1 \le j \le n \text{ and } 1 \le i \le m \end{cases}.$$

3| Main Results

In this section, we will describe our method of solving.

3.1| A New Interval Arithmetic for Triangular Fuzzy Numbers via Intervals Numbers

The aim of this section is to present some notations, notions and results which are of useful in our further consideration.

A number $\tilde{a} = (a^1, a^2, a^3)$ (where $a^1 \le a^2 \le a^3$) is said to be a triangular fuzzy number if its membership function is given by [1]-[3]:

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{x-a^1}{a^2-a^1}, & a^1 \le x \le a^2 \\ \frac{x-a^3}{a^2-a^3}, & a^2 \le x \le a^3 \end{cases}.$$

Assume that $\tilde{a} = (a^1, a^2, a^3) = (a^2|\bar{a}) = \left(a^2\middle|[a^1, a^3] = \langle\frac{a^3+a^1}{2}, \frac{a^3-a^1}{2}\rangle\right)$ and $\tilde{b} = (b^1, b^2, b^3) = (b^2|\bar{b}) = \left(b^2\middle|[b^1, b^3] = \langle\frac{b^3+b^1}{2}, \frac{b^3-b^1}{2}\rangle\right)$ are two triangular fuzzy numbers. For any two triangular fuzzy numbers $\tilde{a} = (a^2|\bar{a})$ and $\tilde{b} = (b^2|\bar{b})$, the arithmetic operations on $\tilde{a}$ and $\tilde{b}$ are defined as:

Addition: $\tilde{a} + \tilde{b} = \left(a^2\middle|[a^1, a^3]\right) + \left(b^2\middle|[b^1, b^3]\right) = \left(a^2 + b^2\middle|[a^1, a^3] + [b^1, b^3]\right)$;

Soustraction: $\tilde{a} - \tilde{b} = \left(a^2\middle|[a^1, a^3]\right) - \left(b^2\middle|[b^1, b^3]\right) = \left(a^2 - b^2\middle|[a^1, a^3] - [b^1, b^3]\right)$; .

Multiplication: $\tilde{a}\tilde{b} = \left(a^2\middle|[a^1, a^3]\right)\left(b^2\middle|[b^1, b^3]\right) = \left(a^2b^2\middle|[a^1, a^3][b^1, b^3]\right)$.

For all the rest of this paper, we will consider the following notations:

Assume that $\tilde{c}_{ij} = (c_{ij}^1, c_{ij}^2, c_{ij}^3)$, $\tilde{x}_j = \left(x_j^2\middle|[x_j^1, x_j^3]\right) = (x_{ij}^1, x_{ij}^2, x_{ij}^3)$, $\tilde{b}_j = \left(b_j^2\middle|[b_j^1, b_j^3]\right) = (b_j^1, b_j^2, b_j^3)$ and $\tilde{a}_i = \left(a_i^2\middle|[a_i^1, a_i^3]\right) = (a_i^1, a_i^2, a_i^3)$ are triangular fuzzy numbers with x_{ij}^1, x_{ij}^3, c_{ij}^1, c_{ij}^3, b_j^1, b_j^3, a_i^1 and a_i^3 are real numbers ($\mathbb{R}$).

3.2| Formulation of a Transportation Problems Involving Interval Numbers (TPIn)

We consider the TPIn as follows [28]:

$$\begin{cases} \text{Min } \bar{Z}(\bar{x}) \approx \sum_{j=1}^{n} \bar{c}_{ij}\bar{x}_{ij} \\ \text{Subject to the constraints} \\ \sum_{j=1}^{n} \bar{x}_{ij} \approx \bar{a}_i , 1 \le i \le m \\ \sum_{i=1}^{m} \bar{x}_{ij} \approx \bar{b}_j , 1 \le j \le n \end{cases}.$$

3.3| Formulation of a Transportation Problem with Triangular Fuzzy Numbers (TPTri)

A TPTri is a linear programming problem of a specific structure. If in transportation problem, all parameters and variables are fuzzy, we will have a fully fuzzy transportation problem as follows. Suppose that there are m warehouses and $\tilde{a}_i$ represents renders of warehouse i and n represents customer and $\tilde{b}_j$ is the demand of customer j. $\tilde{c}_{ij}$ is the cost of transporting one unit of product from warehouse i to the customer j and $\tilde{x}_{ij}$ is the value of transported product from warehouse i to the customer j. The objective is to minimize the cost of transporting a product from the warehouse to the customer.

We consider the TPTri as follows [1]-[3]:

$$\begin{cases} \text{Min } \tilde{Z}(\tilde{x}) \approx \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_{ij} \\ \text{Subject to the constraints} \\ \sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{a}_i , 1 \le i \le m \\ \sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{b}_j , 1 \le j \le n \end{cases}.$$

For all the rest of this paper, we will consider the following TPTri:

$$\begin{cases} \text{Min } \tilde{Z}(\tilde{x}) \approx \sum_{i=1}^{m} \sum_{j=1}^{n} \left(c_{ij}^{2} \middle| \bar{c}_{ij}^{13}\right)\left(x_{ij}^{2} \middle| \bar{x}_{ij}^{13}\right) \\ \qquad \text{Subject to the constraints} \\ \qquad \sum_{j=1}^{n} \left(x_{ij}^{2} \middle| \bar{x}_{ij}^{13}\right) \approx \left(a_{i}^{2} \middle| \bar{a}_{i}^{13}\right) \\ \qquad \sum_{i=1}^{m} \left(x_{ij}^{2} \middle| \bar{x}_{ij}^{13}\right) \approx \left(b_{j}^{2} \middle| \bar{b}_{j}^{13}\right) \\ \qquad 1 \le j \le n \text{ and } 1 \le i \le m. \end{cases}.$$

3.4| Our Method for Solving the Transportation Problem with Triangular Fuzzy Numbers (TPTri)

In this section, a method to find a fuzzy optimal solution of TPTri is presented.

For all the rest of this paper, we will consider the following primal TPIn13:

$$\begin{cases} \text{Min } \bar{Z}^{13}(\bar{x}^{13}) \approx \sum_{i=1}^{m}\sum_{j=1}^{n}\bar{c}_{ij}^{13}\bar{x}_{ij}^{13} \\ \quad \text{Subject to the constraints} \\ \quad \sum_{j=1}^{n}\bar{x}_{ij}^{13} \approx \bar{a}_i^{13}, 1 \le i \le m \\ \quad \sum_{i=1}^{m}\bar{x}_{ij}^{13} \approx \bar{b}_j^{13}, 1 \le j \le n \\ \quad \bar{x}_{ij}^{13} = \left[x_j^1, x_j^3\right] \ge 0 \end{cases}.$$

3.4.1 | Formulation of a transportation problem involving midpoint (TPMi13)

Thanks to the new interval arithmetic and TPIn13, we can write the following Transportation Problem involving Midpoint (TPMi13) [28]:

$$\begin{cases} \text{Min/Max } Z^{13}(x^{13}) = \sum_{i=1}^{m}\sum_{j=1}^{n} m\left(\bar{c}_{ij}^{13}\right)x_{ij}^{13} \\ \quad \text{Subject to the constraints} \\ \quad \sum_{j=1}^{n}x_{ij}^{13} = m\left(\bar{a}_i^{13}\right), 1 \le i \le m \\ \quad \sum_{j=1}^{n}x_{ij}^{13} = m\left(\bar{b}_j^{13}\right), 1 \le j \le n \\ \quad x_{ij}^{13} = m\left(\bar{x}_{ij}^{13}\right) = \frac{x_{ij}^3 + x_{ij}^1}{2} \ge 0 \end{cases}.$$

3.4.2 | Formulation of a classical transportation problem (TP2)

The classical Transport Problem (PT2) is:

$$\begin{cases} \text{Min } Z^2(x^2) \approx \sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}^2 x_{ij}^2 \\ \quad \text{Subject to the constraints} \\ \quad \sum_{j=1}^{n}x_{ij}^2 \approx a_i^2, 1 \le i \le m \\ \quad \sum_{i=1}^{m}x_{ij}^2 \approx b_j^2, 1 \le j \le n \\ \quad x_{ij}^1 \le x_{ij}^2 \le x_{ij}^3 \end{cases}.$$

For all the rest of this paper, we will consider the following notations: $\bar{x}_{ij}^{13} = \left[x_j^1, x_j^3\right]$, $\bar{c}_{ij}^{13} = \left[c_j^1, c_j^3\right]$, $\bar{b}_j^{13} = \left[b_j^1, b_j^3\right]$ and $\bar{a}_i^{13} = \left[a_i^1, a_i^3\right]$.

Thanks to the new interval arithmetic, we can write the following Lemma [28]:

Lemma 1. $x^{13} = \left(x_{ij}^{13}\right)_{m\times n}$ is an optimal solution to the (TPMi13) if and only if $\bar{x}^{13} \approx \left(\bar{x}_{ij}^{13}\right)_{m\times n}$ is an optimal solution to the TPIn13.

Proof. [28]. Assuming that $\sum_{j=1}^{n}x_{ij}^{13} = \sum_{j=1}^{n}\frac{x_{ij}^3+x_{ij}^1}{2} = \frac{a_i^3+a_i^1}{2}$ and $\sum_{j=1}^{n}x_{ij}^{13} = \sum_{j=1}^{n}\frac{x_{ij}^3+x_{ij}^1}{2} = \frac{b_j^3+b_j^1}{2}$ with $w\left(\bar{x}_{ij}^{13}\right) = \frac{w\left(\bar{a}_i^{13}\right)}{N}$ where $N = \#\left\{x_{ij}^{13} \neq 0\right\}$ for $1 \le j \le n \text{ and } 1 \le i \le m$, we can write that $\bar{x}_{ij}^{13} \approx \langle x_{ij}^{13}, w\left(\bar{x}_{ij}^{13}\right)\rangle = \left[x_{ij}^{13} - w\left(\bar{x}_{ij}^{13}\right), x_{ij}^{13} + w\left(\bar{x}_{ij}^{13}\right)\right]$ if and only if $x^{13} = \left(x_{ij}^{13}\right)_{m\times n}$ is an optimal solution to the TPMi13. Then $\bar{x}^{13} \approx \left(\bar{x}_{ij}^{13}\right)_{m\times n}$ is an optimal solution to the TPIn13.

Thanks, the Lemma above, we can write the following corollary [28]:

Corollary 1. If $\bar{x}_{ij}^{13} \approx \left[x_j^{*1}, x_j^{*3}\right]$ is an optimal solution to the TPIn13 and x_{ij}^2 is an optimal solution to the (TP2), then $\tilde{x}^* \approx \left(\tilde{x}_{ij}^*\right)_{m\times n}$ is an optimal solution to the TPTri with $\tilde{x}_{ij}^* \approx \left(x_{ij}^2 | \bar{x}_{ij}^{13}\right) = \left(x_{ij}^{*2} \,\middle|\, \left[x_{ij}^{*1}, x_{ij}^{*3}\right]\right) = \left(x_j^{*1}, x_{ij}^{*2}, x_{ij}^{*3}\right)$.

Notice that TPTri is equivalent to
$$\begin{cases} Min\ \tilde{Z}(\tilde{x}) \approx \sum_{i=1}^m \sum_{j=1}^n \left(c_{ij}^2 \,\middle|\, \left[c_{ij}^1, c_{ij}^3\right]\right)\left(x_{ij}^{*2} \,\middle|\, \left[x_{ij}^{*1}, x_{ij}^{*3}\right]\right) \\ \qquad\qquad \textit{Subject to the constraints} \\ \qquad \sum_{j=1}^n \left(x_{ij}^{*2} \,\middle|\, \left[x_{ij}^{*1}, x_{ij}^{*3}\right]\right) \approx \left(a_i^2 \middle| \left[a_i^1, a_i^3\right]\right) \\ \qquad \sum_{i=1}^m \left(x_{ij}^{*2} \,\middle|\, \left[x_{ij}^{*1}, x_{ij}^{*3}\right]\right) \approx \left(b_j^2 \middle| \left[b_j^1, b_j^3\right]\right) \\ \qquad\qquad 1 \le j \le n \ and \ 1 \le i \le m. \end{cases}.$$

$\sum_{j=1}^n \left(x_{ij}^2 \,\middle|\, \left[x_{ij}^{*1}, x_{ij}^{*3}\right]\right) \approx \left(a_i^2 \middle| \left[a_i^1, a_i^3\right]\right)$ is equivalent to $\sum_{j=1}^n x_{ij}^{*2} \approx a_i^2$ and $\sum_{j=1}^n \left[x_{ij}^{*1}, x_{ij}^{*3}\right] \approx \left[a_i^1, a_i^3\right]$.

$\sum_{j=1}^n \left(x_{ij}^2 \,\middle|\, \left[x_{ij}^{*1}, x_{ij}^{*3}\right]\right) \approx \left(b_j^2 \middle| \left[b_j^1, b_j^3\right]\right)$ is equivalent to $\sum_{j=1}^n x_{ij}^{*2} \approx b_j^2$ and $\sum_{j=1}^n \left[x_{ij}^{*1}, x_{ij}^{*3}\right] \approx \left[b_j^1, b_j^3\right]$.

Moreover $\sum_{j=1}^n \frac{x_{ij}^3 + x_{ij}^1}{2} = \frac{a_i^3 + a_i^1}{2}$ for $1 \le i \le m$ and $\sum_{j=1}^n \frac{x_{ij}^3 + x_{ij}^1}{2} = \frac{b_j^3 + b_j^1}{2}$ for $1 \le j \le n$ and $\sum_{j=1}^n \frac{x_{ij}^3 - x_{ij}^1}{2} = \frac{a_i^3 - a_i^1}{2}$ for $1 \le i \le m$ and $\sum_{j=1}^n \frac{x_{ij}^3 - x_{ij}^1}{2} = \frac{b_j^3 - b_j^1}{2}$ for $1 \le j \le n$.

3.4.3 | The steps of our computational method

The steps of our method for solving the TPTri as follows:

Step 1. Consider a TPTri.

Step 2. Identify TPIn13 and TP2.

Step 3. Ramesh and Ganesan's method [28]: solving the TPIn13 via TPMi13.

Applying the simplex method to the TPMi13 to determine the variables TPMi13:

$x^{13} = \left(x_{ij}^{13}\right)_{m\times n}$ and $\bar{x}_{ij}^{13} \approx \langle x_{ij}^{13}, w\left(\bar{x}_{ij}^{13}\right)\rangle = \left[x_{ij}^{13} - w\left(\bar{x}_{ij}^{13}\right), x_{ij}^{13} + w\left(\bar{x}_{ij}^{13}\right)\right]$ for $1 \le k \le m$ with $w\left(\bar{x}_{ij}^{13}\right) = \frac{w\left(\bar{a}_i^{13}\right)}{N_i}$ where $N_i = \# \left\{x_{ij}^{13} \neq 0\right\}$.

The associated value of the objective function: $Min\ \bar{Z}^{13}\left(\bar{x}^{13}\right) \approx \sum_{i=1}^m \sum_{j=1}^n \bar{c}_{ij}^{13} \bar{x}_{ij}^{13}$.

Step 4. Solving the TP2.

Applying the simplex method to the TP2 to determine the variables TP2:

$x^2 = \left(x_{ij}^2\right)_{m\times n}$ with the associated value of the objective function: $Min\ Z^2(x^2) \approx \sum_{i=1}^m \sum_{j=1}^n c_{ij}^2 x_{ij}^2$.

Step 5. Fuzzy optimal solution of TPTri: optimal solution: $\tilde{x}_{ij}^* \approx \left(x_{ij}^2 | \bar{x}_{ij}^{13}\right) = \left(x_{ij}^{*2} \,\middle|\, \left[x_{ij}^{*1}, x_{ij}^{*3}\right]\right) = \left(x_{ij}^{*1}, x_{ij}^{*2}, x_{ij}^{*3}\right)$ with the associated value of the objective function Min $\tilde{Z}^* = \left(Z^{*1}, Z^{*2}, Z^{*3}\right) = \left(Z^2 | \bar{Z}^{13}\right)$.

4| Advantages of the Proposed Method

Let us explore the main advantages of the proposed method:

The proposed technique does not use the goal and parametric approaches which are difficult to apply in real life situations.

By applying the proposed approach for finding the fuzzy optimal solution, there is no need of much knowledge of fuzzy linear programming technique, Zimmerman approach and crisp linear programming which are difficult to learn for a new decision maker.

The proposed method to solve TPTri is based on traditional transportation algorithms. Thus, the existing and easily available software can be used for the same. However, the existing method [1]-[3] to solve FTP should be implemented into a programming language.

To solve the TPTri by using the existing method [1]-[3], there is need to use arithmetic operations of generalized fuzzy numbers. While, if the proposed technique is used for the same then there is need to use arithmetic operations of real numbers. This proves that it is much easy to apply the proposed method as compared to the existing method [1]-[3].

Moreover, it is possible to assume a generic ranking index for comparing the fuzzy numbers involved in the TPTri , in such a way that each time in which the decision maker wants to solve the TPTri under consideration(s),he can choose (or propose) the ranking index that best suits the TPTri.

5| Numerical Illustration

This section covers the numerical problems to signify the methodology of the proposed algorithm.

Example 1.

Table 1. (TPTri) in triangular balanced form.

	R_1	R_2	Supply ($\tilde{a}_i$)
A	(22, 31, 34)	(15, 19, 29)	(150, 201, 246)
B	(30, 39, 54)	(8, 10, 12)	(50, 99, 154)
Demand ($\tilde{b}_j$)	(100, 150, 200)	(100, 150, 200)	$\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$

Step 1. Consider a TPTri.

$$\begin{cases} \text{Min } \tilde{Z}(\tilde{x}) \approx (22,31,34)\tilde{x}_{11} + (15,19,29)\tilde{x}_{12} + (30,39,54)\tilde{x}_{21} + (8,10,12)\tilde{x}_{22} \\ \text{Subject to the constraints} \\ \tilde{x}_{11} + \tilde{x}_{12} \approx (150,201,246) \\ \tilde{x}_{21} + \tilde{x}_{22} \approx (50,99,154) \\ \tilde{x}_{11} + \tilde{x}_{21} \approx (100,150,200) \\ \tilde{x}_{12} + \tilde{x}_{22} \approx (100,150,200) \\ \tilde{x}_{ij} \text{ are triangular fuzzy numbers} \end{cases}.$$

Kané et al. |J. Fuzzy. Ext. Appl. 2(1) (2021) 89-105

Step 2. Identify TPIn13 and TP2, respectivly.

$$\begin{cases} \text{Min } \bar{Z}^{13}(\bar{x}^{14}) \approx [22,34]\bar{x}_{11}^{13} + [15,29]\bar{x}_{12}^{13} + [30,54]\bar{x}_{21}^{13} + [8,12]\bar{x}_{22}^{13} \\ \quad \text{Subject to the constraints} \\ \quad \bar{x}_{11}^{13} + \bar{x}_{12}^{13} \approx [150,246] = \langle 198, \quad 48\rangle \\ \quad \bar{x}_{21}^{13} + \bar{x}_{22}^{13} \approx [50,154] = \langle 102, \quad 52\rangle \\ \quad \bar{x}_{11}^{13} + \bar{x}_{21}^{13} \approx [100,200] = \langle 150,50\rangle \\ \quad \bar{x}_{12}^{13} + \bar{x}_{22}^{13} \approx [100,200] = \langle 150,50\rangle \end{cases},$$

$$\begin{cases} \text{Min } Z^2(x^2) = 31x_{11}^2 + 19x_{12}^2 + 39x_{21}^2 + 10x_{22}^2 \\ \quad \text{Subject to the constraints} \\ \quad x_{11}^2 + x_{12}^2 = 201 \\ \quad x_{21}^2 + x_{22}^2 = 99 \\ \quad x_{11}^2 + x_{21}^2 = 150 \\ \quad x_{12}^2 + x_{22}^2 = 150 \\ \quad x_{ij}^1 \le x_{ij}^2 \le x_{ij}^3 \end{cases}.$$

Step 3. Ramesh and Ganesan's method [28]: solving the TPIn13 via TPMi13.

$$\begin{cases} \text{Min } Z^{13}(x^{13}) = 28x_{11}^{13} + 22x_{12}^{13} + 42x_{21}^{13} + 10x_{22}^{13} \\ \quad \text{Subject to the constraints} \\ \quad x_{11}^{13} + x_{12}^{13} = 198 \\ \quad x_{21}^{13} + x_{22}^{13} = 102 \\ \quad x_{11}^{13} + x_{21}^{13} = 150 \\ \quad x_{12}^{13} + x_{22}^{13} = 150 \end{cases}.$$

Applying the simplex method to the TPMi13 to determine the variables TPMi13: $x^{13} = \left(x_{ij}^{13}\right)_{m\times n}$. The optimal solution is: $x_{11}^{13} = 150$, $x_{12}^{13} = 48$, $x_{21}^{13} = 0$ and $x_{22}^{13} = 102$ with $Z^{13} = 6276$. We have $\bar{x}_{ij}^{13} \approx \langle x_{ij}^{13}, w(\bar{x}_{ij}^{13})\rangle \approx \left[x_{ij}^{13} - w(\bar{x}_{ij}^{13}), x_{ij}^{13} + w(\bar{x}_{ij}^{13})\ \right]$ with $w(\bar{a}_1^{13}) = 48$ with $N_1 = \# \left\{x_{11}^{13} \neq 0, x_{12}^{13} \neq 0\right\} = 2$ and $w(\bar{a}_2^{13}) = 52$ with $N_2 = \# \left\{x_{22}^{13} \neq 0\right\} = 1$.

We get $w(\bar{x}_{1j}^{13}) = \frac{w(\bar{a}_1^{13})}{N_1} = \frac{48}{2} = 24$ and $\bar{x}_{1j}^{13} \approx \left[x_{1j}^{13} - w(\bar{x}_{1j}^{13}), x_{1j}^{13} + w(\bar{x}_{1j}^{13})\ \right]$: $\bar{x}_{11}^{13} \approx \langle 150,24\rangle = [126,174]$ and $\bar{x}_{12}^{13} \approx \langle 48,24\rangle = [24,72]$. Furthermore $w(\bar{x}_{2j}^{13}) = \frac{w(\bar{a}_2^{13})}{N_2} = \frac{52}{1} = 52$ and $\bar{x}_{2j}^{13} \approx \left[x_{2j}^{13} - w(\bar{x}_{2j}^{13}), x_{2j}^{13} + w(\bar{x}_{2j}^{13})\ \right]$: $\bar{x}_{21}^{13} \approx \bar{0}$ and $\bar{x}_{22}^{13} \approx \langle 102,52\rangle = [50,154]$.

The associated value of the objective function: $Min\ \bar{Z}^{13}(\bar{x}^{13}) \approx \sum_{i=1}^{m} \sum_{j=1}^{n} \bar{c}_{ij}^{13} \bar{x}_{ij}^{13} = [3532,\ 9852]$.

Step 4. Solving the primal TP2.

$$\begin{cases} \text{Min } Z^2(x^2) = 31x_{11}^2 + 19x_{12}^2 + 39x_{21}^2 + 10x_{22}^2 \\ \qquad \text{Subject to the constraints} \\ \qquad x_{11}^2 + x_{12}^2 = 201 \\ \qquad x_{21}^2 + x_{22}^2 = 99 \\ \qquad x_{11}^2 + x_{21}^2 = 150 \\ \qquad x_{12}^2 + x_{22}^2 = 150 \\ \qquad 126 \le x_{11}^2 \le 174 \\ \qquad 24 \le x_{12}^2 \le 72 \\ \qquad 50 \le x_{22}^2 \le 154 \end{cases}.$$

Applying the simplex method to the TP2 to determine the primal variables TP2: $x^2 = \left(x_{ij}^2\right)_{m\times n}$. The optimal solution is: $x_{11}^2 = 150$, $x_{12}^2 = 51$, $x_{21}^2 = 0$ and $x_{22}^2 = 99$ with the associated value of the objective function: Min $Z^2 = 6609$.

Step 5. Fuzzy optimal solution of TPTri.

Optimal solution:

$\tilde{x}_{ij}^* \approx \left(x_{ij}^{*2}|\bar{x}_{ij}^{23}\right) = \left(x_{ij}^{*2}|\left[x_{ij}^{*1}, x_{ij}^{*3}\right]\right) = \left(x_{ij}^{*1}, x_{ij}^{*2}, x_{ij}^{*3}\right)$: $\tilde{x}_{11}^* \approx (150|[102, 198]) = (102, 150, 198)$, $\tilde{x}_{12}^* \approx (51|[0, 96]) = (0, 51, 96)$, $\tilde{x}_{21}^* \approx (0|\bar{0}) = \tilde{0}$, $\tilde{x}_{22}^* \approx (99|[50, 154]) = (50, 99, 154)$, with the associated value of the objective function Min $\tilde{Z}^* = \left(Z^{*1}, Z^{*2}, Z^{*3}\right) = \left(Z^2|\bar{Z}^{13}\right)$. We have Min $\tilde{Z}^* \approx (6609|[3532, 9852]) = (3532, 6609, 9852)$.

Interpretation of results. We will now interpret the minimum total fuzzy transportation cost obtained in *Example 1* by using the proposed methods presented in *Section 3* Similarly, the obtained fuzzy optimal solution will also be interpreted. By using the methods proposed the minimum total fuzzy transportation cost is (3532, 6609, 9852), which can be physically interpreted as follows:

- *The least amount of the minimum total transportation cost is 3532.*
- *The most possible amount of minimum total transportation cost is 6609.*
- *The greatest amount of the minimum total transportation cost is 9852 i.e., the minimum total transportation cost will always be greater than 3532 and less than 6609, and the highest chances are that the minimum total transportation cost will be 9852.*

Example 2. [1]-[3]. Dali Company is the leading producer of soft drinks and low-temperature foods in Taiwan. Currently, Dali plans to develop the South-East Asian market and broaden the visibility of Dali products in the Chinese market. Notably, following the entry of Taiwan to the World Trade Organization, Dali plans to seek strategic alliance with prominent international companies and introduced international bread to lighten the embedded future impact. In the domestic soft drinks market, Dali produces tea beverages to meet demand from four distribution centers in Taichung, Chiayi, Kaohsiung and Taipei, with production being based at three plants in Changhua, Touliu and Hsinchu. According to the preliminary environmental information, *Table 2* summarizes the potential supply available from these three plants, the forecast demand from the four distribution centers and the unit transportation costs for each route used by Dali for the upcoming season.

Table 2. Summarized data in the Dali case (in U.S. dollar).

Source	Destionation Taichung	Chiayi	Kaohsiung	Taipei	Supply ($\tilde{a}_i$) (000 dozen bottles)
Changhua	(\$8, \$10, \$10.8)	(\$20.4, \$22, \$24)	(\$8, \$10, \$10.6)	(\$18.8, \$20, \$22)	(7.2, 8, 8.8)
Touliu	(\$14, \$15, \$16)	(\$18.2, \$20, \$22)	(\$10, \$12, \$13)	(\$6, \$8, \$8.8)	(12, 14, 16)
Hsinchu	(\$18.4, \$20, \$21)	(\$9.6, \$12, \$13)	(\$7.8, \$10, \$10.8)	(\$14, \$15, \$16)	(10.2, 12, 13.8)
Demand ($\tilde{b}_j$) (000 dozen bottles)	(6.2, 7, 7.8)	(8.9, 10, 11.1)	(6.5, 8, 9.5)	(7.8, 9, 10.2)	$\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$

Step 1. The TPTri is given by:

$$\left\{\begin{array}{c}
\text{Min } \tilde{Z}(\tilde{x}) = (8,10,10.8)\left(x_{11}^1, x_{11}^2, x_{11}^3\right) + (20.4,22,24)\left(x_{12}^1, x_{12}^2, x_{12}^3\right) \\
+(8,10,10.6)\left(x_{13}^1, x_{13}^2, x_{13}^3\right) + (18.8,20,22)\left(x_{14}^1, x_{14}^2, x_{14}^3\right) \\
+(14,15,16)\left(x_{21}^1, x_{21}^2, x_{21}^3\right) + (18.2,20,22)\left(x_{22}^1, x_{22}^2, x_{22}^3\right) \\
+(10,12,13)\left(x_{23}^1, x_{23}^2, x_{23}^3\right) + (6,8,8.8)\left(x_{24}^1, x_{24}^2, x_{24}^3\right) \\
+(18.4,20,21)\left(x_{31}^1, x_{31}^2, x_{31}^3\right) + (9.6,12,13)\left(x_{32}^1, x_{32}^2, x_{32}^3\right) \\
+(7.8,10,10.8)\left(x_{33}^1, x_{33}^2, x_{33}^3\right) + (14,15,16)\left(x_{34}^1, x_{34}^2, x_{34}^3\right) \\
\text{Subject to the constraints} \\
\left(x_{11}^1, x_{11}^2, x_{11}^3\right) + \left(x_{12}^1, x_{12}^2, x_{12}^3\right) + \left(x_{13}^1, x_{13}^2, x_{13}^3\right) + \left(x_{14}^1, x_{14}^2, x_{14}^3\right) = (7.2,8,8.8) \\
\left(x_{21}^1, x_{21}^2, x_{21}^3\right) + \left(x_{22}^1, x_{22}^2, x_{22}^3\right) + \left(x_{23}^1, x_{23}^2, x_{23}^3\right) + \left(x_{24}^1, x_{24}^2, x_{24}^3\right) = (12,14,16) \\
\left(x_{31}^1, x_{31}^2, x_{31}^3\right) + \left(x_{32}^1, x_{32}^2, x_{32}^3\right) + \left(x_{33}^1, x_{33}^2, x_{33}^3\right) + \left(x_{34}^1, x_{34}^2, x_{34}^3\right) = (10.2,12,13.8) \\
\left(x_{11}^1, x_{11}^2, x_{11}^3\right) + \left(x_{21}^1, x_{21}^2, x_{21}^3\right) + \left(x_{31}^1, x_{31}^2, x_{31}^3\right) = (6.2,7,7.8) \\
\left(x_{12}^1, x_{12}^2, x_{12}^3\right) + \left(x_{22}^1, x_{22}^2, x_{22}^3\right) + \left(x_{32}^1, x_{32}^2, x_{32}^3\right) = (8.9,10,11.1) \\
\left(x_{13}^1, x_{13}^2, x_{13}^3\right) + \left(x_{23}^1, x_{23}^2, x_{23}^3\right) + \left(x_{33}^1, x_{33}^2, x_{33}^3\right) = (6.5,8,9.5) \\
\left(x_{14}^1, x_{14}^2, x_{14}^3\right) + \left(x_{24}^1, x_{24}^2, x_{24}^3\right) + \left(x_{34}^1, x_{34}^2, x_{34}^3\right) = (7.8,9,10.2)
\end{array}\right. .$$

Step 2. Identify TPIn13 and TP2, respectively.

$$\left\{\begin{array}{c}
\text{Min } \bar{Z}^{13}\left(\bar{x}^{13}\right) = [8, \quad 10.8]\bar{x}_{11}^{13} + [20.4, \quad 24]\bar{x}_{12}^{13} \\
+[8, \quad 10.6]\bar{x}_{13}^{13} + [18.8, 22]\bar{x}_{14}^{13} \\
+[14,16]\bar{x}_{21}^{13} + [18.2, \quad 22]\bar{x}_{22}^{13} \\
+[10, \quad 13]\bar{x}_{23}^{13} + [6, \quad 8.8]\bar{x}_{24}^{13} \\
+[18.4, \quad 21]\bar{x}_{31}^{13} + [9.6, \quad 13]\bar{x}_{32}^{13} \\
+[7.8, \quad 10.8]\bar{x}_{33}^{13} + [14, \quad 16]\bar{x}_{34}^{13} \\
\text{Subject to the constraints} \\
\bar{x}_{11}^{13} + \bar{x}_{12}^{13} + \bar{x}_{13}^{13} + \bar{x}_{14}^{13} = [7.2, \quad 8.8] = \langle 8, \quad 0.8 \rangle \\
\bar{x}_{21}^{13} + \bar{x}_{22}^{13} + \bar{x}_{23}^{13} + \bar{x}_{24}^{13} = [12, \quad 16] = \langle 14, 2 \rangle \\
\bar{x}_{31}^{13} + \bar{x}_{32}^{13} + \bar{x}_{33}^{13} + \bar{x}_{34}^{13} = [10.2, \quad 13.8] = \langle 12, \quad 1.8 \rangle \\
\bar{x}_{11}^{13} + \bar{x}_{21}^{13} + \bar{x}_{31}^{13} = [6.2, \quad 7.8] = \langle 7, \quad 0.8 \rangle \\
\bar{x}_{12}^{13} + \bar{x}_{22}^{13} + \bar{x}_{32}^{13} = [8.9, \quad 11.1] = \langle 10, \quad 1.1 \rangle \\
\bar{x}_{13}^{13} + \bar{x}_{23}^{13} + \bar{x}_{33}^{13} = [6.5, \quad 9.5] = \langle 8, \quad 1.5 \rangle \\
\bar{x}_{14}^{13} + \bar{x}_{24}^{13} + \bar{x}_{34}^{13} = [7.8, \quad 10.2] = \langle 9, \quad 1.2 \rangle
\end{array}\right. ,$$

Kané et al. |J. Fuzzy. Ext. Appl. 2(1) (2021) 89-105

$$\begin{cases} \text{Min } Z^2(x^2) = 10x_{11}^2 + 22x_{12}^2 + 10x_{13}^2 + 20x_{14}^2 \\ \quad 15x_{21}^2 + 20x_{22}^2 + 12x_{23}^2 + 8x_{24}^2 \\ \quad 20x_{31}^2 + 12x_{32}^2 + 10x_{33}^2 + 15x_{34}^2 \\ \quad \text{Subject to the constraints} \\ \quad x_{11}^2 + x_{12}^2 + x_{13}^2 + x_{14}^2 = 8 \\ \quad x_{21}^2 + x_{22}^2 + x_{23}^2 + x_{24}^2 = 14 \\ \quad x_{31}^2 + x_{32}^2 + x_{33}^2 + x_{34}^2 = 12 \\ \quad x_{11}^2 + x_{21}^2 + x_{31}^2 = 7 \\ \quad x_{12}^2 + x_{22}^2 + x_{32}^2 = 10 \\ \quad x_{13}^2 + x_{23}^2 + x_{33}^2 = 8 \\ \quad x_{14}^2 + x_{24}^2 + x_{34}^2 = 9 \end{cases}.$$

Step 3. Ramesh and Ganesan's method [28]: solving the TPIn13 via TPMi13.

Applying the simplex method to the TPMi13 to determine the variables TPMi13:

$$\begin{cases} \text{Min } Z^{13}(x^{13}) = 9.4x_{11}^{13} + 22.2x_{12}^{13} \\ \quad +9.3x_{13}^{13} + 20.4x_{14}^{13} \\ \quad +15x_{21}^{13} + 20.1x_{22}^{13} \\ \quad +11.5x_{23}^{13} + 7.4x_{24}^{13} \\ \quad +19.7x_{31}^{13} + 11.3x_{32}^{13} \\ \quad +9.3x_{33}^{13} + 15x_{34}^{13} \\ \quad \text{Subject to the constraints} \\ \quad x_{11}^{13} + x_{12}^{13} + x_{13}^{13} + x_{14}^{13} = 8 \\ \quad x_{21}^{13} + x_{22}^{13} + x_{23}^{13} + x_{24}^{13} = 14 \\ \quad x_{31}^{13} + x_{32}^{13} + x_{33}^{13} + x_{34}^{13} = 12 \\ \quad x_{11}^{13} + x_{21}^{13} + x_{31}^{13} = 7 \\ \quad x_{12}^{13} + x_{22}^{13} + x_{32}^{13} = 10 \\ \quad x_{13}^{13} + x_{23}^{13} + x_{33}^{13} = 8 \\ \quad x_{14}^{13} + x_{24}^{13} + x_{34}^{13} = 9 \end{cases}.$$

$x^{13} = \left(x_{ij}^{13}\right)_{m\times n}$. The optimal solution is: $x_{11}^{13} = 7, x_{12}^{13} = 0, x_{13}^{13} = 1$ $x_{14}^{13} = 0, x_{21}^{13} = 0, x_{22}^{13} = 0, x_{23}^{13} = 5,$ $x_{24}^{13} = 9$ and $x_{31}^{13} = 0, x_{32}^{13} = 10, x_{33}^{13} = 2$ $x_{34}^{13} = 0$.

$w\left(\bar{a}_1^{13}\right) = 0.8 = \frac{4}{5}$ with $N_1 = \#\left\{x_{11}^{13} \neq 0, x_{13}^{13} \neq 0\right\} = 2$, $w\left(\bar{a}_2^{13}\right) = 2$ with $N_2 = \#\left\{x_{23}^{13} \neq 0, x_{24}^{13} \neq 0\right\} = 2$ and

$w\left(\bar{a}_3^{13}\right) = 1.8 = \frac{9}{5}$ with $N_3 = \#\left\{x_{32}^{13} \neq 0, x_{33}^{13} \neq 0\right\} = 2$.

We get $\bar{x}_{1j}^{13} \approx \left[x_{1j}^{13} - w\left(\bar{x}_{1j}^{13}\right), x_{1j}^{13} + w\left(\bar{x}_{1j}^{13}\right)\right]$ with $w\left(\bar{x}_{1j}^{13}\right) = \frac{4}{10} = \frac{2}{5}$:

$\bar{x}_{11}^{13} \approx \left[7 - \frac{2}{5}, 7 + \frac{2}{5}\right] = \left[\frac{33}{5}, \frac{37}{5}\right]$, $\bar{x}_{12}^{13} \approx \bar{0}$, $\bar{x}_{13}^{13} \approx \left[1 - \frac{2}{5}, 1 + \frac{2}{5}\right] = \left[\frac{3}{5}, \frac{7}{5}\right]$ and $\bar{x}_{14}^{13} \approx \bar{0}$.

$\bar{x}_{2j}^{13} \approx \left[x_{2j}^{13} - w\left(\bar{x}_{2j}^{13}\right), x_{2j}^{13} + w\left(\bar{x}_{2j}^{13}\right)\right]$ with $w\left(\bar{x}_{2j}^{13}\right) = \frac{2}{2} = 1$:

$\bar{x}_{21}^{13} \approx \bar{0}, \bar{x}_{22}^{13} \approx \bar{0}, \bar{x}_{23}^{13} \approx [5-1, 5+1] = [4,\ 6]$ and $\bar{x}_{24}^{13} \approx [9-1,\ 9+1] = [8,\ 10]$.

$\bar{x}_{3j}^{13} \approx \left[x_{3j}^{13} - w\left(\bar{x}_{3j}^{13}\right), x_{3j}^{13} + w\left(\bar{x}_{3j}^{13}\right)\right]$ with $w\left(\bar{x}_{3j}^{13}\right) = \frac{9}{10}$:

$\bar{x}_{31}^{13} \approx \bar{0}, \bar{x}_{32}^{13} \approx \left[10 - \frac{9}{10}, 10 + \frac{9}{10}\right] = \left[\frac{91}{10}, \frac{109}{10}\right], \bar{x}_{33}^{13} \approx \left[2 - \frac{9}{10}, 2 + \frac{9}{10}\right] = \left[\frac{11}{10}, \frac{29}{10}\right]$ and $\bar{x}_{34}^{13} \approx \bar{0}$.

The associated value of the objective function: $Min\ \bar{Z}^{13}\left(\bar{x}^{13}\right) \approx \sum_{i=1}^{m} \sum_{j=1}^{n} \bar{c}_{ij}^{13} \bar{x}_{ij}^{13} = \left[\frac{12081}{50}\$, \frac{21689}{50}\$\right]$.

Step 4. Solving the primal TP2.

$$
\begin{cases}
\text{Min } Z^2\left(x^2\right) = 10x_{11}^2 + 22x_{12}^2 + 10x_{13}^2 + 20x_{14}^2 \\
\quad 15x_{21}^2 + 20x_{22}^2 + 12x_{23}^2 + 8x_{24}^2 \\
\quad 20x_{31}^2 + 12x_{32}^2 + 10x_{33}^2 + 15x_{34}^2 \\
\quad \text{Subject to the constraints} \\
\quad x_{11}^2 + x_{12}^2 + x_{13}^2 + x_{14}^2 = 8 \\
\quad x_{21}^2 + x_{22}^2 + x_{23}^2 + x_{24}^2 = 14 \\
\quad x_{31}^2 + x_{32}^2 + x_{33}^2 + x_{34}^2 = 12 \\
\quad x_{11}^2 + x_{21}^2 + x_{31}^2 = 7 \\
\quad x_{12}^2 + x_{22}^2 + x_{32}^2 = 10 \\
\quad x_{13}^2 + x_{23}^2 + x_{33}^2 = 8 \\
\quad x_{14}^2 + x_{24}^2 + x_{34}^2 = 9 \\
\quad \frac{33}{5} \le x_{11}^2 \le \frac{37}{5} \\
\quad \frac{3}{5} \le x_{13}^2 \le \frac{7}{5} \\
\quad 4 \le x_{23}^2 \le 6 \\
\quad 8 \le x_{24}^2 \le 10 \\
\quad \frac{91}{10} \le x_{32}^2 \le \frac{109}{10} \\
\quad \frac{11}{10} \le x_{33}^2 \le \frac{29}{10}
\end{cases}
$$

The optimal solution is: $x_{11}^2 = 7$, $x_{12}^2 = 0$, $x_{13}^2 = 1$ $x_{14}^2 = 0$, $x_{21}^2 = 0$, $x_{22}^2 = 0$, $x_{23}^2 = 5$, $x_{24}^2 = 9$ and $x_{31}^2 = 0$, $x_{32}^2 = 10$, $x_{33}^2 = 2$ $x_{34}^2 = 0$ with Min $Z^2 = 352\$$.

Step 5. Fuzzy optimal solution of TPTri.

Optimal solution:

$\tilde{x}_{ij}^* \approx \left(x_{ij}^{*2} | \bar{x}_{ij}^{23}\right) = \left(x_{ij}^{*2} \middle| \left[x_{ij}^{*1}, x_{ij}^{*3}\right]\right) = \left(x_{ij}^{*1}, x_{ij}^{*2}, x_{ij}^{*3}\right)$:

$\tilde{x}_{11}^* \approx \left(\frac{33}{5},\ 7,\ \frac{37}{5}\right), \tilde{x}_{12}^* \approx \tilde{0},\ \tilde{x}_{13}^* \approx \left(\frac{3}{5},\ 1,\ \frac{7}{5}\right)$ and $\tilde{x}_{14}^* \approx \tilde{0}$;

$\tilde{x}_{21}^* \approx \tilde{0},\ \tilde{x}_{22}^* \approx \tilde{0},\ \tilde{x}_{23}^* \approx (4,\ 5,\ 6)$ and $\tilde{x}_{24}^* \approx (7,\ 9,\ 10)$;

$\tilde{x}_{31}^* \approx \tilde{0},\ \tilde{x}_{32}^* \approx \left(\frac{11}{10},\ 10,\ \frac{29}{10}\right),\ \tilde{x}_{33}^* \approx \left(\frac{91}{10},\ 2,\ \frac{109}{10}\right)$ and $\tilde{x}_{34}^* \approx \tilde{0}$.

Kané et al. |J. Fuzzy. Ext. Appl. 2(1) (2021) 89-105

With the associated value of the objective function Min $\tilde{Z}^* = \left(Z^{*1}, Z^{*2}, Z^{*3}\right) = \left(Z^2 \middle| \overline{Z}^{13}\right)$.

We have Min $\tilde{Z}^* \approx \left(352\$ \middle| \left[\frac{12081}{50}\$, \frac{21689}{50}\$\right]\right) = \left(\frac{12081}{50}\$, 352\$, \frac{21689}{50}\$\right)$.

Interpretation of results:

We will now interpret the minimum total fuzzy transportation cost obtained in *Example 2* by using the proposed methods presented in Section 3 Similarly, the obtained fuzzy optimal solution will also be interpreted. By using the methods proposed the minimum total fuzzy transportation cost is $\left(\frac{12081}{50}\$, 352\$, \frac{21689}{50}\$\right)$, which can be physically interpreted as follows:

The least amount of the minimum total transportation cost is $\frac{12081}{50}\$$.

The most possible amount of minimum total transportation cost is 352$.

The greatest amount of the minimum total transportation cost is $\frac{21689}{50}\$$ i.e., the minimum total transportation cost will always be greater than $\frac{12081}{50}\$$ and less than 352$, and the highest chances are that the minimum total transportation cost will be $\frac{21689}{50}\$$.

5| Concluding Remarks and Future Research Directions

These days a number of researchers have shown interest in the area of fuzzy transportation problems and various attempts have been made to study the solution of these problems. In this paper, to overcome the shortcomings of the existing methods we introduced a new formulation of transportation problem involving Triangular fuzzy numbers for the transportation costs and values of supplies and demands. We propose a fuzzy linear programming approach for solving Triangular fuzzy numbers transportation problem based on the converting into an TPIn and a classical TP. To show the advantages of the proposed methods over existing methods, some fuzzy transportation problems, may or may not be solved by the existing methods, are solved by using the proposed methods and it is shown that it is better to use the proposed methods as compared to the existing methods for solving the transportation problems. Finally, we feel that, there are many other points of research and should be studied later on. Some of these points are discussed below.

The solid transportation problem considers the supply, the demand, and the conveyance to satisfy the transportation requirement in a cost-effective manner. Thus, research on the topic for developing the proposed method to derive the fuzzy objective value of the fuzzy solid transportation problem when the cost coefficients, the supply and demand quantities and conveyance capacities are interval-valued triangular fuzzy numbers, is left to the next research work.

Further research on introducing a new formulation of interval-valued triangular fuzzy numbers transportation problem that lead to a method for solving this problem based on the classical transportation algorithms is an interesting stream of future research.

From both theoretical and algorithmic considerations, and examples solved in this paper, it can be noticed that some shortcomings of the methods for solving the fuzzy transportation problems known from the literature can be resolved by using the new methods proposed in *Section 3*.

Acknowledgements

The authors are very grateful to the anonymous referees for their valuable comments and suggestions to improve the paper in the present form.

Conflicts of Interest

The authors declare no conflicts of interest.

References

[1] Kaur, J., & Kumar, A. (2016). *An introduction to fuzzy linear programming problems*. Springer.

[2] Nasseri, S. H., Ebrahimnejad, A., & Cao, B. Y. (2019). Fuzzy linear programming. In *Fuzzy linear programming: solution techniques and applications* (pp. 39-61). Cham: Springer.

[3] Kaur, A., Kacprzyk, J., Kumar, A., & Analyst, C. F. (2020). *Fuzzy transportation and transshipment problems*. Springer International Publishing.

[4] Zimmermann, H. J. (1978). Fuzzy programming and linear programming with several objective functions. *Fuzzy sets and systems, 1*(1), 45-55.

[5] ÓhÉigeartaigh, M. (1982). A fuzzy transportation algorithm. *Fuzzy sets and systems, 8*(3), 235-243.

[6] Chanas, S., Kołodziejczyk, W., & Machaj, A. (1984). A fuzzy approach to the transportation problem. *Fuzzy sets and systems, 13*(3), 211-221.

[7] Chanas, S., Delgado, M., Verdegay, J. L., & Vila, M. A. (1993). Interval and fuzzy extensions of classical transportation problems. *Transportation planning and technology*, *17*(2), 203-218.

[8] Chanas, S., & Kuchta, D. (1996). A concept of the optimal solution of the transportation problem with fuzzy cost coefficients. *Fuzzy sets and systems, 82*(3), 299-305.

[9] Jiménez, F., & Verdegay, J. L. (1998). Uncertain solid transportation problems. *Fuzzy sets and systems, 100*(1-3), 45-57.

[10] Jiménez, F., & Verdegay, J. L. (1999). Solving fuzzy solid transportation problems by an evolutionary algorithm based parametric approach. *European journal of operational research, 117*(3), 485-510.

[11] Liu, S. T., & Kao, C. (2004). Solving fuzzy transportation problems based on extension principle. *European Journal of operational research, 153*(3), 661-674.

[12] Gani, A., & Razak, K. A. (2006). Two stage fuzzy transportation problem. *Journal of physical science, 10*, 63–69. http://inet.vidyasagar.ac.in:8080/jspui/handle/123456789/720

[13] Li, L., Huang, Z., Da, Q., & Hu, J. (2008, May). A new method based on goal programming for solving transportation problem with fuzzy cost. *2008 international symposiums on information processing* (pp. 3-8). IEEE.

[14] Lin, F. T. (2009, August). Solving the transportation problem with fuzzy coefficients using genetic algorithms. *2009 IEEE international conference on fuzzy systems* (pp. 1468-1473). IEEE.

[15] Dinagar, D. S., & Palanivel, K. (2009). The transportation problem in fuzzy environment. *International journal of algorithms, computing and mathematics*, *2*(3), 65-71.

[16] Pandian, P., & Natarajan, G. (2010). A new algorithm for finding a fuzzy optimal solution for fuzzy transportation problems. *Applied mathematical sciences, 4*(2), 79-90.

[17] Kumar, A., & Kaur, A. (2011). Application of linear programming for solving fuzzy transportation problems. *Journal of applied mathematics & informatics, 29*(3_4), 831-846.

[18] Gupta, A., Kumar, A., & Kaur, A. (2012). Mehar's method to find exact fuzzy optimal solution of unbalanced fully fuzzy multi-objective transportation problems. *Optimization letters, 6*(8), 1737-1751.

[19] Ebrahimnejad, A. (2015). A duality approach for solving bounded linear programming problems with fuzzy variables based on ranking functions and its application in bounded transportation problems. *International journal of systems science, 46*(11), 2048-2060.

[20] Shanmugasundari, M., & Ganesan, K. (2013). A novel approach for the fuzzy optimal solution of fuzzy transportation problem. *Transportation*, *3*(1), 1416-1424.

[21] Chandran, S., & Kandaswamy, G. (2016). A fuzzy approach to transport optimization problem. *Optimization and engineering, 17*(4), 965-980.

Kané et al. |J. Fuzzy. Ext. Appl. 2(1) (2021) 89-105

[22] Ebrahimnejad, A. (2016). Note on "A fuzzy approach to transport optimization problem". *Optimization and engineering, 17*(4), 981-985.

[23] Kumar, A., & Kaur, A. (2014). Optimal way of selecting cities and conveyances for supplying coal in uncertain environment. *Sadhana, 39*(1), 165-187.

[24] Ebrahimnejad, A. (2015). An improved approach for solving fuzzy transportation problem with triangular fuzzy numbers. *Journal of intelligent & fuzzy systems, 29*(2), 963-974.

[25] Kaur, A., & Kumar, A. (2011). A new method for solving fuzzy transportation problems using ranking function. *Applied mathematical modelling, 35*(12), 5652-5661.

[26] Kaur, A., & Kumar, A. (2012). A new approach for solving fuzzy transportation problems using generalized trapezoidal fuzzy numbers. *Applied soft computing, 12*(3), 1201-1213.

[27] Ebrahimnejad, A. (2014). A simplified new approach for solving fuzzy transportation problems with generalized trapezoidal fuzzy numbers. *Applied soft computing, 19*, 171-176.

[28] Ramesh, G., & Ganesan, K. (2012). Duality theory for interval linear programming problems. *IOSR journal of mathematics, 4*(4), 39-47.

Publisher: Ayandegan Institute of Higher Education, Iran.
Director- in-Charge: Seyyed Ahmad Edalatpanah
Editor-in-Chief: Florentin Smarandache

Journal of Fuzzy Extension & Applications (JFEA) is an international scholarly open access, interdisciplinary, fully refereed, and quarterly free of charge journal as a non-commercial publication. The publication start year of JFEA is 2020 and affiliated with Ayandegan Institute of Higher Education. The JFEA Journal has been indexed in the well-known world databases mainly; Directory of Open Access Journals (DOAJ), China Academic Journals (CNKI), WorldCat, Islamic World Science Citation Center (ISC), EBSCO, EuroPub, Google Scholar, Ulrichsweb, and Scientific Information Database (SID). The JFEA has partnered with Publons, the world's leading peer review platform, to officially recognize your peer review contributions. For more information about Publons or to sign up, visit our webpage at Publons. Moreover, JFEA utilizes "Plagiarism Detection Software (iThenticate)" for checking the originality of submitted papers in the reviewing process. This journal respects the ethical rules in publishing and is subject to the rules of the Ethics Committee" for Publication (COPE) and committed to follow the executive regulations of the Law on Prevention and Combating Fraud in Scientific Works.